Communication Networks
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Module - 08
Media Access Control
Lecture - 37
Slotted Aloha (contd.)

So, so far we have actually discussed this Slotted Aloha. We have seen the system model, we have seen all the approximations, we have tried to build our DTMC and we have got basic ingredients of that DTMC. So, let us try to we have already defined the state model, and we have also started defining the transition matrix. So, let us try to see that definition.

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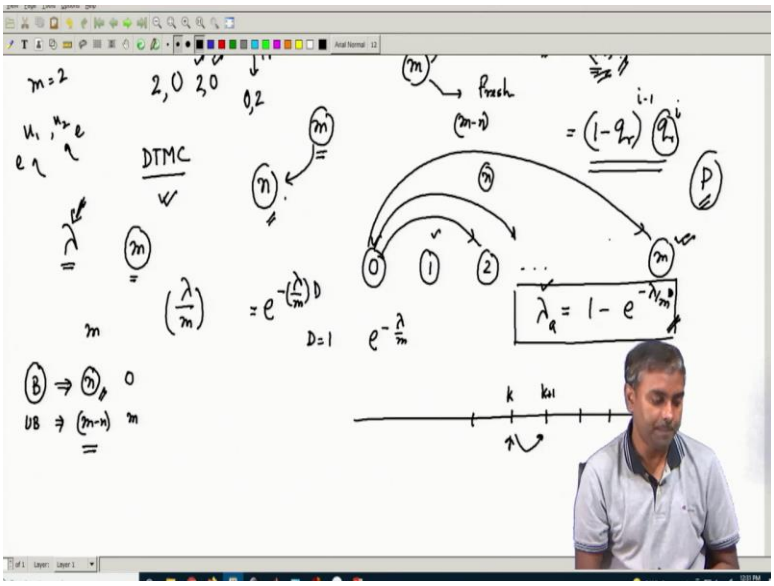

So, that was our last discussion. So, what we have seen is that there are two types of node: one is backlogged and the other one is unbacklogged ok, new node ok, or new arrival node. So, we have defined that out of total m nodes, n nodes are backlogged and the rest m minus n are not backlogged and from there we have also got two probabilities that are in the next slot. So, suppose he is in this particular state.

So, in time frame if we see that time frame are all slotted. So, at this time frame k, he is in this state n backlogged and m minus n not backlogged. So, what will be happening in k plus 1, that is what we are trying to attempt, trying to see. The states are given like this 0 means n equal to 0, n equal to 1. So, 0 backlog, 1 backlog, 2 backlog, and so on up to m backlog can be there. Now, we want to see the transition probability from this k to k plus 1.

So, what might happen? Over here in the kth one, it might happen that this m minus 1, they might attempt transmission ok, and some of them might attempt transmission according to their arrival. What is the probability? That is given by this. Now with this probability that in the earlier slot, he had arrival, 1 arrival, and from there, we can calculate whether he will be attempting; because new nodes mean he has arrival, he will be definitely transmitting.

So, that is the probability that he will be attempting, like this every node who is independent of each other might attempt transmission. So, each of these m minus n nodes might attempt transmission with this probability, with this given probability; 1 minus reward minus lambda by m, a if D is taken to be 1, if that is not the case D must be also multiplied over here lambda by m into D, it should be something like that ok.

So, right now, we are taking the assumption that D is equal to 1 ok, unit time we are taking. So, that is one possibility. And there is a possibility that all those nodes that are backlogged with value q r will be also this q r-value, they will be also attempting transmission. All this taken into account, we have to now decide whether it will be a successful transmission it will be idle or it will be a kind of collision ok. And accordingly, we have to see where it will go.

Let us try to understand that particular part ok? When he is in 0 let us try to see; that means, 0 backlogged nodes, all m nodes are. So, basically, backlog nodes are 0, and all m nodes are backlogged. If this is the case what will happen to him? Ok. Can he now we ask ourselves can he go to 1? 1 means after this next state over here, there will be 1 backlog node. What does that mean? From 0 there will be 1 backlog node, is that something that can physically happen? No, this cannot happen.

As you can see for him to be backlogged there must be a collision and in a collision at least two or more have to be involved, two or more stations must transmit then only there will be a collision. So, if there are 0 backlog nodes, no nodes are attempting ok from their retransmission reserve because there is nobody in the backlog.

So, that is not happening. So, it can be only backlogged nodes or fresh arrival nodes, who can attempt this? At least more than 1 node has to be attempted, then only there will be a collision, and then only there will be a backlog. So, the 0 to 1 transition is not possible, it can go from 0 to 2, if 2 are simultaneously transmitting, it can go from 0 to 3, and so on. It can go up from 0 to m, all transitions are possible ok.

So, let us try to see we will be we will be generating two kinds of probability that will actually characterize this whole transition matrix. So, this is just to give an example of what can happen and what cannot happen.

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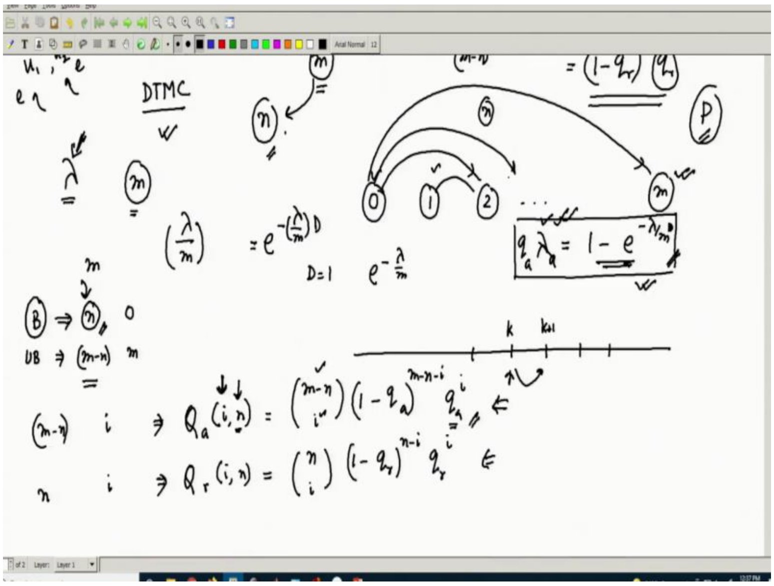

So, let us try to see, I will first define them and then I will explain. So, Q I, n; this is one particular probability I will be expressing, and Q r i, n. What do I mean by this? First I will write the expression. So, this is m minus n c i and 1 minus this whatever this lambda a is there. So, that lambda is a whole to the power, or maybe I can just because the other one has been given definition as q.

So, I can write it q a. So, then accordingly I will modify this; so, that we have a consistency of symbol. So, this can be written as q a to the power m minus n minus i and q a to the power I will, I will explain this later. And, the next one is n c i 1 minus q r whole to the power n minus i into q r to the power i.

Now, let us try to see what this says. This says if I am in a situation where n number of backlogs are there, therefore, m minus n will be non-backlogged. So, out of m minus n, i nodes are attempting transmission.

So, basically, I am in state n, and in that I nodes, I new nodes who have new packet arrival are attempting transmission in the next slot in k plus 1st slot. So, what is the associative probability? That is a binomial distribution, out of m minus n I will have to choose i with means what is the probability? That I of them will be successfully transmitting; so, Q a is this, this says that he will be attempting. So, i of them are transmitting, and m minus n nodes are there which are new means they are unbacklogged.

So, among them m minus n minus i nodes will not be transmitting. So, this is exactly the probability that out at state n or out of m minus n node, i node will be attempting transmission. So, this is the probability Q a says that out of m minus n new node i node will be transmitting, attempting transmission in the next slot k plus 1. Similarly, this one is telling that out of n backlog nodes, i nodes will be attempting transmission in the next slot. So, these two probabilities we have now characterized.

Now, let us try to see how I actually define this state transition. So, now, I will write the state transition and then we will explain it. So, let us try to write the ok.

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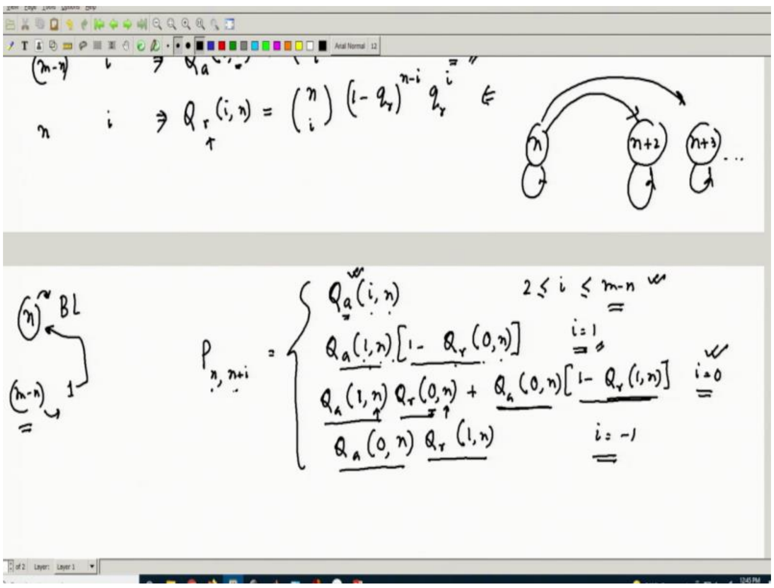

So, let us try to write the state transition equation. So, that is P n to n plus i ok. So, from n nth state, it goes to n plus i, and then its definition is given as this ok. It will be Q a i, n; I will explain everything, I will first write this for i it will be Q a 1 n into 1 minus Q r 0, n or i equals to 1. And, this will be Q a 1, n; Q r 0, n plus Q a 0 n into 1 minus Q r 1 n ok for i equals to 0 and the last one is Q a 0, n; Q r 1, n for i equals to minus 1 ok. Let us try to see.

So, n to n plus I, I am giving a definition over here if I equal 1; that means, it is just going to the next state. So, over here 1 to 2, 2 to 3, and so on ok. As you can see I start from 2 ok. So, therefore, as you can see n, it cannot increase to the immediate next stage, it goes to 2. So, that is why we are we have already explained that it cannot just increase 1 ok. So, that is something we have already described so, the same thing is happening over here.

So, basically n to n plus i, i can be going from 2 to m minus 1 n. So, this might happen. So, this is one description we have given. Then there is the next one where it can only go from n to n plus 1 ok, i equals 1 for that we have a special definition. And, then I equal to 0; that means, it comes back to the same state n to n and this is I equal to minus 1; that means, n to n minus 1, 1 state behind ok. So, what is the associated probability?

So, let us first characterize this one. What does this mean? That he is in state n and then he goes to some upper state starting from n plus 2 to n plus 3 and so on dot dot dot. What is the probability? When he will be? So, he is in state which means there are n backlog states right? So, from there he wants to go to this state. What is the possibility? The possibility is if he goes to n plus 2 or n plus 3 or any n plus I; that means, I nodes are in backlog and I nodes are actually now, again additionally going into the backlog.

Because he is going from n to n plus i. If additional nodes are going to be in backlog, what can happen? Those who are already in backlog if they attempt and they are unsuccessful will remain in backlog. So, that is not the thing we should think about. Only backlog can you increase if new nodes are attempting transmission and they are also collided, that is the only time when there will be additional backlog. So, that can happen only when fresh arrivals are colliding.

How many of them should collide? I of them. So, out of n ok; so, he is in n state and I am coming. So, new nodes are coming that come by this. So, that is why we are putting Q a i, n. So, whenever I put and I have a restriction that I must be greater than equal to 2 or at least more than 2 should attempt, then only there will be backlogged. So, if more than 2, any value up to m minus n because m minus n stations are there that are not backlogged.

If all of them attempt immediately it will go from n to n plus i. So, I have already characterized that probability. So, with this value, this value I know from here ok, that is good; one thing we have characterized. So, basically, what has happened? For this one, if you try to see this one from any node to all other higher nodes, these probabilities are all characterized ok. So, that has been done.

Now, let us try to see that this backlog is increased just by 1 ok from n to n plus 1. How that might happen? That might happen if one station is attempting transmission, fresh transmission, and another sum of means either one or more than one backlog station is also attempting. Then there is a possibility that one improvement, one increment will be happening because there are n stations that are backlogged and m minus n station.

Now, among these only one station is attempting, he potentially included in the backlog if these guys are also attempting and they collide. Then they will remain in backlog plus this fresh one who is trying to transmit, he will also go into backlog. So, that probability I will have to find out.

So, what is the condition? The condition is that one fresh station will be attempted. So, in state n only 1 as you can see i equals 1, Q a i equals 1 that is probability multiplied by the probability that all backlog stations. At least more than means one or more than one backlog station also simultaneously should transmit.

So, what is the probability that none of the backlog stations transmit? That is Q r 0, n; so, out of n backlog station none of them are transmitting; Q r 0, n over here I if you put 0. 1 minus that is the probability that at least more than 0; that means, 1, 2, 3 anybody is transmitting, whoever transmit that will create a collision with the fresh transmission. So, this probability if I multiply with this that is the probability that the backlog will be increased by 1. So, n will go to n plus 1. So, that is characterized very nicely.

Now, let us try to see the condition that it will remain in the same state when he remains in the same state. So, let us try to see that. The first condition is that he will remain in the same state if there is a new attempt that is successful; that means, you have only one new attempt and it has been successful.

When that will happen? This Q a 1, n; that means, out of at state n among non-backlog station only 1 has attempted. So, that they will not collide. So, only 1 has attempted, and among the non-backlog, sorry backlogged station nobody has attempted; so, 0 out of n Q r. So, if these two probabilities are multiplied this is a condition of successful transmission. So, that is the condition of successful transmission.

Now what is the other condition? Another condition is that the new nodes do not want to go into backlog so; that means, the new node should not have collided. So, this was a successful one.

So, a new node came and he has successfully transmitted, the also backlog number will not be increased, over here we will try to see that the new node has not attempted. If he has not attempted then when the backlog station will remain the same, if the other nodes who are in backlog either do not attempt ok or if they attempt more than 1 attempt and are not successful; then they will all remain in the backlog. So, what is the possibility that the 1 backlog will be cleared? If only 1 station attempt in the backlog; so, Q r 1, n; out of n only 1 is attempting.

So, if that condition is happening then the backlog will be at least 1 backlog will be cleared, then the backlog will come down. So, 1 minus that if I do, this condition will give me that backlog will not be cleared; either they will none of them will attempt. So, that is Q r 0, n or multiple will be attempting they all collide and they will all remain in the backlog. So, 1 minus this will be giving that condition.

At that condition if none of the new nodes are attempting, then we do not have any backlog improving increasing, or decreasing; so, therefore, i is equal to 0. So, this is the probability that they will remain in the same state, that also we have now characterized. Now, the last one they might come back by 1 because every successful thing will reduce the backlog by 1 if it is coming from backlog ok. So, that particular condition comes over here.

What should happen? At that condition, 1 backlog must be attempted. So, which is Q r 1, n and none of the new nodes should attempt which is Q a 0, 1 n. If these two conditions simultaneously happen then from a backlog 1 guy will be successfully transmitting and he will come to a new arrival node. So, 1 of the backlog nodes will be reduced; so, that is why it goes from n to n minus 1. So, this is the whole transition matrix as you can see.

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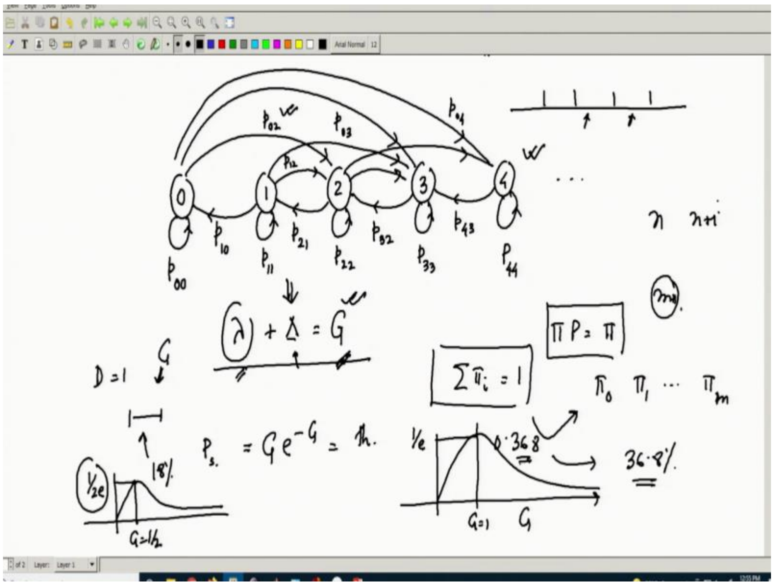

With this, the overall probability state transition will remain like this 0, 1, 2, 3, 4, and dot dot up to m it will go. So, as you can see this will be there p 00; so, 0 to 0, this will be there p 11, p 22, p 33, p 44 ok, only 1 backward state is possible because n to n minus 1, only 1 successful can mean possibly being done in one particular stage from k to k plus 1.

So, therefore, it can only go backward by 1 state which is p 10, similarly from here p 2 to 1, p 3 to 2, p 4 to 3, that is possible. Now upward. So, n to n plus I ok. 0 to 1 is not possible, we have already seen that 0 to 2 is possible, p 0 2, 0 to 3, p 03 and so on all the states ok. Similarly, 1, 1 to 2 is possible that we have seen that the state is just increased by 1, 1 to 2 is possible ok.

Only 0 it was not possible; so, that is why that restriction was given, 1 to 2 is possible, p 1 to 2, similarly 1 to 3, 1 to 4, and so on, all those states will be possible. Similarly, from 2 also 2 to 3, 2 to 4, and so on from every state these transitions are possible. So, all these values are already characterized by these equations, and all these Q r, and Q a are already known by this set of equations. So, basically, you have completely characterized the state transition matrix.

Now, from this example, you can see how a Markov chain discrete-time Markov chain is formed and how the probability transition matrix you get. Once you have got that, it is just solving the set of linear equations pi P equals pi because there are m states; so, there will be m such equations ok. You will get m linear equations with this pi 0, pi 1 up to sorry there are m plus 1 actually, if there are m then it starts from 0 up to pi m.

So, it will have m plus 1 such equation plus there will be one more equation which characterizes that summation of pi i is equal to 1. So, this is the normalization. From this set of equations, you actually solve them, each of which this pi 0 pi 1 pi 2. So, you will be able to; so, how many stations are in backlog and what is the associated probability that you will be able to solve ok, and that describes the system.

So, basically, you can solve the whole system equations or the system dynamics ok? So, once you have done that from there we can actually try to see how to analyze the whole system ok. So, now, let us try to understand how I analyze the whole system from this particular, this particular Markov chain development.

So, one means very simple analyze means analyze analysis I will be giving is that without doing all these things. We will just give a very simplified assumption that overall due to this what will happen to the arrival strength? So, what has happened? I had this lambda ok, I will do an analysis like Aloha. So, this is just to give an analogy and give a simplified assumption that will drive some particular insight we will see.

So, I have an overall arrival rate of lambda and on top of that, I am all also doing this retransmission, due to that some extra things will be also coming, extra arrival rate and I call this G ok, whatever that G might be. So, I have this lambda plus some extra arrival rate due to this retransmission and overall I get G.

Now, this G we approximate as Poisson. This will be happening because if I take this let us say this it is happening in discrete time of course, this retransmission attempt will be discrete. But, if we take those time instances means this particular retransmission attempt to be very small, then the overall time it will take to have new means this retransmission attempt probably will be quite big, the inter-arrival time.

So, with that, we can take that time to be almost continuous, and then with that, we take an assumption that there is a Poisson arrival. So, can we make that assumption ok? In that condition, we can have a Poisson overall Poisson arrival rate of G and from there we can try to see what is the vulnerable period.

So, the vulnerable period is our D or D equals 1. So, within one slot with a G arrival rate, what is the probability that there will be no collision? Ok, now we are trying to just analyze that. So, with the G arrival rate; so, now we are not doing all these analyses. We just try to do a simplified almost similar Aloha-like assumption and analysis, just to get some insight. So, if we take that G and now try to see that within one slot no other arrival means will be happening. So, that that particular thing we want to take.

So, what is the possibility that only 1 arrival happens, because then it is successful transmission and what is the possibility that no arrival happens? So, this this something we want to analyze. So, let us see what will be the success probability, the success probability will be that within this only 1 arrival happens which is actually G into e to the power minus G because time is 1.

So, the probability that 1 arrival will be happening with rate G is G into e to the power minus G divided by factorial 1 because it is a Poisson arrival. Factor value 1 is 1, so that is why this is and time unit is 1.

So, this is the probability that a successful transmission will happen, whenever there is 1 arrival I know that in the next slot, it will be a successful transmission. And, I have now taken this whole retransmission attempt plus the fresh arrival altogether as G. So, therefore, that is the only arrival that is there. So, I can now just do this and we can get this to be my success P success.

So, basically, P success means at every slot whether there is a successful transmission or not. So, that is actually the overall throughput also. If I get the P success, suppose out of this 90 slot I get the P success probability, whatever probability I get, suppose out of a very large number of this one how many slots will be populated? So, that is the throughput; so, therefore, this is also a throughput.

So, this throughput if I now plot with respect to this G will be seeing a remarkable improvement. This has a similar curvature, but it goes up to 1 by e, now this peak will be 1 by e at G equals 1, and 1 by e means it is 36, 40 0.368 or I can see 36.8 percent because that is throughput, 36.8 percent is now, successfully transmitted at G equal to 1.

Earlier what has happened? In Aloha it was like this, at this was at 1 by 2 e at G equals to half. So, the highest achievable throughput was 1 by 2 e which is half of this. So, that is why it was like 18 percent or ok. So, the throughput was 18 percent over here. Whereas, just by doing this slotted Aloha, so far we have not done the detailed analysis which is this Markov chain of course we will do that later, but right now we are just trying to give a sense of what will be happening.

So, we can see that the overall throughput can be enhanced up to 36 percent, which is a huge improvement that we can see. Double throughput we can get by just employing this slotted time slot.

So, as we have earlier the vulnerability period was reduced by half, and we can see the enhancement in throughput, we can get a double throughput just by doing this. So, as you can see this is how people have done the analysis and from the analysis, they could take the inside and put that inside the intuition of protocol development and immediately could get a very good return in the protocol performance. So, this is this is what has happened in Aloha to slotted Aloha transition. You can now understand why people have done that and what is the approximate benefit that you get out of this.

But so far, we still have not done the analysis of this thing. We have constructed that Markov chain and we will try to see what is the implication of that more accurate analysis. This is not an accurate analysis, this is an approximate analysis.

We are told that the actual arrival rate is lambda and that retransmission was taken in a more approximate wise. We have taken that delta, taking the approximation that is also our independent Poisson arrival with rate delta. So, those things we have taken, but that is not the exact thing. So, the exact scenario will be analyzed through this method and we will do that in the next class ok?

Thank you.