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NPTEL ONLINE CERTIFICATION COURSE

Course
on
Analog Communication

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Lecture 57: Sampling Theorem (Contd.)

Okay, so we have started pulse modulation that something we have already started and we have told that we wish to actually establish the theory behind this pulse modulation. So pulse modulation thus take discrete samples of time and signal right. So if you wish to do that we have already proven the Nyquist Theorem that yes there is a possibility that I can take just samples and that will still with the low pass filter if we took some criteria on the sampling.

We have a possibility of getting the signal back original signal back from the sampled signal this is something we have already proven. And next what we wish to see that yes those samples which are means following the Nyquist criteria are actually the true representative of the signal, this is called the reconstruction theorem or the interpolation theorem. So we will try to see that those samples are good enough in certain way if we present them they actually represent the whole signal okay. So let us try to see those things.

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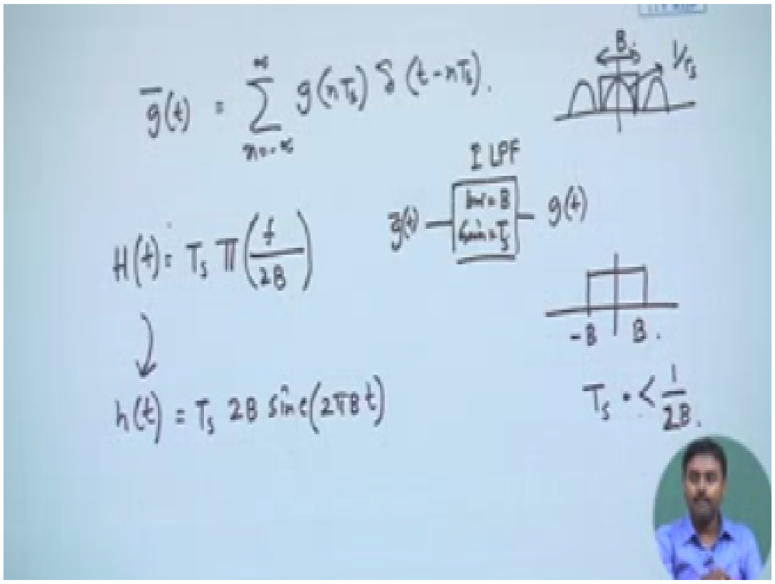

So what we have so far said that this gtbar right, which is the sampled version, what is that, that was actually at that instance we had some sample okay. So that sample multiplied by Δ at that point okay. So let us call that sample at nth instance as g(nTs) okay. So this just represents that at nth instance what is the sampled signal of that particular G okay.

This should be multiplied by Δ(t-nts) okay. So at nts what is the sample, this form, and we are multiplying there with the Δ, so that must be this gbar, because gbar was nothing but that sampled with that strength there was a Δ function at that location. So that is what we are doing, so this must be a representation and goes from -∞ to +∞ right. So this is our gbar right.

Now what we have said that if it is following Nyquist criteria that is happening, then I can actually filter this, that is what we have said that gbar T if we try to see the frequency domain representation that looks like this as long as Nyquist criteria is fulfilled. If I just put a ideal low pass filter which is having a band of B okay, this must give me back my original signal okay.

And also we need to see that this low pass filter that I will be putting if I wish to go back to actual original G, what was the strength of this one, that was 1/Ts. So low pass filter must have a gain of Ts, so that they cancels each other okay. So this has already gets in terms of power gets reduced by 1/Ts if we can amplify them in terms of sorry, amplitude gets reduced by 1/Ts, if we amplify them by Ts factor this will originally go to our original Gf.

So basically this pass through a low pass filter must give ne GF as well as Nyquist criteria is fulfilled right. So what we have to do, this signal has to be passed through a ideal low pass filter of bandwidth B and gain Ts right. So this is a ideal low pass filter, so at this side if we put Gbar T I must be getting GT, this is something we have understood as long as the sampling has been done with Nyquist criteria okay.

That means that Ts is less than 1/2B okay, so that condition has been already satisfied okay. So what is my HF then, that is ideal low pass filter which has a gain of Ts, so it must have a gain and that is a box function, so like in box function if we represent by π, so it must be a box function of let us say F frequency with the box width of 2B, because it should look like this from –B to +B.

So it is a box of width 2B, so this is a box function with a gain Ts and this one right. So what is the corresponding Ht which is the impulse response of this filter okay, I know the HF corresponding Ht, we know for a box function the corresponding Fourier Ht will be Fourier inverse transform, so that is a sinc function we already know that. So that sinc function should be Ts remains over here that should be 2B because it is a 2B box function, so 2B sinc sin C, 2πBt right.

Now let us say we have just satisfied the Nyquist criteria okay that means our Ts what have to happen that must be less than 2B right. Just satisfied means I also take the equality condition. Now there is a case that when I can take the equality condition if the highest frequency, the signal we are considering highest frequency of that is not a impulse okay, there is no impulse in the highest frequency and it almost goes to 0 okay. If this criteria is fulfilled then I can always sample it, so what we are trying to say is like this.

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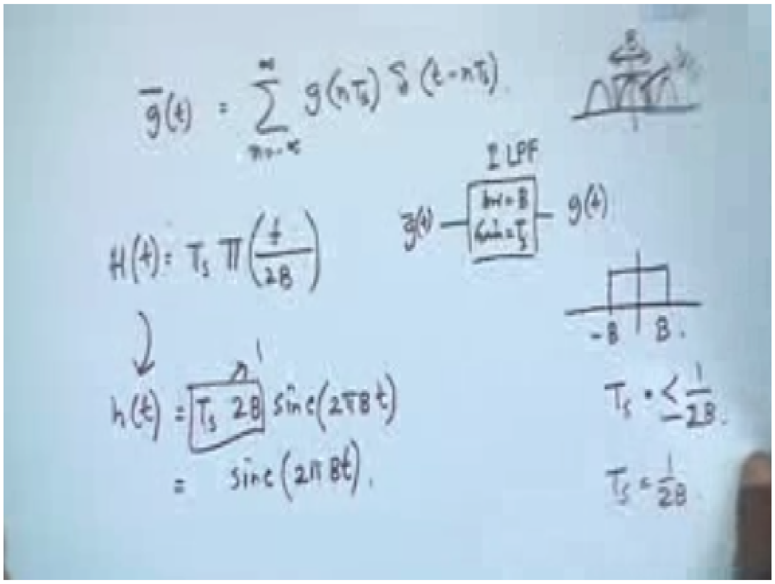

Suppose I have a signal whose frequency response is this, what is this, this is a cost megacity, if I now sample it there is a problem, if I just do it with a Nyquist frequency of means that sampling frequency is just matching with the frequency of the sinusoidal. If that is the case then I will have a problem, because this is a bad limited signal of course, with the bandwidth suppose this is omega M let us say.

If this is omega M the band or if this is FM at FM we have something okay. So this is FM, so this is a band limited signal of highest frequency FM, but at FM it has a impulse. There if we start doing sampling probably there will be super imposition of things okay. So as long as there is no sample at the end of the band, we can always even do Nyquist sampling at that frequency okay.

So basically instead of writing this less than I can also write less than equal to and the best I can do is Ts=1/2B. So let us say we have done this sampling just with the Nyquist sampling bit. So then Ts=1/2B, so this becomes 1, so I can write this as sinc 2πBT right.

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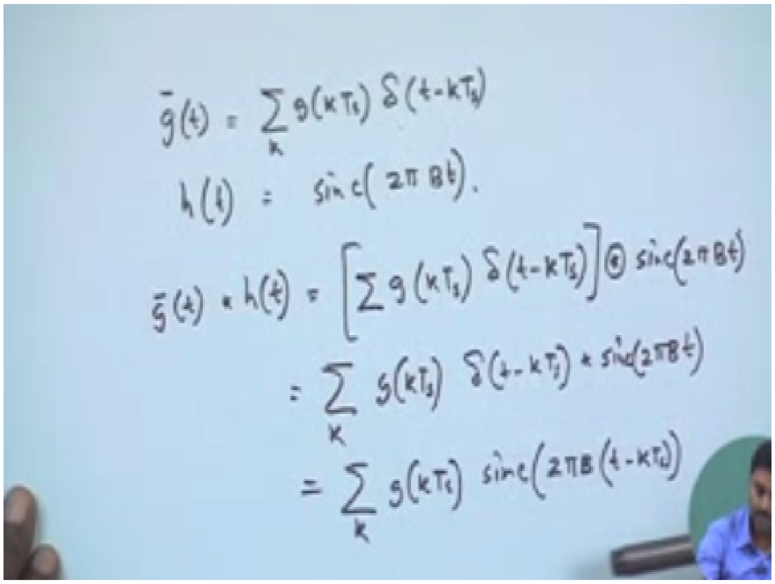

So this is all fine now what I have to do is I had a g(t) right which is represented has this Σ let us say if have k or n which every you put k gKTs δ t – kTs you can even put n earlier I was writing n so you can also do that okay so this I the gT and my ht is sinc 2ΠBt right so what will be the out it is convolution we know it is a linear time in variant system because it is ideal this one so the output will be convolution of ht and g^t so g^t convolution of h(t) what is that this convoluted with this one this one convoluted with this one but this is a δ function.

So convolution is just keep this signal in tacked put it will just go at that δ location right so what will happen it will be just have this g(kTs) δt – kTs this whole thing convoluted with sinc(2ΠBt) right because this Σ has nothing to do no k factor over here so I can take this inside the Σ so I can write k g(kTs) so this is just a δ convolution with this so δ(t – kTs) convolution sinc (2ΠBt) this is just a sinc function but at that δ location so this must be sinc2ΠB insisted of t it will be (t - kTs) right.

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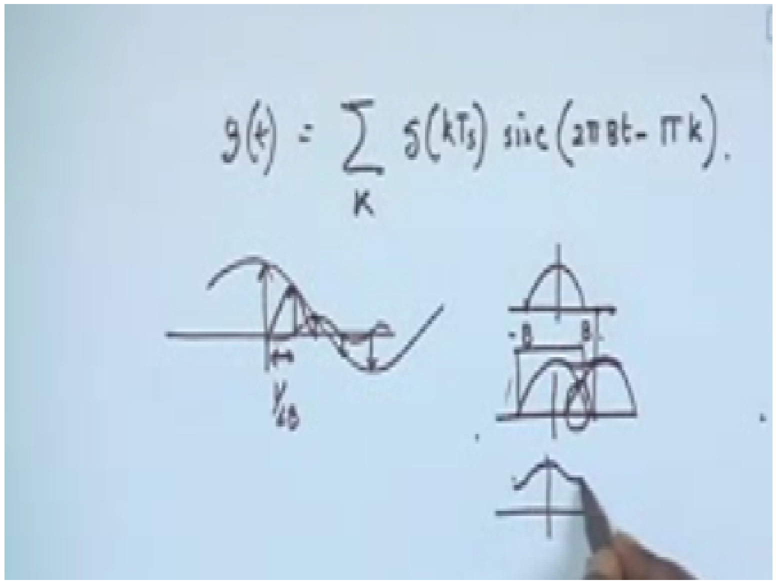

So what is that this is actually Σ k g(kTs) sinc (2ΠBt - (2ΠB x kTs) B x 2Bts we already know it is one so this will remain as Πk right so what exactly is happening we already know that if we do inquest sampling and the from the frequency domain we have a understanding that if we put this filter will be at the output will be just getting out the signal so this must be gT according to understanding of the frequency domain so if this is gT what is actually happening let us say this is my gT I have now taken those samples at gkTs right.

And at that sample we are putting a sinc function okay so at that position we are putting a sinc function with this parameter so basically that sinc we will putting a sinc which goes to 0 over here and then keeps on going and here also there will also be sinc which goes to 0 over here because this is actually just what is the separation that is ½ 2B and this sinc function with 2ΠBt this is just a shifting factor just exit to next this one okay.

So basically they goes to at ½ B they goes to 0 so it is just those sinc function if you just put one after another they will reconstruct the signal so basically what happens we have understood now that a signal is nothing but you take those samples and put a sinc function over there okay so that is all you will have to do so basically it is nothing but addition of all those sinc which recreates the signal.

So that is why it is called that this particular thing is the interpolation of the same signal gT can be represented as a infinite sum like this one infinite sum of sinc function so sinc becomes the bases almost like Ferrier series so Ferrier series we are saying that any signal can be if it is a periodic signal can be represented as the infinite sum of those exponential things here also it is a infinite sum of delayed sinc function have in strength where they are also every frequency term was having strength.

So basically this sinc function becomes a bases function and having strength which is exactly the sampled valued at that location wherever that sinc is different okay so this I called the reconstruction theory of sampling so basically why we are able to do sampling because underlying we have this understanding w know that if I do sampling with incuse criteria I can always all those samples I can represent with the corresponding sinc function and I will be able to represent it.

This passing through a idea low pass filter is nothing but that every sample is now getting that sinc function because the filter has ideal filter has impulse response of sinc so every pulse that comes and falls on that at that location there will be sinc function which will be getting created, so basically this passing through ideal low pass filter just recreates those sinc function for every impulse that you give at the input and that is what happens.

Because the Σ of all those sinc function are the reconstructed message signal so it gets reconstructed so reconstruction of a message signal is not a big deal and that is why we know that sampling signal will always give us opportunity to gain reconstruct it later on okay so this is something we should have in mind whenever we are doing it but remember it can only be done if we are following inquest criteria.

Otherwise there will be distortion okay so that particular distortion is called a VLC in the literature so basically whenever you have a band limited signal okay of let us say –B to + B and when your sampling the sampling frequency is less than 2B so what will happen the Fs might be it will not be this is to B it should be beyond that but we do not suppose deliberately but it over here so what will happen this will get repeated at a every Fs.

So at this location there is a huge amount of aliasing which will be actually creating the signal like this because they will added and then after that even if you put a low pass filter so what will happen the signal will not be the original one it will more look like this, this is the earlier is in so basically whenever you do sampling you have to careful that you are employing like ways criteria otherwise your sampling will, will not give you opportunity when if you put ideal low pass filter they will not give you opportunity to re create your original message so that is very important so and that is why sampling theorem is there okay.

So now what will try to do before you can going towards the modulation like amplitude modulation or all sampling modulation falls position modulation or means the with modulation what will try to do we will try to see some kind of sampling which are more practical okay, so we will start discussing about some practical.

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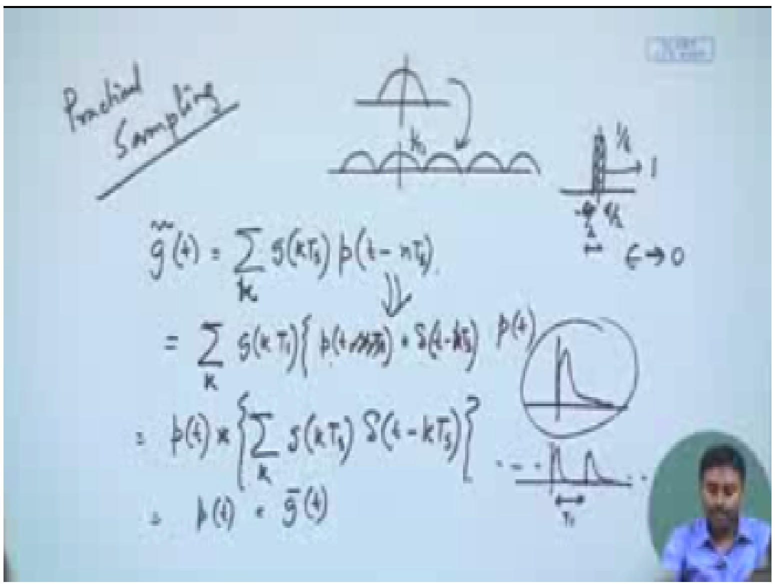

Form of sampling about practical sampling means see as you might have seen that I did sampling of a signal is the band limited sequence, so once it is band limited the pass spectral density is something which is also band limited and we integrate it from – into + we get a finite power right but whenever we sample it after sampling what is happening within this 1 / ts times it is overall amplitude spectrum is getting reduce but this is getting repeated of two ∞ right, so overall powr is becoming infinite.

So where from this just by doing sampling we are getting infinite power where from we are getting this power so basically what is happening if you think about the sampler which was that train of δ function we have already earlier proven then there are δ function has means single δ function as infinite energy, if that is the case a δ function train will have infinite power that is the source of power, so basically we will never be able to achieve or δ function or δ train or train of δ function.

This is something which is not possible because that if you wish to get that kind of signal that will give you means that will be actually generating infinite power which is not possible so this ideal sampling which was proposed by like this is not possible in practice, so we have to now think about some sampling which are actually valid in practical scenario so what will happen the pulse is that will be creating that will b or finite to it, it is not infinite scenario small like δ function δ function the pulse with we cannot talk about that width that is infinite small we have already given one example.

In our earlier class that they have said that is the pulse switch let say like it – ε / valid from – ε/ 2 2 + ε / 2 and the pulse width will b 1/ ε then the area under it is remains 1 and then we make ε tensed to 0 so what happens is pulse with almost vanish it because the pulse rate is ε that goes then means goes towards 0 and this goes ∞ so that is actually in pulse so impulse the pulse width is infinite symbol is small it tends to 0 whereas actual pulse in our finite duration, so now whatever modulation will be discussing or whatever.

Means sampling will be discussing will be discussing with respect to pulse or finite width or arbitrary shape pulse which are practically genera table and other level okay, so therefore suppose we multiply the signal gt with respect to that kind of pulse let say we have a pulse Pt which looks like this okay it goes to 0 there is here, so this is our Pt now what are impulse so if this is the case then when I do suppose I take a train of this pulse so how that will look like, so there will be a pulse again after Ps.

There will be another such pulse and so on which will get repeated from - ∞, so if my signal original signal gt I start multiplying with this particular pulse it will look like this n or K whichever you put KTs now it will be multiplied not with δ but this pulse right so this is what will be happen, but what we can say is any pulse is nothing but the convolution of that pulse with the δ so Pt – nts I can always write mathematically as this, this is nothing but Pt – NT_s convoluted with or sorry Pt.

Convoluted with δt – NTs okay so whenever we do this convolution and this will be creating a pulse at same similar shape pulse at t and t – NTs so this to our equivalent representation that is something we know from the theory of δ function because this Pt has nothing to do it does not have sorry this will nothing to do with K so this Pt can go out with convolution so some and convolution can be basically fit, so this happens to be VKT_S into $\delta t - KT_S$ right.

And we term this as g δt okay, so because this is not original sampling so what we are not writing it at as g^ or g sorry g bar t which was original sampling with δ so this sampled with that pulse and then we could prove this so I can write that it is nothing but PE that pulse is pulse convolution with this is actually again we are going back to the sampling with δ train okay for mathematically of presentation so this we can write as g / t so that what happens whenever we actually sample it with any arbitrary pulse, so we are actually theorizing it so that we can sample it with any arbitrary pulse then we can take some finite with pulse probably okay. So now if we just do furrier transform of this.

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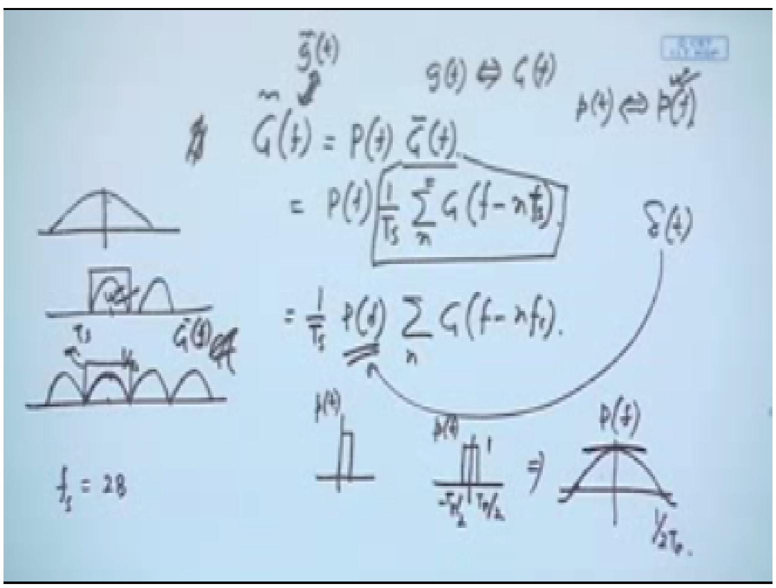

So basically if we write that as let us say capital G till f that is the furrier transform of g till the t let us say these are furrier pare and we also know G t as a furrier pare of gf and pt and furrier pare of pf suppose we know that, so for the pulse also we know the furrier transform if that is the case what will be g ~ t g ~f because g ~f we have already proven that is p t convolution g hat or g bar /t, so it must be in frequency domain multiplication so that should be pf which is the furrier transform of the pulse itself multiplied by this g bar f right.

Now this g bar f we have already evaluated earlier while doing the sampling theorem, so we can write this ef this should be this we have already proven that, that is 1/ts ∑ if we just write n or k let us write n so this is g and it is f- nt or fs right. So this is something we have already proven that this g hat t is this equal to this is something we have proven in our sampling theorem by doing the furrier series and then followed by a furrier transform, so that is what happens.

So basically what we get is 1/tsp f I can pf inside right so if the something wishes so or otherwise I can keep pf over here, so basically it is ∑ n g f – nfs right that is all that is happens right. Whenever we have any arbitrary pulse now for any arbitrary pulse we have a representation right, so if this pulse was ideally δ t then what will be the corresponding pf can be constant so if this is a constant immediately what I get I get by earlier representation okay.

And of course that ts can has to be given so that this ts gets cancelled so basically if the pulse is something like this I can always put our low pass filter with ts again then ts will be cancelled out with the frequency ideal low pass filter and then because this gf – nfc just look like this okay it is repeated and then ideal low pass filtering will just give me back gf but now what is happening this if I see this gf – fc that remains the same let us say we have just turn the this Female Speaker: has ben chosen so that they are just equal to this fs is just equal to 2b so then this will be just repeated from there with the strength of each of them are one / ts right.

So this is my g means this is actually my g hat or g dot f this if I pass through a ideal low pass filter with the gain of ts so that ts and 1/ts gets cancelled and I get this, original gf but now because I have done the sampling with the pulse this pf should be multiplied with this function. Now any non ideal case of pf suppose let us say my pf look like this or my pt looks like this a pulse of certain duration or my pt looks like this okay.

So it is defined from –tp / 2 to + tp / 2 okay, strength is one so that is my pt, immediately what will become the pf? Pf will look like a because this is a box function so it should be a sync function because the width is tp this should be 1/2tp okay so that should be my pf now this should be multiplied with this one so what will happen? One side multiplied this particular gf also will get a multiplication with this pf so basically the gf whenever it gets multiplicities the frequency spectrum of the gf gets little bit distorted.

So this is what happens if I give a instead of ideal pulse if I give some other non ideal pulse I will always get a distortion, but this distortion I can restrict by what method if I make this tp smaller and smaller what will happens this 1/2tp will go away further away and this will almost become flatter in the region of interest from –b to + b there it will not get any distortion due to that due to the presence of the spectrum of pf.

So that is what you will have to do whenever your pulse is not a impulse function we can understand that but what you can always do that instead of having a impulse function you can have a finite duration pulse but lesser the width of the pulse better you will be in terms of distortion of the original signal. And then if this is flat enough so this will remain almost intact I can now put a ideal low pass filtering and I can get my signal back without much distortion.

So this is the theorem when we do not put impulse or let us say impulse function or if a train of impulse for our sampling because pulse will means pulse represented by impulse cannot be generated or recreated using any general or day to day circuitry because it required infinite energy so if we give a finite duration pulse the lesser the duration is falter its frequency response will be and less amount of distortion you will be getting due to the presence of that pulse okay.

So in the next class what we will try to do we will try to see different kind of pulse amplitude modulation by employing this means practical means pulse train okay so that something will try to study and will try to see what is the effect of them and then probably will have a brief discussion about what is pulse sampling modulation how we can employ pulse position modulation and pulse width modulation okay, thank you.