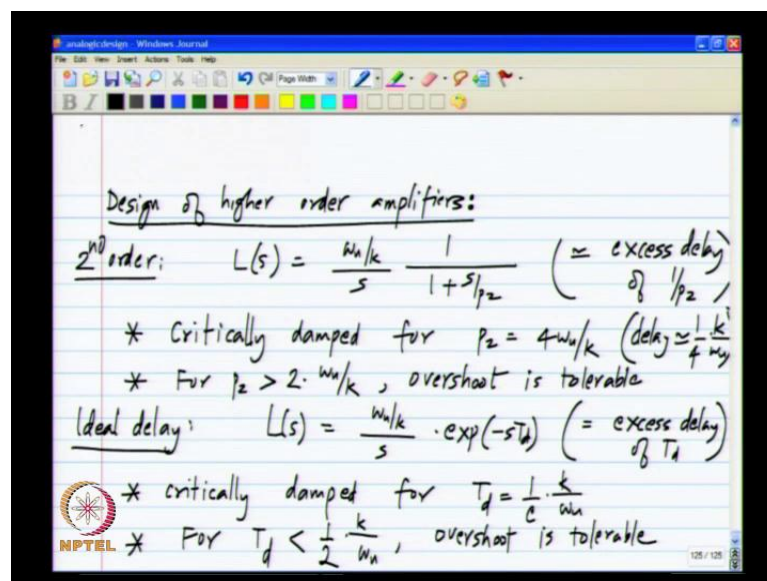


**Analog Integrated Circuit Design**  
**Prof. Nagendra Krishnapura**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Madras**

**Lecture No - 11**  
**Phase Margin**

Hello and welcome again in the previous class, we were discussing stability of amplifiers and in particular how to make higher order amplifiers are all also well behaved.

(Refer Slide Time: 00:27)



So, as they have seen, we can do the analytical calculations of the step response for the case where you have an integrator and an extra pole that is a second order system. For higher order system, when you have many extra poles, it is not possible to do a analytical calculations, it gets very complicated.

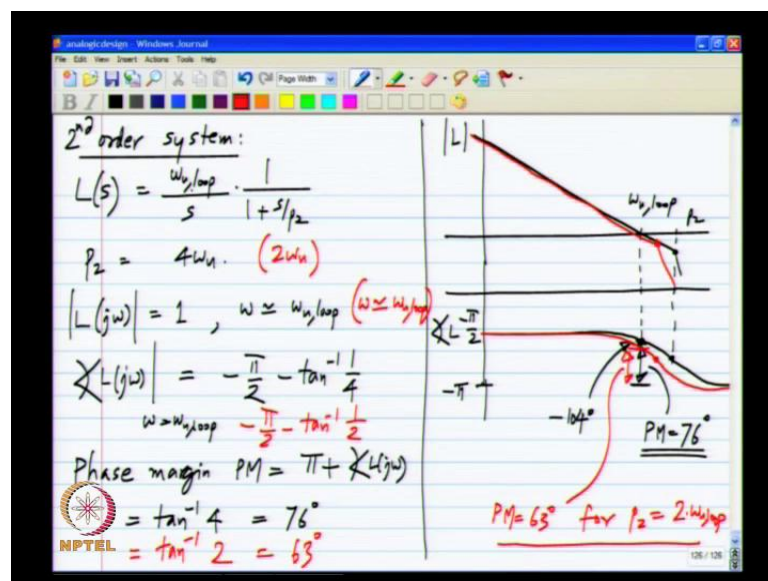
Therefore, what we do is we get some idea of how the loop gains should look like from what we know about second order system and the system with ideal delay and then apply the same to higher order systems. Just as a quick summary, when we have a second order system, what we mean by this is that the loop gain is of the form of an integrator with a certain unit loop gain frequency. An extra pole at  $P_2$ , we have also seen that this is approximately like having an excess delay of  $1/p_2$  meaning if you had only the integrator the step response of the loop gain would be a ramp of slope  $\omega u$  by  $k$ .

If you have an extra pole, you will still get the ramp of the same slope, but it will be delayed with respect to the original one by an amount  $1/p_2$  approximately. For this particular system, we know that the system is critically damped for  $p_2$  being  $4\omega_u$  by  $k$ . Again, this corresponds to a delay this is equivalent to a delay of  $1/4$  times the time constant of the integration. So, this is what we know and if  $p_2$  falls below this, we get a certain amount of overshoot in the step response and for a system with an ideal delay, the loop gain is again of the form of an integral with some extra delay.

So, and this corresponds to I mean this is exactly equal to an excess delay of  $T_d$ . So, in this case we know that the system is critically damped for  $T_d$  being  $1/e$  times  $k$  by  $\omega_u$  and if the value  $T_d$  exceeds this, we get a certain amount of overshoot, but we have earlier discussed that for  $T_d$  is less. Let us say half of  $k$  by  $\omega_u$ , the amount of overshoot is limited and it may be tolerable.

Similarly, when you have the one extra pole  $p_2$  and  $p_2$  is greater than about 2 times  $\omega_u$  by  $k$  when  $p_2$  is 2 times  $\omega_u$  by  $k$ , there will be certainly be overshoot, but it will not be significant and the overshoot is tolerable. So, what we will do is we will look at the loop gain magnitude and phase response corresponding to these values, which we know our reasonably good and then make the loop gain of any higher order system correspond to the same.

(Refer Slide Time: 05:07)



So, first let us consider the second order system poles loop gain is  $\omega_u$  by  $k$  divided by  $s$ , and in general it is some unity loop gain frequency  $\omega_u$  loop  $\omega_u$  loop by  $s^2$ . The magnitude of the  $L$  of  $j\omega$ , let us first take the case where  $P_2$  is  $4\omega_u$  and the magnitude of  $L$  will be one when  $\omega$  is approximately  $\omega_u$  loop there will be some contribution to the magnitude from the second term, but it is small enough that we can ignore it. So, the phase angle of  $L$  at the unity loop gain frequency is given by  $-\frac{\pi}{2}$  due to the integration minus  $\tan^{-1} \frac{1}{4}$  due to the second pole and the phase margin  $PM$  which is  $\pi$  plus the angle.

So, this tells you how far you are away from the critical point of  $-1, 0$ , this gives a measure of how far away you are from that and this is equal to  $\tan^{-1} 4$  and this corresponds to the phase margin of  $76$  degrees, this we have already seen. So, what does this mean is if I plot the Bode plot magnitude of  $L$  and the angle of  $L$ . The magnitude of  $L$  will have a  $20$  dB per decade slope and it crosses unity approximately at  $\omega_u$  loop.

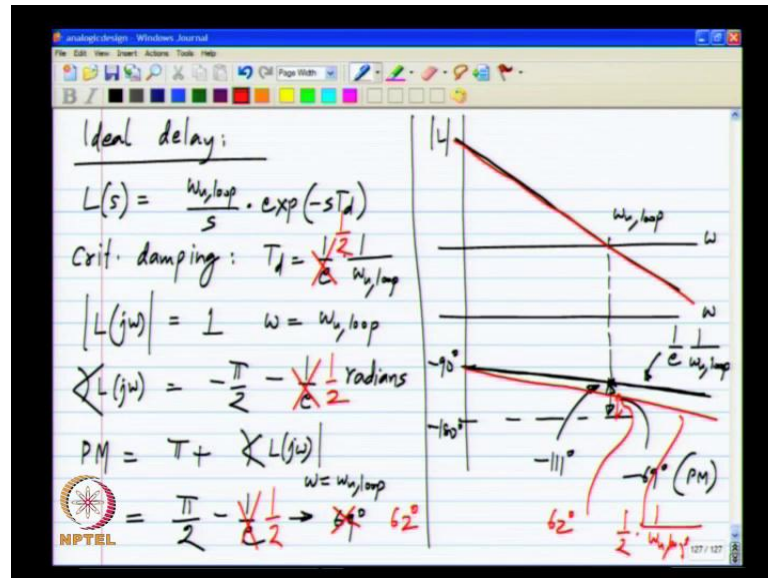
At  $P_2$ , it has another breakpoint and it falls down, further the phase starts from  $-\frac{\pi}{2}$  due to the integrator and it starts changing due to  $P_2$  at  $P_2$ . It will be  $-\frac{3\pi}{4}$  and finally it goes off to  $-\pi$  and the phase angle at this point is  $-\frac{3\pi}{4}$  and the distance from  $-\pi$  which is a phase margin is  $76$  degrees. Similarly, we can evaluate for what happens when  $P_2$  is  $2\omega_u$ , instead I will write it here in a different color and in this case the magnitude of  $L$  is 1 not at  $\omega = \omega_u$  loop, but at a slightly lower frequency.

We will still approximate the same because just for is of and calculations this is not as good an approximation any more, but will still do that. Now, I will calculate the same thing, so instead of  $-\frac{\pi}{4}$ , I get  $-\frac{\pi}{2}$  and the phase margin will be  $\frac{\pi}{2}$  and which corresponds to  $63$  degrees. So, in this case as I said, there will be little bit overshoot, but not too much. So, in this case, the loop gain in magnitude will look like that and starts going down before because the value of  $P_2$  is only 2 times the unity loop gain frequency.

The phase also starts lagging a little bit before, so in this case, the phase margin is reduced and the phase margin is only  $63$  degrees for  $P_2$  being 2 times  $\omega_u$  loop. So, both of these are reasonably good values and we say that our phase margin as to be within

this range. So, in general if you are not given any further information and asked to design an amplifier, you should design an amplifier whose loop gain as a phase margin of about 60 degrees or more. So, that will ensure that the amount of ringing in the step response is not significant.

(Refer Slide Time: 10:30)



Now, just to confirm our findings, we will do it for the case with ideal delay for which the loop gain is  $\omega_u$  by  $s$ , the ideal loop gain times exponential of minus  $s T_d$  which denotes an extra delay of  $T_d$ . We know that the critical damping occurs for  $T_d$  being  $1 / (e \omega_u)$ , now we see that the magnitude of  $L$  of  $j \omega$  is 1 for  $\omega$  equals  $\omega_u$ . In this case, it is exact because the magnitude of the second part of the function, here it is unity and the angle of  $L$  will be minus  $\pi / 2$  due to the integrator minus  $\omega T_d$  and  $\omega T_d$  is nothing but 1 by  $e$ , keep in mind that this is in radians.

So, the phase margin which is  $\pi$  minus the angle of  $L$  of  $j \omega$  at  $\omega$  being  $\omega_u$  is  $\pi / 2$  minus  $1 / 2$  by radian which corresponds to 69 degree. So, you see that you will get almost the same value, in the other case it was 76 degree for critical damping for second order and in this case it is 69 degree. It is a small difference and such a small difference is due to the difference in the detail of the response because having an extra pole not exactly and having an ideal delay, it is only approximately like that. So, let us again draw the magnitude and phase response to get

some experience with this the magnitude of  $L$ , in this case it simply goes off as minus 20 degree per decade and it crosses unity at  $\omega_u$  loop and the phase. It starts from minus  $\pi/2$  at very low frequencies, keep in mind that this is on a log  $\omega$  scale cannot show 0 and then the phase goes down linearly. For the critical damping at  $\omega = \omega_u$  loop, you have a phase angle of minus 100 and leave degree.

This is approximately minus 90, so this is on a log scale. So, on a straight line on a log scale does not actually look like a straight line have drawn it approximately, and the distance from minus 180 degrees is minus 69 degrees. This is the phase margin and when we first discussed the system with ideal delay in the negative feedback we also said that we also tried different amounts of delay and. So, what happened, we saw that for a delay of  $1/e$  times  $1/\omega_u$  loop. It was critically damped for  $\pi/2$  times  $1/\omega_u$  loop, it was unstable and for delay of approximately half of  $1/\omega_u$  loop or less. There was ringing, but not too much, let me make this instead of  $1/e$ , I will make it  $1/2$ .

So, what happens, now the magnitude of  $L$  of  $j\omega$  becomes 1 at still at the same frequency, the second part of the expression does not change the loop gain magnitude, but the phase. Instead of being minus  $\pi/2$  minus  $1/e$  will be minus  $\pi/2$  minus half radians. So, the phase margin will be  $\pi/2$  minus half which corresponds to you know that half of the radian is 28 degrees. So, this corresponds to 62 degrees. So, again what we got from the second order system, we saw that in a second order system when the pole was at a term two times, the unity loop gain frequency the phase margin was 63 degrees and the pole being at the twice the unity.

The loop gain frequency is approximately like having a delay of half of  $1/\omega_u$  loop gain frequency and in the ideal delay case that particular delay gives you a phase margin of 62 degrees. So, the magnitude will look exactly the same as before, the phase angle will be a little more and the new phase margin will be 62 degrees. So, this is for an excess delay of  $1/e$   $1/\omega_u$  loop and this is for an excess delay of half of  $1/\omega_u$  loop and this is a good value to remember.

So, let us say you are trying to make a negative feedback system, whose integrators as a unity gain frequency of  $\omega_u$  loop that is the loop gain supposed to be an integration. It has a unity gain frequency of  $\omega_u$  loop or a time constant of  $1/\omega_u$  loop, then the

excess delay can at most behalf of the time constant. That is a reasonable thing to remember as well, so from this, we conclude that phase margin of 60 to 70 degrees are good. If we have more, it is even better, because it approaches a first order system, but in practice it may be more and more difficult to arrange it. It may be more and more conservative design if your attempt to do so, but like I said you should try to aim for a phase margin of around 60 degrees to avoid a significant ringing in the step response.

(Refer Slide Time: 16:50)

Order	$L(s)$	closed loop instability?	critically damped	Phase margin (crit. damp)
1	$\frac{\omega_{cl}}{s}$	NO	—	always $90^\circ$
2	$\frac{\omega_{cl}}{s} \cdot \frac{1}{1+s/p_2}$	NO	$p_2 = 4\omega_{cl}$	$76^\circ$
3	$\frac{\omega_{cl}}{s} \cdot \frac{1}{(1+s/p_2)^2}$			

So, now, what does it mean for a higher order systems, let us see. So, will make a table with order and let me start with first order the simplest case and the loop gain in that case will be simply omega u loop by s. We can ask the question whether it is our unstable here by instability, I mean poles in the right of plain, when you put this in a negative feedback loop.

We know that there is unconditionally stable and it will never happen like that and we can also ask when I said critically damped and for this particular system, it is irrelevant there is never any overshoot in the step response. The step response is a first order and the phase margin is always 90 degrees. Now, we will look at the second order case against something, we have already analyzed and this also is never unstable in that. If you put this in a close loop, the poles never go into the right half plane. Also, they can get very close to it and this is critically damped for P 2 being 4 times omega u loop and for this the phase margin.

Let us say they look at phase margin for critical damping and this will be 76 degrees. So, now you can look at higher and higher in order system, now remember I am looking at a two extra poles behind the integration and those two poles can be anywhere, but for these of analysis all right that both the poles are identical. So, in general we can have a factor of the form  $1 + s + P_2$  and  $1 + s + P_3$ , but it becomes harder and harder to handle analytically. Later, from stimulation we can see that all those cases are equivalent. So, we will draw a conclusion from this particular one  $\omega_u$  loop by  $s + 1$  by  $1 + P_2$  whole square and can this be unstable in closed loop.

(Refer Slide Time: 19:48)

$$V_i \rightarrow \begin{array}{c} \oplus \\ \ominus \end{array} \rightarrow \text{Summing Junction} \rightarrow K \rightarrow V_o$$

$$V_o \rightarrow H \rightarrow \text{Summing Junction}$$

$$\frac{V_o}{V_i} = \frac{K}{1 + KH}$$

$$1 + \frac{1}{KH} = 1 + \frac{s}{\omega_{u,loop}} \left(1 + \frac{s}{P_2}\right)^2 = \frac{1}{H} \cdot \frac{1}{\left(1 + \frac{1}{KH}\right)}$$

$$= 1 + \frac{s}{\omega_{u,loop}} + 2 \cdot \frac{s^2}{\omega_{u,loop} P_2} + \frac{s^3}{\omega_{u,loop} P_2^2} \quad | \quad s = j\omega$$

$$= \left(1 - \frac{2\omega^2}{\omega_{u,loop} P_2}\right) + j\omega \frac{1}{\omega_{u,loop}} - \frac{j\omega^3}{\omega_{u,loop} P_2^2}$$

So, if we have a feedback system like this  $V_o/V_i$  is of the form  $g/(1 + gh)$  alternatively  $1/h(1 + gh)$  and a poles will be on the  $j\omega$  axis, if for some particular value of  $s$  equals  $j\omega$ , the denominator is 0. So, it will examine the denominator for our case and remember our loop gain is given by this particular expression. So, we have  $1 + s/\omega_u$  plus  $2 \cdot s^2/(\omega_u P_2)$  plus  $s^3/(\omega_u P_2^2)$ . If you evaluate it at some sinusoidal frequency  $s$  equals  $j\omega$  we get  $1 - 2\omega^2/(\omega_u P_2)$  plus  $j\omega/\omega_u$ . Remember, these two terms gives real part and this two term give the imaginary part, now for the denominator to be 0 both of these real part and the imaginary part have to be 0.

(Refer Slide Time: 22:23)

The image shows a digital whiteboard with the following handwritten content:

$$= \left( 1 - \frac{2\omega^2}{\omega_{cl,loop} \cdot p_2} \right) + \frac{j\omega}{\omega_{cl,loop}} - \frac{j\omega^3}{\omega_{cl,loop} \cdot p_2^2} \quad \frac{j\omega}{\omega_{cl,loop}} \left( 1 - \frac{\omega^2}{p_2^2} \right)$$

$$1 - \frac{2\omega^2}{\omega_{cl,loop} \cdot p_2} = 0 \quad \omega^2 = \frac{\omega_{cl,loop} \cdot p_2}{2}$$

$$1 - \frac{\omega^2}{p_2^2} = 0 \quad \omega^2 = p_2^2$$

$$p_2 = \frac{\omega_{cl,loop}}{2} ; \omega = \frac{\omega_{cl,loop}}{2}$$

The NPTEL logo is visible in the bottom left corner of the whiteboard.

Let me copy over this expression were equating the real part to 0, I get 1 minus 2 omega square by omega u loop times P 2 to be 0 and this says that omega square is omega u loop times P 2 divided by 2 and the imaginary part being equal to 0. This can be factored out into j omega by omega u loop 1 minus omega square omega u loop P 2 square. This part has to be 0, 1 minus omega square by omega u loop P 2 square is 0. So, omega square is omega u loop times P 2 square.

So, we have two variables here the actual frequency at which the denominator close loop gain becomes 0. So, the gain becomes infinity and also the value for P 2 for which it happens. So, by equating these two, we see that if P 2 is omega u loop divided by 2, these two equations are satisfied and also omega will be equal to, sorry I had made a mistake here in the parenthesis. I should had only omega square by P 2 square is also omega square by P 2 square. So, omega squared will be exactly equal to P 2 square. So, P 2 will be omega u 2 divided by 2 and omega is also equal to omega u loop divided by 2, so I made a slight mess in the algebra, but this is the final answer.



(Refer Slide Time: 24:44)

Order	$L(s)$	closed loop instability?	Critically damped	Phase margin (crit. damp)
1	$\frac{\omega_{cl}/s}{s}$	NO	—	always 90°
2	$\frac{\omega_{cl}/s}{s} \cdot \frac{1}{1+s/p_2}$	NO	$p_2 = 4\omega_{cl}/s$	76°
3	$\frac{\omega_{cl}/s}{s} \cdot \frac{1}{(1+s/p_2)^2}$	$p_2 > \frac{\omega_{cl}}{2}$		$p_2 = 8.1 \omega_{cl}/s$ for 76° PM
4	$\frac{\omega_{cl}/s}{s} \cdot \frac{1}{(1+s/p_2)^3}$	$p_2 > 1.25 \omega_{cl}/s$		$p_2 = 12.2 \omega_{cl}/s$ for 76° PM $p_2 = 16.3 \omega_{cl}/s$ for 76° PM

So, what does it mean is that there is closed loop instability there can be closed loop instability if  $P_2$  equals  $\omega_{cl}/s$  divided by 2 and I will not show the other analyses. Here, it turns out that if  $P_2$  is more than this the poles will be definitely in the right half plane. So, this corresponds to excess delay that is very large and in this case. So, it can be unstable that is the poles can be in the right half plane the poles of the closed loop system. That will happen  $P_2$  being  $\omega_{cl}/s$  by 2, please notice that in this case the non dominant pole has come inside the unity loop gain frequency.

If you think of it as a equal and delay, the delay will be 2 by  $\omega_{cl}/s$  or twice the time constant of the integration. This we know is too much of a delay and in this case, it give you instability were should the pole be so that we can get no ringing. So, again as I said repeatedly, it is not is not easy to calculate the time domain response. So, what we do is, we make sure that the phase margin is 76 degree based on the guidelines we get from second order systems.

What we mean is that when the magnitude of the loop gain becomes 1, the angle of the loop gain should be minus 90 minus 14 degrees that leaves 76 degrees of phase margin. If you compute it for that we get  $P_2$  to be 8.1 times  $\omega_{cl}/s$  for 76 degrees phase margin. So, now, this two problems are not separate in that we are not exactly calculating the time domain response without overshoot, but we are looking at a particular phase margin 76 degrees and saying that  $P_2$  is 8.1 times  $\omega_{cl}/s$ . So, there are couple of

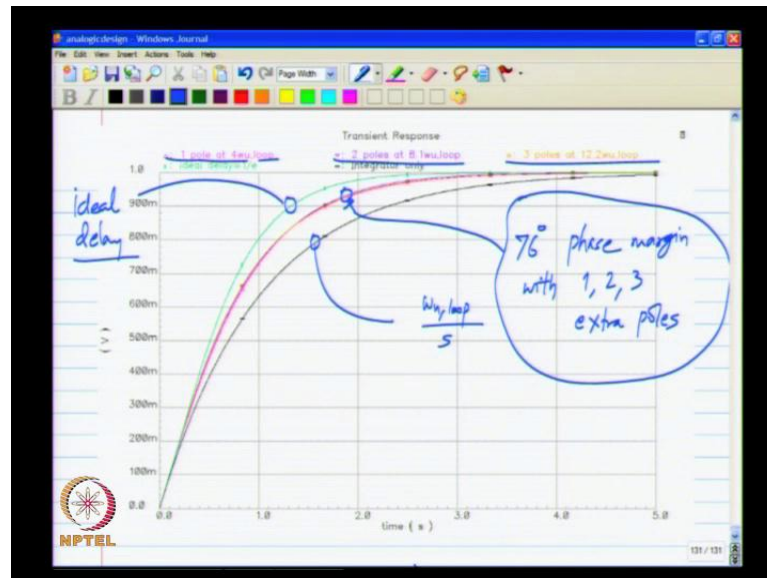
things to notice here, so first is when there are two extra poles, each of those has to be further away from the unity loop gain frequency compared to when there is one extra pole.

So, when there is one extra pole, it has to be a 4 times the unity loop gain frequency when there are two they have to be a 8.1 times the unity loop gain frequency approximately twice are small. So, the combined effect is roughly the same the same exercise can be carried out for higher order system. In this case, we have three extra poles all at  $P_2$  and this is again for analytical convenience and  $P_2$  has to be greater than I think  $1.125 \omega_u$  loop notice also that here  $P_2$  had to be half of  $\omega_u$  loop and here it is more than  $\omega_u$  loop. Then, it will be unstable because the more poles you have each one contributes an excess delay and the excess delay contribution due to each pole can be smaller while still driving it to instability.

Of course, the instabilities and academic concern, we do not even want to get there while designing the amplifier for a good design. We would like to have  $P_2$  to be about 12.2 times  $\omega_u$  loop for 76 degrees phase margin and similarly, we can calculate for fifth order where instead of a  $1 + s/P_2$  whole cube. We have  $1 + s/P_2$  to the power 4, so in this case it turns out that  $P_2$  should be 16 times  $\omega_u$  loop for 76 degrees phase margin. So, as you have more and more poles and we assume identical poles.

So, each of those poles has to be further and further away from the unity loop gain frequency. In fact, you can see that if you think of each pole has adding a delay here, the you have a single pole adding a delay of one-fourth the time constant of the integration that is in the loop gain and here each one has to had approximately one-eighth. So, two of them combined will add one-fourth here, three of them have to add one-twelfth and three of them combined will be one-fourth. Similarly, here four extra poles each has to add one-sixteenth and the combined effect will be at one-fourth. So, this also should conform to you that the each pole acts like a delay and the delay contributes to ringing and even more delay contributes to outright instability. So, just as a quick example, we can see we can see what happens.

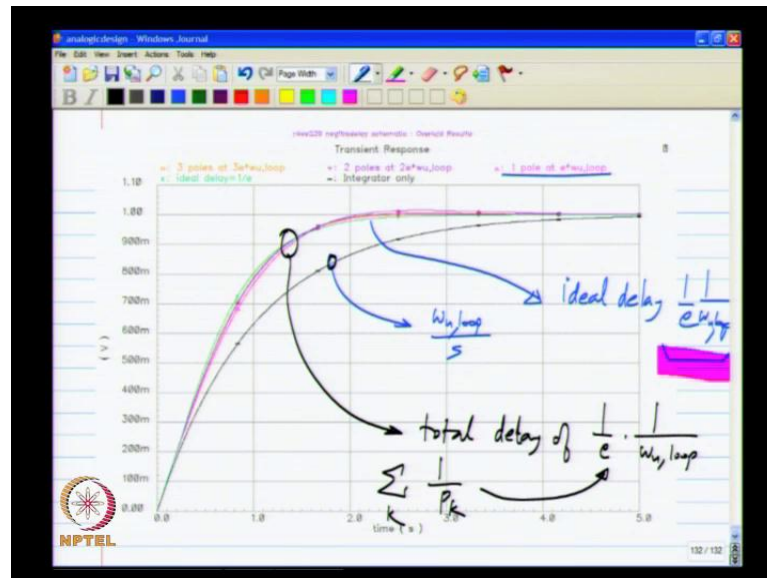
(Refer Slide Time: 29:52)



So, I have shown the response to the first order system, this is simply  $\omega_u$  loop by  $s$  and this it follows the familiar first order response and I have also shown what we first calculated with the excess delay of  $1$  by  $e$ . It is critically damped and this is with an ideal delay and all the other cases are in between. In fact, if you look at it a plotted with a single extra pole at four times  $\omega_u$  loop, two poles at  $8.13$  poles at  $12.2$ .

So, all of these things which corresponds to  $76$  degrees of phase margin, we can see all of them are Bows together some are here all of this corresponds to  $76$  degrees phase margin with  $1, 2$  and  $3$  extra poles. So, the only lack of generality here is that when you have three all three poles are at the same frequency. So, this also again should give you a confirmation that is  $76$  degrees phase margin is a good number to have. Then, you will not have any overshoot, we can do a similar exercise based on what we learned from the response with an ideal delay, so let me paste that here.

(Refer Slide Time:31:25)



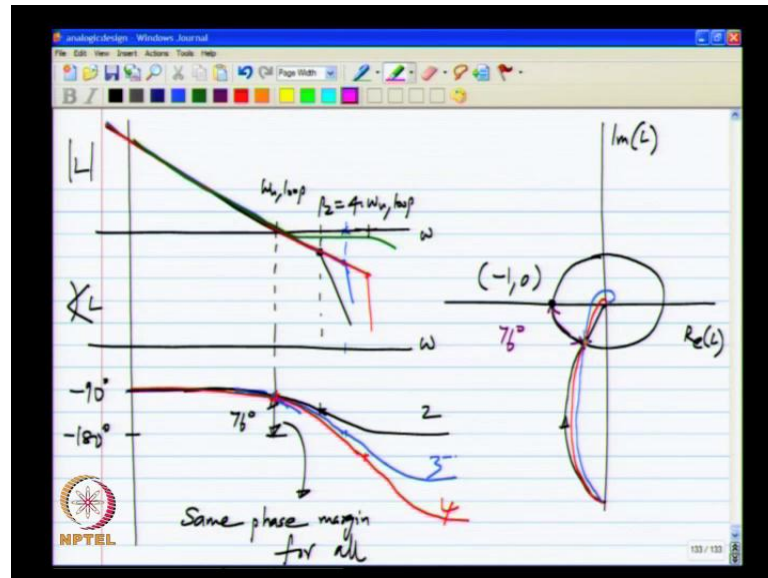
So, again here what I have shown is the first order the loop gain is simply  $\omega_u$  loop by  $s$  and that follows the first order response. The critical dam stuff here is shown in green with the excess delay with the ideal delay of  $1/e$  times  $1/\omega_u$  loop. Now, what I have done is I have taken one extra pole like for instance one extra poles at  $e$  times  $\omega_u$  loop. So, roughly speaking this is this corresponds to delay of  $1/e$  times  $1/\omega_u$  loop, basically the same delay as in this case and you see that while the response is not exactly the same.

It is the pink curve here it is in the same range and as you go to the higher and higher order system, when you have two poles at  $2e$  times  $\omega_u$  loop. So, each one corresponds to the delay of  $1/e$  times  $1/\omega_u$  loop. So, the two of them combined for my delay of one over  $e$  times  $1/\omega_u$  loop. So, again the same value has this, so the response is quite close to over the green curve, which is with the ideal delay and I have taken three extra poles and four extra poles. So, all of them are bunched together in this area. In fact, the higher the order, it becomes closer to the response with an ideal delay. So, all this corresponds to a total delay of  $1/e$  times  $1/\omega_u$  loop, so when I mean by total delay, if you have multiple poles the summation, so delay due to each pole, so all of them corresponds to the same case.

So, this again should tell you that if you have a delay of a  $1/e$  times  $1/\omega_u$  loop regardless of the order of the system, you will get a reasonably well behaved step

response. So, we have seen that this translates to in the ideal case a phase margin of about 69 degrees saw again confirming that phase margin of about 69 degrees. So, again confirming that phase margins of 60 degrees are more are good numbers to have and just to complete the discussion.

(Refer Slide Time: 33:55)



We have looked at what should be the response of different order systems. So, if you have a second order system for a critical damping the second pole should be at 4 times the unity loop gain frequency and if you have two identical poles, there should be at eight times the unity loop gain frequency. So, I am sorry here it becomes stiffer and if you have three extra poles it falls down even stiffer, but for the same phase margin, we computed that there should be at 12 times the unity loop gain frequency. So, if you look at the phase plots the phase, of all of them will start from minus 90 and this goes to like that when you have only a single extra pole.

So, that corresponds to a phase margin of 76 degrees or a phase slag of 104 degrees at omega equals omega u loop. So, this is the magnitude of the loop gain this is the angle of the loop gain. Now, the next one the third order system with two identical poles, also starts like this, and then it has a similar phase margin. Finally, it goes to a larger phase slag, close the larger phase slag of minus 270, but at this point it has the same phase margin. So, let me draw it little more accurately at 4 times omega u loop, it should touch

minus 180, and then and if you look at the red one. It will do the same here and then will do this and it goes even for that.

So, the phase at very high frequency is very different, but the phase when the loop gain crosses unity is the same for all of them and we are seen that that correspond to the same kind of step response. So, this is the second order system that is a third order system to extra poles and this is a fourth order system. Now, we can look at the Nyquist plots and let me draw the unit circle nice and big here. So, the loop gain of the second order system was that and the angle corresponding to this the angle from here to here. This is 76 degrees if you look at a third order system, so I cannot show the details very clearly here, but the point is intersects at the same point.

Then, it goes and cuts the negative real axis and then intersects, but the point is this the Nyquist plot never comes close to this minus 1, 0. If it comes close to the minus 1, 0 point, it comes close to the minus 1, 0, it becomes close to instability. This is the imaginary versus  $\omega L$  of  $L$  the usual Nyquist plot and similarly if you have higher order system, it does something ignore the detail before cuts the unit circle. After it cuts the unit circle, it does that and goes around, but it is still keeping far away from the minus 1, 0 point if you have the phase margin of 76 degree.

So, that is what the Nyquist plot picture looks like, so the ultimate test of stability is the Nyquist plot because the poles. The condition of poles being in the left half plane can be map to the Nyquist the Nyquist plot not encircling the minus 1, 0 point in the clockwise direction, but assuming that the magnitude is well behaved. That is the magnitude reduces after reduces monotonically, then it goes inside the reason, I am mentioning this is it is always possible to make a  $\omega L$  system with the poles and 0 such that lets say, it is has a 76 degree phase margin. After it goes inside, it can come back towards minus 1, 0 and then go to 0.

It is possible to come up with system like that and if you do that it will be unstable even if the phase margin is 76 degree, but we are looking at relatively simpler system. Here, you have either extra poles or some extra poles and some extra 0s in that case, it is safe to say that if the phase margin 76 degree inside the unit circle it kind of goes straight towards the origin and does not come close to minus 1, 0. So, the magnitude more or less goes down monotonically, so the real problem will be if the magnitude after falling

below unity stay close to unity. The phase angle increases, then it will come closer to minus 1, 0, but most of the system that we deal with will not do this.

This is something that you have to watch out for in the magnitude response, you should not have a case where you have something like this. Then, the magnitude is close to unity and the phase angle increases, if you do that, you will end up with the system that you have a lot of ringing, but usually we do not encounter this particular case. So, we just ignore that, so the bottom line is the stability there should be no encircled mind, no clockwise encircled mind of minus 1, 0 point by the Nyquist plot by encircled mind.

I mean including both the Nyquist plot for the positive and the negative frequency, the plot for the negative frequencies will be the mirror image of that for positive frequency. Now, we do not want we do not want the poles nearly in the left half plain, there should be far away for enough away from the imaginary axis. So, the step response does not have significant ringing for that the condition is that the Nyquist plot steer clear of steers sufficiently clear of the minus 1, 0 point and for most system. It can be expressed by a single number that is the phase angle the angle at which Nyquist plot cuts the unit circle and how far it is away from negative real axis. It is far enough away from the negative real axis, then you are safe the Nyquist plot will not come to close to minus 1, 0.

So, we examine some cases for which we could find the solution analytically that is a negative feedback system with an ideal delay and a negative feedback system with whose loop gain is an integration plus an extra pole. In both these cases, we can evaluate the condition on the delay or the pole. So, that the step response is well behaved and for those well behaved step response is that we find that this angle here is from 60 to 70 degree. We also know that if we do not have any excess delay at all, the angle is that angle is 90 degrees, because in that case, the Nyquist plot simply a with the imaginary axis.

So, we can make a statement then that any phase margin 60 degrees or more is good and when you have no further information, you should design for a phase margin of about sixty degrees. So, like I said there are also some exceptions you can also come up with a v rd loop gain function where which maintains the phase margin of a 60 degrees or more, but just after it crosses unity that is after the magnitude falls below unity. This means that

the Nyquist plot inside the unit circle because of some 0s, it comes close to the minus 1, 0 the critical point, but we are assumed that in most of our cases it does not happen.

So, it is enough to look at the phase margin, but it is always a good practice to look at the actual magnitude plot and see that after it crosses unity it decreases monotonically or does not over around the 0 dB point for a wide range of frequency. So, this is a summary of a discussion of the stability for a higher order systems, now any amplifier that you design is not likely to be first order or even second order. It is going to have a lot of parasitic poles, it can be due to the integrator implementation, it can be due to the feedback network and it can also be due to the distributed nature of capacitors in a resistance.

So, when you take a register it is not like there is a capacitor attached to it one end or the other it is distributed throughout the register, which corresponds to a higher order system. In fact, infinite order system, so if you rely on analytical calculations only, it will be impossible to design it for any step response. So, we take guide line from this system for which we can calculate and draw the guide lines for other system and the guide line is that the phase margin should be more than 60 degrees. So, we confirmed by looking at the step responses which can be obtained from the simulation and we also draw the magnitude and phase response of this system. So, you know that if you design an amplifier, your magnitude and phase should look like this.

If you do that, you will be safe, now just before we close a quick note on why we should avoid overshoot there are many reasons. So, first of all let us say one of the places where feedback loops use more frequently is power supply a the purpose a regulated power supply is to hold the voltage to be a constant in spite of the load that you connect. So, if you connect a heavy load or light load, it should maintain the output voltage to be a constant and it has a system like this it as an integrator and because the implementation of the integration is complicated there will be extra poles.

Now, what happens is let us say you change the load to the system. So, that is like investigating the step response a change in load is similar to some extra stimulus to the system and there will be a step response. Now, let us say there is an overshoot this means that the voltage can go behind the respect regulated value. So, that is if you are supposed to regulate the 6 volt, the voltage can rise up to let us a 6.5 volts. So, now, the load the



circuit that you connect the circuited that you are powering from the regulated power supply is getting old days that is more than 6 poles and sometimes it can be damaging to the system. Similarly, if it goes below 6 volts can lead to some other corruption like for instance, it could be a memory chip.

If the voltage slips a well below the regulated voltage, what can happen is that you have a ram chip and the ram does not have a enough supply voltage and it lose its data. So, that it is always good to limit the amount of overshoots and undershoots in the transience. So, for this, you have to design the system for a particular phase margin and that is why it is useful, now one more note about this phase margin. So, from our investigation, we said that phase margin of 60 degree or more is good, but this is not a secret number this assumes that the overshoot is an important criterion in the step response of the system. There may be cases where you can tolerate a little more ringing at the little more overshoot.

So, there phase margin thirty degree may be adequate, so we will not talk about that as a general criterion, but in specific cases may be a phase margin of 30 degrees is adequate. So, do not think of 60 degrees as a sacred number and you should absolutely never ever design it for a phase margin of less than 60 degrees. So, generally to have a well behaved step response this is the phase margin, but there may be a lot of cases where the ringing in the step responses not of that much concern.

It could be that designing for 60 degrees or 70 degrees of phase margin is too expensive in terms of the complexity of the circuit or may be the power dissipation of the circuit in that case you may well settled for lesser phase margin because it is not critical. So, I hopes that part is clear, I hope you now understand why this phase margin of 60 or seventy degrees is considered appropriate how it comes about. So, as I said for the higher order systems, the same phase margin is used, because if you have a large number of pose beyond the unity loop gain frequency. You can think of each of them as contributing to certain amount of delay and the total delay as to be limited to a certain value.

(Refer Slide Time: 47:54)

The image shows a digital notepad with the following handwritten content:

$$L(s) = \frac{\omega_{u,loop}}{s} \prod_{k=1}^{N-1} \left(1 + \frac{s}{p_k}\right)$$

Below this, it states:  $p_k > \omega_{u,loop}$

Then, it sets the magnitude to 1:  $|L(j\omega)| = 1, \quad \omega = \omega_{u,loop}$

Finally, it calculates the phase margin:  $PM = +\frac{\pi}{2} - \underbrace{\sum_{k=1}^{N-1} \tan^{-1} \frac{\omega_{u,loop}}{p_k}}_{14^\circ} = 76^\circ$

The notepad also features an NPTEL logo in the bottom left corner and a page number '138 / 137' in the bottom right corner.

So, to conclude in general the loop gain will be of the form  $\omega_{u,loop}$  by  $s$  and a number of extra poles which may not be at the same frequency. I have taken  $n$  minus 1 extra poles this gives you  $n$  order system. So, now I will assume that all the poles are sufficiently greater than  $\omega_{u,loop}$ . So, this means that the magnitude of  $L$  is 1 for  $\omega = \omega_{u,loop}$ .

The phase angle of this when the magnitude is 1 is given by minus  $\pi/2$  due to the integration and minus summation of  $\tan^{-1} \omega_{u,loop} / p_k$  for  $k$  being 1 to  $n$  minus 1 and the phase margin is  $\pi/2$  plus this is that much. Let us say, we want this to be 76 degrees and this means that this entire sum has to be above 14 degrees. So, this is one way to think about it, alternatively we can also think about it in terms of delay.

(Refer Slide Time: 49:41)

The image shows a digital notepad with the following handwritten content:

$$L(s) = \frac{\omega_{u,loop}}{s} \prod_{k=1}^{N-1} \left(1 + \frac{s}{p_k}\right)$$

Below this, it states:  $p_k > \omega_{u,loop}$

Then, it sets the magnitude to 1:  $|L(j\omega)| = 1, \quad \omega = \omega_{u,loop}$

Finally, it calculates the phase margin:  $PM = +\frac{\pi}{2} - \sum_{k=1}^{N-1} \tan^{-1} \frac{\omega_{u,loop}}{p_k} = 76^\circ$

There are annotations under the phase margin calculation: a bracket under the sum is labeled  $14^\circ$ , and the final result  $76^\circ$  is underlined with the text "more than  $60^\circ$ ".

So, the total excess delay here is the summation of all this poles and this has to be limited to some reasonable value, which is either one by  $e$  times  $1$  by  $\omega_u$  loop or if you are willing to relax it a little bit more half of  $1$  by  $\omega_u$  loop and. Similarly, the phase margin criterion is not absolute, so this could be more than  $60$  degrees. So, whatever it is you see that when you have a number of extra poles all the poles have to be well beyond the unity loop gain frequency. So, either the phase margin is more than  $60$  degrees or the excess delay is less than half of  $1$  over  $\omega_u$  loop. So, that ends the lecture, so in the lecture, we will look at how to implement Op amps and how excess delay can come in even in that situation.