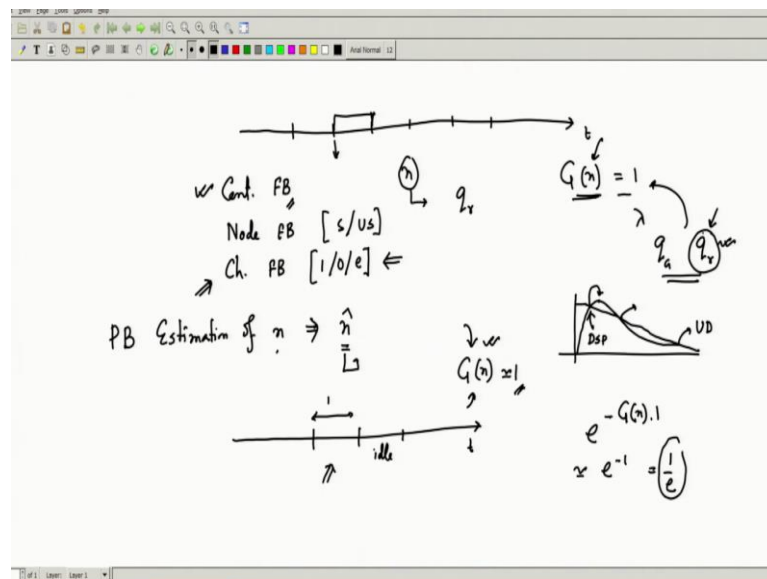


Communication Networks
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Module - 09
Media Access Control Protocol
Lecture - 40
Stabilized Slotted Aloha- Bayesian Estimation

Ok. So, so far we have started discussing our stabilization of slotted ALOHA right and in that particular thing, we have also started analyzing what kind of feedback it can get from the channel. So, we will just briefly summarize that and then we will start on how to stabilize a slotted ALOHA. So, that is something we will be trying to investigate today. So, let us try to see what other feedback methods that we have.

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So, we have told that in a particular slotted ALOHA within a slot or just at the end of the slot nodes are capable of getting feedback ok. So, started discussing three kinds of feedback, one is centralized feedback. So, that was one centralized feedback that meant every node whatever is happening will know. So, that means all the nodes that are backlogged whether they have attempted transmission whether their transmission was successful or not.

So, all this information will come to every node. So, the centralized node is observing all these things, he is capable of observing all the taking all these data, or maybe he might be just taking feedback from every node just after the transmission. He takes feedback and he then broadcasts that feedback to everybody, that is too much to ask for but this is probably the ideal case.

Then you will be able to keep track of how many nodes are backlogged, and what is the current status of them, so all those things you will be able to track. So, this is the ideal or best possible feedback that can happen. There was another feedback method which is called just it will get a feedback of success or unsuccess in the transmission.

So, it is feedback from an individual node perspective. So, every node if he has transmitted in a particular slot, so, if he has transmitted in a particular slot at the end of this one he gets the feedback on whether his transmission was successful or not by some method he can get that feedback.

So, this is node-wise feedback. It is very local and it is only giving two states, success or unsuccess in his transmission, he does not know about anybody else. There is another thing which is called channel feedback that is almost like from the channel you are getting feedback what is the status of the channel in the last slot.

So, in the last slot there might be 1, 0, or e, what does that mean? 1 means there was a transmission and it was successful, 0 means the channel was idle nobody has attempted transmission so it was remaining idle and e means there were multiple transmissions and they all collided; so, it was erroneous.

So, this is the channel feedback that a station might all stations might get from the channel or there might be a mechanism we will see that later on. They might be able to then see what is happening in the channel. Everybody will be doing it individually, but they will be able to detect that. So, these are the three possible feedbacks, which are possible. Depending on this feedback the stabilization methods will be differing ok.

So, the first thing that we will be trying to do is a stabilization where we have some kind of channel feedback ok if we have centralized feedback then we know everything. So, basically estimation of this particular how many nodes are in the backup. So, this state n

that estimation will be possible and if we know this at state n what is happening, then automatically we will be able to update this q_r probability that I will be transmitting.

So, this G_n remains 1 because at G_n equal to 1 slotted ALOHA gets the highest throughput which is 36.8 percent right. So, we want to keep this G_n 1, so for that G_n is a composite factor right? It has the factor of λ ok which is the arrival rate or we can call that, that means that for the stations that have not actually gone into backlog whatever attempt they will be making.

So, we had that means that probability whether they will be attempting in a slot that probability we can get and also we can get this. So, there was q_a and q_r , we can in such a manner we can adjust them because our G_n we have already seen. That is a function of this q_a and q_r , in such a manner we adjust our q_r because q_a we do not have hands-on because q_a is completely dependent on how many means what whatever the arrival rate λ is.

So, q_r we have our control, so we can adjust this q_r . So, that G_n becomes 1 because we want to keep the G_n around. We have already talked about how we want to keep it around this stabilized and its desired stabilization point. So, if you remember our graph, so, this was a desired stabilization point, this was another stabilization point or equilibrium, but that was an undesired equilibrium or stabilization point and this is an unstable point.

So, this is the unstable equilibria this is stable equilibrium, but this is the desired one because here the throughput is higher. Also, we want to keep this over here where G_n equals 1, and that we can do by adjusting q_r that was our stabilization understanding right? So, this is what we want to do.

So, if we have centralized feedback then we have exact knowledge of n and accordingly, we will adjust q_r so that we get our $g_r g_n$ equal to 1, so this is something that can be done in a centralized one. That is why I said centralized is the best one. We can always push our throughput towards that 36.8 percent. For channel 1 I have difficulty ok.

The difficulty is how do I estimate this n because I only know the channel feedback. I do not know exactly what is happening in all the other nodes. I know the channel feedback.

So, what we will try to do is now devise a mechanism so that from this channel feedback also we can somehow estimate this n . So, that is called the Pseudo Bayesian estimation of n . So, we can call that as this estimation as \hat{n} . We would like to ideally get \hat{n} closer to n . So, then the estimation error will be less and then we can actually put G_n as close as possible to 1 and we can achieve the best throughput.

So, we will try to see from this channel feedback how I get this nice estimation of n , ok. So, for that let us try to see when G_n becomes 1, what is actually the average status of a slot remaining idle or a slot remaining in collision, or slot remaining successfully means a slot being utilized for successful transmission. So, let us try to see that part ok.

So, when G_n is almost equal to 1 ok, so at that time what will happen? So, with G_n which is almost equal to 1 is it fresh, or is this the packet arrival rate right, g_n equals 1 is our packet arrival rate now in a particular slot ok? So that means, on average one packet arrives per slot, this is what is happening and we have also seen that this can be approximated as Poisson arrival.

So, we have already shown that with means the slightly lesser value of q_a or this q_a and q_r . So, this must be less than 1 sufficiently. So, we get this to be a Poisson-approximated arrival. So, if that is the case, so, somehow suppose I have done my estimation correctly. So, this is one assumption I am taking. Suppose I have done this estimation correctly and then with that G_n I could adjust I could adjust my q_r , so, this one so that I get my G_n approximately 1.

Now, let us try to see on an average what will be happening, so, this is very important. So, let us try to see in a slot so, I have this in a particular slot arrivals are happening or attempts will be happening with respect to this G_n ; G_n equal to 1 ok. So, this is a mixture of fresh arrival as well as backlog retransmission. So, overall with G_n equal to 1, this is happening. So, one slot is of duration length 1 and it is Poisson we have also assumed approximately Poisson.

So, now let us try to see what is the probability that there will be no arrival. If no arrival happens over here; that means, no fresh arrival happens and no retransmission attempt also is happening. So that means, in the next slot, it will remain idle. So, what is the probability? That is a slot duration of one with a packet means an arrival rate of G_n equals 1, no arrival will be happening.

So, that is equal to the power minus whatever is the arrival rate which is G_n into the slot duration. G_n is 1 approximately. So, this will be approximately e to the power minus 1 or it is 1 by e ok. So, this is on average the probability that the slot will remain idle. So, if I now take a sufficiently long duration what is the probability that the slot remains idle? That will be 1 by e . Remember this scenario, where I can keep my G_n always equal to 1 ok?

So, if the estimation was correct, accordingly I have adjusted my q_r properly then this will be happening ok, very nice. Now let us try to see, what is the probability that there will be a successful transmission, ok. And then we will calculate probably, what is the probability that there will be a collision. What is the probability that successful transmission will happen?

So, again with Poisson arrival single arrival has to happen. If over here only a single arrival either from the fresh node one arrival happens, but no other backlog node attempts, or from the fresh node no arrival happens and only one backlog node attempts. So, these are the two possibilities that successful transmission will happen.

So, if that composite new arrival and this one composite is Poisson and the rate is G_n equals to 1, then with this rate in one slot duration what is the probability that only one arrival will be happening? Again Poisson I can put.

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The slide shows a handwritten derivation for the probability of success and collision in a Poisson arrival scenario. It includes a diagram of a slot duration D and the following equations:

$$P_B \text{ Estimation of } n \Rightarrow n = \lambda D$$

$$G(n) \approx 1$$

$$P_{\text{Success}} = \frac{e^{-G(n)} \cdot [G(n)]^1}{1!} = e^{-1} = \frac{1}{e}$$

$$P_{\text{coll.}} = 1 - \frac{1}{e} - \frac{1}{e} = \left(1 - \frac{2}{e}\right)$$

The derivation also shows the general Poisson distribution formula: $e^{-\lambda x} \frac{(\lambda x)^n}{n!}$ and the specific case for $n=1$: $e^{-1} \cdot \frac{1^1}{1!} = \frac{1}{e}$.

e^{-G_n} and the time duration is 1. So, this will be happening and this will be divided by $e^{-\lambda x}$. So, what is your Poisson on this one? $e^{-\lambda x}$ into λx whole to the power 1 only 1 arrival divided by factorial 1, so same thing we will be putting. So, λx is G_n over here and x is the time duration which is this D which is 1 ok.

So, this will be that and G_n into 1 whole to the power 1 divided by factorial 1. So, that is the probability that a successful transmission will happen ok or this might be also we can say that on average this much percentage of time successful transmission will be happening. What is this? G_n equal to 1, this is 1, so this is e^{-1} , this is $1/1!$ is also 1. So, it is again 1 by e .

What is then the probability of collision? That will be $1 - e^{-G_n}$ which is it remains idle minus p success, so this will be $1 - e^{-G_n}$; so that is the collision probability. So, now, we have characterized if I can keep my G_n equal to 1 then, this e^{-1} is the success probability, this is the ideal probability and this is actually the collision probability.

Now, if we try to calculate the percentage ok, so, that we will be able to calculate ok; how much percentage of time it will remain, either in success mode or in collision mode, or in idle mode, so this is something that is possible. Why we are doing that? Because this will be helpful in creating our estimation. So, right now we will give the description of the estimation that they do. So, let us try to see what kind of estimation they do. So, what they are trying to do? Every time I will be observing the channel.

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$$p_{\text{success}} = \frac{e^{-G(n)} \cdot [G(n)]^n}{n!} = e^{-1} \left(\frac{1}{e}\right)^n \frac{e^{\lambda x} (\lambda x)^n}{n!}$$

$$p_{\text{cd}} = 1 - \frac{1}{e} - \left(\frac{1}{e}\right) = \left(1 - \frac{2}{e}\right) \frac{1}{e}$$

$$G(\hat{n}_k) = 1$$

$$q_r = \frac{1}{\hat{n}_k}$$

$$q_r(\hat{n}) = \min\left[1, \frac{1}{\hat{n}}\right]$$

$$\hat{n}_{k+1} = \begin{cases} \max\{\lambda, \hat{n}_k + \lambda - 1\} & \text{if Idle/Success} \\ \hat{n}_k + \lambda + (e-2)^{-1} & \text{if Cd.} \end{cases}$$

So, the channel will tell me, whether a success has happened or a collision has happened or the channel was idle. These three things I will be able to know and at the beginning of that slot I had some estimations. Suppose that is a kth slot, so I have this \hat{n}_k , this is my estimation; let us say that is the current estimation.

How do I update, the next estimation of how many nodes are black backlogged? So, what I want to do is $\hat{n}_k + 1$ in the next slot I want to estimate the n value, the number of stations which has been backlogged; that means, actually the state. Once I know this I will be able to immediately calculate the q_r so that I can keep my G_n or the estimated one, $G_{\hat{n}_k + 1}$ because that only one I will be using.

So, if this remains almost 1 that will be my target by updating q_r because that is the stabilization we have talked about. So, now, let us try to see the updated equation. This is a kind of Bayesian because we will take this probability that we have calculated how much with how much probability it is successful, how much probability it is idle, and how much probability it is collision.

So, taking that we will be doing an estimation ok. So, how do you do that? So, that is like this. I will first give the equation and then I will talk about why that equation is like this. So, the maximum of lambda or the earlier state plus lambda minus 1 will be happening, if idle or successful. So, channel feedback if channel feedback is idle or successful I will

be doing this updateation. We will talk about that otherwise I will be doing this if a collision has happened.

Let us try to intuitively understand why this is so, why this will keep my G_n that ok almost towards 1 ok. So, it is like this. First, let me give a description of this protocol, and how this protocol actually works. So, in this protocol what happens? We think that this is something also we discussed in the previous class; we think that there is nothing called backlogged or fresh arrival. Everybody at every station which is fresh arrival also is taken to be backlogged.

So, the protocol is described like this at any this things ok. So, what I want to see is that suppose there are n number of nodes or let us take m number of nodes ok, out of them n are in backlog in the k th stage, so this is my k th one. In the k th stage, n are in backlogged ok already. So that means that was reported over here ok in the boundary of the previous slots.

Now in this slot if 1 or 2 makes an arrival there will be immediate; so, let us say that in the number of nodes that make an arrival, they will be immediately taken as backlog. So, now, what will happen? My status will be m minus n minus n dash will be fresh and n plus n dash will be in backlog.

So, fresh arrival unlike the previous description of unstabilized slotted ALOHA they are not taken to be fresh arrival will be immediately transmitted and backlog will be done with this q_r probability they will attempt transmission. It is not like that so everybody is considered to be backlogged and with probability q_r they will be attempting.

So, when that happens then what will be happening? As you can now see if it is in state n ok then whatever q_r they put this $n q_r$ that has to be 1 because this is the overall attempt that will be made. So, that is actually G_n , this is by G_n now.

I will adjust by q_r in such a manner that $n q_r$ becomes 1 if that happens because that is the average attempt. Each one is trying with probability q_r and there are n stations. Be it fresh arrived station or be it backlogged overall n station, so this n is actually that n plus n dash. So, each one is attempting with q_r no fresh arrival are there now. So, overall G ; that means, an overall attempt will be n into q_r on an average ok. So, that is what we want to make.

So, therefore, the q_r update becomes very easy $1/n$. Now, if I can estimate n properly then this $1/n$ will make in q_r , and automatically $n q_r$ will be 1. So, always G will remain at 1 which is the correct condition for achieving the highest throughput in slotted ALOHA, which makes the entire ALOHA stabilized.

So, therefore, this estimation is so important now and we have through the protocol considering the new arrival to be backlogged we have made the equation very simplified. So, what will be happening? This q_r/n is that new estimation as we have been told that will be $1/n$, but sometimes what might happen is this n might go to 0.

So, this is the situation that we do not want, that this will become less than means my G/n will become less than 1. So, that is why this updateation will be done like this minimum of 1 and $1/n$ ok. So, if n that is greater than 1 or equal to 1 then this will remain $1/n$ ok.

If it is less than 1 then we still want to keep it 1 so that over all this arrival rate still remains something ok. So, we can keep the arrival rate; which means, we do not want to the problem if this is a probability actually. This probability should not go beyond 1. So, this restricts that, this condition restricts that. If n somehow becomes less than 1, I should not keep the probability greater than 1 that is that does not make sense. So, that is why we do this updateation.

So, most of the time if n is greater than equal to 1, this will give you proper updateation of q_r so that $n q_r$ which is our G remains 1 ok, so that is exactly what we are trying to do. Now with this updation let us try to see how this updation works, and why this will be giving you almost shearly a good tracking of n . So, let us try to see what we are trying to do. If it is let us say there is a successful transmission.

So, in successful transmission what will be happening? So, earlier you had this k ok? Now, $n k$ what will be the addition? Whoever is making new arrival they are immediately taken to be in the backlog. So, $n k$ plus if on an average λ , arrival happens per unit time, so, λ on an average we must add with $n k$.

So, for every slot, we will be adding this λ . This might every time it might not give me the correct result, but over a time duration on average, this will be tracking n properly. So, that is something we should do and when I see feedback that successful

transmission has happened from the channel, then I know that at least one backlog must go away.

So, minus 1, so that is why we are doing this update equation. But remember this updateation if it goes below lambda then I have trouble because every time I know on an average lambda will be arriving and they will be automatically taken into backlog according to the new description of the protocol. So, at least it should not go any slot it should not go less than lambda. So, that is why I have this maximum of these two things whichever is the maximum I will be taking that is ok.

So, if this is maximum ok, but if this goes beyond lambda then lambda will be taken. So, every time updateation at least lambda will be there. So, the arrival rate is always taken into account in n_k new n_k hat ok or n_k plus 1 hat. So, this is good for success we have understood. Why for idle also we are subtracting 1?

As you can see success and idle these two are with equal probability. Now, if a long chain of idle happens ok. So, then what generally is happening? Let us say I am at some n_k , long chain of idles is happening; that means, nobody is transmitting and no new arrival has happened ok this is supposed. Then what will be happening? This NK will remain the same.

So, this will be the case. So, basically what is happening is that a lot of idles are there my n_k is in this one if I do not do anything, I do not subtract anything then n_k will remain the same. But there is a problem because if that is the case I will be keep on actually means if I do not reduce it unnecessarily this will remain high. But, generally, in idle what is our assumption?

Our assumption is it is remaining idle means nobody is for a long duration of time it is remaining idle means actually it is not it does not have any transmission request. So, basically, I should in idle I should know that there is a possibility that I can transmit. So, I should give a chance to whoever can transmit right.

So, basically what I need to do is accordingly I must update my q_r accordingly. So, basically, my q_r must be increased it must be enhanced whenever there are multiple idles, but if I do not subtract it in idle period I will not be doing that. So, that is why deliberately I am subtracting 1. And why this 1? Because idle and success are

equiprobable as you can see $1/e$ and $1/e$ with equiprobable they come. So, on average they must have a similar effect. So, that is why I am always subtracting 1 from there.

To keep that keep having a similar effect, so that is exactly what we are doing. So, it was having $1/e$, and from there I took that factor 1. So, it is almost similar to an idle case. Now, similar collision. What is the probability of collision? As you can see if this is $1/e$ this is e^{-2} right.

So, for $1/e$ we have taken 1, so for e^{-2} I should take this $1/e^{-2}$. So, that is exactly what we are taking for updateation. So, every time there is a collision. I must increase it because I know that it will be increased. So, this will account for that average value, average increment it will keep that in the same line or same alignment.

So, that is why whenever there is a collision what I am doing? I am of course, adding λ because I know that every slot on an average λ arrival λ extra number of arrivals will be happening and with every collision what is the average increment of this one that is estimated by this e^{-2} $1/e^{-2}$.

So, with that, we are now updating for collision and this has been proven that this very nice heuristic way of updating $n_k + 1$, actually stabilizes. This can easily mean if you run the algorithm it can be easily shown that this stabilizes it and keeps the G_n close to 1.

So, that is one way of updating our n and correspondingly our main target was updating this q_r which is the parameter in hand. The only thing is that we know that this q_r depends on n , so that is why we could do this updateation. So, this is one way of doing this update if I have the channel feedback.

Now, you will see this channel feedback also is an assumption not always you will get this channel feedback. We will try to show where you can get channel feedback and where you cannot. So, we will also show those physical media where channel feedback is possible where channel feedback is not possible, but if channel feedback is possible this is the way we can do it.

If channel feedback is not possible in that case we will in the next class we will discuss what we can do so that in a heuristic manner, we can almost update the QR according to our requirement to stabilize the whole slotted ALOHA. So, this is something that we will be targeting next and that is the initiation of the binary backoff algorithm.

That is a very famous one which has been used in Ethernet which has been used in Wi-Fi. This binary backoff algorithm will be the one where I only know the station feedback and or the node feedback and from there I try to generate a backoff algorithm ok. So, this is our next target. We will discuss this next.

Thank you.