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NPTEL ONLINE CERTIFICATON COURSE

Course
On
Analog Communication

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Lecture 08: Fourier Transform

Okay so welcome to analog communication course so far we have done some discussion about Fourier series and the representation of any signal any periodic signal let us say with respect to some known basis periodic signals like a sinusoidal or exponential sinusoidal so this is something we have done we have also done a simple example in the last class I just recapitulate that.

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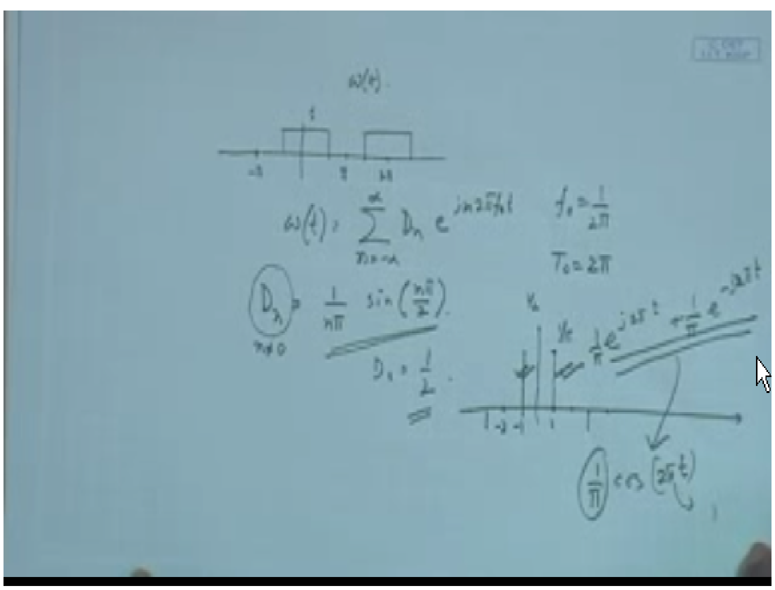

So what we have done is we had a signal like this which has a period of 2π so define from –π/π and it is basically centered around 2π again and so on so it just repeats after every 2π and the strength is 1so this is our general signal which is wt okay so while representing this WT we could represent WT as exponential Fourier series so n→ - ∞ I am just recapitulate it the same result so it is DN $e^{Jn2\pi}$ F_0T where of course $F_0=1/2\pi$ which is meaning that T_0 is 2π so that the period.

And we have also evaluated our DN so that happens to be whenever n is not equal to 0 so DN becomes 1/nπ sin nπ/2 so this was our DN and e_0 was just ½ so this is something we have already done so eventually what we have got is something like this a spectra which was defined in frequency domain so it was having value of 1/2 or 0.2 at 0 then at every F_0 $2F_0$ $3F_0$ and there are values so at F0 it is if you just put sorry at means frequency a component 1 so that is n =1.

So there if you just put this you get 1/π and so on so at 2 so this was at 1 at 2 we got 0 and so on it was going into negative again 0 and so on and symmetric so that is -1 -2 something like this so this is what we have got if you see carefully that our this TN term is now a real number all DN including d0these are real numbers so initially what we have told that DN can be a complex number and the amplitude of that is plotted in amplitude spectrum and the phase of that is plotted in phase victim.

Because the DN are all real and so the D and D-n so basically what happens the phase part is always 0 so the phase spectrum will nothing but will be just 0 everywhere so it does not have any phase part right for all the frequency component so for every other frequency component starting from means the DC value which is at frequencies 0 so it has a ½ so that means 0 you if you wish to plot this it should be something like this all the basis function.

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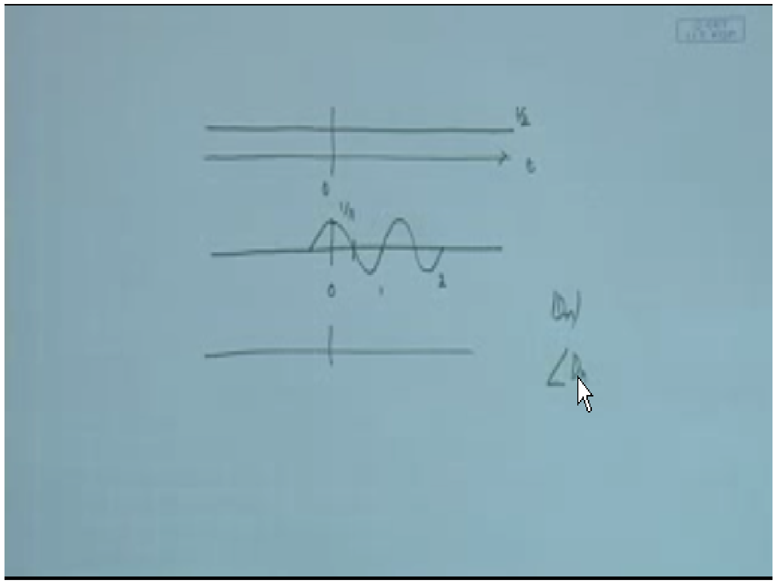

So D0 so that is in time if we just plot so that is actually half strength DC that the strength is ½ times 0 -∞+∞ defined as this; next is at frequency one arch so the it is a sinusoid or we should say co sinusoidal with amplitude 1 /π so all we have to do is we have to draw a co sinusoidal of frequency 1 that means the period is 1 so basically so this part if this is 0 this must be 1 and soon it repeats after the radius 1.

So the period is 1 and what happens basically the strength should be 1 /π as we have got and remember because the phase is 0 so the cost actually starts from here on the 0 phase co sinusoidal so always whenever you are actually constructing these signals you have individual constituent of this same signal so basically signal has been decomposed into multiple either DC value and all other orthogonal co sinusoidal.

So we just try to plot those co sinusoidal now what is happening you should also remember that this particular co sinusoidal if you see it from here because it is a exponential Fourier series so this part is actually making the $e^{J2\pi}$ that F is now 1 okay so basically it is at 1 this is defined so F will be 1over here so it is this right and you have a corresponding co sinusoidal at the negative frequency so that also you will have to put so and because the DN are for both the cases 1 /π.
So it becomes 1 /π $e^{-J\ 2\pi t}$ and then if you just add these two what you get is actually a cost with strength 1 /π okay so that means the frequency of it is 1 and the strength is 1 /π so basically whenever I am plotting this co sinusoidal you need to understand that we are taking a positive frequency and the corresponding negative frequency taking both the amplitude as well as the phase information.

So if you see over here carefully we were actually taking the amplitude information and putting it over here because the amplitude is even symmetric amplitude spectrum so we will have same value over here so modulus of DN will be put over here and the phase of DN will be since put over here + and - so that is the phase part because here the phase is 0 so we do not see the effect of phase.

But otherwise the phase effect should be there and the corresponding co sinusoidal that will be happening then will give you exactly the amplitude as well as the phase information so cos will be either lag or leading by that amount of phase so this is what you have to do whenever you are trying to drew means draw the corresponding basis signals you have to take from the amplitude signal the amplitude and you have to take phase from the phase spectrum okay.

So these two information you have to take and you have to draw the basis signal similarly the second one will also come over here if you see the second one the second one has amplitude 0 at frequency 2 right so of course the time period will be at1 / 2 so basically what will happen this has a higher frequency so it will actually repeat within this okay so the thing will be at a higher frequency but because the amplitude is 0 you do not see this.

Because it has its echo sinusoidal where the amplitude is 0 so the third constant is non-existent the 4^{th} one if you see is that the amplitude which is negative so accordingly co sinusoidal will have a negative sin in front of that so the co sinusoidal will start at a π phase shift so it will be looking like this frequency of that will be θ so you start putting the θ frequency within this and it keep doing that as you go along the frequency line.

Your frequencies will be doubled 2 or 4 times and accordingly the repetition period will be synced and you will have to draw those things with exact phase and amplitude once you draw this and if you add all of them up to ∞ you will see that exactly the signal that we have targeted will be generated so this is this is the significance of it so basically what we have done is taken a periodic signal we could represent them with respect to corresponding co sinusoidal or DC value and the specification that is required is completely contained in that DN.

So DN basically gives you two information one is the amplitude information and other one is the phase information for every frequency whenever you specify n that means it is nxf0 that fundamental frequency so n x F0 at that frequency what is the corresponding amplitude that will be the modulus of DN and the phase of DN will be the corresponding phase so you get these two and then you plot it and whenever you are plotting you are actually putting a co sinusoidal.

So you are taking two values from the spectrum as we have already explained that it should be always because it is a real signal so it should always take a exponential Fourier series one from positive and a corresponding negatives so these two together will generate your co sinusoidal θ that is what is happening so and you will be getting as long as you are having that complete amplitude spectrum and complete phase spectrum for every frequency term you will be getting corresponding amplitude.

And phase and accordingly you draw the sinusoidal and you add them up if your phase and frequency sorry phase and amplitude information are correct that means DN you have correctly evaluated you will always if you sum all of them you will always get your signal back this is what is happening that every single now is decomposed into all the harmonics of it and every harmonic is defined for this particular targeted signal is defined by its specific amplitude and specific phase okay.

So far so good we know now what is the utility of Fourier series and how Fourier series actually represent a signal in a different domain so it is actually we are representing the signal in frequency domain so we are almost seeing the frequency component or the sinusoidal component of that signal which constituent this particular signal okay so this is all good now we have talked about a measurement of a signal right initially in the first few classes so we have talked about energy of a signal.

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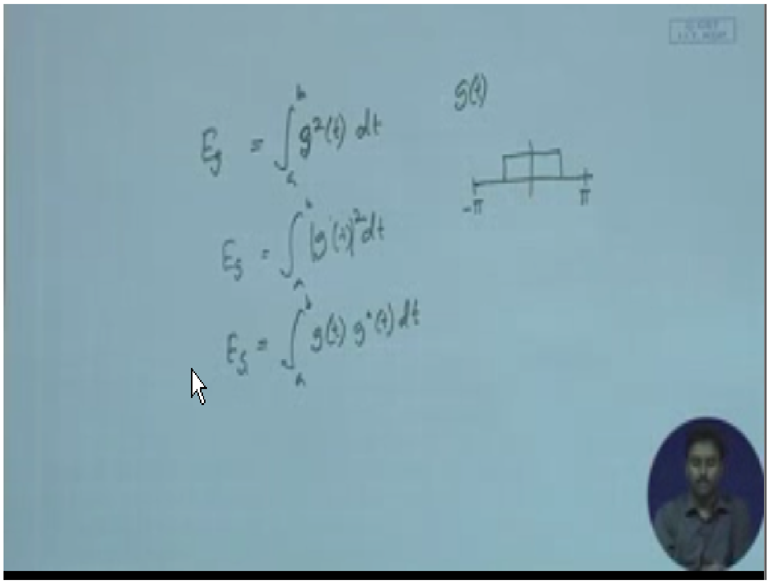

So if the signal was real we have told that the signal has to be squared and integrated over the time that it is defined so let us say it is defined from A to B so as long as signal GT is a real signal this is what we will have to do and we get the corresponding energy of the signal this is something we have defined that squaring the signal and then integrating over the time that it is defined now if the signal is not real that it is a complex signal.

Then we have to specify another term that means you have to take the modulus of that signal and square it so basically you have to do this mod GT square DT and you have to integrate so same thing you have to doubt for calculating valued into energy because it is a complex signal and energy is a real number so we need a real number so we have to do or we have to take the signal we have to do a complex conjugate of that. This is all something we have defined by definition this must be energy right.

So there is something we are told already that is a measurement of signal like we have done that vector analogies are in vector it is the distance of the vector or the strength of the vector which is also means measured by distance^2 right so this is how it is being measured now if we just say for any signal we want to basically get the energy value of that signal right it is very easy take that signal integrate it over the time that it is defined.

So suppose we have that square pulse right so remember it is a periodic signal so definition of energy we have already talked about periodic signal where it becomes a power signal right so the definition of energy is not there but if we just take the definition from -π to π then it is a energy signal and then within this we can still define the energy of this signal so if you wish to calculate the energy what you can do you can just integrate from - π to π G ^{2} right.

If this is my signal you can get it is there any other way or in the frequency domain can we also talk about this energy representation or can we just say that this particular signal we have already identified that it is means defined by multiple basis signals and their linear combination and the coefficient of them are already known via Fourier series right so if we just take this signal and take those constituent is there a possibility of defining the energy from that perspective.

So in the frequency perspective every component how much energy it actually provides to add up to give this energy is it possible so that something will be now exploding so let us try to see what happens to this.

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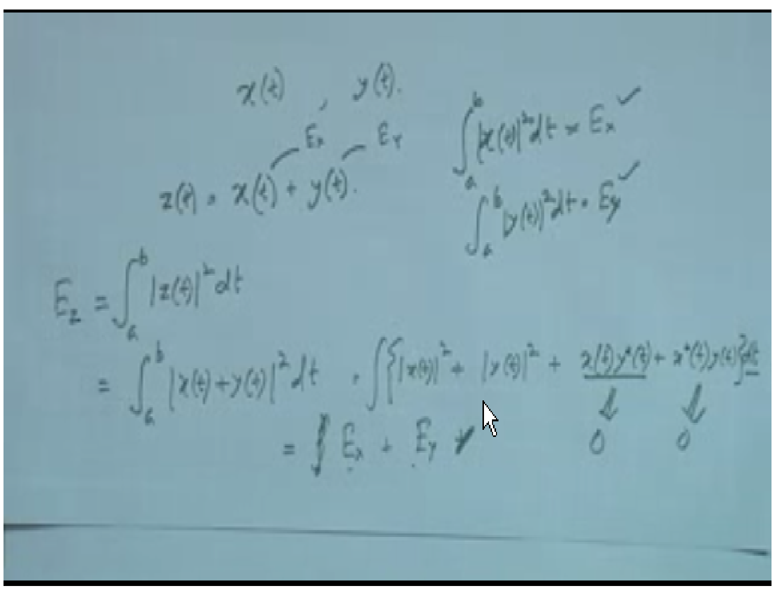

So before that let us say I have two signal X(t) and Y(t) okay so my definition these two signals are orthogonal to each other so I am just trying to exploit that orthogonal property and energy calculation okay so we are just assuming that two signals very simple two signals which are orthogonal to each other it might be just like cos ω CT and sin ω CT we have already proven these two signal are orthogonal within a particular period right.

So if it is defined from- pie - pie okay so take these to signal and let us say I wish to generate another signal JP which is just an addition of these two signals now my target is so I am almost going towards linear combination of orthogonal signals so right now I am just giving a very simple example that I have got two orthogonal signal and just adding them it is a linear combination with coefficient 1 +1 so I am just adding these two signal now we wish to see suppose for X(T) and Y(T) I have already evaluated the energy of them.

So this is suppose E X and this is EY I have evaluated the energy of that so how I have evaluated X & Y if means I do not care if they are complex or if they are real so I just have done this XT modular square integration BT over the period that it is defined okay so A to B so for our case if XT is cos ω CT so -π to+1 okay so this is something I have evaluated and this gives me EX similarly for y also I have done the same thing so I have done a to be Y T modular square DT that gives me EY.

Suppose I know these two parameter EX and EY both of them can I now calculate the energy of JT without actually means adding them integrating and all those things so does orthogonality gives me some idea of these things so let us try to see so energy of JT what does that mean a to be modular JT^2 DT right so that actually means AB now it should be XT +YT modulus whole square DT right now expand this so that should be mod XT whole square plus mod yT whole square plus xty *t + X*T YT right.

This whole thing integration over DT right so now you can see this particular part mod XT whole square DT that is actually Ex for us this part is the Ey for us now the orthogonality plays a big role what is the integration under t over that period as long as XT and YT are orthogonal signal we know that that term should be 0 that is the property of orthogonal any orthogonal signal if you take that signal and take the complex conjugate of that multiply that and integrate over the period that you are targeting or where the signal is defined.

It will be always as long as XT YT are orthogonal that should be always 0 so this term happens to be 0 this term happens to be 0 so basically what we get is the energy of Z is just the summation of energy of x and y is a very fundamental result which has big implications okay so we will try to use this and try to prove a very fundamental theorem which is called Percival theorem so let us try to see if now from this if you just go to a linear combination of orthogonal signal.

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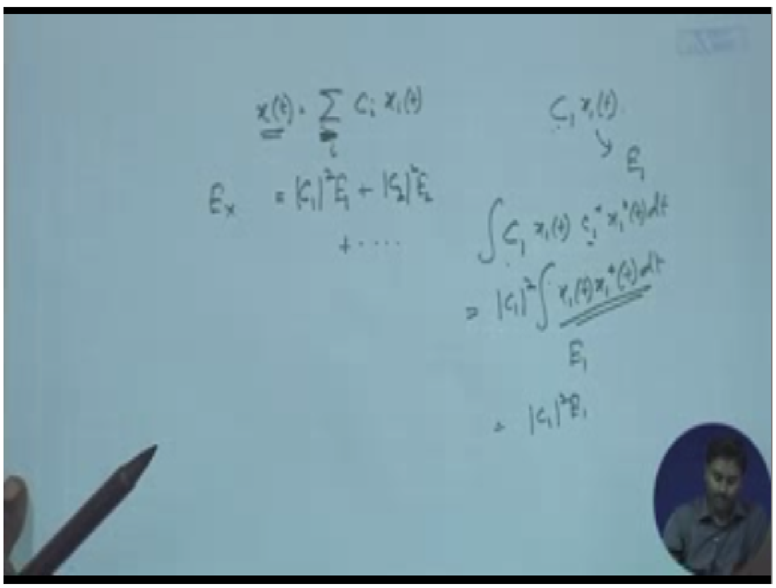

So suppose I have a signal let us say XT that is defined as some linear combination with coefficients CI X_i (t) where all the X_i are orthogonal to each other okay so that means that if you take cross product so X 1 with X 2 T star and you integrate over the particular targeted period you will get 0 okay so these are all excited are all individual orthogonal signal and as I goes from in to its limit as many eyes are there okay.

So over I you have to solve ok so if this happens what we can get if you see so the energy of XT if you just try to calculate this that must be we know that it is a just take the previous example if you have two orthogonal signals to each other it will be individual energies square or individual energy addition of that right so if I try to calculate the individual energy the individual constituent is suppose let us say I start from 1 so or 0 so let us say I start from 1.

So c1 x1P what is the energy if we know that X_1 P has an energy of E_1 okay already therefore that c1X_1 t will have an energy of you have to integrate modulus of this so C_1 XC_1 star so that should be mod C1 ^{2} that will go out of the integration so if I just do that C1 X 1T C 1* X1*T DT integration so C1C1* will be mod $C1^2$ this integration X1pX1*t DT that is the energy of X1 which is E1 so it should be mod $C1^2$ E1 so that is the first term.
So it should be the first term should give me the first energy component and similarly all other energy component so there will be C2 mod square E2+… up to as many I s are there all cross components because the signals are orthogonal to each other will be just vanished so I can always evaluate as long as I can represent a signal with respect to the constituent orthogonal signal and they are in linear combination.

If I know all those coefficients I can always evaluate the energy of the original signal. This is what was targeted and I can do that now let us apply this to Fourier series let us see what happens.

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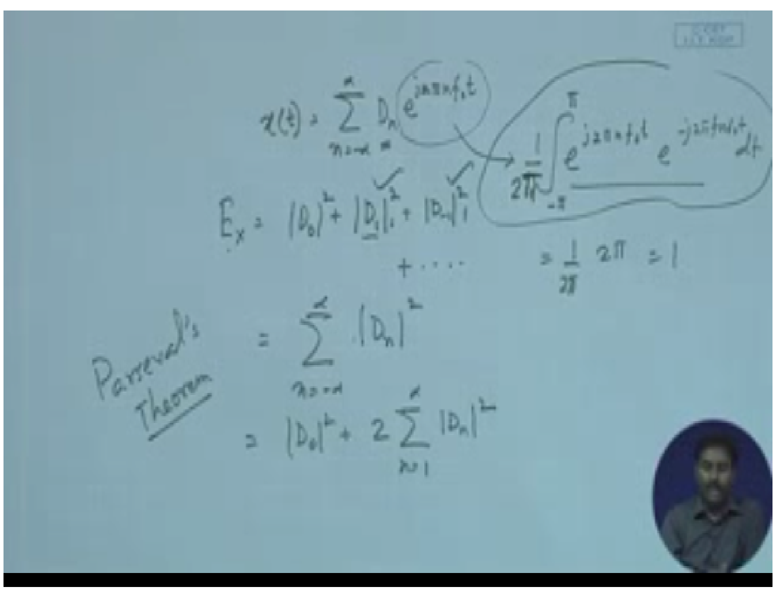

So in Fourier series what has happened any signal XT a periodic signal of course that has been represented as n=-∞+∞ we are taking the exponential Fourier series formula so that should be DN E^- it was sorry+ J2π and F0T right so that is the representation we have already seen that and

BN must be evaluated as done by Fourier series right so this is something we have already seen now what we are saying is this that here as long as we know DN we know that for every end these are those orthogonal components right.

So therefore the energy of X must be the individual energy where the coefficients are it is a linear combination of these orthogonal signal right each of those orthogonal signal what is their corresponding energy so if I just take $e^{J 2\pi n f0t}$ and I wish to evaluate its energy so what I have to do I will take this signal I have to do a complex conjugate 2π n F0t DT integrate over a period let us say -π to π if I do not do this will give me 1 so I get 2π right so this is what I get immediately.

So I know that if this is a particular signal and I wish to evaluate its energy so I can always evaluate it like this and of course whenever we are evaluating this energy it also has 1/t0 if you remember the signal means the way the signal has been evaluated so the corresponding DN term will always have 1/T0 so that 1/ T0 will come so 1/ 2π will come this will give me 1 so that will be always giving me 1 because this 1 /T0 or 1/T 0 means 1/2π that will be there okay.

So I have to evaluate this way the energy 1 / 2π -π to π you have to do this right and I get individual energy of those signals as right so this is something I get so once I have got this then I know that the overall energy should be just individuals suppose the 0F term is t0 so mod $D0^2$ + then I have to get D1D -1everybodies mod square so $D1^2$ + D -1^2 +… now I also know that of course remember that this coefficient square into the energy of the first one okay.

Now the energy of the first one is for any value of n that is 1 so that is why all those values are coming to be 1 right so what we get what all energy is just the summation of this coefficients so I can either write them as n =-∞ to +∞ mod Dn^2 or I can because I know these two are symmetric so I can write it as mod $D0^{2+}$ 2 x n going from 1 to ∞ mod En2 I can also write this way this is the famous possible theorem of computation of energy of a particular signal.

So basically what has happened if I had a signal XT I was aware of its energy so that I can do in a very simple way that okay take the signal integrate it from within a period so let us say –π 2π I can do that that is one way of doing it but because I know the possible theorem what I can do if I know the Fourier series representation of this signal that just means that all that DNs I know I have evaluated all those DN and so once I know all those DN the energy of the signal is also can be written or can also be represented in a different way.

That means all constituent signals all constituent means orthogonal signal you take their coefficient and take a modulus of that next square and all of them you will get the energy of the signal so basically it is just very simply understandable that a signal if it is already there you have decomposed into multiple basis signals and each of those basis signal has their own energy okay so the energy is dependent on the corresponding coefficients that is the N.

So as long as you know that DN you can take modular square you get your energy representation of that particular constituent signal and we call the signal is linear combination you can just take linear combination of this energy or addition of this energy I should not say linear combination addition of these energies that will create the overall energy what exactly is this if you just take it from vector analogy so we have already done a vector analogy to signal so what is happening if you have in vector we already know.

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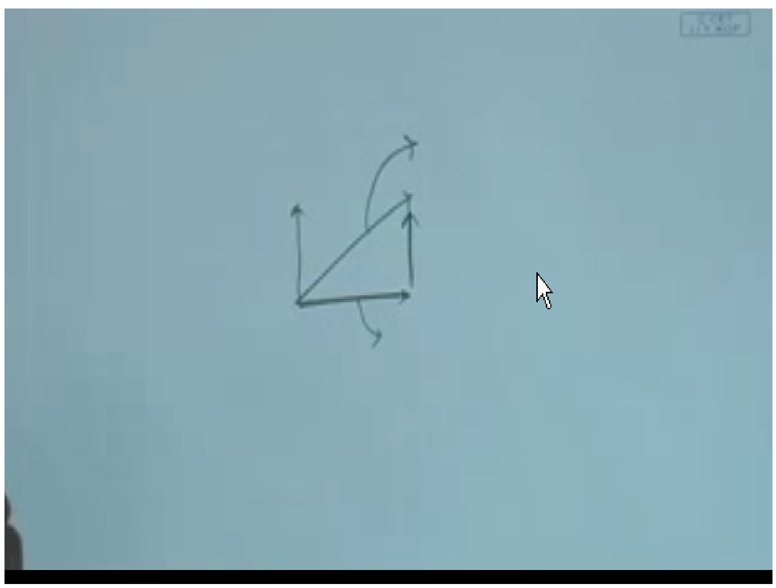

That if a particular vector is represented by linear combination of two vectors then what should be the strength of this vector that must be through Pythagoras theorem we know because these two are orthogonal so that must be this square plus this square right and this square is the actually strength or measurement of that vector so measurement of this vector plus measurement of this vector must be the measurement of this vector as long as the measurement is distance square okay.

So if that is the case and if we are talking about Euclidean space we can always see vector measurement which is distance square is just the vector measurement of the corresponding component in means when represented in orthogonal vector space right same thing is happening over here if you see what is happening this DN so whatever D and we are putting over here this D ends are actually the measurement or the coefficients of the corresponding orthogonal signals that constituent the signal.

So as long as we know this D ends we can get corresponding energy which is actually again the measurement of that particular signal or the energy of that particular signal so measurement means energy over here so energy of that particular signal so we get energy of all constituent signals and it is just the addition of those energy which gives me the overall energy of the signal that's the strength of possible system.

So basically Percival theorem state almost similar thing as we have got in vector so all the cross terms are vanished because they are orthogonal similar thing happening like vectors and only the corresponding measurement add up to give me the measurement of the signal and our measurement is energy so in the next class what we'll try to do after getting this sense of signal we'll try to think about a signal which is continuous in time that true means in a way but most importantly if it is a non periodic signal or energy signals what to do what kind of treatment we should take for those kind of signals will concern means will concern ourselves about that on.