**Analog Integrated Circuit Design**
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**Lecture - 24**
**Noise in Resistors**

Hello and welcome to lecture 24 of analog integrated circuit design. In this lecture, we deal with one of the very important non-ideal features of any real circuit that is random noise. It turns out that dissipative components like resister or a mosfet have random voice, and components which do not have any power loss like capacitor and inductors do not have.

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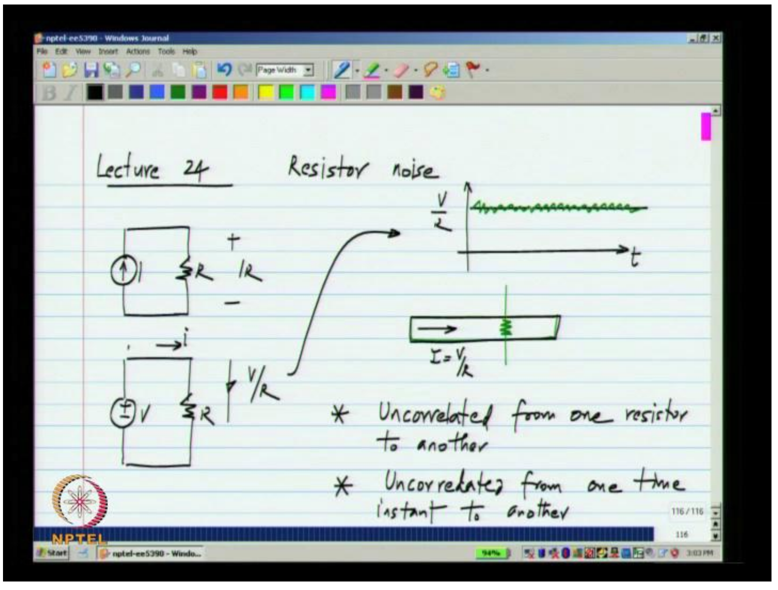


So, in this lecture, we will look at random voice in a resistor says if you pass current five through a resister R voltage across, that should be I times R and if you apply a voltage source V across resistance R, the current through that well of course V divided by R in reality because of random motion of carriers. Through the register, the current is exactly V by R. That is why you take the circuit and measure the value of I very precisely versus time.

You will see that the current has the mean value of E by R, but they are variation around this mean value, and that happens because if you take any cross section of resistor, let us say the current I is flowing, that way I equals V by R. That means that on average, this much charge is flowing every second through the resistor, but if you do take a cross section and see the charge flowing across the cross section in addition to this average value, there will be charges randomly moving back across the cross section. This is because random formal of charges as long as the temperature is above absolute 0, this is going to happen.

Now, we need a way to quantify this and also, to estimate it at any given circuit. Clearly because of this, that current, any circuit is not exactly what we calculate based on the devise relationship. So, how can you specify something that is random as such? First of all, the resister noise, it turns out if you do this experiment with two different resistors, the noise wave form that you see here will not be the same.

They will be uncorrelated from one resistor to another. It is a random phenomenon, and what is having in the resisters does not have anything to do with this. What happens in another resistor, it also turns out that in particular case over resistor, it is uncorrelated from one time instant to another. So, if you take the value at the particular time and another time in the same resistor, they will also be uncorrelated from each other.

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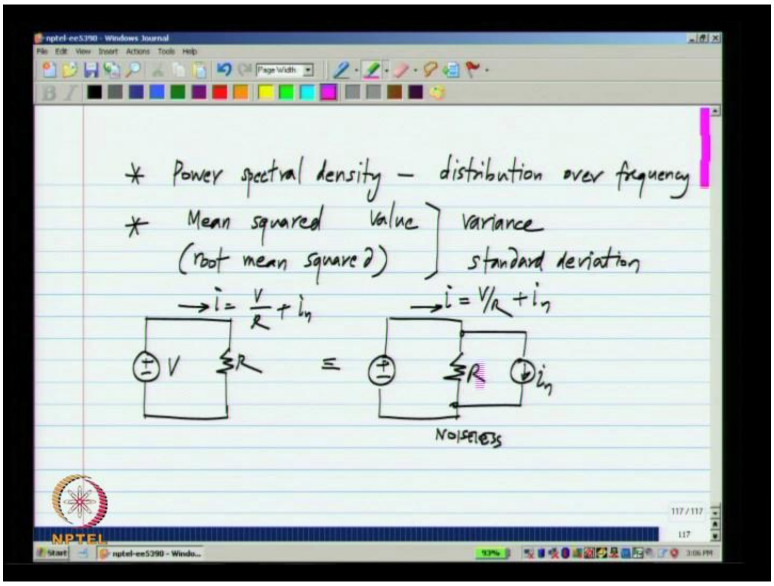


How can we quantify such random quantity? There are many ways. From your study of probably random process you will know that. You can specify the power spectrum density or in general, the spectral density which relates to how the energy in the random is distributed over frequency.

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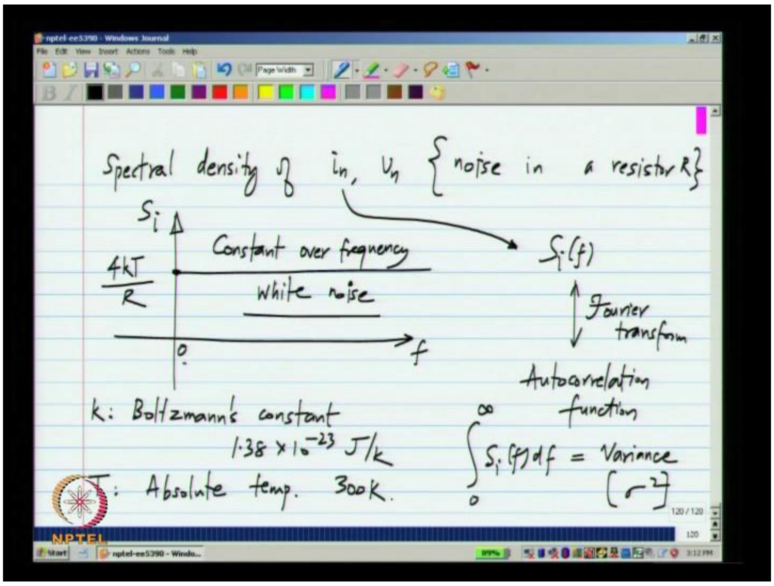


You can also specify the mean square or the root mean square value. One of these things and for zero mean, the random processor is these are the same as the variance, and the standard deviation. Now, in this case, we assume that the average current is V by R and rest of it is by definition, it will have zero mean. So, we can either call it root mean square value of standard deviation or the mean square value or variance.

We have to specify one of these things to specify the size of the noise because noise is random. We cannot specify amplitude. In fact, theoretically the amplitude of such turns out to be infinity, but we can specify the size by specifying the mean square value of noise. We need to learn how to do this for a resistor. First of all, let me go back to two pictures I drew earlier.

If I have a voltage source across resistance, the current i equals V by R plus sum noise current, and this can be modeled by assuming that there is noise less resistor and a noise current source across it of the average just to symbolize that it is a current source. It does not mean the current is always in this direction. As I said the mean value of i n is 0 and the current in both direction clearly, in this case also i will be equal to V by R due to the noise resister plus i n.

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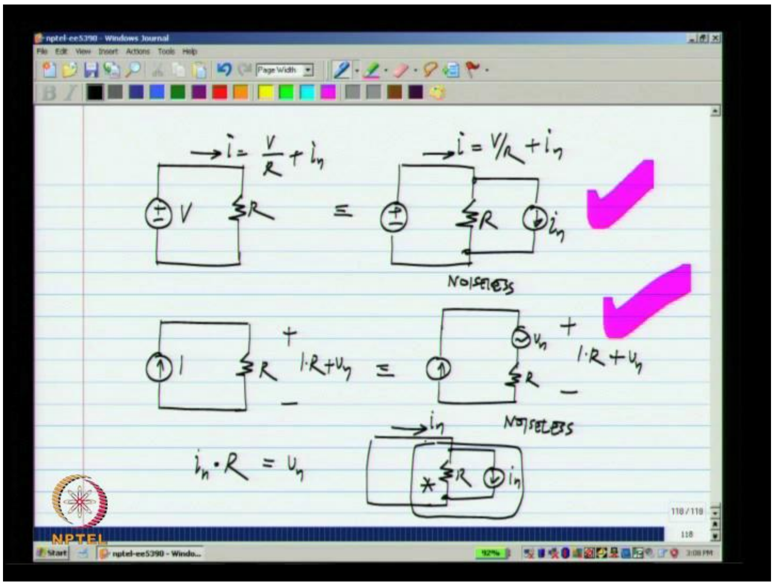

So, we model the noise in the resistor as a current source. If I look at the alternative description, if I pass a current i, throw the resistance R, the voltage across will be i times R plus A noise voltage and this again can be part of the current flowing to a noise less resistor and series voltage source which is equal to the noise voltage. In this case also, the voltage cross i times R plus V n.

So, in the pictures on the right side, what is that means? The noise itself is factor out as a source and this is very common thing. You assume that there is a noise component which follows the V i relation that you know it is home flow of resister or a square loss of a mosfet and so on, and the noisy part of is modeled out as either the voltage source or a current source whichever is convenient.

Finally, if we are going to use this model or that model for the resistor, the two have to describe the same thing and that will happen if i n times are equals V n. This we can see very easily by assuming that let us say we have applied the zero voltage across the resistor and by the way this is noise less. So, the current following here is i n. This entire thing, the combination is what modules the resistor.

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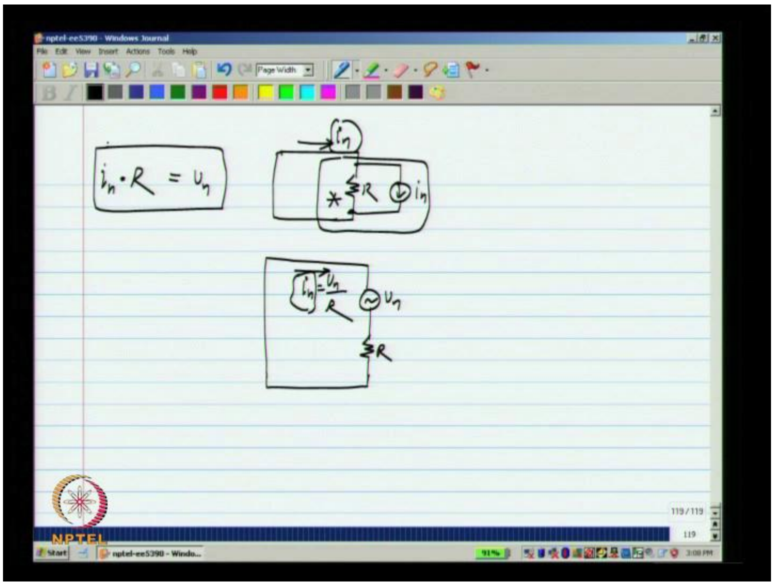


Now, I should get the same result if I use the other model for the resistor noise which is a resistor noiseless resister series voltage source V n and i short circuited just like clearly the connect will be V n divided by R, and the currents here and here have to be the same. So, that means the equivalent voltage source of a resistor and equivalent current source or a resistor are related by ratio which is equal to the resistance R. As I mentioned before, we cannot specify the instance value of either i n or V n. So, what we will specify instead is the spectrum density. That is how the energy in e V n or i n is distributed over frequency i n and V n are noise in a resister R.

Now, first of all, what is the spectrum density? Spectrum density is some function of frequency and as common in circuit design, we take the one sided spectrum density. That means, that specify the spectrum density for positive frequencies and what does the spectral density tells you is some function.

The spectral density of i n will be some s i of f. What is this? This is related to the auto-correlation function of this process i n to the courier transform of the out correlated function and in particular, the interior of s i overall frequency zero to infinity will be equal to the outer correlated function at A times shift of zero or the variance sigma square.

So, this is how the spectrum density is useful in specifying the magnitude of the noise. Now, for a description of auto-correlated function spectral density, you can refer to any basic book on random processes. It turns out that in the particular case of a resistor, if I plot the spectral density of the current, it is constant for all frequency and such a function is known as white noise.

If the spectrum density of certain noise process is constant for all frequencies, it is known as white noise analogues to white color which has all frequencies in it and the magnitude of the spectral density is 4 k t divided by R, where K S Boltzmann's constant 1.38 times to minus 23 joule per Kelvin and p is the absolute temperature and room temperature corresponds to 300 Kelvin. Now, this is the spectral density of the noise current of a resistor. It is constant over frequencies and it has magnitude equal to 4 k t divided by R.

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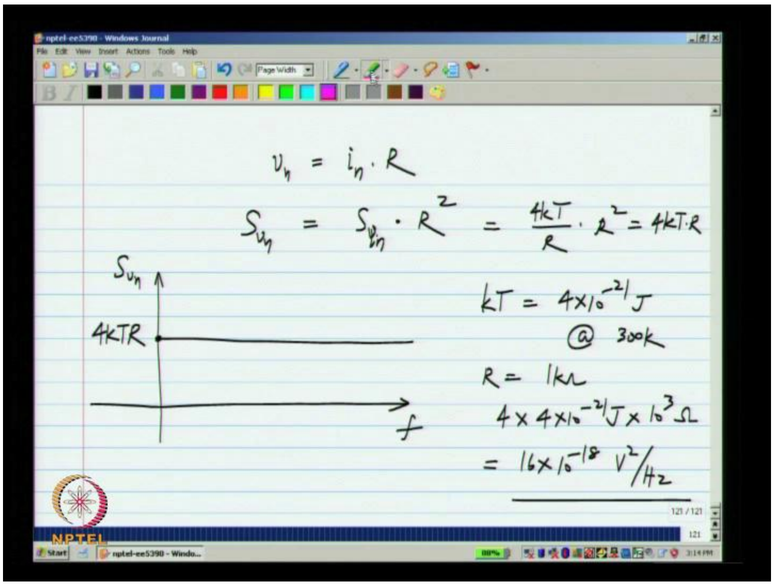


Now, you also know that the noise voltage relate to the noise current by multiplier factor R. So, the noise spectral density to the voltage is related to the noise spectral density of the current by the square of multiplying the factor. This is because basically the spectral density relates to the square of these quantities. So, that means that the noise voltage resistor as the spectral density which is k T divided by R square equals 4 k T R.

Naturally that is also constant with frequency and it is 4 k T R. My good numbers remember that k T equal 4 times 10 to minus 21 joules at 300 Kelvin and if you calculate, let R equal 1 kilo times. What we will get is 4 times 10 to the power 21 joules times 10 to 3 ohms, and this gives you 16 times 10 to the minus 18 volts square per hertz.

The voltage noise is specified in volt square of hertz. The old region, it is related to the square of the voltages and the spectral density obviously relates to a density over frequency. So, it is given as old squares divided by hertz or old squares per hertz.

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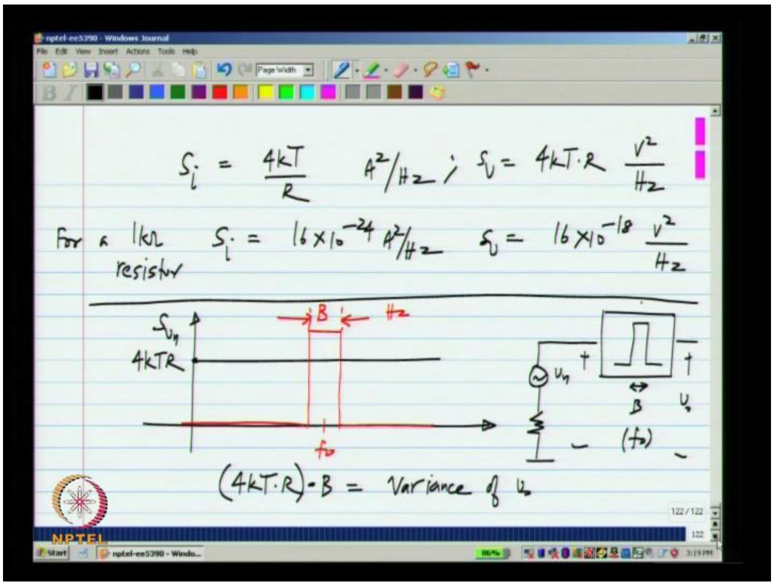


Similarly, the current noise spectral density is given by units of ampere square per hertz, and as I mentioned for 1 kilo ohm resistor, the straight answer to be 16 times into 18 old square per hertz and clearly from that we can also calculate S i which is 16 times 10 to the power minus 24 ampere square per hertz. Now, we did not derive any of these results. We just took them for granted.

These results were derived long ago and they have been used widely and confirmed experimentally and so on. What does that mean? I mean we have this obstruct description of noise in terms of the spectral density and so on, and we said something about it being related to distribution of this quantity over frequencies, but when we deal with the voltages current, I have 1 volt signal here and I have 10 mili volt signals there.

So, now the noise is something that is random and I should like to specify also in terms of voltages or currents. Then, I can compare my signal to the noise and see if the signal is sufficiently above noise of common experience. You know that if you are trying to (()) some signal or let us say you are trying to listen something and there is lot of noise, you will not be able to if little noise is there. So, you have to be able to quantify the amount of signal to the amount of noise and further, we need to be able to describe the signal and the noise in the same units, and we describe the signal in terms of voltages and currents.

Similarly, we have to do the same for the noise as well. So, what is the meaning of this spectral density? What it means is that let us take the voltage noise spectral density of a resistor and a resister modeled by a noise class resistor and series with a noise voltage source. So, across this, there is some voltage, although you will expect there to be no voltage because no current is flowing. Because of noise, there will be some voltage and let us say you apply these two and ideal band pass to filter off some band width and at the output, there will be some other voltage. Let me call that B. Now, how to use the spectral density is that spectrum density as a certain volume 4 k T R.

In this case, it happens to be with frequency and let us say we have the band pass filter b at the centre frequency of f naught. So, that means that at f naught, I have ideal band pass filter and band width is b hertz. So, that means that in the range of frequencies, it will pass the input voltage as it is to the output and outside it does not, ok. So, then it passes the noise energy in the pass band of the filter and blocks all of it in the stop band of the filter, ok. Then, the spectral density times the band width because inside the band, the transmission is constant. It simply passes the noise. This will be equal to the variance of V naught. That is what it means.

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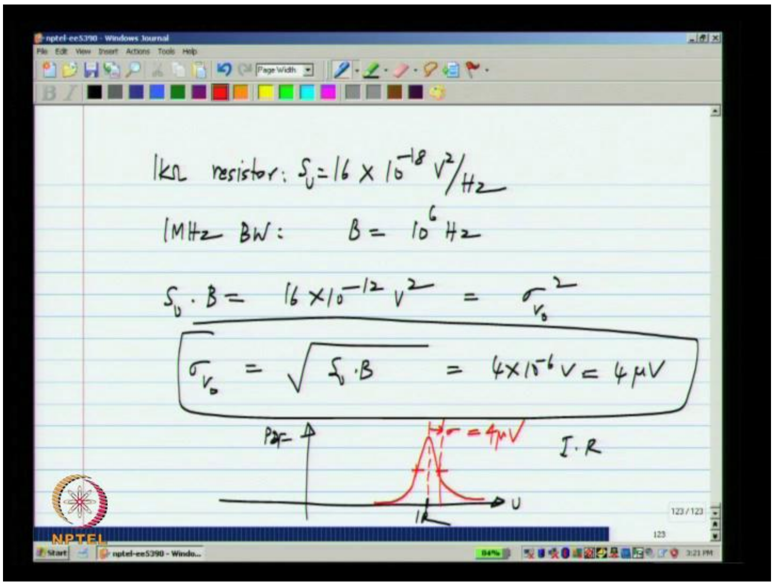

So, for instance, let us again take 1 kilo ohm. Resistor has a noise spectra density voltage of 16 times 10 to the minus 18 volts square per hertz and if I pass it through a band pass filter, ideal band pass filter of 1 mega band width, this means that B is 10 to the power 6 hertz. So, this spectral density times B will be 16 times 10 to the minus 12 whole square and this is the variant of the output voltage. Now, we would like to specify RMS value and that is nothing, but standard deviation and sigma V naught will be square root of V times B which will be equal to 4 times 10 to the minus 6 volts or 4 micro volts. So, this is how the voltage noise spectral density or current noise spectral density can be used for calculating the amount of noise.

When you say that amount of noise, we have to able to quantify and here we have said that the standard deviation of noise or the root means square value of the noise is 4 micro volts. Now, it turns out that like many other random phenomena commonly seen in nature, distribution of this noise is also Gaussian. So, what we would say is, let us say we have the current I flowing a resistor R. It will have the mean value of I R and if I plot the density function, probably density function vertices, the voltage will have the mean I R and it will have some distribution around it. Now, what it means by having a distribution? Let us say you make thousand of this example circuit where you pass a current. I throw a resistor R and you measure the voltage across all of them at the same time. Then, you will find that all of them will not be equal to I times R. They will have mean value of I R, but they will have the distribution around it and the standard deviation of that distribution will be 4 micro volts.

Now, it also turns out that you take one of these circuits, you have passed a current. I throw a resister R and you measure the noise at different time instants. Let us say you do it some 10000 time instants. Then, they will also have an average value of I times R, but they will have the distribution around I times R, and the standard deviation that distribution will also be 4 micro volts. That is what it means. You can specify the noise at any instead of time, but you can specify the root mean square value and draw whatever conclusion from that. Now, given that the distribution is we can say that 99 percent of the time, the values will lie with the minus 3 sigma of the mean. This is something that we know from the knowledge of the distribution. So, we can expect that most of the time the voltage that you measure across the resister will be within plus minus micro.

Volts of the means value I times R, whether this is sufficient closed or not, that depends on the application. We will later say the example of how to calculate that, but we will know that if you pass a current I resister R, where resistance is 1 kilo ohm and the band width of the interest is 1 mega hertz, you will have standard deviation of 4 micro volts or 99 percent of time. You will measure the voltage within the plus minus 12 micro volts of I times R.

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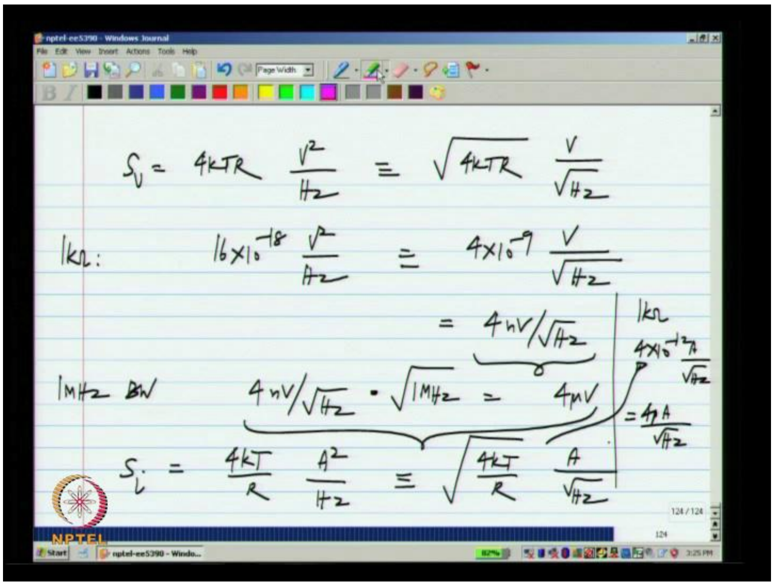

Now, because we deal with voltage and currents, sometimes the spectral density like A is 4 k T R volt square per hertz and this is equality specified as square root of 4 k T R volts per square root hertz. The reason to do this is we preferred to have units of volts square, that is all and we have the square root of band width under it. So, what does it mean? Again for kilo ohm resistor, we have 16 times to the 18 old squares per hertz and this equivalently represented as the square root of this value which is 4 times 10 to the minus 9 old square root hertz which is the same as 4 nana volts square root hertz. Now, this kind of specification is given, so that you can quickly relate to the voltages.

Now, again when you were talking about 1 mega hertz band width, an ideal band 1 mega hertz through which this noise is passed, all you have to do is multiply the spectral density in volts per square root hertz times the square root of band width square root of 10 to 6 is 10 to 3. So, we will get 4 micro volts. By the way, sometimes it is easier to specify in terms of volts instead of volts square. Similarly, the current noise spectral density which is 4 k T by R ampler square per hertz can be equally represented as square root 4 k T by R ampere for square root. Again for 1 kilo ohm resistor, this will turn to be 4 times into the minus. Well, ampler or square root hertz or 4 per square hertz because we are so used to dealing with the voltages and currents and not squares of currents square root hertz, but other description is equally valid and also, use equally often.

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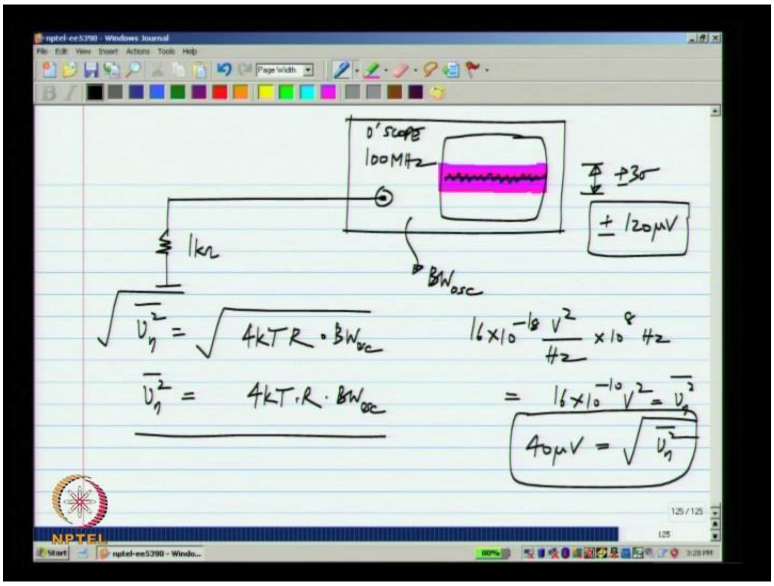


Now, just to give you a little more feel for this resistor noise, let us say I take the 1 kilo ohm resistor and connect it. What will I see according to ohms law, there is no voltage cross because there is no current flowing through it, but we do know that the random functions carries inside this will cause some voltage. How much will that voltage be? As usual we cannot specify the exact value at a given instant, but we can specify sort of the size.

So, what we will actually see on the oscilloscope, it will itself have some band width. Let me call it BW osc and the resistor has the voltage noise spectral density 4 k T R will assume that as usual the voltage across the resistor has sense without disturbing it by the noisily scope. So, the variance is that much and the root means square value that much. I will use that some times to denote the root mean square value of the signal, and this denotes the means square value of the signal and that is 4 k T R times the band width of the oscilloscope.

So, given that this is the 1 kilo ohm 4 k T R 16 times 10 to the minus 18 volts square per hertz and let us have the noisily scope which has the 100 mega hertz band width. So, this times 10 to the 8 hertz equals 16 times 10 to the minus 10 volts square. That is the mean square value and if you take the square root of this, we get 40 micro volts. It is quite a small value to be measured with an oscilloscope, but let us do. We have the noisily scope with such a fine resolution.

What you will see is, you will see band of some thickness and because this is a distribution, the amplitude is actually infinite. It can take values plus infinity or minus infinity, but 99 percent of time, it will be within plus minus 3 sigma band and that I will draw as the band that you will see on the oscilloscope which is displaying noise and that transferred to be a within band of plus minus 120 micro volts.

So, if you have a sufficient oscilloscope and you connect up to a resistor, this is what you will see. Alternatively, if you want to see larger noise, you know that the voltage noise spectrum density is directly proportional to the resistance value. So, what you can do is to use the larger and larger resistor, so that you will see larger noise from the oscilloscope.

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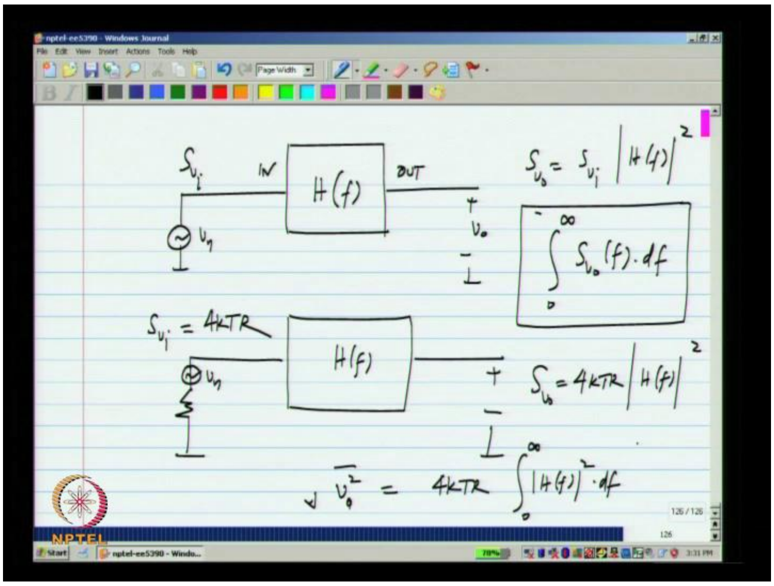

What we did so far was to define the spectral density, that is the distribution noise over frequency and if you take a certain band of width, let us say you are selecting using ideal band pass filter which passes the certain frequency and completely blocks others, what you will see is the spectral density times that band width because the spectrum density is the constant. Now, when the noise voltage of a resistor passes through some circuit, it may not experience the transfer function which is constant over all frequencies.

So, in general, let us say we have some transfer function which varies with the frequencies and to its input; we connect the voice source, some noise source which is spectral densities. Let us say S vi. Just to denote the input, what we will see at the output is again this is known from basic random process theory that at the output we will see another noise process whose spectral density is the input spectral density times the magnitude square root of the transfer function H. Now, if you want to calculate output variance, you have to integrate the output. The spectral density from 0 to infinity, we will quickly see an example of this one.

So, from the input density, you can calculate the output spectral density. Now, let us have some H and to its input, you connect resistor. We know that now the noise spectral density of the noise source here equals 4 k T times R. The output noise spectral density will be 4 k T R times mod H of f square and the mean square value of the output will be 4 k T R integral mod H of f square d f from 0 to infinity. Now, just like any other voltage or current in the circuit, the noise voltage or the noise current experiences some transfer function when it goes to the output.

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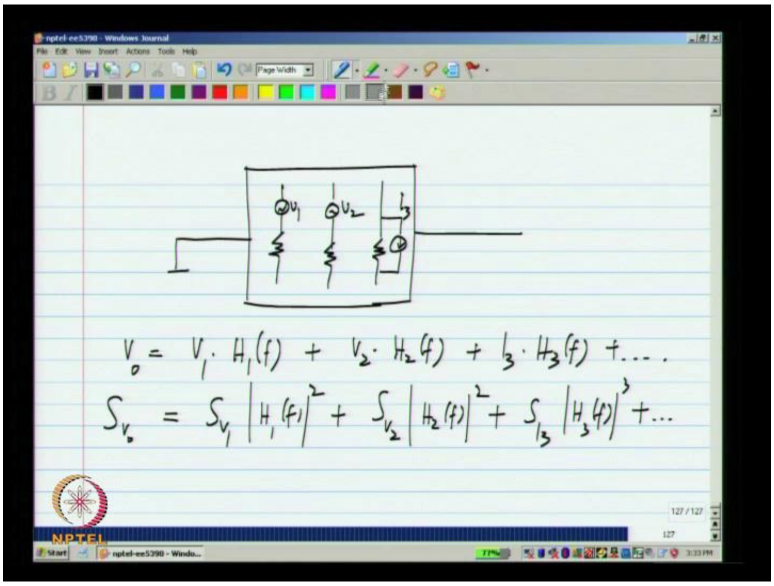

Now, when the noise goes to some transfer function, the output also have some noise and the spectral density of that noise equals the density of the input noise times the magnitude square of the transfer function. Now, if you have the multiple noise sources in a circuit, what you have to do is you calculate the transfer function from every noise source to the output.

Now, this is very common scenario that you have the circuit with a number of resistors. Let me call it V1, V2 and some time you may choose to model the noise with the current source instead of the voltage source, and let us say the input is 0 as usual. Because of the noise, the output will not be 0 in a liner time. In variant network, the input is 0, the output will be 0, but because of the noise source, it will not be 0. So, how do we calculate the output? We calculate the transfer function from each source to the output. So, let us say V 0 SV 1 times H1 which is the function of frequency plus V2 times H2 of f plus I3 times H3 of f dimensional, it does not matter times and so on. We could have all these things.

Now, the spectral density of the output will be of the spectral density of V1 times the magnitude square of H1 plus spectral density of V2 times magnitude square of H2 and spectral density of I3 magnitude square of H3 and so on. Now, the first one comes from just super position activate each noise at a time. Find out transfer functions and add them up. Now, the noise is affected to it in a very small quantity. So, any calculation that you do with noise will be linear. Now, even if you have circuit, you are familiar with the concept of small signal linearity around the operating point. If you have small disturbances, they can be represented by equivalent linear circuit.

Now, the noise will be small and it can be handled by the small signal linear equivalent circuit. So, the super position always applies. Now, to go from first one to the second one to calculate the spectral density, they make some assumption that this VN 1 and VN 2 and IN 3 are uncorrelated from each other. That is why we can add up the spectral densities of individual contribution. If they are correlated, you have to take the spectral density of the whole thing and if they are not correlated, it turns that the out spectral density will simply be some of individual spectral densities, and this is a correct assumption because it turns out that if you have physically different components, the noise from those things will be uncorrelated, right. If you have physically two different resistors, the noise from each resistor is correlated-uncorrelated from each other. The noise from the two resistors will be uncorrelated from each other. In fact, we simply do a quick calculation to verify that is indeed the case.

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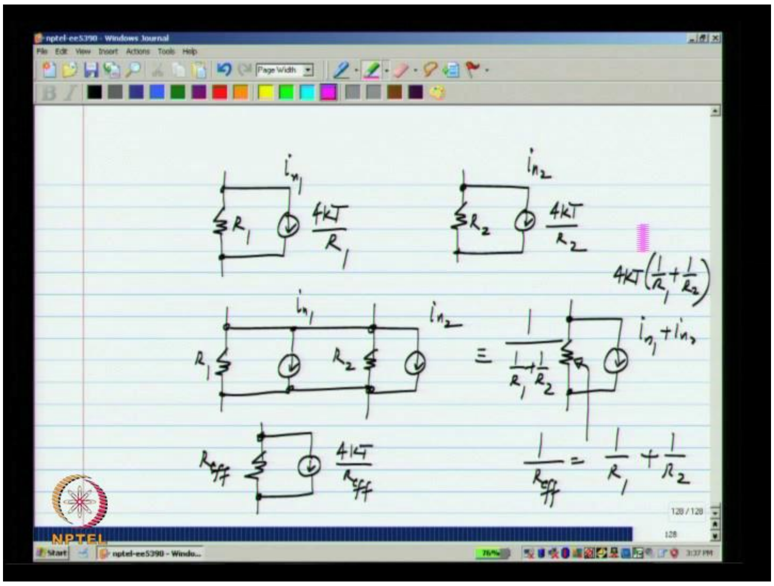


So, let me use the current source model. If I have the resistance R1, it will have a current source whose spectral density is 4 k T divided by R1. This is not the value of current. This is the value of the spectral density. If I have a resistor R2, I will have 4 k T divided by R2. Let me call this I 1 and I 2. Now, if I have these two parallel, what do I get? I get the parallel combination of the resistors which is the reciprocal of 1 over R 1 plus 1 over R 2 and I will have I 1 plus IN 2, and because these are correlated, respected density will be simply the sum of two spectrum density. The current source adds up in parallel.

So, we will have 4 k T, the density 1 over R 1 plus 1 over R 2 and you can see that this is consistent because the effective resistance here is such that it is 1 over R 1 plus 1 over R 2. The parallel combination of these two is the single resistor equal to R effective and you expect that there will be current source corresponding to each noise that will be 4 k T divided by R.

Now, if you assume that the noise from this and that is uncorrelated, we do get the consistent result. This is quick verification. It is very well known that the noise from different component will be uncorrelated from each other. Now, what we will do next is to take the simple circuit which has the transfer function that is not uniform with frequency. So far we have delta with the ideal band pass filter which had a uniform transfer function that is either 1 in the pass band or 0 in this pass band.

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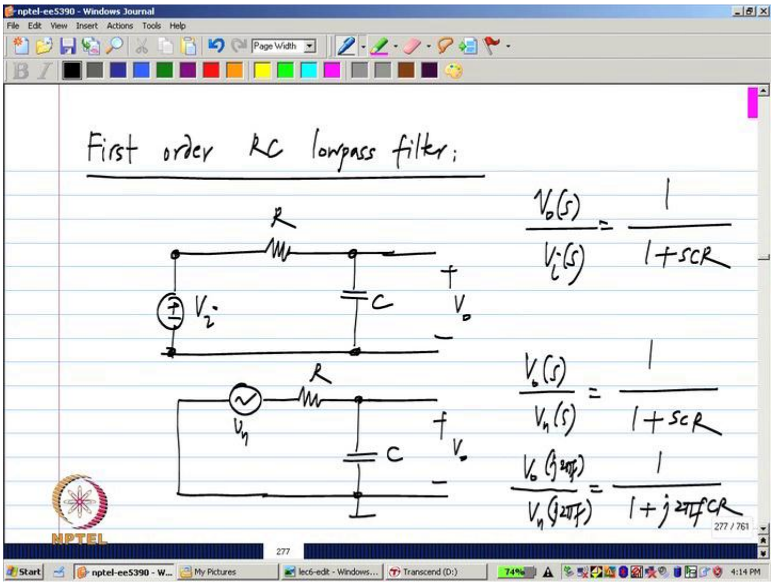


Now, we will look at another circuit and the circuit is also important for other reasons because its basic results are used in a lot of other contacts of this. Circuit will look at is first order RC low pass filter and if we apply input P1, we get an output V0 and we know that the transfer function of this circuit in terms of S will be 1 by 1 plus SCR. This is the transfer function from this input to the output. Now, in this circuit as usual there will be noise because of the resistor.

The capacitor does not add any noise. What we would like to do is find out the output noise. Now, as I mentioned earlier, noise is calculated from the near equivalent circuit and we can do that without the signal because the linear circuit follows super position and if you want to find out the effect of noise and signal together, what I will do is, I will calculate the effect of signal separately and noise separately and add up two. Of course, there are circuits. This is not valued behavior of noise is influenced by the presents of the large signals, but most of the common amplifier circuit operate in the linear mode, whether you have the signal plot, the noise will behave in the same way.

So, we will set the signal to 0 and calculate the noise. If I set the signal to 0, and I represent the noise by the equivalent source VN and I want to calculate V0. Now, you very easily see that this VN is in the place of this input. So, quite obviously, the transfer function VN to the output is the same as from VI to the output. So, V naught of S by VN of S is 1 by 1 plus SCR. Now, when you are doing the calculation with noise, usually we represent everything as function of the frequency f and not omega that is f n naught in radium per second. So, we will calculate this as the function of f and that will be 1 by 1 plus j 2 pi f CR.

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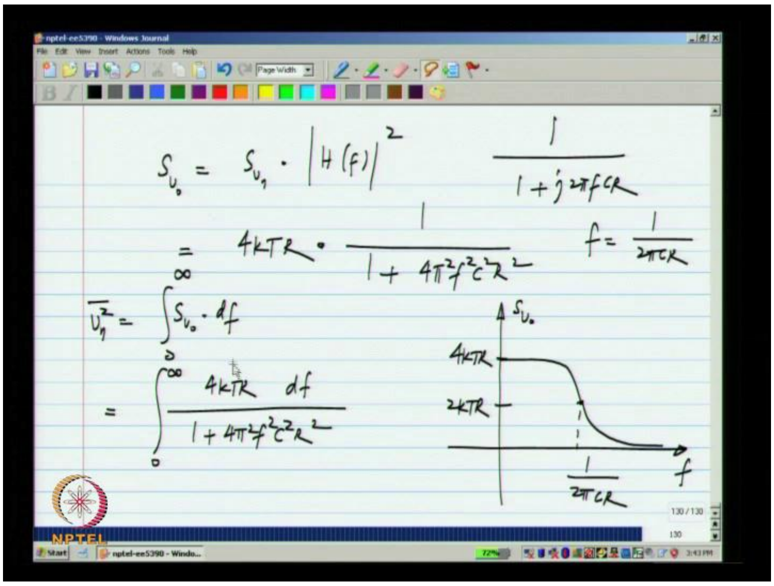


So, from the earlier discussion, we know that SV naught is SVN times magnitude H of f square which is 4 k T R respective density resistor noise times the magnitude square of 1 by 1 plus j 2 phi f CR. It is very easy to calculate this. It has 1 over 4-5 square f square C square R square. Now, what is the shape that this will have added low frequencies? It has the magnitude of 4 k T R and at high frequencies because of f in the denominator, it will drop down to 0 and we also see that the band width of the filter when f is 1 over 2 phi CR. This denominator is 2 and we will have spectral density of 2 KTR and this is at the frequency of on over 2 pi CR and after it drops down to 0 like that and this is SV0 and this is very much the expected behavior. When this is after all low pass filter, the noise voltage is low pass filter when it appears from output at low frequency.

What happens is you simply have (()) a resistor and all of the noise, the resister appears at the output and very frequency the capacitor assess the short circuit and no voltage appear across it. That is simply what is described by the spectral density plot I drew just now. I would also like to calculate the mean square value or the variance of the output noise. I note this is SB0 integrated over all frequencies and this is very simple integral from 0 to infinity and there are so many ways of doing it.

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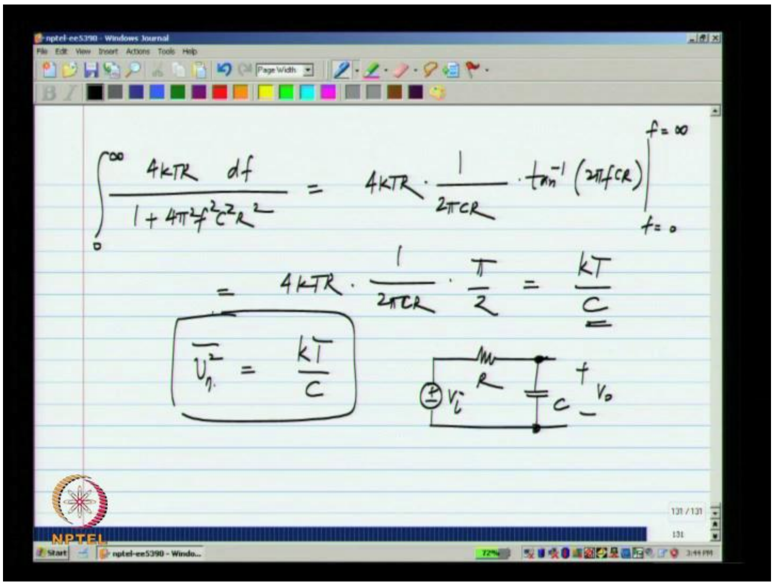

This gives you a tan inverse function. So, that is what the result turns out to be and that is nothing, but 4 k T R 1 over 2 phi CR times Y by 2 and it turns out to be k T by C. So, the means square value of the output noise is k T by C in this particular circuit. Now, what does that means? First of all, when you have the signal, also the output signal will be dependent on the filter transfer function. So, that is why you have at the band width of the filter, the output will have the sinusoidal which has an amplitude reduced by factor of square root of Q and phase shifted by 45 degree. In addition to that, you will have the random noise which is not white because the spectral density is not constant with frequency, but which will have the variance of k D by C.

So, the first thing, this is an example circuit for calculating the noise and if you have more complicated circuits, this is what you do. You calculate the transfer function from the resistor noise to the output and if you have from each resistor noise to the output and now there is another interesting thing about this that the noise is independence of R. So, the noise happens to be just k T by C and in fact, after you become little familiar with circuit design and interacting with circuit designers, you keep hearing k D by noise. First of all, why does it come out to be independent of R? This is because if you increase the value of R in the circuit, the spectral density in the noise in the register increases. It is 4 k

T R. The voltage noise density will increase, but the band width which is 1 over 2 pi CR will decrease in the same proportion.

Now, the integrated noise is nothing, but the spectral density times the band width. Of course, the filter is not abrupt filter which allows everything within a band width and stops everything outside the band width, but still the principle is the same. You have spectral density which is increasing in proportion to the resistance and the band width which is decreasing in direct proportion to the resistance. So, the product happens to be constant and that is equal to k T by C.

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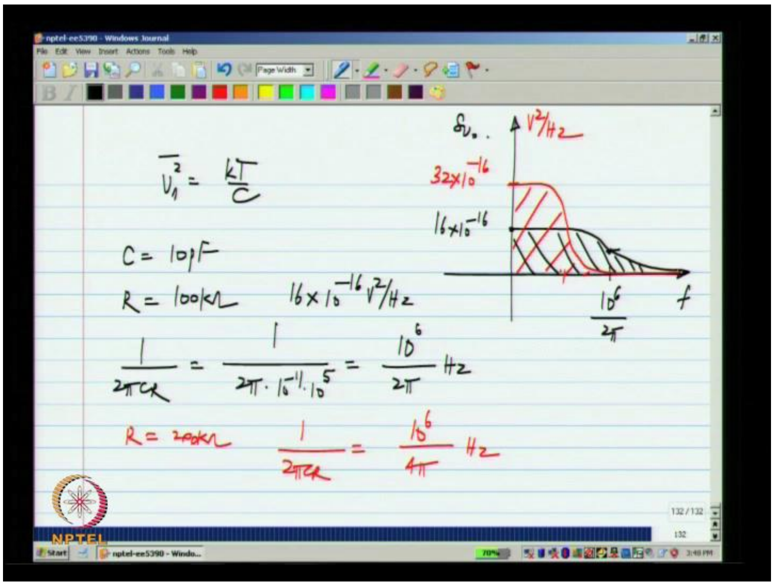

So, let us take some values. Let me take C2 10RH and let me take R to be let us say 100 kilo ohms. So, 1 over 2 pi CR equals 1 over 2 pi 10 to the minus 11 into the pi equals 10 to the 6 pi 2 pi hertz and 150 kilo hertz also, and for this 100 kilo ohm, the noise spectral density 4 k T R will be 16 times 10 to the minus 16 whole square per hertz. Let us say somewhere here and then, it drops down to 8 and then, it drops out like that. Let us make my R equals 200 kilo ohms. What happens is the band width drops to 10 to the 6 y 4 pi hertz. So, it goes here and the spectral density increases at low frequencies, but then it drops out more quickly. When it does that, what are the variants? After all it is the area under the spectral density curve from 0 to infinity.

The spectral density here is that 0 times 10 to 16 and this is of course, this is in volts square hertz. Now, just turns out that area under this red curve is exactly the same as the area under this black curve. So, regard less value of the resister in a low pass filter, you will have integrated k T by C. Now, this also tells you which will see the examples later that if you have the circuit with the resister capacitor and so on will lower the noise. You will end up with having to increase the value of the capacitors which typically increases the area, and we will also see later the increase power distribution that what we learnt from this particular analysis.

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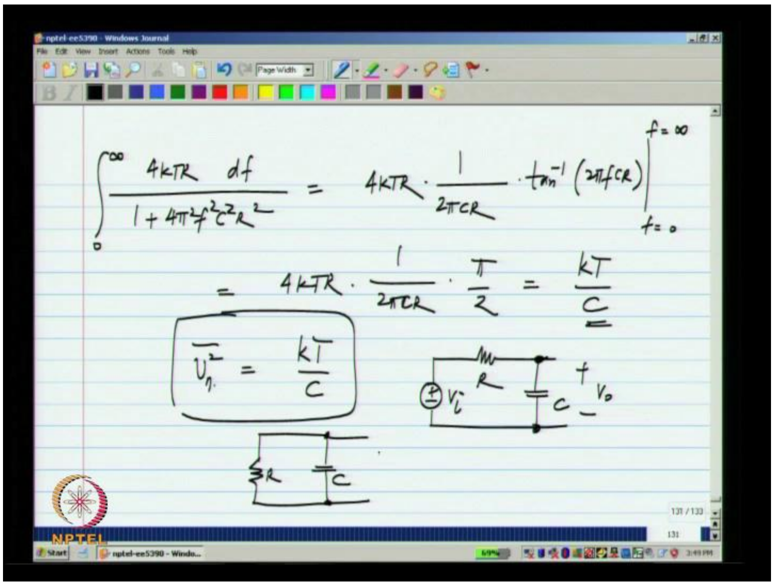

Now, also this analysis closes up other interesting questions like I have resister across the capacitor. This is simply the same circuit re-drawn. What has calculated is the noise of this circuit, right and we see the variance of the capacitor voltage k D by C independent the value of the resistor. So, the question is what happens when R goes to infinity? So, we have only the capacitor, but we seem to say that the noise across that is k D by C independent of the resistor. So, mathematically the result is valued even when R goes to infinity. So, it is an interesting problem. You can think about it. You can think about what interpretation it leads to. What it means to have R equal to infinity in the circuit and still have a variance of the k D by C for the capacitor voltage. Now, there is another way of arriving at result which I will not deal with in detail.

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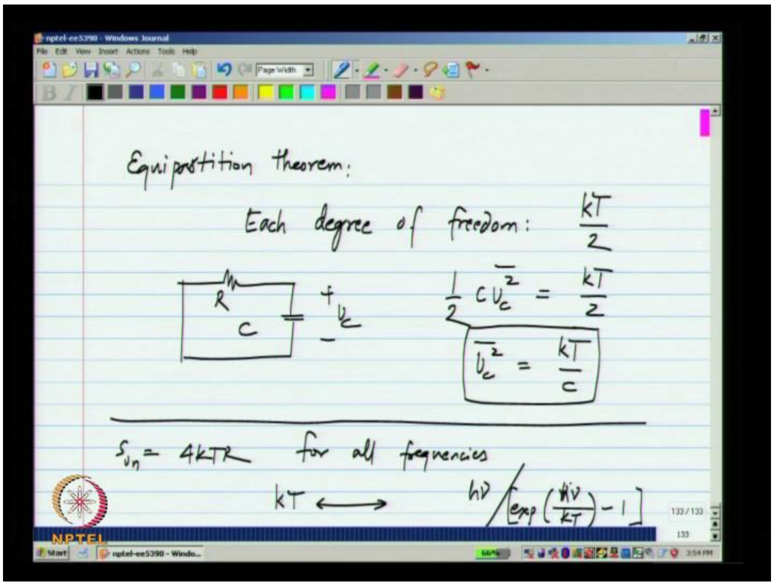


From the Equipartition theorem which you may remember from statistical mechanics, each degree of freedom has the energy of k T by 2. Now, if you have the circuit with one capacitor like this, there is one degree of freedom, the capacitor voltage and the energy of that is nothing, but half c v c square and that will be equal to k T by 2 and from this, you can see that the mean square value of the capacitor voltage become k T by C. Now, the result of this Equipartition part theorem is the fundamental result and that is how noise of the resistor is derived in the original paper, but this is other abstract and very hard to make circuit calculation from this very easily. So, we will use the spectral density of the resistor and from that we will calculate the transfer functions, and from that we will calculate the variance, but this is of something that you should know and you can follow the literature for more details from this one and finally, the noise spectral density of a resistor is 4 k TR for all frequencies, ok.

Now, what does this mean? The variance of this will be infinite because the band width is infinite, right. 4 k TR integrated from 0 to infinity will be infinite. So, if I take the resistor and measure the voltage across, will I measure the infinity? What do you think? Well, clearly not because if you do have the infinite voltage across the resistor, none of the small signal assumption would be true and no circuit will ever work. So, many things happen. First of all, you are measuring instrument will have finite band width. So, that will limit the amount of voltage that it measures.

So, that is one thing and the resistor itself will have something parasitic capacitance across something because between any two terminals, there will be some capacitance and across every resistor will have some capacitor. However, if you make the resistor physically smaller, the capacitance will be smaller and so on. So, there is another thing that limits the noise, but even more fundamentally, the noise itself turns out to be note white. This again I will not elaborate on, but from your first year physic course, you may remember what is known as ultra violet that lets to the introduction of quantization and quantum mechanics.

So, initially let us assume that the distribution of energy across frequency uniform and then, it led to this prediction that the black body will have the infinite power and that was resold by who postulated the energies only in discreet packets, and that is proportional to the frequency of the signal proportional to the frequency of the radiation. That gives you result that make since that it has the finite energy and the result of all that is, basically this k T will be replaced by H mu divided by exponential of H mu by k T minus 1.

So, this k T is something which is independent of frequency, but this function is very much dependent on the frequencies and HS constant and mu is the frequency, and you can very easily see that when the frequencies are very small, H mu is much smaller than k T. This reduces to k T. So, essentially what we have is the low frequencies appropriation to the reality k T minus 1 appropriation k T low frequencies and the low frequencies includes frequencies up to (()) hertz and so on. So, we can safely use this white assumption for all our circuits. In the next lecture, we will deal with noise and other components like the mosfet.