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NPTEL ONLINE CERTIFICATION COURSE

Course
On
Analog Communication

By
Prof. Goutam Das
G S Sanyal School of Telecommunications
Indian Institute of Technology Kharagpur

Lecture 07: Fourier Series (Contd.)

Okay, now we will try to give some interpretation on this negative frequency a concept of negative frequency. So let us first try to see whenever we are saying the concept of negative frequency what exactly we are getting? We are getting a as you might have seen, so we are having this at positive frequency we are having this mode Cn.

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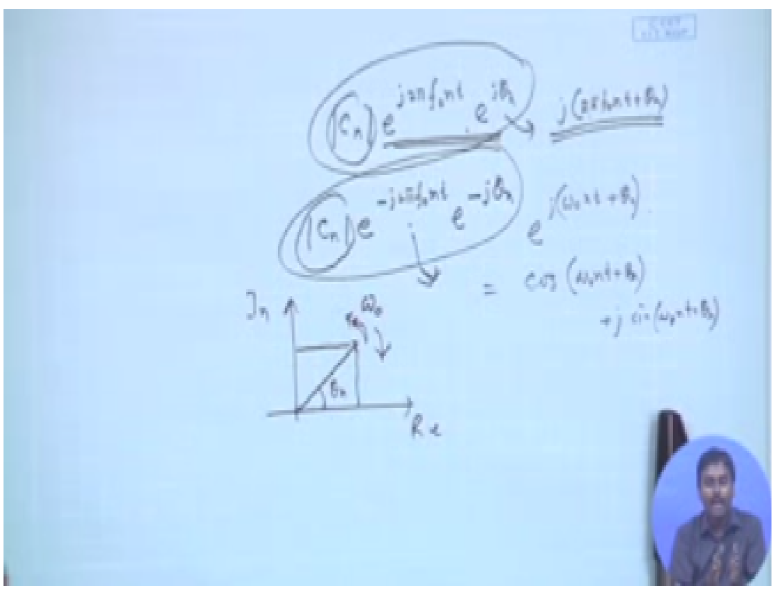

And then some $e^{j2\pi f0nt}$ x $e^{j\varphi n}$ right. And at the negative I have same Cn $e^{-j2\pi f0nt}$ $e^{-j\theta f}$ this is something we have right, if you carefully see these two things those two are nothing but 2 phasor right okay. So if just ignore this mode Cn what do you have we have this, this is the unit length phasor in the complex plane so if have this is the real axes this is the imaginary axes right, so this just a unit length single length phasor which is actually rotating as see here the variables t with respect to t if I start increasing t what is happening this overall angle so suppose of this one it is actually J 2πfont + θn.

So θn must be some constant okay which does not depend on t and after that it is just with respect to this phasor this particular unit thing is rotating okay and then only we will be getting this phasor representation because ej this one has a real part and a imaginary part when I put ej2πf0 I can put as Ω 0, Ω0 nt + θn what does this means actually it is cos Ω 0nt + θ n + j sin Ω0 nt+ θn okay. We all know that if with uniform angular velocity Ω0 if w rotate f phasor the projection to a axes along which it has a watt along that axes that is what is happening the projection actually follows a sinusoidal or co sinusoidal right it is something we already know.

So basically what is happening whenever I am actually rotating this with angular velocity Ω0 if I trace this projection with respect to time I will get at the real axes or co sinusoidal and at the imaginary axes or sinusoidal this is actually the representation in complex to mine right, so because we have representing that as complex signal now what we also know that eventually by our furrier series representation there was a positive frequency and there was a negative frequency, how that negative frequency looks like?

Negative frequency also is having just this representation minus some or in minus of j 2πf0nt + θn okay. So it also has that concept of θn but whenever we are giving the rotation it is actually if we have to trace back it is rotating in the other direction. So this is rotating in the positive direction whereas as we increase this θ basically it will be if I see in the co sinusoidal it is actually initially first it will be increasing as I basically increase this t whereas here if I increase t I will see that there will be a reduction.

So basically if it rotates on this side the other one is rotating on this side okay. So these two phasor just the positive one is representing a rotating phasor which is rotating in anti clock wise direction right, and the negative phasor is something which is rotating in the clockwise direction right so the negative frequency actually means these two phasor and means clock wise rotating phasor gives me the negative frequency.

And what is eventually happening if you see why the signal is becoming real I have this representation now if you are interested what you can do, you can start putting this positive phasor and trace the signal imaginary signal. And if you represent this rotating one which is clock wise rotating and if you how you trace this two signal at these two up with respect to time every time instants you take those values and add them.

What eventually you have see that this thing will get cancelled out so in the imaginary axes if you add these two you will get nothing it will be always 0 for every value of t it will be 0 and ink the real axes for every value of t will be getting occur different co sinusoidal okay the co sinusoidal we were producing this is exactly what is happening. So that means my co sinusoidal I can actually have a equivalent representation it just mathematically equivalents a equivalent representation where I can say that it is just a addition of two phasor rotating clock wise and counter clock wise and that counter clock wise is actually the positive frequency and clock wise one is the negative frequency.

If I just take these two things and then plot the frequency domain representation that gives the due to exponential Fourier series whatever representation we get so and in that the frequency is just this which is in terms of this complex rotating phasor frequency or that phasor rotation frequency angular ration okay that the phasor has okay and that is why because there are two counter rotating phasor so I have automatically on getting a concept of plus and minus.

A general sinusoidal does not have this direction okay because general sinusoidal is just represented with respect to Ω0 right we just write sin some 2π f0nt so it is just having a frequency I do not have a concept of negative frequency let us whenever we start representing it in the complex domain then we a representation because there is a direction of rotation. The phasor generates sinusoidal but it actually generates due to its rotation in which direction different direction generated different kind of complex means a components okay.

So that is in actual that is the means that is the realization of negative frequency why negative frequency comes and we have already seen that these two are equivalent representation here in phasor also you could immediately realize that it is nothing but a real one if I take this counter rotating phasor the imaginary part gets cancelled out you can just trace them in time at them you will see that they get vanished.

They will just negate each other and you will see that it is getting cancelled out whereas the co sinusoidal one there will be a real path and that is actually the signal that you are getting. So any real signal it must have a equivalent phasor rotating phasor representation and because he rotating phasor representation has direction. So that is why we are getting this positive and negative frequency concept so it is just a concept as long as we are representing it in exponential.

So you might be asking why do I need then the exponential representation that might be a natural portion that we have a equivalent trigonometry representation already the exponential one is actually giving me that trigonometric representation on this whereas the exponential one is actually a complex one which is originally not existent. However my, this one this one the actual co sinusoidal representation that is the real one those are the signal which exist, so why we are actually looking for this exponential representation.

The reason is that in the trigonometric you have a sinusoidal as well as the co sinusoidal representation okay which is double representation and they have different coefficient. Instead of that if I do just exponential representation yes I get complex number C_n but I get a unique thing okay of course you know that at every C_n has two component the amplitude as well as spectrum but sorry amplitude as well as the phase but we know that if I just inter predate in complex domain it is just a unique representation C_n I do not need t A_n and B_n it just C_n okay.

So that is the utility of representing it in exponential Fourier series, and you will later on see that exponential Fourier series also at zeo towards realizing another thing where any signal can be represented that is call the Fourier transform. Well next probably not in this class we will try to appreciate what happens when we try to represent any signal not just periodic signal with Fourier series we have already presented periodic signal.
Later on we will try to see that if we have any non periodic signal also that can be represented and they are you will see that exponential Fourier series becomes extremely helpful because that gets extended towards the Fourier transform okay. So with this may be we can give two very important examples, so let us see the first example.

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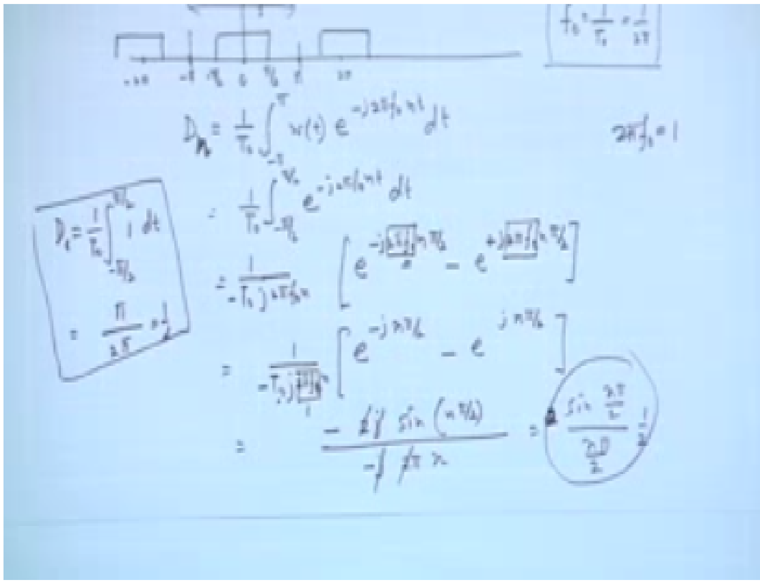

So is the first example what we are trying to do we are actually just whatever we are leaned we are trying to apply them okay, so we have a signal this is something I have already means given you now I just try to solve that so this signal is something like this it has a period of 2 π and it is represented as this, this is π this is 2π so after every 2 π it gets repeated same in the negative also. So at -2π so basically this is eventually –π/ 2 this is π/2 this amplitude value is one so this is my gt if you just take okay.

So all I want to do is I wish to see how this signal can be decomposed in to known signals okay. So what we have to do we have to evaluate first let us say D0 okay so D0 is nothing but 1/t0 integration now I have to do it over a period that should be – π to + π that can be one period it is 2π because this thing gets repeated at every 2π okay so –π to + π I have to put the signal itself okay so which is ψ t x $e^{-j2\pi f0nt}$.
So what is f0? F0 is 1/t0 and t0 is just 2π so 1/2π right, so this is what we get. Now evaluate this integration so all we have to do is evaluate this means actually Fourier series we are try to evaluate. So if you evaluate this integration what we get see –π to π – π to –π/2 it is not 0 actually so whatever happen is integration in that period will be 0 and + π/2 to π it is 0 so it is only define between – π/2 that π/2 and the value is 1 right so it is just e^{-j}. So just do the integration it should be t0 – j2 πf0n and $e^{-j2\,\pi f0n}$.

So we have to put the limit so one will be π/2 – right again – π/2 so – π/2 become + okay, so this is all that we get. Now you know that f0 is 1/2 π so this is actually becoming 2 πf0 that becomes 1 so this entire part you can put as one. So 1/t0 okay, because at d0 it should be already n =0 so it becomes just 1 so if I just evaluate d0, d0 it should be again 1/t0 integration from – π/2 to + π/2 only defined and ψ t becomes one that is alright but this becomes one already so it should be one dt.

So this becomes π/t0 is 2 π so this become ½ so d0 is ½ already okay this one so how much we get it is $e^{-jn\,\pi/2}$ - $e^{jn\,\pi/2}$ right this is fine okay. So now all we have to there is a minus so this is just again put all as theorem will be immediately seeing that the j part get cancelled and this becomes -2cos right sorry the j part is actually because it is minus so the j part will be there son it will be -2jsign inside a part right so it becomes -2j sin n π/2 okay and then again you have this becomes one t0 is 2 π and so basically you have –j is already there which gets cancelled t0 is 2 π and n right, so 2 get cancelled so we have sin n π/2/ n π so this our answer right.

Eventually if you plot it is actually almost like a sin x/ x kind of thing so I can put this as by 2 and I can multiply in to 2 ½ it is just half sin sink function of sin c function right. So what is happening?

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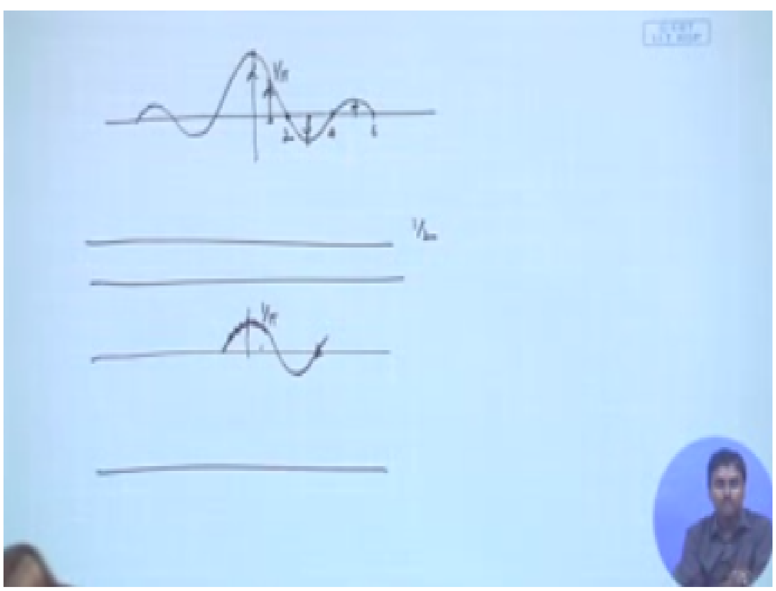

If I just plot it, it is same function at d0 it is ½ and then it just goes to again 0 at 2 goes to again 0 at 4 and so on right 6 so I just comes down like this goes up something happens over here that is the envelope of course and at every value this is 0 so this is the plot we get D0 and corresponding Dn do you have a phase over here? You do not right, because you have seen that all dn it is no longer complex so they are all real values so basically the phase plot will be all 0 it does not have any phase plot just amplitude plot what eventually is happening at these the sinusoidal are getting mixed.

The sinusoidal firs at Dc term of ½ so basically if you just take a signal of strength half okay. next a sinusoidal of period one sorry period that two pull okay and the amplitude is at this point probably the amplitude will be 1/ π, so with 1/ π and period that 2 π if you just put a sinusoidal sorry it should not be cos it should be see the phase part is not there it is real part so it must be the cos part right so it is the cos part and the amplitude is 1/ π.

So basically what is happening like this you keep the second one which is 2 x 2 π okay? That is not there that is not present, so that coefficient is 0 so keep putting all those sinusoidal you add them together you will see that the same signal is being generated, as long as the coefficient are right if the coefficients are not right they will probably not reproduce the same signal. So this is what exactly is happening so any signal if you do Fourier analysis it will give corresponding sinusoidal okay nothing else.

The next sinusoidal probably the next one will not be there but the next to next one that is the thrice the frequency so it should be repeated here the sinusoidal is repeating within this period only once okay so it should be repeating thrice within this period with a amplitude of this much if just put that and you keep adding those all those sinusoidal up to infinity you will see your signal is getting represented.

So this is what exactly happens in Fourier's why we have done that? We will see that later this is a typical example which will be required. But next will be doing a particular means analysis for a particular signal which is immensely important you will see the sampling theorem is based on this particular Fourier series representation we will come back to that but before doing that we need to understand something called the Dirac Δ function.

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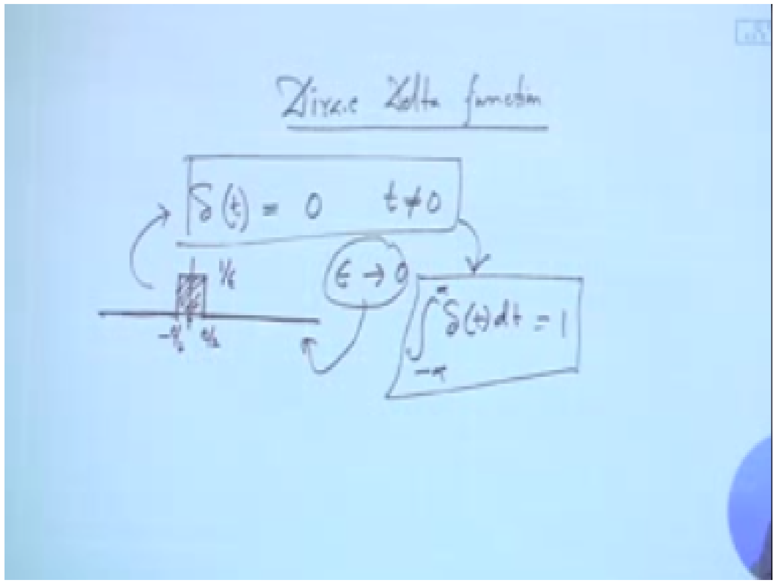

So we will try to first defined what do we mean by Dirac delta function so Dirac delta function by definition it is like this is defined it was of course defined by Dirac and it is one its name, so δ t is have a value 0 as long as t is not equal to 0 that is the definition of a Dirac delta function okay it has some more definition we will come back to that later okay. So the definition is that rather than 0 it is 0 everywhere it is just at 0 it almost undefined it can take very huge value okay.

A equivalent representation of δ t is something like this it is the box function okay defined from –ε / 2 to + ε/ 2 and the strength of the box is 1/ε okay, so what does this means? This will become δ or this will approach to δ Dirac delta the way we have defined it if ε tens to 0 so what will happen? As ε tens to 0 the separation will be this is the box function so it is 0 everywhere as ε becomes 0 and 0 so all other values it will have eventually 0 only at value 0 it has value 1/ε, ε tens to 0 so this goes towards infinity right.

So it happens to take the position of δt okay. So this is one typical example of δ function and whenever you give this definition it has a added valuation, think about this thing from this definition if I integrate δ t from – infinity to + infinity according to our definition is a box function so it should be area under this box all of the plus it is 0 so this is this area is one so this must be one so that is the added thing added definition of δt.

So δt must be defined with respect to this and this one should be added okay which test almost test δt towards that box function, if these two definition or the definition of δt then I can give a definition of δt in this manner where I have to put additional condition that ε ten to 0. So this is in away our definition of δt. Now let us try to see some property of this δt oaky suppose I have function let us say xt.

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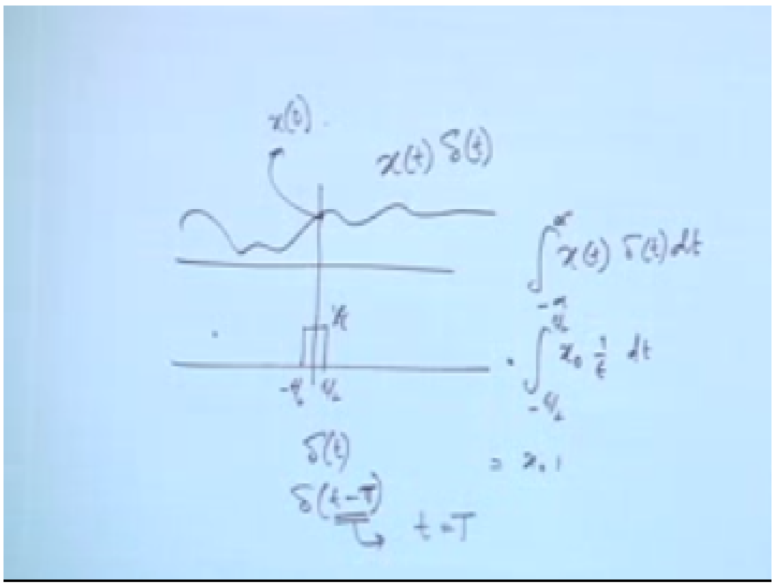

I multiply this function with δt okay, what do I get, so what is happening? Xt might be having any value think about that box definition of δt so at t = 0 it has around t = 0 it has this definition right –ε/2 ε/2 and it is 1/ε, so now what will be the value of this it will actually this value can only be realize if I start first integrating it okay let us say I can if I can integrate this thing why I am doing that because δt integrated fashion it has some definition right. So if I integrate it now what is happening on top of this I am putting this definition that ε tens to 0 okay so if ε is tending towards 0 so what do I get?

When I integrate as ε become 0 I have a this function is there this function at x0 as some value okay the ε is very small and infinite signal is small then x0 around that will almost not be varying as long as it is a band limited signal that means it is not change arbitrary that this point immediately next point it can go to anywhere, as then it is a continues differentiable analytic function with all those nice property then I know that very small ε it cannot change it cannot deviate.

So it will be as long as by ε is very small within this –ε2/2 to + ε/2 it will be always taking value x0 okay. So what I can do? I can write that it will be taking x0 value so it should be so everywhere it is taking x0 value and this integration has to be done from –ε/ 2 to +ε /2 okay and within this what is the value of value of δt? That is 1/ ε, so this should be the integration x0 is now a constant because I have already said that if ε is sufficiently small x0 is not means xt is not varying around that value okay so x0 goes out and again I get one.

So what is eventually happening? Whenever I multiply a particular function with δt and if I integrate it I will be getting that particular value back at that point. So the functional value at that point I will be getting back this is called the sampling property of δt so basically whenever I multiply a function with δt and integrate it, it will just sample out the value where δ t is defined okay. So whenever I say δ t it is defined at 0 whenever is say δt – T it is defined at t = because this has to be 0, it is defined at t = T.

So at T whatever the value of that function it will just take out that value so this is why δ as a very nice property mathematically but it picks up the sample value at that point wherever that δ is defined. Rest of the values it will just make it 0 okay so this is very important property of δ and this is some property which will be heavily using especially you will see the entire sampling theorem is based on this, this particular understanding of δ.

So what will do in the next class that will try to explode this particular sampling condition and some other property of δ to get a compulsive understanding of a particular signal, so this is something we will do next.