Analog Integrated Circuit Design

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Lecture - 1
Course Introduction: Negative Feedback Control

Hello everyone. Welcome to the course analog integrated circuit design. It is a video course under the national program for technology enhanced learning. To introduce myself, I am Nagendra Krishnapura from the department of the electrical engineering at IIT Madras. I have been working here for 5 years. You can find my website here and also, my email. Here the website has some reference material problem sets from other analog courses which you may find useful and other resources. Now, I just give quick introduction to this course. This course is entitled analog integrated circuit design.

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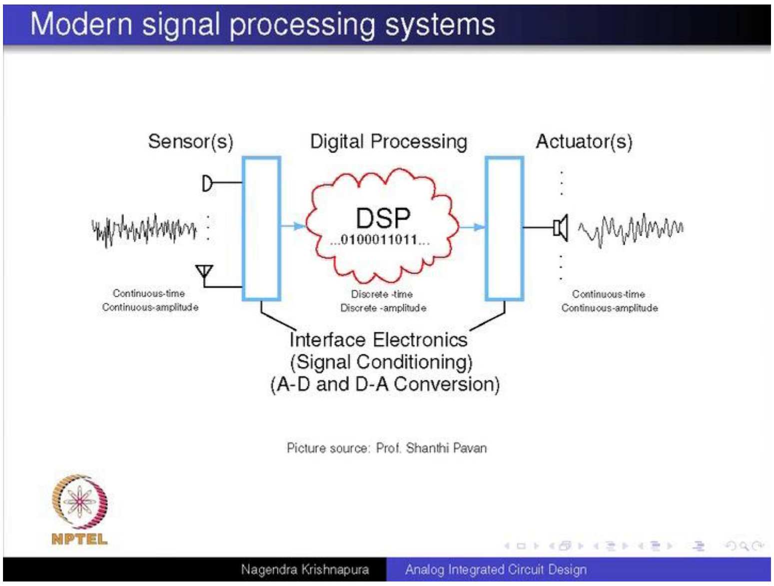

Modern signal processing systems

Sensor(s)

Digital Processing

Actuator(s)

DSP

...0100011011...

Continuous-time
Continuous-amplitude

Discrete -time
Discrete -amplitude

Continuous-time
Continuous-amplitude

Interface Electronics
(Signal Conditioning)
(A-D and D-A Conversion)

Picture source: Prof. Shanthi Pavan

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So, first we need to understand why analog integrated circuit design? Why we steady these things at all? So, if you look at any modern signal processing system, it looks something like this. Concepts can be depicted this way. It typically has core of digital signal processing, and digital signal processing is used because digital storage is easy and digital processing is very easy. To interface with the real word for instant with the voice signal as I am speaking with the radio signals, you have to interface with the analog world. All the real world signals are analog signals like the voice or the radio signal.

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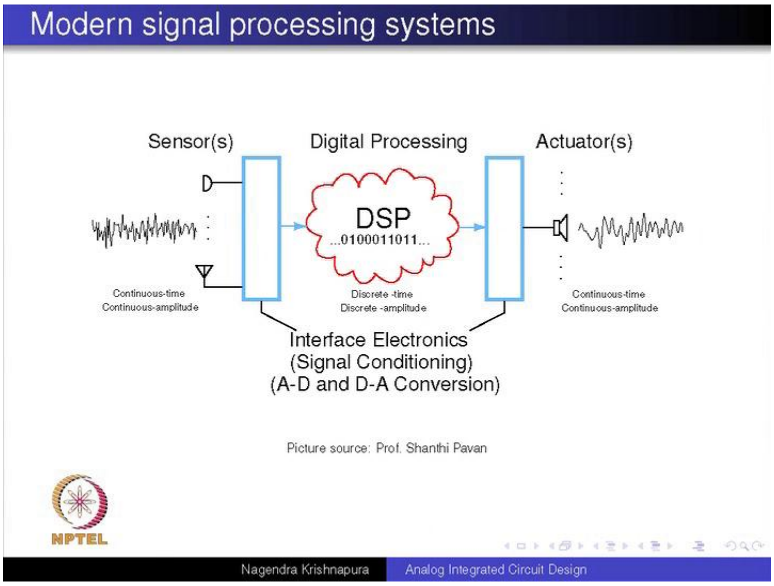

Modern signal processing systems

Picture source: Prof. Shanthi Pavan

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So, you receive the audio or the radio signals through what are known as sensors, and convert them to the electrical signals and convert them to do some analog signal processing, and convert them to the digital domain. Similarly, on the other side, you take the digital signals; convert them to analog to some analog signal processing and actuator the natural world signals. This could be word instant, the signal coming out of your loudspeakers when you are hearing my lectures.

So, in this entire system which is rather complex, every system today can be represented like this. It could be a music player, and then the digital signal processing consists of the storage and the processing. Inside the player, it could be a mobile phone in a system like this. You will have an outer layer of an analog signal processing and a inner digital core, and it is in the interface electronics that you will use analog circuits. You will use analog circuits for signal conditioning. You also use analog circuits for analog to digital and digital to analog conversion. So, the circuit that we will be studying in this course will be related to this circuit that you will be using in this interface blocks.

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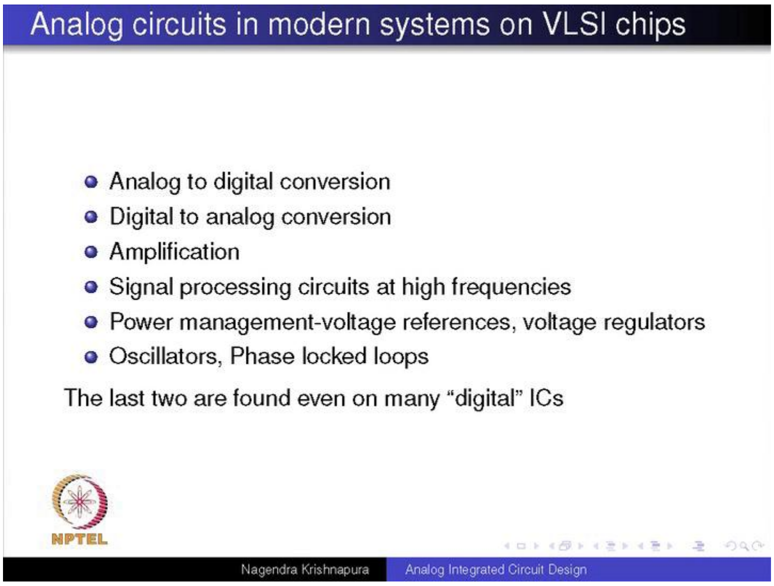

Analog circuits in modern systems on VLSI chips

- Analog to digital conversion
- Digital to analog conversion
- Amplification
- Signal processing circuits at high frequencies
- Power management-voltage references, voltage regulators
- Oscillators, Phase locked loops

The last two are found even on many “digital” ICs

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So, like I mentioned where do we use analog circuits in modern system? On VLSI chips you have analog to digital conversion that is done by an analog circuit. We have digital to analog conversion. Similarly, that is an analog block and you have amplification which for instant could be audio amplifier. You have a small audio signal, you amplified to the level that it can be driven by a loudspeaker, and that is done by analog circuits and you also have signal processing circuit at high frequencies. It is where more convenient to do it using analog than digital, and these two are common to lot of circuits. We have power management which means that you have voltage references, voltage regulators.

I am sure all of you have used the power supplies in the lab at some point. They give you precisely adjustable DC voltages and their generated using voltage references, and voltage regulators and finally, you have oscillators which for instant in a digital system. Also, you need a clock and that is given by the oscillators and you use a phrase lock loop to precisely address the frequency of oscillators. If you look at the last two which I have highlighted here, they are found even on many so-called digital ICS. For instance, you will have power management blocks, voltage references and regulators and oscillators and PLLS even in a microprocessor, and these are actually analog circuits which are useful for variety of application.

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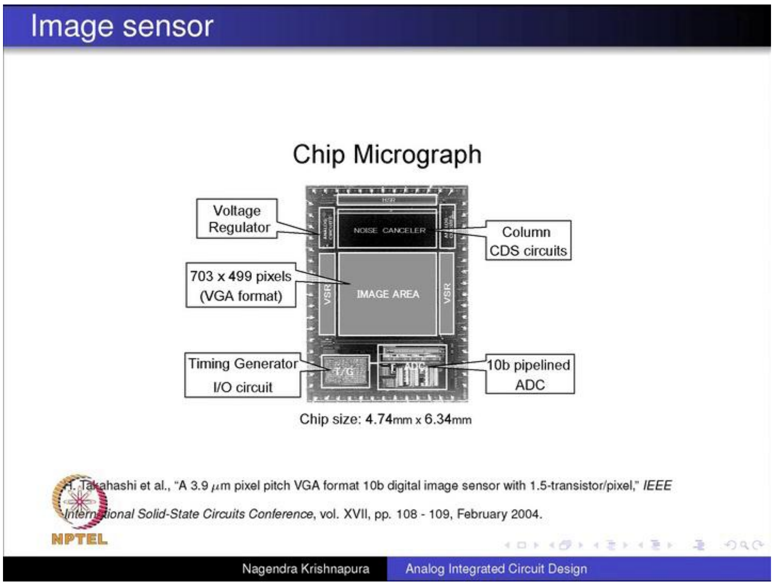

Image sensor

Chip Micrograph

Chip size: 4.74mm x 6.34mm

H. Takahashi et al., "A 3.9 μm pixel pitch VGA format 10b digital image sensor with 1.5-transistor/pixel," *IEEE International Solid-State Circuits Conference*, vol. XVII, pp. 108 - 109, February 2004.

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So, we will be studying these circuits as well in this course. Just to give you some quick example, I have taken a photograph of a chip which was population conference. Few years back, the chip itself is an image sensor, but you see that you have the image sensing area in the middle, but you have voltage regulator on top which is an analog circuit. You have what are known as sensing circuit on the top which are also analog circuit.

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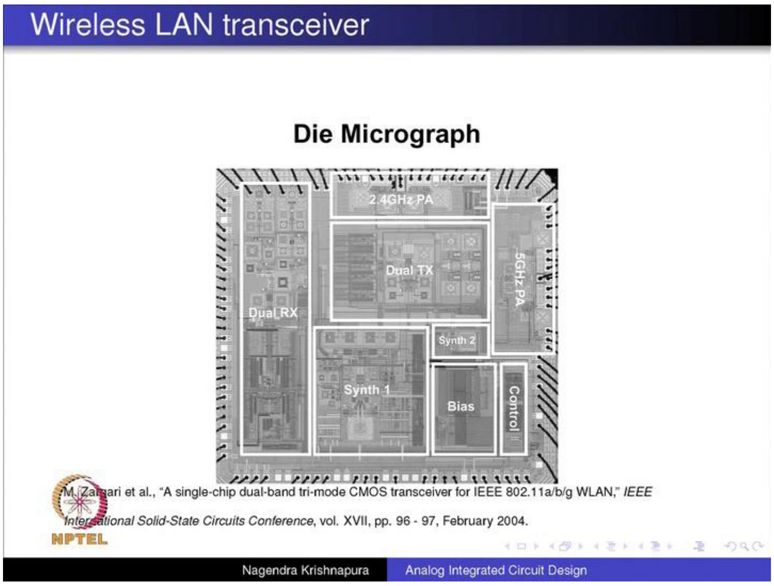

Wireless LAN transceiver

Die Micrograph

M. Zargari et al., "A single-chip dual-band tri-mode CMOS transceiver for IEEE 802.11a/b/g WLAN," *IEEE International Solid-State Circuits Conference*, vol. XVII, pp. 96 - 97, February 2004.

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Similarly, you have timing generators which as oscillators and other clocks which is an analog circuit and you have analog to digital converter which is also an analog circuit. Just to take another example, here is a wireless LAN transceiver. This is what would be in your wireless LAN chips inside your laptop.

So, this also you see that there are number of blocks, and each of these blocks denotes a different analog circuit. You have high-frequency power amplifiers here and here, and you have a frequency synthesizer which generates, carry of frequency very precisely in this part, and you have a wire circuitry over here and eternal are example is a D RAM which we mostly think of completely the digital blocks, but there are higher circuits and there are decoders etcetera which form and sense amplifier, which form analog circuitry inside a D RAM.

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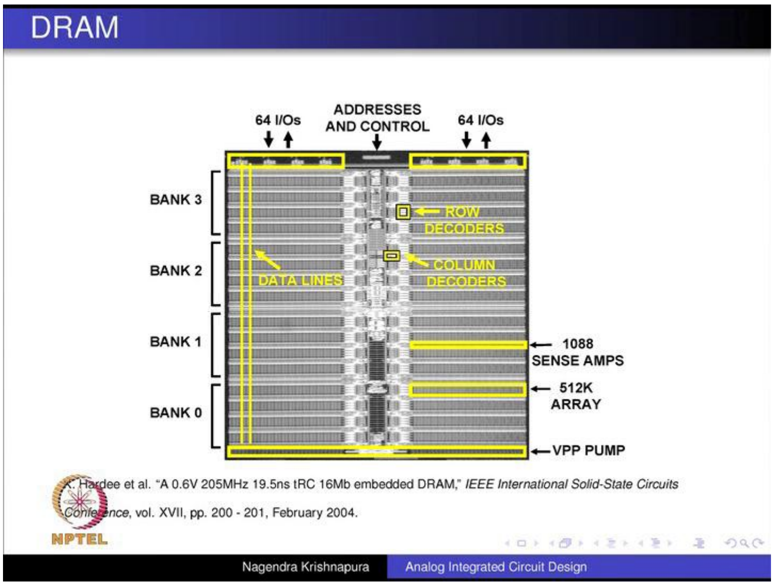

DRAM

K. Hardee et al. "A 0.6V 205MHz 19.5ns tRC 16Mb embedded DRAM," *IEEE International Solid-State Circuits Conference*, vol. XVII, pp. 200 - 201, February 2004.

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So, these are some example where analog circuits are used in a modern VLSI chips with that background. Let us move onto the next thing and one of the other thing is what can you do after you learn analogic design. If you look at analogic design in India, there is a number of companies which are starting analog circuit design activity which includes multinationals and Indian start-up. So, there is a big demand for skilled designers in the area of analog integrated circuit design.

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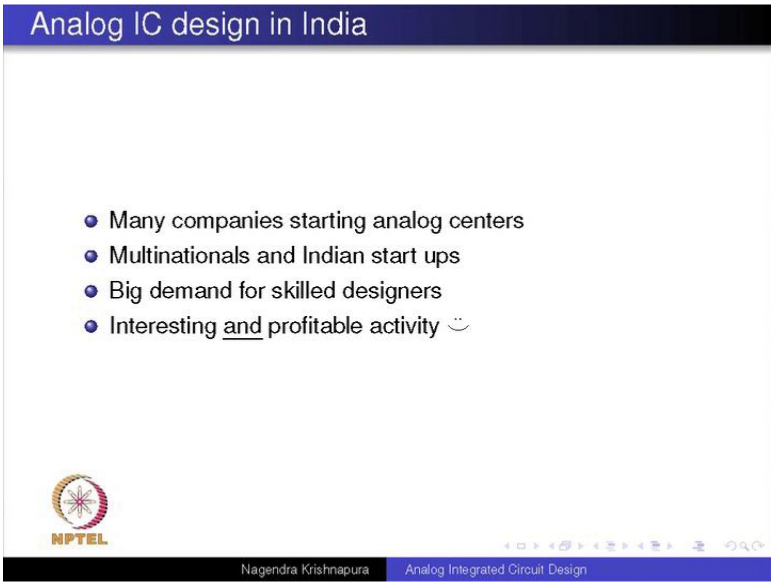

Analog IC design in India

- Many companies starting analog centers
- Multinationals and Indian start ups
- Big demand for skilled designers
- Interesting and profitable activity ⌣

So, it is not only interesting, it is also a profitable activity for many of you. So, with that background, we know now why it might be interesting to learn analogic design. So, let us look at what are we going to do in this course. So, here is the list of the course goals. Basic goals can be summarized in the first line which says that you learn to design negative feedback circuits on CMOS integrated circuits.

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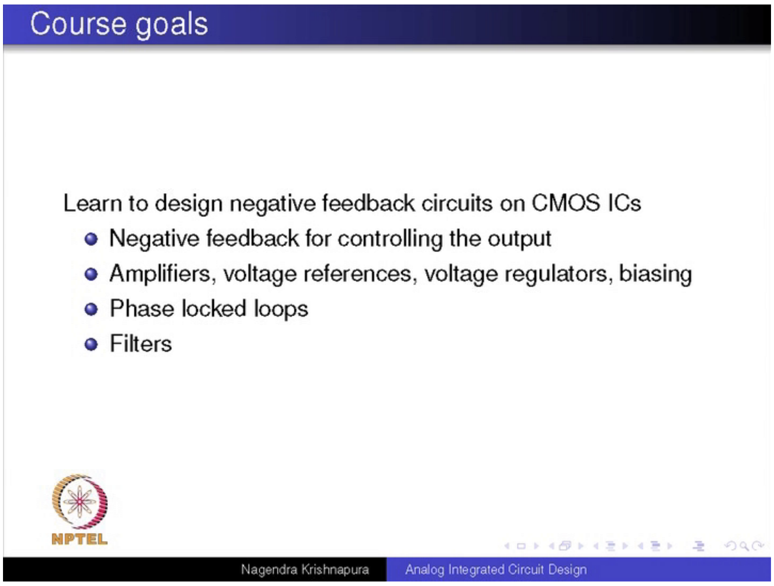

Course goals

Learn to design negative feedback circuits on CMOS ICs

- Negative feedback for controlling the output
- Amplifiers, voltage references, voltage regulators, biasing
- Phase locked loops
- Filters

So, what is negative feedback? You use negative feedback for controlling the output. In some way will come to the details of this as we go along and using that, you will learn to make amplifiers, voltage references, voltage regulators, other biasing circuit. We will also see how to make phrase lock loops and then, we will see some application circuit such as filters as we go through the course. We will learn about the details of all of these blocks, and these are the prerequisites for the course, that is these are the things that you need to know if you want to understand what is going on in this course.

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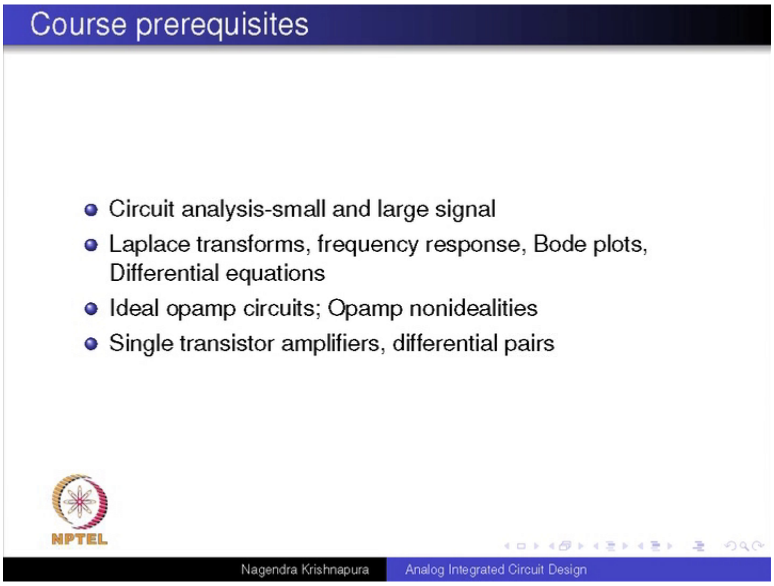

Course prerequisites

- Circuit analysis-small and large signal
- Laplace transforms, frequency response, Bode plots, Differential equations
- Ideal opamp circuits; Opamp nonidealities
- Single transistor amplifiers, differential pairs

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So, first of all, we need to know circuit analysis in both small signal and large signal domains. I am sure you have learned these things in your basic circuit course, and you will need to be fluent with laplace transforms which is used for frequency response, analysis circuits, sinusoidal steady states response, bode plots and some knowledge of differential equations. These are required for analyzing circuits which have capacitors and conductors. In addition to this, you need to know something about ideal opamp circuits. Basically you need to how to analyze ideal opamp circuits, and have some idea of the non-idealities of the opamp.

We will be discussing these in detail in this course and you will need to know single transistor amplifiers blocks like common source, common emitter, common collector, and common base. Similarly, common gate and common drain amplifiers and also differential pairs because we will be using these basic amplifier blocks to make our more complicated circuits. So, here is a slightly more detailed overview of the course contents. The first part of the course we will deal with negative feedback amplifiers.

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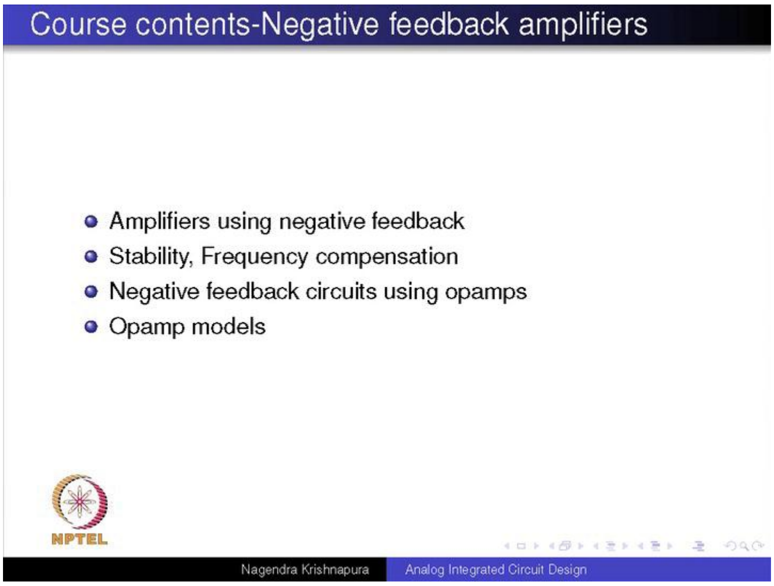

Course contents-Negative feedback amplifiers

- Amplifiers using negative feedback
- Stability, Frequency compensation
- Negative feedback circuits using opamps
- Opamp models

So, this means, first we will see how to make an amplifier using negative feedback. In the first place, when you use negative feedback as your familiar from control system courses, you will run into constants with stability and you will have to frequency compensates the negative feedback loop. Those things we will go through and will see how to make negative feedback circuits using opamps. We will also see various levels of opamps models. One of the things that you will learn in this course is that you do not use a single model for every occasion. You will use models with varying degrees of sophistication for different occasions. So, we will see opamp models of various levels.

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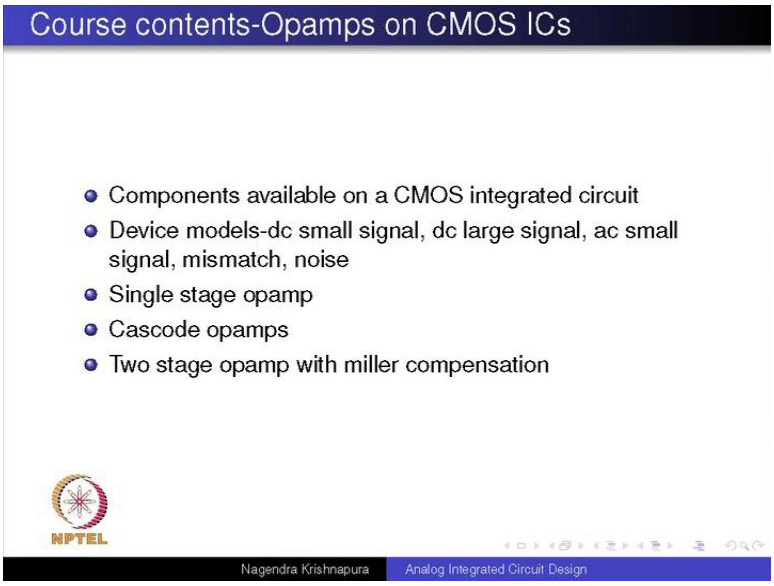

Course contents-Opamps on CMOS ICs

- Components available on a CMOS integrated circuit
- Device models-dc small signal, dc large signal, ac small signal, mismatch, noise
- Single stage opamp
- Cascode opamps
- Two stage opamp with miller compensation

Then, after we are learning about the opamps at a block level, we will go to how to make the opamps at the transistor level. So, in particular we will be looking at how to make opamps on a CMOS integrated circuit. For this we will first look at what components are available on CMOS integrated circuits, so that we can make opamps with it and we also look at concepts like device model that is small signal DC models of the components of the CMOS IC. Similarly, DC large signal model, AC small signal models and two thinks have unfamiliar to you mismatch and noise models, and using these components, we will make a single stage opamp what are known as cascade opamps as well as multi stage opamp with frequency compensation.

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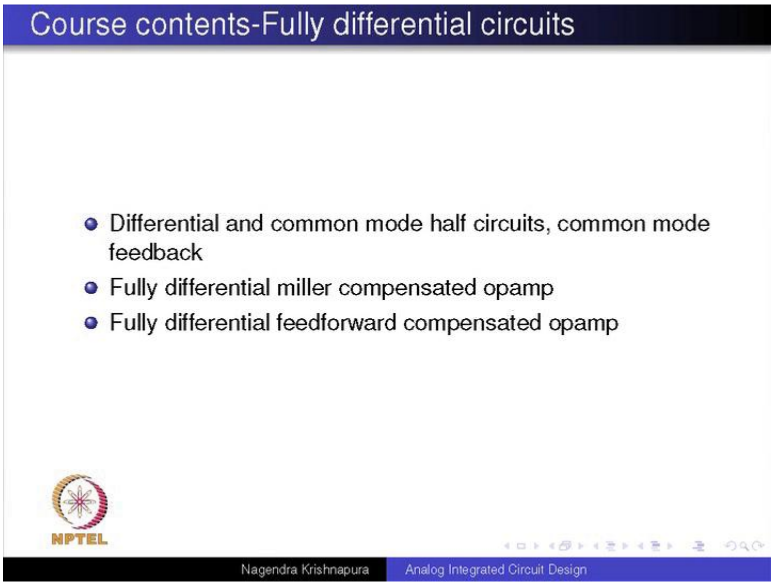

Course contents-Fully differential circuits

- Differential and common mode half circuits, common mode feedback
- Fully differential miller compensated opamp
- Fully differential feedforward compensated opamp

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We will also look at what are known as fully differential circuits. These are the circuits where every signal are represented by a pair of signal equal and opposite to each other, and studying this, we will come across differential in common mode half circuit, common mode feedback circuit and we will make fully differential miller compensated opamp, and a fully differential feed forward compensated opamp. These are just varieties of the opamps which are suitable for use in different applications.

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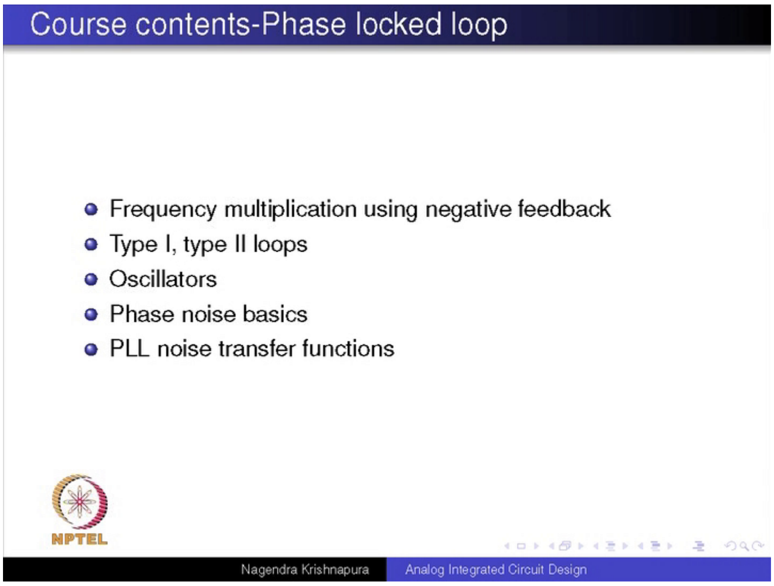

Course contents-Phase locked loop

- Frequency multiplication using negative feedback
- Type I, type II loops
- Oscillators
- Phase noise basics
- PLL noise transfer functions

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Then, we will also look at the phase locked loop which is really important building block of analog integrated circuit. First, we will try to make frequency multiplication circuit using negative feedback just like we make an amplifier using negative feedback. That will lead us to a phase locked loop, and it turns out there are two types of phase locked loop. The type I loop is of practical importance. Then, we look at the components of the phase locked loop which is the oscillator and then, we also look at the basics of phase noise and also, look at different noise transfer function inside a phase lock.

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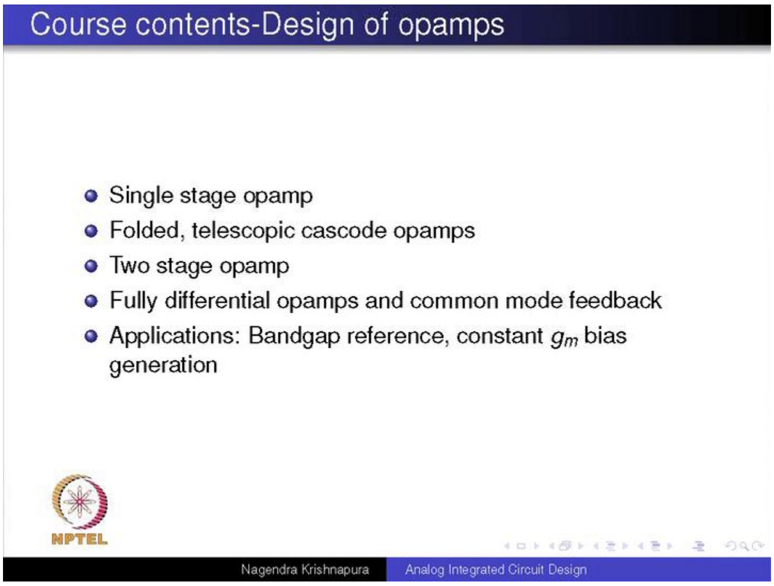

Course contents-Design of opamps

- Single stage opamp
- Folded, telescopic cascode opamps
- Two stage opamp
- Fully differential opamps and common mode feedback
- Applications: Bandgap reference, constant g_m bias generation

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So, as I mentioned earlier, we will also be going through designs of different opamps which are listed here. Single stage opamp, folded, telescopic cascade opamps, two stage opamp, and fully differential opamp and common mode feedback and we will look at some application circuits such as bandgap reference and constant g m bias generation.

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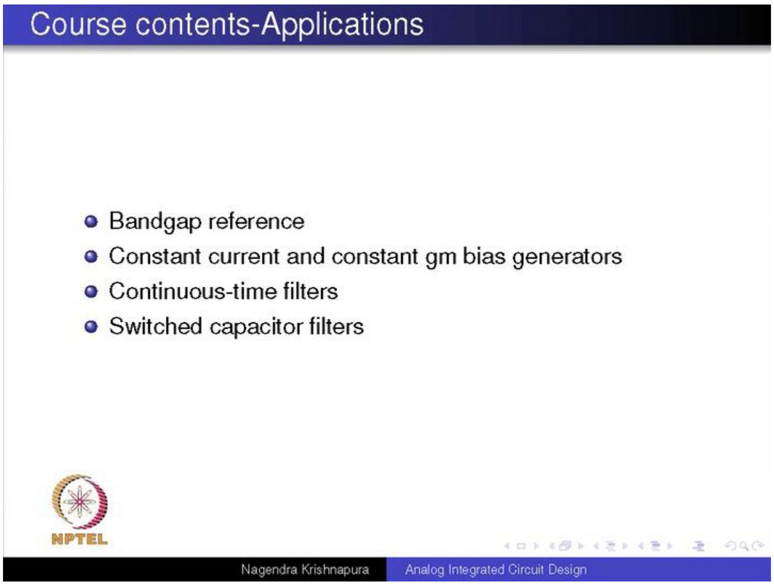

Course contents-Applications

- Bandgap reference
- Constant current and constant gm bias generators
- Continuous-time filters
- Switched capacitor filters

NPTEL

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This lists applications and some more details. These are the applications we will be looking at. A bandgap reference is a kind of circuit that maintains a constant voltage regardless of the temperature at which it is operating. We will see later that most circuits are sensitive to the temperature, but this band gap reference circuits put out a voltage which is quite insensitive to temperature. We will look at how to make constant current and constant gm bias. This is for biasing transistors.

We will look at continuous time filters and then, switched capacitor filters also. So, that is about what is going to be in this course. Of course, all of that will be covered in great detail in the rest of the classes. Now, if you look at the title, it says analog integrated circuit design. Now, what is designed and how do I learn it? Many of you or half are asking this question of yourselves.

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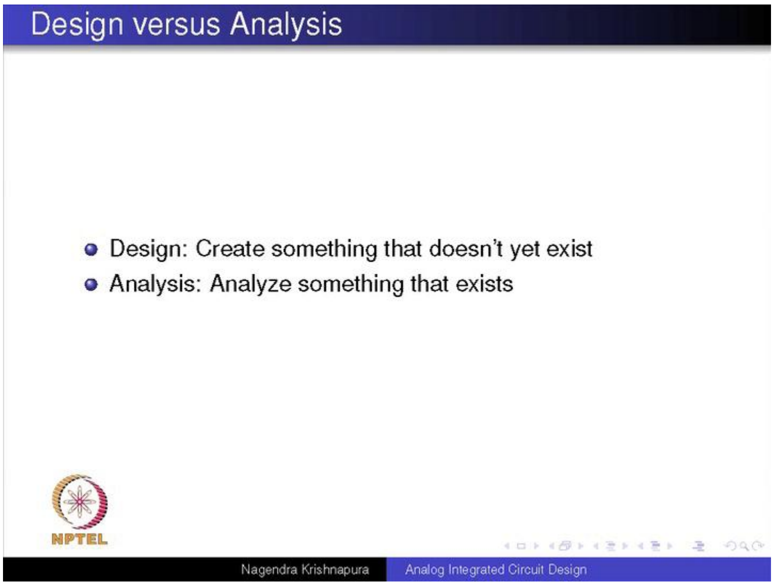

Design versus Analysis

- Design: Create something that doesn't yet exist
- Analysis: Analyze something that exists

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Here is a quick distinction between design and analysis. Essentially, design is creating something that does not exist here whereas, analysis relates to studying something that already exists. So, because of this that is some difference between the ways you approach design and analysis, and typically analysis is easier.

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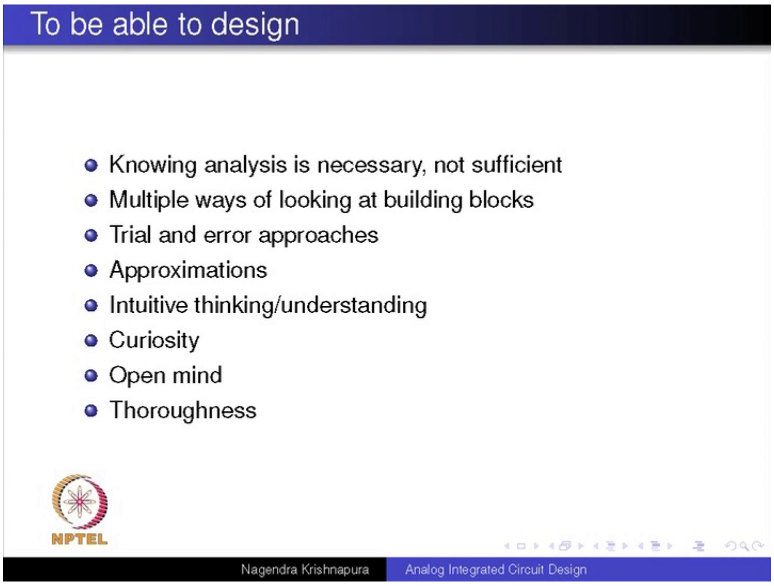

To be able to design

- Knowing analysis is necessary, not sufficient
- Multiple ways of looking at building blocks
- Trial and error approaches
- Approximations
- Intuitive thinking/understanding
- Curiosity
- Open mind
- Thoroughness

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Usually what happens is you study is a method of analysis and then, you apply it. There is a sequence of steps you apply that and you will be able to analyze the circuit or system or whatever that is you are analyzing. Now, to be able to design, you have to know a little more than that. You of course have to know analysis that is prerequisites. It is necessary, but not sufficient, but in addition to this to be able to design something, you will need to have multiple ways of looking at building blocks because you do not know exactly in what combination to put things together to be able to design. You have to be comfortable with trial and error approaches. It is not a fix sequence of steps like analysis, but sometimes you try some combinations if it does not work.

You try some other combinations and it should be guided by the knowledge that you have already acquired with. Analysis should also be comfortable with approximations, and we will see down this course that we will use approximation a lot because many real world situations, many real situations if you try to calculate the exact answer, the answer look very complicated and more importantly does not lead to any insight. Now, for design what is needed is insight. You may not need to know the exact answer, but need to know where the answer is going to give. If you change a particular variable for that approximation are crucial.

Now, it looks as though the approximation is the shortcut to a solution. It is not actually making a judicious approximation. It is more difficult than doing exact analysis using the fix sequence of steps. This is because you do not know exactly what approximation is good and what approximation is valid. In a particular situation that you have to judge based on the situation, you analyze and you also have to be able to think about situation intuitively. Intuitively means that you have to be able to get an idea of the answer without going through the complete analysis of the situations and of course, last three are extremely important for creativity.

You have to be curious about what you are studying and then, you have to have an open mind when you coming up with your solution and of course, you have to be through that is you cannot design something that meet one particular aspect and then, ignore everything else. That will be a very bad design. For a good design, you have to thoroughly investigate every possible implication of the design.

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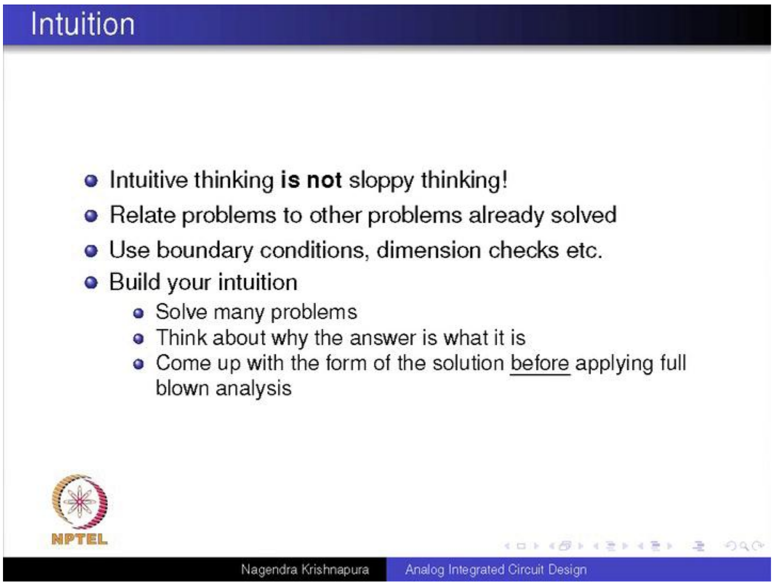

Intuition

- Intuitive thinking **is not** sloppy thinking!
- Relate problems to other problems already solved
- Use boundary conditions, dimension checks etc.
- Build your intuition
    - Solve many problems
    - Think about why the answer is what it is
    - Come up with the form of the solution before applying full blown analysis

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I will just give a quick thing what is about intuition. Now, sometimes intuitive thinking is confused with sloppy thinking that is you do not think through the problem thoroughly that is not what it means. What it means is basically look at a new problem. You try to let us I guess or analyze the problem without actually going through every step of the analysis. Now, how do you do this? You can relate the problem to the other problem which you have already solved.

This is the very common thing that we do look at a problem and then, you say this is exactly what the other problem that I solved yesterday and then, you are analyze it using the same method, and sometimes you use some boundary condition dimensions checks to get an idea of what the answer should look like, but without calculating the exact answer and intuition is something you can build up.

So, the way to do that is you solve many problems. When you solve a lot of problems, you will start to get an idea of what the solution would look like. What is to be done here is mindless solving of hundreds of problem, but every time you solve the problem, you need to think about why the answer came out the way it is. It did that is you get an answer and then, you should think about the answer and see how it came about. Similarly, when you solve another circuit, you do the same. After you do this for a number of problems what happens is, you will start recognizing the nature of the solution even before doing the analysis that is what I mean by intuition at least you should be able to guess the form of the solution before applying full blown analysis. So, intuition and analysis complement each other.

So, after you get sufficiently experience, you will be able to intuitively tell the solution to many things and there will be many situations where your intuition is not up to you by getting not up to arriving at the solution, but that is then you use your analytical skills to come up with your solution and then, build up your intuition based on time up. So, that is about the contents of the course and also, some insides into the design. Now, one quick word on using these lectures that a lot of you will be using these on your computer that is watching the lecture and it is somewhat different from sitting in a classroom from where you interact with the professor directly.

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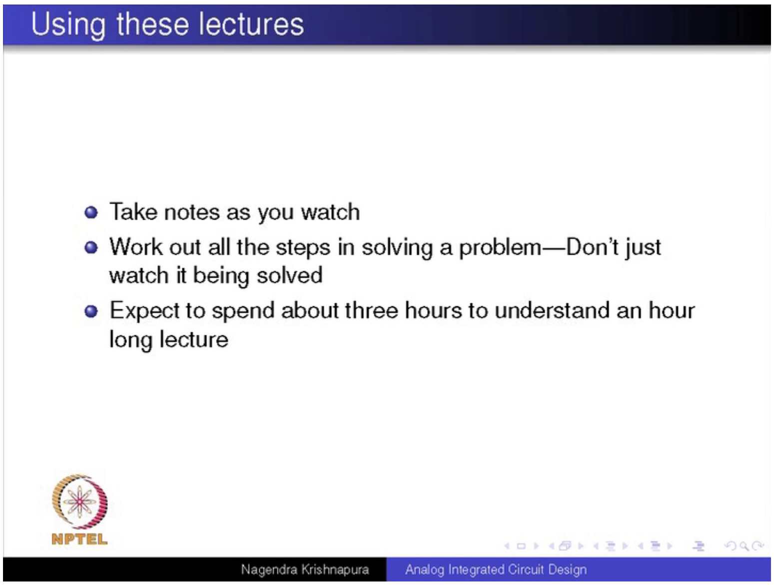

Using these lectures

- Take notes as you watch
- Work out all the steps in solving a problem—Don't just watch it being solved
- Expect to spend about three hours to understand an hour long lecture

So, one of the most important thing is just like you do in class, you take notes as you watch because watching me solve the problem or do an analysis will not help you get the same skill. You have to do it yourself because when you are watching the lecture, you may think that I understand this, I understand this and so on, but the whole point of an analog IC design course is that you have to be able to go back and do the design yourself, and the only way you be able to do is if you do the analysis also as I do it or maybe oppose the lecture and do the analysis yourselves. The second important thing is to have to work out all steps in solving a problem. I mean you do not just watch me or somebody else solving it. You have to work out every step of it and then, more importantly the answer you get in every step.

So, that is the way to build up your skill and doing all this, you can expect to spend about three hours to understand one hour of this lecture. So, you have to be able to devote the time to be able to fully understand it. So, a lot of you will be watching this on the computer. So, there will be facility to pause and repeat and so on. You have to please make full use of that one and secondly, in many cases I will be putting down a problem and then, demo studying the solution. If you can just pause at that point when I present the problem, you try to solve it yourself and then, after you try to solve it, you see what I do that is the way I solve the problem.

Now, a brief overview of the prerequisites. This is not meant to be a detailed treatment of all the prerequisites. You need to be able to understand this course. This is just a quick overview. You have to go back to the relevant textbooks and then, practice these skills and become fluent with it. Basically, the prerequisites are circuit analysis. In this course, we will be designing analog circuits and we will be using analog circuit analysis. As a matter of routine, circuit analysis will be used at every step. So, you have to be very fluent with it.

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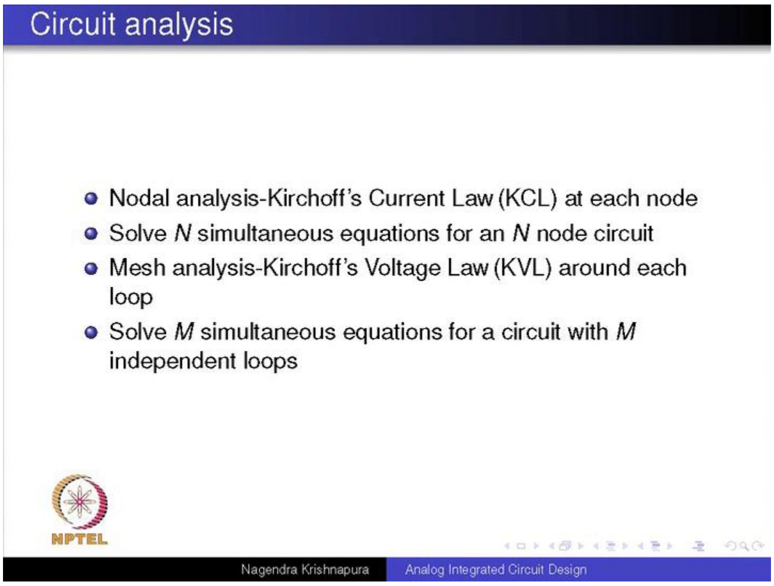

Circuit analysis

- Nodal analysis-Kirchoff's Current Law (KCL) at each node
- Solve N simultaneous equations for an N node circuit
- Mesh analysis-Kirchoff's Voltage Law (KVL) around each loop
- Solve M simultaneous equations for a circuit with M independent loops

Now, you have learnt circuit analysis, various methods of doing that. You can do nodal analysis, you can do mesh analysis. Then, most of the cases in this course will be using nodal analysis. You can of course use mesh analysis equally. Well, the nodal analysis consists of N simultaneous equations for N node circuit.

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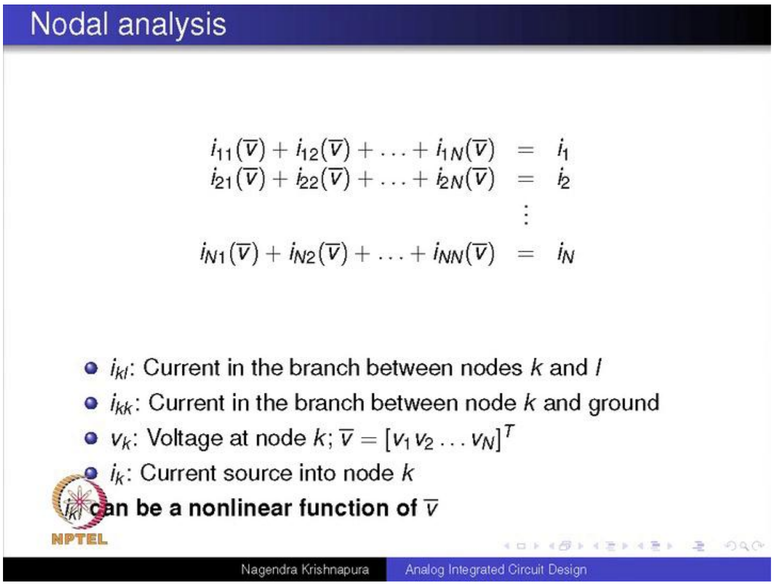

Nodal analysis

$$\begin{array}{rcl} i_{11}(\overline{v}) + i_{12}(\overline{v}) + \ldots + i_{1N}(\overline{v}) & = & i_1 \\ i_{21}(\overline{v}) + i_{22}(\overline{v}) + \ldots + i_{2N}(\overline{v}) & = & i_2 \\ & \vdots & \\ i_{N1}(\overline{v}) + i_{N2}(\overline{v}) + \ldots + i_{NN}(\overline{v}) & = & i_N \end{array}$$

- i_{kl}: Current in the branch between nodes k and l
- i_{kk}: Current in the branch between node k and ground
- v_k: Voltage at node k; $\overline{v} = [v_1 v_2 \ldots v_N]^T$
- i_k: Current source into node k

i_{kl} can be a nonlinear function of $\overline{v}$

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This is the general form of nodal analysis, that is the sum of current flowing out of every branch equal the current source which is connected to that branch and in general, this current can be non-linear functions of voltages, but the situation we are most familiar with and we will be using the most is linear circuit analysis, which of these currents is a linear function of voltage.

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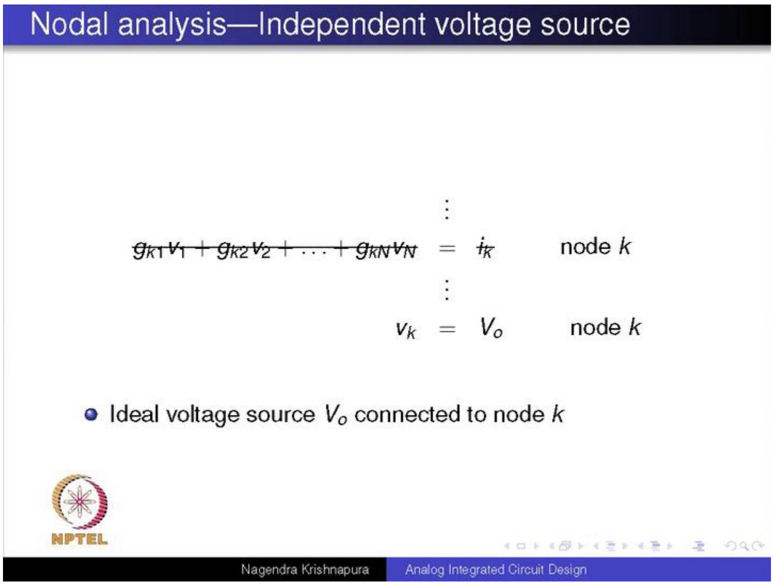

Nodal analysis—Independent voltage source

$$\begin{array}{rcll} & \vdots & & \\ \cancel{g_{k1}v_1 + g_{k2}v_2 + \ldots + g_{kN}v_N} & = & \cancel{i_k} & \text{node } k \\ & \vdots & & \\ v_k & = & V_o & \text{node } k \end{array}$$

- Ideal voltage source V_o connected to node k

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So, this type of set of equations can be solved by matrix inversion. Here j kl stands for the conductance between node k and l and j kk stands for the conductance between node k and ground. This depicts a situation where you have a number of conductance and current sources only as you know there are some special cases.

If you have an independent voltage source, you cannot use the nodal question, and that node you have to use the equation for the voltage source. Similarly, for the controlled voltage source, you cannot use the node equation because you do not know what current is flowing through the controlled source. Instead of that you use the equation for the controlled voltage source if you have a current controlled voltage source.

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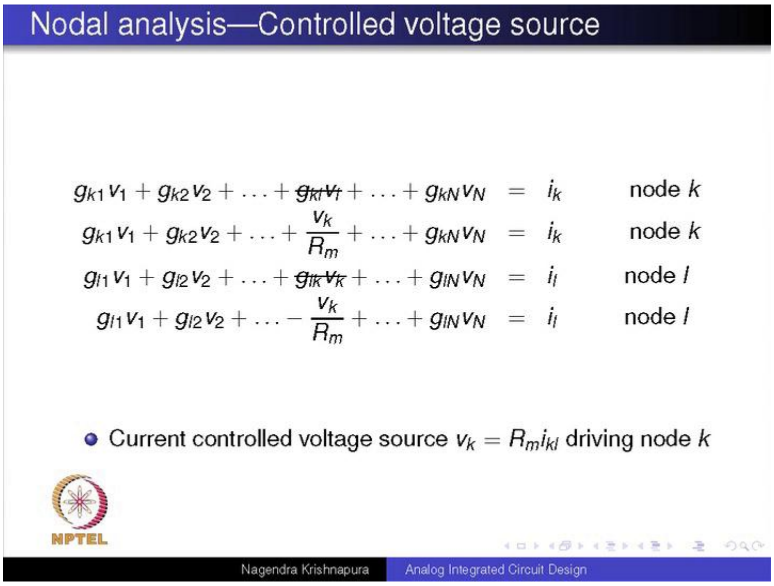

Nodal analysis—Controlled voltage source

$$\begin{aligned} g_{k1}v_1 + g_{k2}v_2 + \ldots + \cancel{g_{kl}v_l} + \ldots + g_{kN}v_N &= i_k && \text{node } k \\ g_{k1}v_1 + g_{k2}v_2 + \ldots + \frac{v_k}{R_m} + \ldots + g_{kN}v_N &= i_k && \text{node } k \\ g_{l1}v_1 + g_{l2}v_2 + \ldots + \cancel{g_{lk}v_k} + \ldots + g_{lN}v_N &= i_l && \text{node } l \\ g_{l1}v_1 + g_{l2}v_2 + \ldots - \frac{v_k}{R_m} + \ldots + g_{lN}v_N &= i_l && \text{node } l \end{aligned}$$

- Current controlled voltage source $v_k = R_m i_{kl}$ driving node k

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Similarly, you replace the equation of the node k with something that relates to the current controlled voltage source. Finally, the situation that we will see that very often in integrated circuit design because all transistors are voltage controlled current sources. The equation at the node which has a voltage controlled current source will have an extra term which has to be absorbed into the left-hand side. As I mentioned, this is not meant to be a detailed treatment of circuit analysis. This is just a refresher of what you have learnt earlier, and what you should be fluent with, so that you can follow this course effectively.

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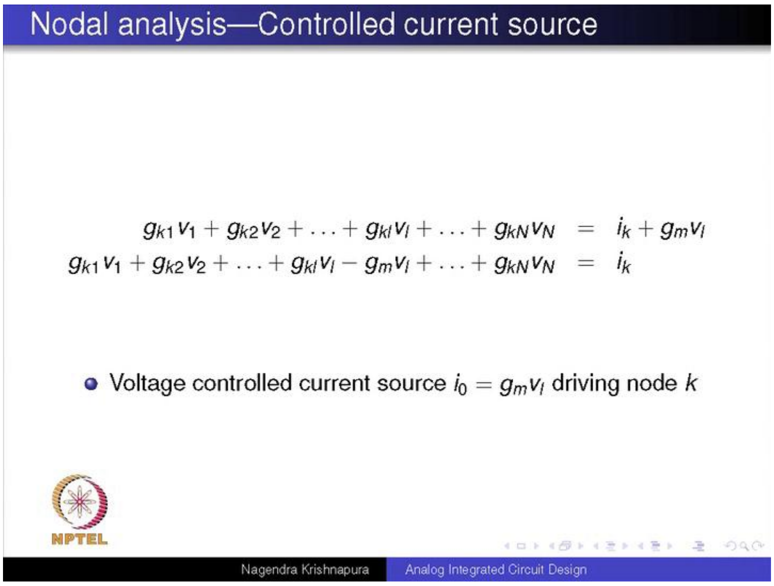

Nodal analysis—Controlled current source

$$\begin{aligned} g_{k1}v_1 + g_{k2}v_2 + \ldots + g_{kl}v_l + \ldots + g_{kN}v_N &= i_k + g_m v_l \\ g_{k1}v_1 + g_{k2}v_2 + \ldots + g_{kl}v_l - g_m v_l + \ldots + g_{kN}v_N &= i_k \end{aligned}$$

- Voltage controlled current source $i_0 = g_m v_l$ driving node k

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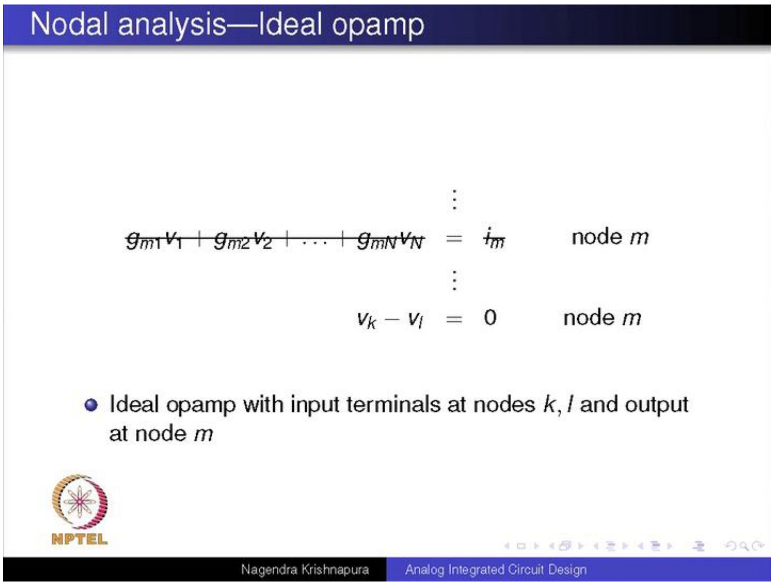

Nodal analysis—Ideal opamp

$$\begin{aligned} &\vdots \\ \cancel{g_{m1}v_1 + g_{m2}v_2 + \ldots + g_{mN}v_N} &= \cancel{i_m} && \text{node } m \\ &\vdots \\ v_k - v_l &= 0 && \text{node } m \end{aligned}$$

- Ideal opamp with input terminals at nodes k, l and output at node m

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One last element which needs a special treatment is ideal opamp. If you have an ideal opamp, again you cannot use the Kirchhoff’s current law to usefully analyze the output of the opamp because again you do not know what current is going through output of the opamp. You substitute that with an equation of the input the opamp which says that the input terminal of the ideal opamp are at the same voltage k, and l are input terminal and v k minus v l equal to 0 v k minus v l equal 0 at the input of the ideal opamp. So, with this you will get n equations in m node and you will be able to solve all the variables in the circuit.

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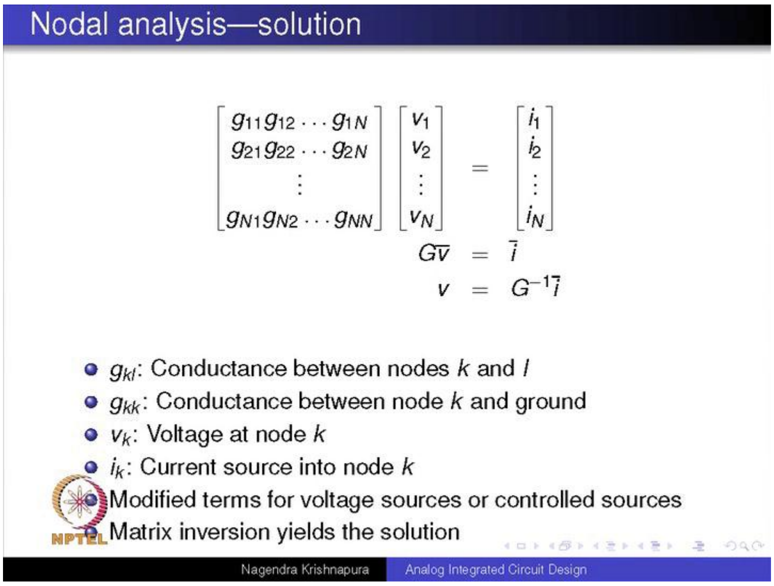

Nodal analysis—solution

$$\begin{bmatrix} g_{11} g_{12} \cdots g_{1N} \\ g_{21} g_{22} \cdots g_{2N} \\ \vdots \\ g_{N1} g_{N2} \cdots g_{NN} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix}$$

$$G\overline{v} = \overline{i}$$

$$v = G^{-1}\overline{i}$$

- g_{kl}: Conductance between nodes k and l
- g_{kk}: Conductance between node k and ground
- v_k: Voltage at node k
- i_k: Current source into node k
- Modified terms for voltage sources or controlled sources
- Matrix inversion yields the solution

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How do you actually solve this system of equations? You have to invert the matrix. This g matrix will be able to get the solution.

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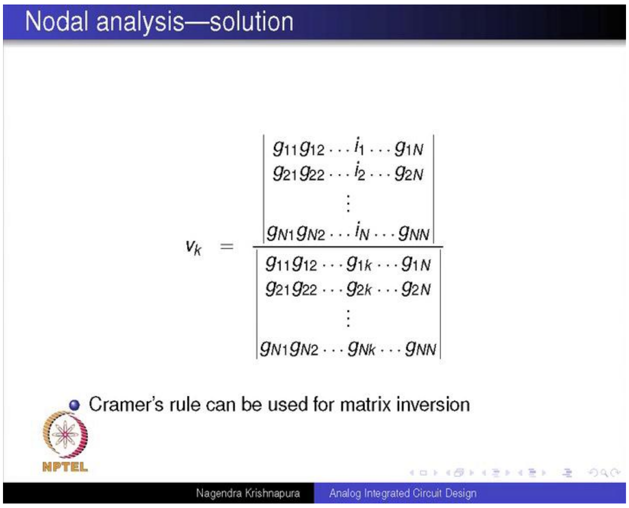

Nodal analysis—solution

$$v_k = \frac{\begin{vmatrix} g_{11} g_{12} \cdots i_1 \cdots g_{1N} \\ g_{21} g_{22} \cdots i_2 \cdots g_{2N} \\ \vdots \\ g_{N1} g_{N2} \cdots i_N \cdots g_{NN} \end{vmatrix}}{\begin{vmatrix} g_{11} g_{12} \cdots g_{1k} \cdots g_{1N} \\ g_{21} g_{22} \cdots g_{2k} \cdots g_{2N} \\ \vdots \\ g_{N1} g_{N2} \cdots g_{Nk} \cdots g_{NN} \end{vmatrix}}$$

- Cramer's rule can be used for matrix inversion

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Again, this is a quick reminder that one of the ways of doing this is Cramer’s rule where you substitute the right hand side victim into the appropriate column of the matrix, and take the ratio of determinants.

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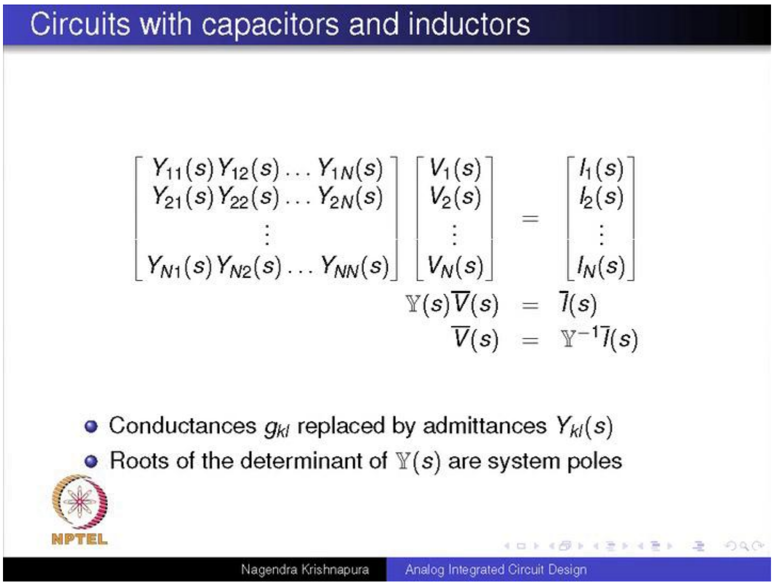

Circuits with capacitors and inductors

$$\begin{bmatrix} Y_{11}(s)\,Y_{12}(s)\ldots Y_{1N}(s)\\ Y_{21}(s)\,Y_{22}(s)\ldots Y_{2N}(s)\\ \vdots \\ Y_{N1}(s)\,Y_{N2}(s)\ldots Y_{NN}(s)\end{bmatrix}\begin{bmatrix}V_1(s)\\ V_2(s)\\ \vdots\\ V_N(s)\end{bmatrix} = \begin{bmatrix}I_1(s)\\ I_2(s)\\ \vdots \\ I_N(s)\end{bmatrix}$$

$$\mathbb{Y}(s)\overline{V}(s) = \overline{I}(s)$$

$$\overline{V}(s) = \mathbb{Y}^{-1}\overline{I}(s)$$

- Conductances g_{kl} replaced by admittances $Y_{kl}(s)$
- Roots of the determinant of $\mathbb{Y}(s)$ are system poles

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Exactly the same scheme holds when you have capacitors and inductors. The earlier situation implied that we had only conductances or resistances when you have capacitors and inductors. Exactly the same situation arises except that instead of conductance, they are talking about admittances and you have to invite admittances matrix to get the solution.

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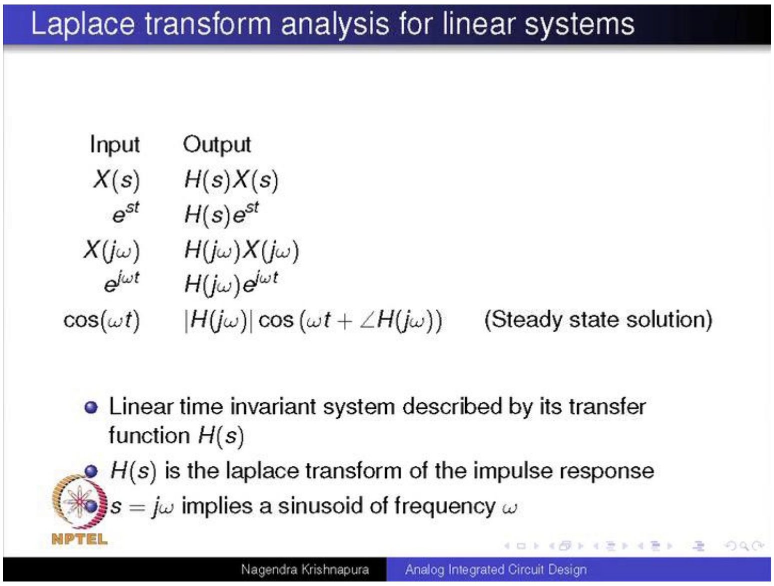

Laplace transform analysis for linear systems

Input	Output	
$X(s)$	$H(s)X(s)$	
e^{st}	$H(s)e^{st}$	
$X(j\omega)$	$H(j\omega)X(j\omega)$	
$e^{j\omega t}$	$H(j\omega)e^{j\omega t}$	
$\cos(\omega t)$	$\lvert H(j\omega)\rvert \cos(\omega t + \angle H(j\omega))$	(Steady state solution)

- Linear time invariant system described by its transfer function $H(s)$
- $H(s)$ is the laplace transform of the impulse response
- $s = j\omega$ implies a sinusoid of frequency ω

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Now, another thing that you need to refresh yourself on is Laplace transform analysis for linear systems. So, it is a very convenient tool for analysis. You know that a linear system, linear time invariant system can be described by its transfer function H of s. In that case, if the input is H of s, the output will be H of s times H of s and also, you know that the input is in e to the minus s t. The output will be h of s e to the s t and this is most commonly used with sinusoid steady-state substituting s equal to j omega implies a sinusoid of frequency omega is applied to the input to the linear time invariant system. So, if you have an input of X of j omega, the output will be H of j omega times of j omega.

So, if you apply an input e to the j omega t which basically corresponds to the sinusoid of frequency omega, the output will be H of j omega which is the complex number times e raised to j omega t. You know what happens in practice is you will have to apply a real sinusoid cos omega t to the system. If you apply cos omega t to a linear time invariant system which has a transfer function H of s, the output will have a magnitude H of j omega, that is the magnitude of transfer function at the frequency of the sinusoid and it will also be phase shifted. It is cos omega t plus angle of H of j omega, that is a phase shift and the amount of phase shift equals the phase angle of the complex number H of j omega at the frequency of the sinusoid.

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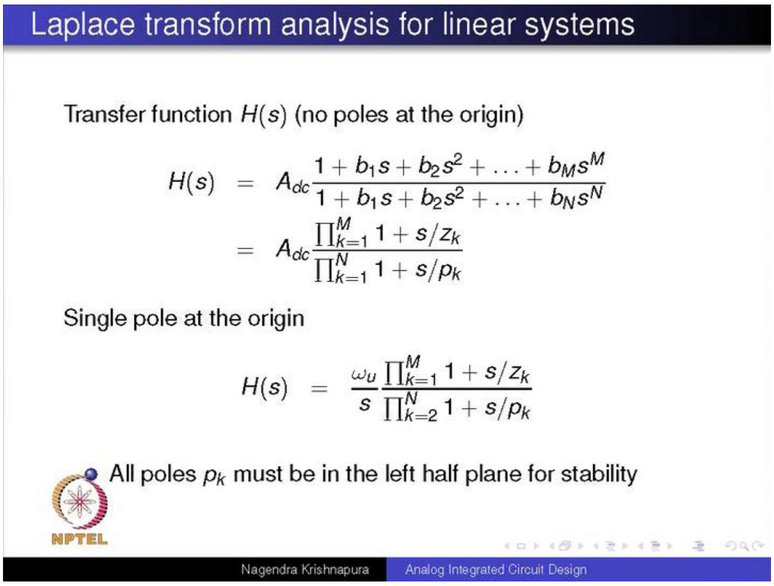

Laplace transform analysis for linear systems

Transfer function $H(s)$ (no poles at the origin)

$$\begin{aligned} H(s) &= A_{dc}\frac{1 + b_1 s + b_2 s^2 + \ldots + b_M s^M}{1 + b_1 s + b_2 s^2 + \ldots + b_N s^N} \\ &= A_{dc}\frac{\prod_{k=1}^{M} 1 + s/z_k}{\prod_{k=1}^{N} 1 + s/p_k} \end{aligned}$$

Single pole at the origin

$$H(s) = \frac{\omega_u}{s}\frac{\prod_{k=1}^{M} 1 + s/z_k}{\prod_{k=2}^{N} 1 + s/p_k}$$

- All poles p_k must be in the left half plane for stability

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Now, this is what is known as a steady state solution. What it means is that is why you apply cos omega t to the stable linear time invariant system, and wait for some time until all the transients settle. The output that you get will be a sinusoid at the same frequency, and its amplitude and phase will be changed by the transfer function of the linear time invariant system. So, again please refresh your basics of sinusoidal, steady state solution, Laplace transform because again we will be using that widely in through this course.

So, the transfer function H of s of any linear time invariant system can be put in this form, where A dc is the dc gain and you have a numerator polynomial and a denominator polynomial, and instead of describing it as a polynomial like this, it can also be represented as a product of terms where each terms has corresponding roots which are zeros for the numerator and poles for the denominators.

I have taken a special case, where there is a single pole at the origin because this is a case that we will be using quite heavily with our negative feedback system. So, in this case H can be represented as a term omega u by s. This is the dimensional term and it has a pole at the origin that is s equal to 0 and then, there can be a product of zeros and numerator product of terms containing poles in the denominator, and as we know for stability, all of the poles P k must be in the left half plane.

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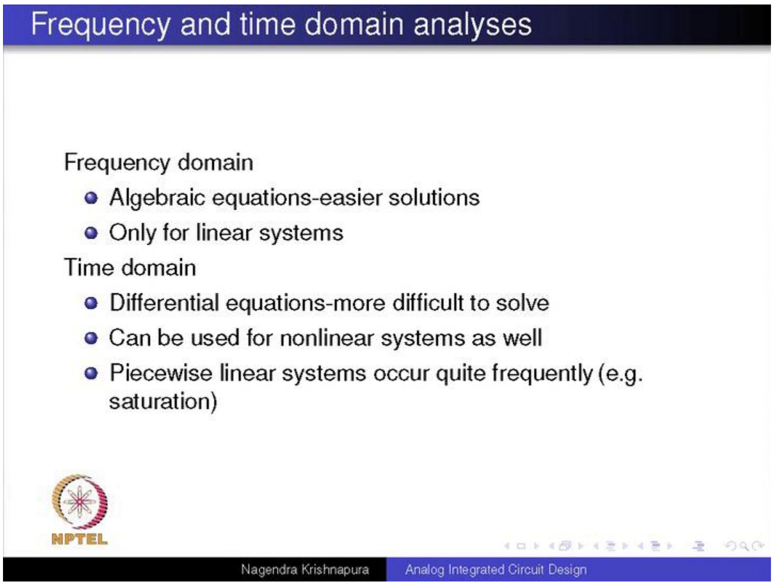

Frequency and time domain analyses

Frequency domain
- Algebraic equations-easier solutions
- Only for linear systems

Time domain
- Differential equations-more difficult to solve
- Can be used for nonlinear systems as well
- Piecewise linear systems occur quite frequently (e.g. saturation)

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Now, the two kinds of analysis that we are already familiar with is our frequency and time domain analysis. The advantage of the frequency domain analysis that you end up with an algebraic equations instead of differential equation, and this is much easier to solve, but the limitation is that it is applicable only for a linear system. So, we will be using this heavily, but time domain analysis is also very important in the same system.

So, you will have to solve differential equation and these are more difficult to solve and usually, we take the shortcut of using the frequency domain solution and we will be using algebraic equation instead of differential equations, but the advantages of time domain analysis is that it can be used for non-linear systems and then, you frequently encounter non-linear situations, right. For instant piecewise, linear systems are quite common like many times you have saturation and circuits, and this is an example of a piecewise linear system.

Now, one of the other things about time domain analysis is that it can be more intuitive. Many times you can figure out what is happening in the time domain more intuitively than what is happening in the frequency domain. So, for intuition, lot of times we use time domain examples and for the exact analysis frequency domain examples, sometimes also time domain analysis. So, it is important to be fluent with both frequency and time domain analysis to be able to understand circuits very well.

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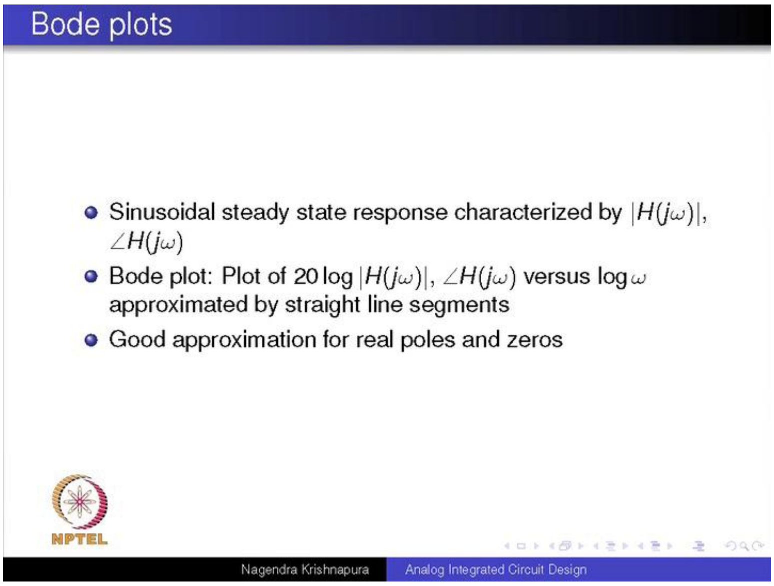

Bode plots

- Sinusoidal steady state response characterized by $|H(j\omega)|$, $\angle H(j\omega)$
- Bode plot: Plot of $20\log|H(j\omega)|$, $\angle H(j\omega)$ versus $\log\omega$ approximated by straight line segments
- Good approximation for real poles and zeros

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Another concept that we use widely is bode plots. Bode plots are nothing, but sinusoidal steady state response of linear time invariant circuits. As we know the sinusoidal steady-state response is characterized by a simple complex number H of j omega this. H of j omega has a magnitude and also phase. The magnitude and the phase are plotted separately and these plots are known as bode plots.

Basically, you plot the logarithm of the magnitude and the angle of the transfer function versus logarithm of the frequency and more importantly, you approximated by straight-line segments. Now, these things, these approximated plots can be quite good. It is a good approximation when you have only real poles and zeros. Lot of the situations that we analyze, we will have real poles and zeros and for that bode plots will be extremely useful. It is a quick way of looking at the response of the circuit.

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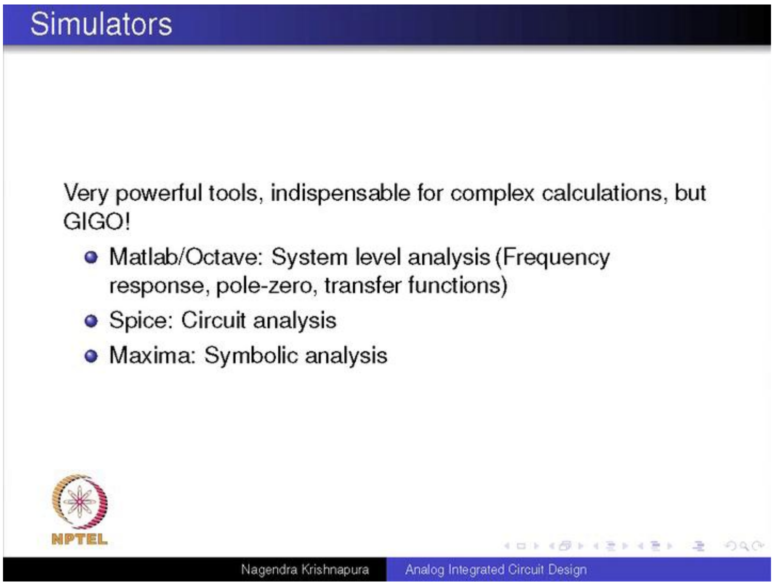

Simulators

Very powerful tools, indispensable for complex calculations, but GIGO!

- Matlab/Octave: System level analysis (Frequency response, pole-zero, transfer functions)
- Spice: Circuit analysis
- Maxima: Symbolic analysis

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Finally, you can use simulator to verify some of the results that we obtain in this course because our circuits become more complicated. It becomes more and more difficult to analyze and you will have to resort to simulators, but before we use the simulators, we should really understand what you are doing because the simulators are powerful tools and they are also indispensable for complex circuits, but as you know with any computer analysis garbage in and garbage out.

So, in the initial stages whenever you simulate any circuit, you make sure that you analyzed it by hand. Before you take simple circuit, you analyze it by hand and make sure that what you know the answer that you are going to and then, simulate and verify it. As you go on, you will be able to use the simulator in the appropriate way and then, you can use it to analyze more and more complex circuits.

Simulators are also very powerful because finally everything has to be done with an experiment, but in many cases, there are many things that cannot be measured with the experiments, but that can be measured using simulators for instant extremely small currents and then, currents in any branch of the circuit, these are all things that are very difficult to measure in a real experiment, but quite easy to do in a simulator. So, simulator can be a good learning tool as well, but you have to use it appropriately and there are different levels of simulations.

So, we can use tools like Matlab and Octave for system level simulations, that is analysis for frequency response pole-zero transfer functions and you can use spice for circuit analysis at the transistor level including non-linear circuit and sometimes, you may want to be in the middle of a very complicated calculations, and you may want to use a symbolic manipulator. You make sure that the complexity of algebra does not lead you to a wrong solution. When you have too many terms, you may end up making mistakes. In those situations, you use a tool like Maxima for symbolic analysis.

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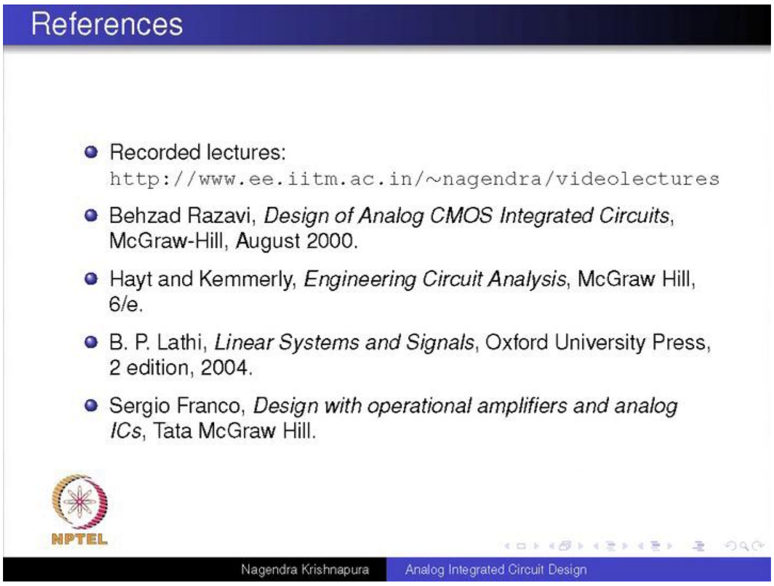

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These are some references that can be used for this course. So, the first one refers to the video lectures from our website and rests are the reference books. This course does not follow a particular reference book in the same order, but there are lots of examples. Problems from the second reference relate to the design of CMOS integrated circuits and the third one is for basic circuit analysis. The fourth one is for linear signals and systems which is useful for laplace transform and analysis like that and the last one is a book on opamps which have lot of example problems on opamps that you can look at.

So, that is about the introduction to the course and the prerequisites that you need to have. If you feel that you need little more practice on prerequisites, I suggest that you go to the appropriate textbook or perhaps your notes from the old courses, and solve a lot of example problems and become fluent with it. Once you do that, you will be able to very easily follow this course because the actual amount of mathematics that we use in this course is rather small and it is quite easy to follow. So, hopefully that I have convinced you of the fact that analog circuit design is interesting and it is useful for designing present integrated circuits.

So, what will we do here onwards is to do the actual integrated design, we will get into the course itself and start. As I mentioned earlier, we will start by analyzing negative feedback amplifiers throughout most of this. Of course, I will be using the tablet in this mode that is I will be writing it as a whiteboard and I mentioned this earlier as well. So, through the course I will be posing certain problems and then, I will do the analysis. It will be very beneficial for you if you pause the video at that point and do the analysis yourself and then, compare it to what I do because the thing that you remember best is the thing that you do yourself and not the thing that you watch somebody else doing.

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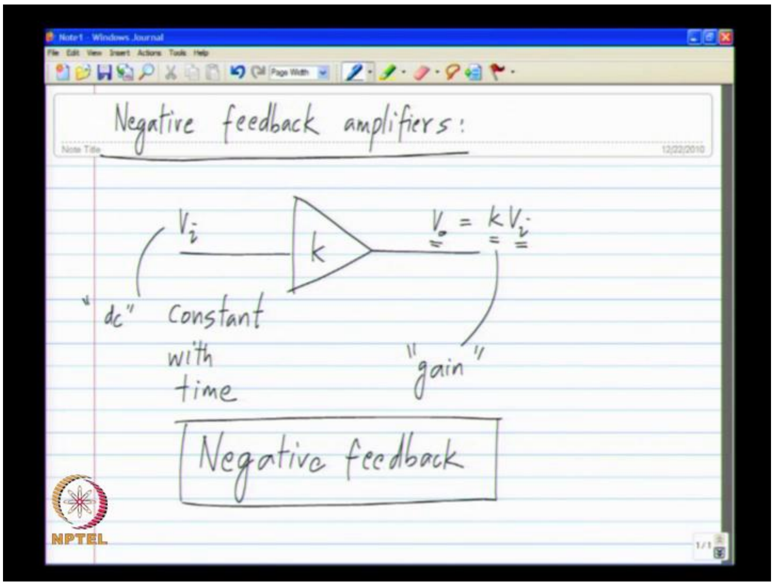

So, what we will start off with is negative feedback amplifiers that is let us say I am interested in amplifier. That means, that I have an input V i and I liked output to be k times V i and for now assume that V i is A dc that is to say it is constant with time. That means we are assuming that V i is constant with time, it is not varying with time. Now, I would like to make a circuit that gives me an output voltage V naught k times V i and what is k is the gain that I desire. So, let us say it could be 5 or 10 or whatever value that you want and also, I would like to realize this using negative feedback. How do I do this is the first part of the problem.

So, first we have to understand what I mean using negative feedback. What is meant by negative feedback? I am sure all of you have an intuitive idea of what negative feedback is because we all the time if not make amplifiers for other things. So, using negative feedback simply means that using feedback means that you look at what the output is. This is the actual output and you compare to the desired output and based on the result of comparison, you control the output. So, this is what is meant by feedback system which you look at the output and then, you compare with desire output and you control the output, so that it becomes the appropriate value. This is negative feedback system in general.

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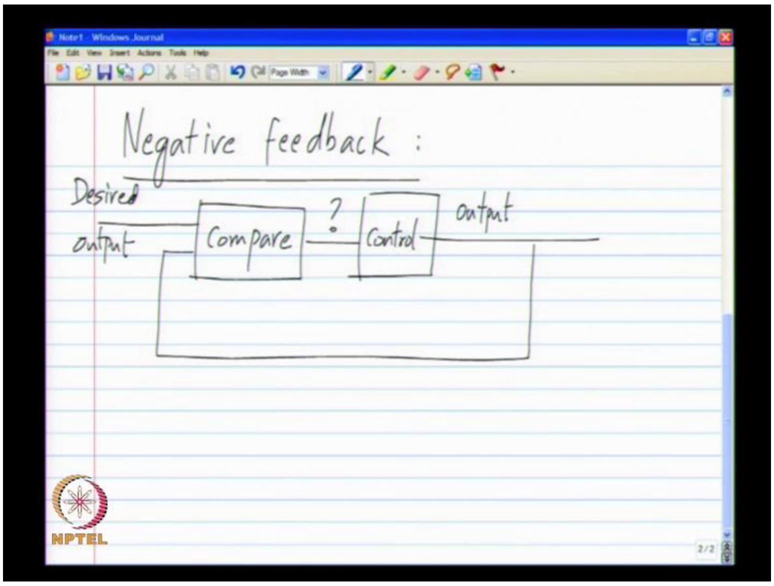

Negative feedback :

We can have a lot of obvious example that you use in everyday life. For instance, let us say you are driving a car or motorcycle and you want to adjust the speed to a certain limit. So, let us say you want to reach the speed of 50 kilometers an hour. For this system, the desire speed is 50 kilometers an hour and let us say how you actually do this.

What you do is you look at the actual speed and you sense it. Here, sensing the actual speed simply means you look at the speedometer.

So, you sense the actual speed, you compare it to the desired speed and then, you do something, you control the actual speed. What is this something that you do? That is the most important block here and that lead directly to our negative feedback amplifier. So, before we go there, what do I mean by compare? Essentially by compare I mean that you take the difference between the desired speed and the actual speed. So, when you are looking at the speedometer, let us say the desired value is 50 and the then, needle is showing 40, essentially acting on a difference between 40 and 50 that is what you do, right.

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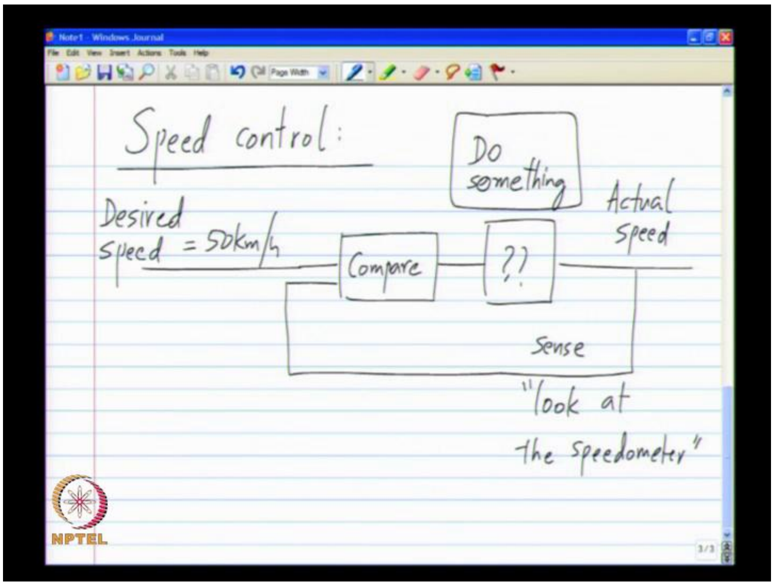

So, in other words, a negative feedback system for speed control. Here I am talking about your manually controlling the speed of the motorcycle the way most people drive. So, if you want to go at a certain desired speed, you take the difference between the desired speed and the actual speed. Of course when you are driving, you do not do this. Computation is simply look at whether the speedometer is at the right level or not and based on this difference which is the error, you do something and you do something in a way that finally the actual speed reaches the desired speed.

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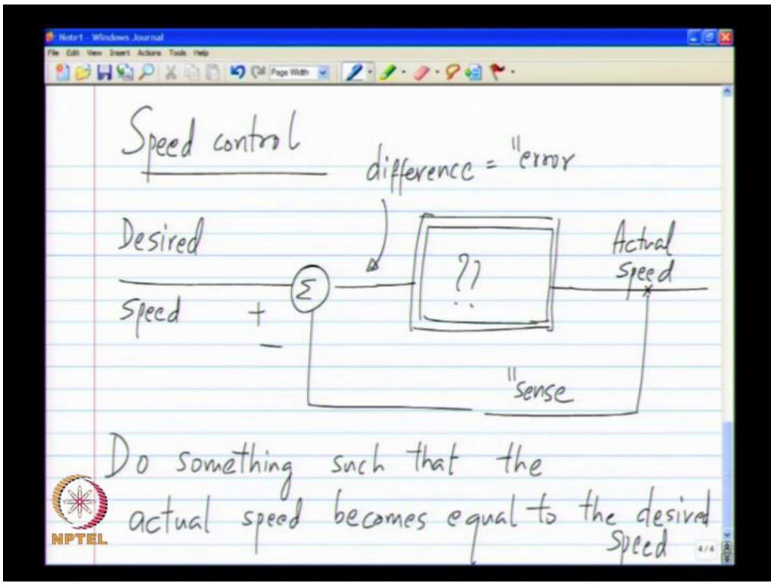

Speed control

difference = "error

Desired Speed

+

–

Σ

??

Actual Speed

"sense

Do something such that the actual speed becomes equal to the desired speed

So, what is that something that you do? This is something that we use all the time. I am in I am sure. Some of you would drive a motorcycle everyday and then, you do this. You want to go to a certain speed and then, you are able to reach the desired speed, but what is the action that you take so that you reach the desired speed. What we have to do is to find out the nature of this block because this system is very effective if I tell you to go out 50 kilometers in an hour, you will reach 50.

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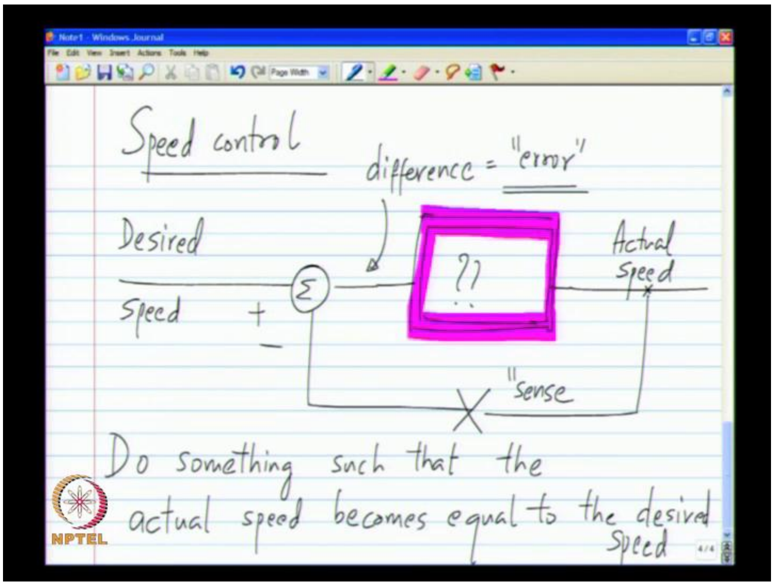

Speed control

difference = "error"

Desired Speed

+

–

Σ

??

Actual Speed

"sense

Do something such that the actual speed becomes equal to the desired speed

If I will tell you to go at 40 kilometers an hour, you will go out 40. So, you look like the system works. We just have to find out how this system works and replicate the same thing with circuits, so that amplifier does the same thing. The amplifier output has to be k times V i. So, it has to reach k times V i starting from any value, it has to mimic the action that you carry out on a motorcycle or your car.

So, we have to find out the nature of the block which is marked by question mark, this one. How do we do that? The way to do that is by looking at what happens if the sensing fails. Why is this easy to do? Because if the sensing fails, there is no feedback and this error will be constant. What I mean by this is for instant let us say you have a stuck speedometer. So, let us say the desired speed is 50 kilometers an hour and the speedometer is stuck at 30 kilometers an hour. If this is the case, you will always think that the error is 20 kilometers an hour.

That is to say you will think that you are going 20 kilometers an hour less than where you should be going. You will think that you are going at 20 kilometers less than the speed at which you should be going. Then, what happens to the actual speed because you think that you are constantly going at 30 kilometers an hour. What happens is that you go on accelerating and the actual speed will go on increasing with time.

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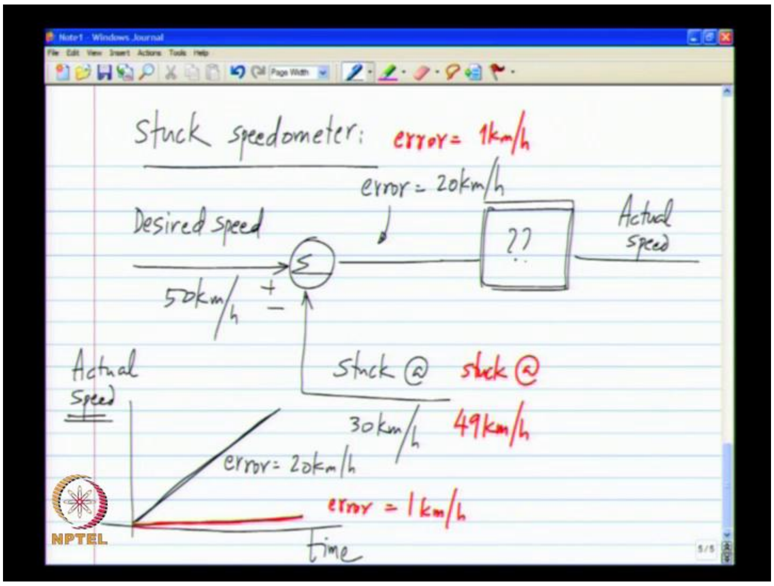

This is exactly what happens, right. Let us say you have no idea what speed you are going at, that is you have no sense, no other vehicles or no other reference point you have, only your speedometer to look at.So, if your speedometer is stuck at the fixed speed, what you will do is you will accelerate continuously and the actual speed will go on increasing continuously with time. So, this curve gives you a clue of the behavior of this block which I have marked by question mark. What is the speed? What is this block? You can get another clue to this by assuming that instead of being stuck at 30 kilometers an hour that side, it is stuck at 49 kilometers an hour.

So, in this case again the speedometer is stuck, but you think that you are going very close to the desired speed. You are only just 1 kilometer below the desired speed. So, what you do again is you will accelerate, but this time you will do very gently. So, what will happen is the speed will again go on increasing with time, but it will do so at a much lower rate.

So, the behavior of the system marked in question marks is this red curve when the error is only 1 kilometer an hour, and it is the black curve when the error is 20 kilometers an hour. From this we can you tell what the nature of the system is. It looks pretty obvious that the actual speed is an integrated version of the error. That is why the error keeps increasing monotonically with time. So, we will use this principle to derive the negative feedback amplifier in the next class.