

Analog Integrated Circuit Design
Prof. Nagendra Krishnapura
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 25
Noise

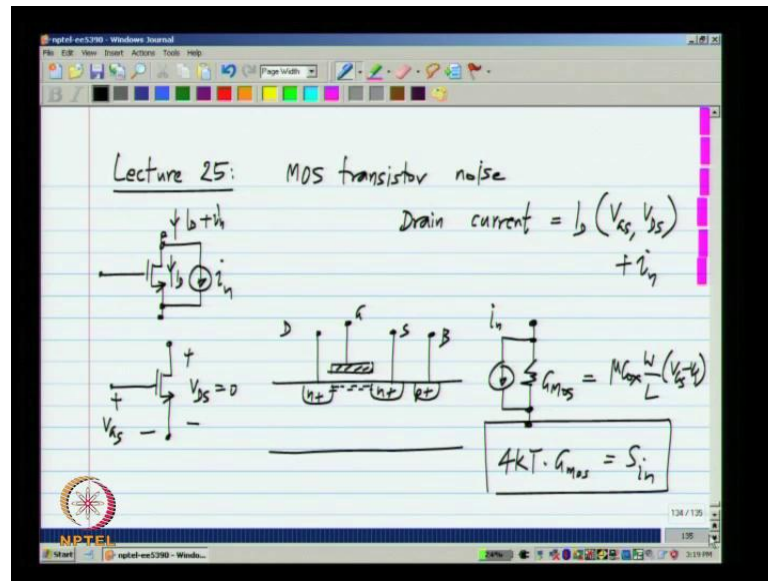
Hello, welcome to the lecture 25 of Analog Integrated Circuit Design. In the previous lecture we looked at random noise in resistor, which happens due to random thermal motion of charge carriers. So, in addition to the current that is predicted by the home slog will have some noise, it can be represented has an equivalent to noise current across the resistor or equivalent noise of voltage in series with the resistor.

Both of this will be described by their noise spectral densities, and in case of resistor thermal noise, the noise spectral density is wide that means, it has the constant magnitude to all frequencies, and it is proportional to the absolute temperature. The voltage noise, spectral density is $4 K T$ times R whole square per Hertz, and the current noise spectral density $K T$ divided by R ampere square per Hertz. The two are equivalent descriptions and either one can be used in any circuit for calculating the asset of resistors noise.

Now, the noise is the random phenomena, and it can be described by quantity such as the mean square value or the root means square value, and to do that they as to be some back limiting. Since, the resistor noise is white and uniform over all frequencies, within some band width the variance or the mean square value is given by the integration of the spectral density over that band width. So, they the spectral density is nothing but, if you pass the noise through an ideal 1 Hertz band pass filter, the variance will be equal to the value of the spectral density at that frequency.

We also elaborated this by calculating the spectral density and the integrated noise in an RC filter, if you have the capacitor across the resistor, it will lead to a finite the a means square value of noise and that is equal to $K d$ by c , which is the fundamental result that appear repeatedly in circuit design. And as I mentioned in the previous lecture, the Mosfet also has noise just like the resistor noise, and that is what we are going to take up in the today's lecture.

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In today's lecture we will deal with the noise of the MOS transistor, when we have a MOS transistor either an n MOS transistor or P MOS transistor, we have the current that is related to V_{GS} and V_{DS} , and in case of the P MOS transistor V_{SG} and V_{SD} . Now, the basic model that we have assumes that I_D is related by the square root of v_{gs} , but even if law is more complicated, there is some formula that gives you the current based on V_{GS} and V_{DS} .

Now, what does it mean to the noise is the MOS transistor, the drain current will be I_D which is the function of the V_{GS} and V_{DS} , as calculated from the formula plus from some noise. That is just like we had in the resistor the current predicted by the Ohms law plus some noise, here we have current predicted by square law plus some noise, and as with the resistor we will describe the spectral density of this noise. And from that spectral density, we can calculate mean square of noise and the different circuits and so on.

So, I will take the n MOS example, so as usual we represent the noise by assuming that there is noise less transistor, which gives you the current predicted by the formula plus some noise i_n . And here we have the current predictor by the formula plus the noise current, now what does the spectral density of the noise current look like, the transistor as operates in different region in the linear region, or the triode region and in the saturation region.

Now, it is easier first to think of what happens in the triode region, let us imagine that the transistor is operating with the certain V_{GS} and with V_{DS} equal to 0 that means, it say operating symmetrically. And if you look at the picture of the device, under these condition there will be a uniform channel going from the drain to the source, this is the gain. And we of course, have the substrate which is connected to some potential, the mechanism of conduction of Mosfet is exactly same as the mechanism in the resistor, it is due to drift of carriers.

And when V_{DS} equal to 0 the Mosfet is simpler resistor, now the charge are due to the channel that is form by inversion, but the operation is very similar to that of the resistor. So, it turns out that the noise is also exactly the same as what we would have in a resistor, in the drip triode region the transistor is nothing but, a conductance which I will call G_{MOS} .

And the conduct itself is given by the equation in the triode region, which is $\mu C_{ox} W$ by L , V_{GS} minus V_T , the conductance happens to be controllable by V_{GS} , but it is a conductance from the less. Now, if we had the resistance, so it is conductance was G_{MOS} , the noise spectral density would have been $4 K T$ times G_{MOS} , this would be the noise spectral density of the current noise i_n , turn out this formula exactly true for the Mosfet in the triode region.

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MOS transistor's current noise in the triode region

$$S_{i_n} = 4kT \cdot G_{MOS} \quad G_{MOS} = \mu C_{ox} \frac{W}{L} (V_{GS} - V_T)$$

In the saturation region,

$$S_{i_n} = \frac{2}{3} kT \cdot g_m \quad g_m = \mu C_{ox} \frac{W}{L} (V_{GS} - V_T)$$

OPERATING POINT

$$S_{i_n} = \frac{4kT \cdot |Q_I|}{L} \quad \left. \vphantom{S_{i_n}} \right\} \text{in all regions}$$

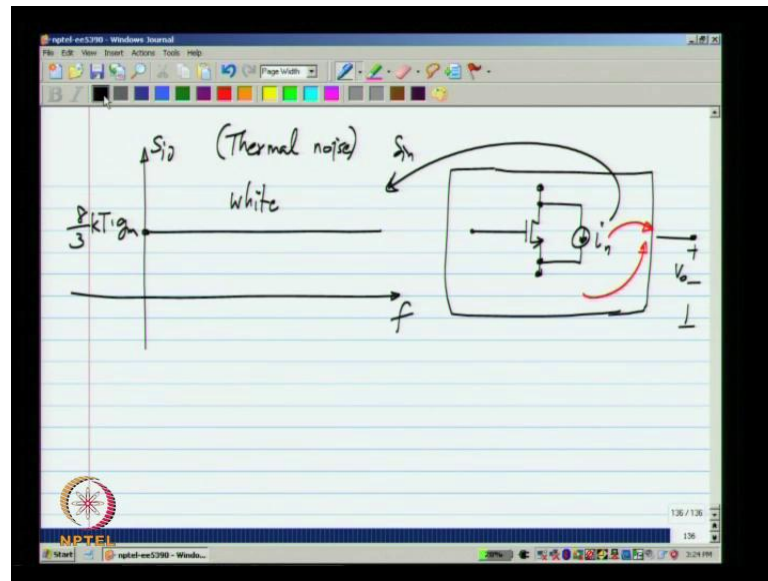
It is $4 k T$ times G_{MOS} where G_{MOS} the conductance at V_{DS} is equal to 0, that is $\mu C_{ox} W$ by $L V_{GS} - V_T$, in the saturation region the noise turns out to be $\frac{8}{3} k T$ times g_m at the operating point. Whatever the operating point the transistor result, just like all the small signal parameter of the transistor depend on the operating point, the noise also depends on the operating point and it happens to be $\frac{8}{3} k T$ times g_m . And you can quickly verified that, the current noise in the triode region or the current noise of the resistor is given by some formula it is looks like $k T$ times a conductance, and here it happens to be $k T$ times some trans conductance.

And if you assume that the transistor obey square law, g_m is given by $\mu C_{ox} W$ by $L V_{GS} - V_T$. And from this expression and that expression you can see that, the noise all formula, is in fact the quite similar in the triode region and saturation region, the dependence as $V_{GS} - V_T$ similar, expect that fact of $\frac{2}{3}$ in the saturation region. In fact, the noise of the Mosfet turns out to be given by this particular formula $4 k T$ times the magnitude of the charge in the inversion layer divided by L square, this is true in all regions.

We know that from the formula for C_{GS} in the triode region and saturation region, when the transistor operating in de triode region, the total charge is given by $C_{ox} W L$ minus $V_{GS} - V_T$. And when the transistor is operating in saturation region, their charge in the channel is given by $\frac{2}{3} V_{GS} - V_T$, so this formula covers both the triode region and the saturation region, and all the region in between. The transition is the usual is never discontinuous, but it is continuous.

If you minus to find the charge a formula for the charge in all the regions, you can use this formula for accurately predict the noise spectral density. Now, for hand calculation we assume that is either in the deep triode region or in the saturation region, and use these formulas, the two formulas that I have written down here. And most of the time an amplifiers will operate in the saturation region, and these is the formula that we use.

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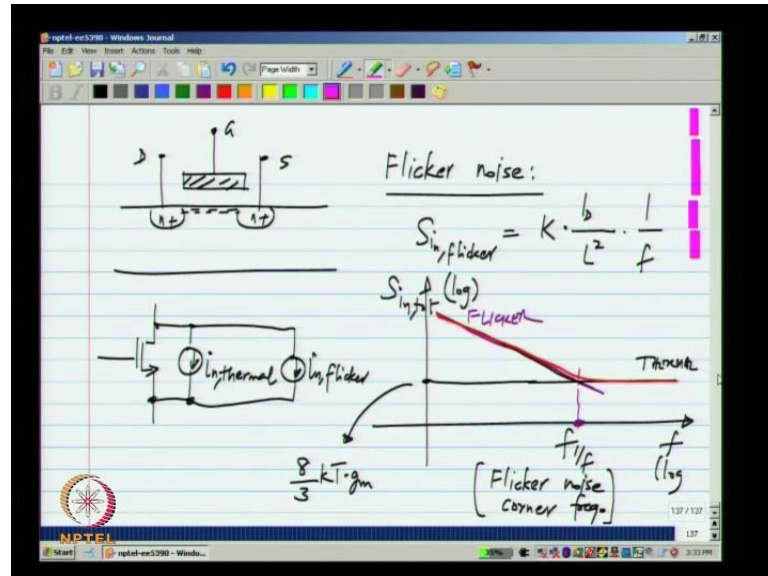
The spectral density S_{iD} is white, the noise in the MOSFET is also due to thermal fluctuation of carrier, this is the thermal noise of the MOSFET, just like we have the thermal noise of the resistor. And that happens to be white with the spectral density equal to $\frac{8}{3} kT g_m$ in the saturation region. Now, how do we use this in the circuit, in parallel with every MOSFET transistor will have a current source representing the noise of that MOS transistor, and the spectral density of this is given by that expression.

Now, if you have the circuit with the number of MOS transistor, and you have a certain output voltages, let us say V_{out} , you find the transfer function from every noise current source to the output. And you will have the output to be the superposition of all these noise current source, and because the noise from different devices, are uncorrelated from each other. The output will be simply the noise spectral density of the source times the transfer function magnitude square, plus noise spectral density of the second source times the transfer function magnitude square and so on, exactly what we did with the resistance.

So, if you have a number of component including resistors, you have to do this for every noise source from every MOSFET and every resistor, we so and seen an example of how to do this noise calculations, when you have MOSFET and resistors. Now, it turns out that in addition to the thermal noise, there is also noise flicker noise in a MOS transistor, and this noise happens to have different spectral density. Now, like thermal noise is

caused by random fluctuations in the motion of charges, due to some non zero temperature, flicker noise is caused by a completely different region.

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When you have MOS transistor and the charge in the inversion layer is what is available for conduction, and the charge in the inversion layer equals the charge on the gate. Let us assume that there are no fixed charges in the dielectric, it turns out that at the interface between the dielectric and the semiconductor, there are trap charges. And these trap charges are randomly trapped and released with different time constants, there are different theories about flicker noise, one of the theories is that the charges are trapped and de-trapped with the different time constants.

And some of all those things gives you some noise which is flicker noise, because the amount of charge in the inversion layer is balanced by the charge on gate, and also fixed charges that we may have in between two plates. We will have some randomness in the number of charges and consequently randomness in the current as well. And this phenomenon is known as flicker noise, the flicker noise as thermal noise is described by some spectral density.

We can think of the MOS transistor as having thermal noise, and also additionally a flicker noise factor is of course, and in practice of course we represent by single source which represents the some MOS these two noise currents. Now, it turns out that the flicker noise is related to some proportionality constant, and the operating point current

divided by L^2 , and also the spectral density is constant with frequency. This is the important distinguishing things between white noise and flicker noise, the white noise spectral density is constant with frequency, whereas the flicker noise is not constant with frequency; it is inversely proportional to the frequency.

So, the sum of thermal noise and flicker noise will be the total noise of the MOSFET, now as you can expect this flicker noise spectral density increases, when the frequencies are low, because of the inverse proportional due to frequency. So, it is particularly ((Refer Time: 15:20)) some in low frequency circuits, so the MOSFET if I plot the total cross spectral density, total spectral density noise current, what I will have is the thermal noise part, it depends on the operating point as $\frac{8}{3} k T g_m$.

And the flicker noise part which I will represent as straight line, it is inversely proportional to the frequency, but I will assume that the spectral density is plotted on a large scale, and the frequency is also on a large scale. So, this is the flicker noise part of it, that is like that, at low frequency the flicker noise will be more than thermal noise spectral density, at high frequency will dominate. The frequency at which the two spectral densities are equal to each other, is usually denoted by $f_{1/f}$, the flicker noise corner frequency.

So, the total noise spectral density will be some of the two, it will look something like that if you are making the low frequency circuit, you are probably going to be limited to the flicker noise and if you have the high frequency probably going to be limited by thermal noise. Now, one other difference between thermal noise and flicker noise is that, thermal noise is most fundamental phenomena, as you can just say it is related to the absolute temperature, and the fact that you have some current flowing fluid.

When you have some trans conductance you will have some thermal noise, and even in the triode region you will have some thermal noise. Now, flicker noise it is related to these inter phase tapes, and consequently also related to how clean the fabrication is and so on. And now if you take two MOSFETs, let us say from two different vendors and buyers then in same trans conductance, they will have the same thermal noise in saturation region, but they will have widely different flicker noise spectral density, because that depends very much on the fabrication forces that used and so on.

That is why we need the empirical constant in K , in the formula for the flicker noise, it is entirely possible that if you take similar processor, let us say 0.1 process from boundary one and boundary two, one of them much less flicker noise than the other one. And that may also influence your selection of process especially when you are operating at the low frequencies. So, here when I say low and high frequencies I mean, whether it is lower than the flicker noise corner frequency or higher than the flicker noise corner frequencies.

Now, flicker corner frequencies is again is not some hard and fast number, depending on process it can be high or low meaning, it could be 100 of kilo Hertz range, if you take a long channel technology like 0.5 micron also. And in the really deep some micron process, which are really common today, the flicker noise corner frequency could be 100's of Megahertz as well. But, once you have the process information, you will be given the flicker noise information and you have to design your circuit, so that the total effect of flicker and thermal noises are sufficiently small, so that your circuit operates without being affected by noise.

Now, just one more remark on flicker noise, as I said the out of a relation function as the inverse Fourier transform of the spectral density, and the mean square values are the variance is the integral of the spectra density from 0 to infinity. Now, if you integrate flicker noise 0 to infinity you will also get infinity, now this is the fundamental problem with the theory itself, it is not clear what happens to flicker noise at very very low frequencies.

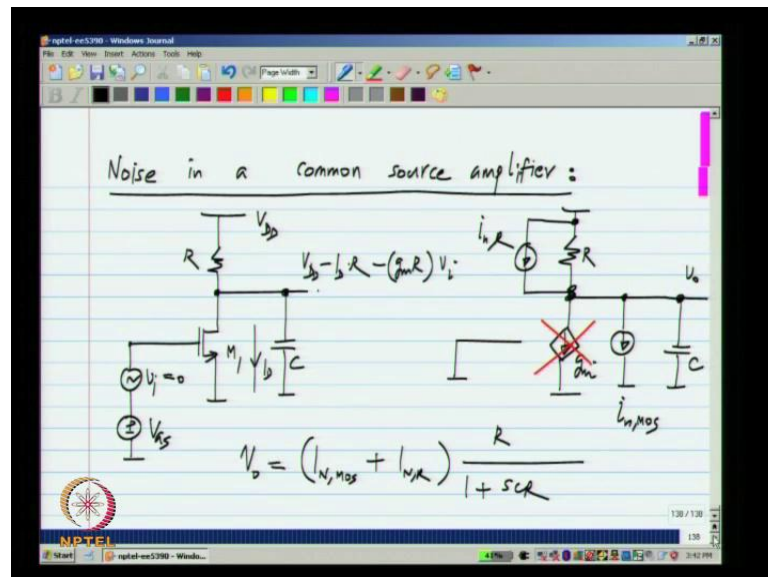
So, you have to impose some lower limit on the limit of integration, you integrate not from 0 to infinity like you do based on definition of variance, but you have to impose some lower limit based on how long a time period you will be observing the noise. Now, the noise of the P MOS transistor is exactly same as the n MOS transistor it will be given conductance in the triode region, and the trans conductance in the saturation region. And just like with the n MOS transistor is simply add a current source in parallel of with the P MOS transistor and module it, when you have many transistors you will add up the effect of them.

If you know P MOS and n MOS transistors have same single mode and all amplifier can be realize either with n MOS or P MOS transistor, there are many criteria for choosing

either P MOS or m MOS. And one primary thing is mobility, generally the mobility of the m MOS transistors with higher, so that can be used for the high speed applications and so on. But, one more parameter which may influence your choice of P MOS and m MOS transistor flicker noise.

As I said earlier thermal noise is fundamental and m MOS transistor and P MOS transistor balanced at the same transistors will have the same thermal noise, but they can have very different flicker noises, even in the same processors. So, if you are particularly worried about the flicker noise, you could choose the P MOS instead of m MOS or vice-versa depending on flicker noise in your processes.

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Now, let us look at the example of noise calculation in a common source amplifier, this is the very simple circuit which you will be familiar with from basic electronic sources. And what is it in a simplest form, it has only the transistor M 1 and the resistor R, now in reality you will have arrangements and so on, I will just assume that it is biased some V_{GS} state, so that a certain quiescent current I_D flows to the MOS transistor. And I add my single V_i to that point, now we know that the output quiescent value will be V_{DD} minus I_D times R .

And in addition to that you have the effect of the small signal input, and that will be minus $g_m R$ times V_i , we know that the gain of the common source amplifier is $g_m R$ and that times V_i will be the output. And g_m of course, is calculated at the operating

point of I_D minus V_{T1} could represent for instance of it as $\mu C_{ox} W$ by L , the quiescent value is V_{GS} minus the threshold voltage. I would like to calculate the noise in the circuit, I will assume that there is the certain capacitance C at the output of the amplifier.

Now, I introduce this, because first of all in any real amplifier you will have some capacitance, if not an intentional load at least the parasitic capacitance. And also as we saw earlier to be able to calculate the mean square the value of the variance, we should have a finite band width. If you integrate thermal noise or white noise over a infinite band width, you will get infinity which is the meaningless result, in reality there will be always some band limiting, and I will assume that it will come capacitance C at the output of the amplifier.

I am interested in calculating only the noise, we already we know what happens to the signal it gets amplified by factor minus $g_m R$ in addition to that, we will have noise and to calculate only the noise as usual, what I do is assume that the input signal is 0. I will said V_{I} to be 0, I will use the small signal model of the MOS transistor, this is a g_m times V_{GS} . But, since my V_{GS} 0 this will be just an open circuit and I will assume that there is no g_{ds} , we know that there is small signal of output conductance just a simplicity of calculation, I will assume that no g_{dc} .

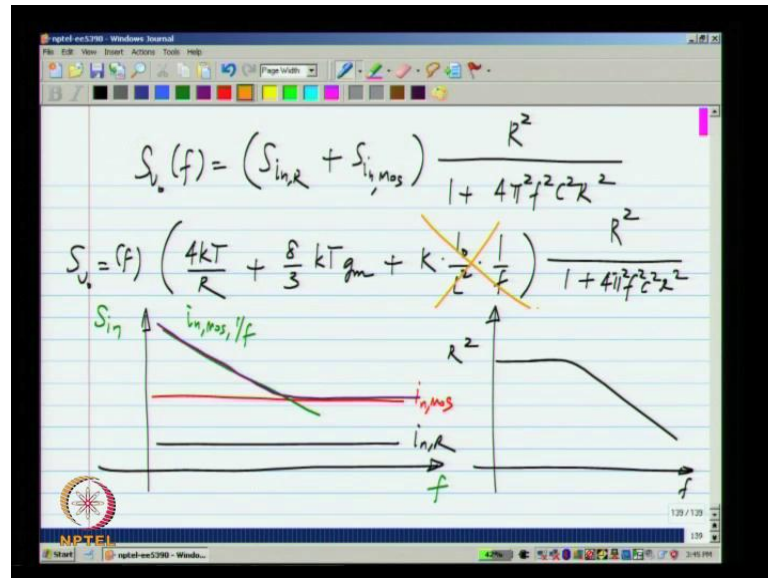
But, even if you want to included it is very easy, it can be combined parallel with the load resistance R , and I have my resistance R going to small signal around here, and I have the load capacitance C . And most importantly I have the noise Mosfet, and I have the noise of the resistor and in this case I choose represent by current sources, and what I want to find out is the output V_{out} due to noise.

How do I do this, all I have to find out the do is transfer function from $i_n R$ to the noise and i_n MOS noise, now the polarity of noise also do not matter, they have random quantities, so I just show it as having some polarity. Now, you can see that this noise of the resistor and the noise of the Mosfet are in parallel, so the simply add off with the right polarity, so the output V_o is nothing but, in the $1/N$ plus domain, the MOS transistor noise plus the resistors noise times the impedance.

Remember, this control current sources in active because V_{GS} 0, so all we have is pare of current source going into parallel combination of r_c , and we also know that the

impedance of the parallel is r_c is R by $1 + S_c r$. Now, we just have to express this in terms of f that is will as the spectral density, and we can integrate that to get the mean square value of the variance.

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So, this spectral density is nothing but, because it is spectral density is add up, because these two are un co-related to quantity and when we add them up the result is simply the sum of the spectral densities, by the way with these polarity this should be a minus, it is not plus. But, that does not make any different, because once there are uncorrelated spectral densities times 1 by $1 + 4\pi^2 f^2 C^2 R^2$, so this is what will have.

Now, what is the noise spectral density of the resistor $4kT$ by R plus the spectral density resistor noise current plus the noise spectra L density of the most transistor $8/3rd$ g_m plus I could also include flicker noise, the whole thing multiplied by 1 by $1 + 4\pi^2 f^2 C^2 R^2$, this is the spectral density of the output noise. What will it look like resistors noise will have some value, the MOS transistor thermal noise will have some value, and the MOS transistor flicker noise will have some value like that.

Let us assume that this is the noise spectral density the current versus frequency both on a large scale, and the total would be something like that, and that has to be multiplied by this particular function. And this function will have a shape which start from R^2 at

low frequencies and then, it goes off like that, so we have to sum this spectral density and multiplied by that to get the output spectral density. And it will have some shape of this order ((Refer Time: 28:53)), it will have some shape like that depending on where this band width of this particular function is compare to the flicker noise frequency and so on. So, we can calculate the spectral density, now I will calculate the integrated noise or the variance, the variance of mean square value is also called the integrated noise, and just for simplicity I will ignore the flicker noise.

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The image shows a digital whiteboard with handwritten mathematical equations. The equations are as follows:

$$S_{v_o}(f) = \left(\frac{4kT}{R} + \frac{8}{3} kT \cdot g_m \right) \cdot \frac{R^2}{1 + 4\pi^2 f^2 C^2 R^2}$$

$$= \frac{8}{3} kT g_m \left(1 + \frac{3}{2} \cdot \frac{1}{g_m R} \right) \cdot \frac{R^2}{1 + 4\pi^2 f^2 C^2 R^2}$$

Arrows point from the terms in the second equation to labels: "MOS" points to $\frac{8}{3} kT g_m$ and "RESISTOR" points to $\frac{3}{2} \cdot \frac{1}{g_m R}$.

$$\overline{V_o^2} = \int_0^{\infty} S_{v_o}(f) df = \frac{8}{3} kT g_m \left(1 + \frac{3}{2} \cdot \frac{1}{g_m R} \right) \cdot \frac{R^2}{4RC}$$

$$= \frac{2}{3} \cdot \frac{kT}{C} \cdot g_m R \left(1 + \frac{3}{2} \cdot \frac{1}{g_m R} \right)$$

I will approximate the noise spectral density by considering only the thermal noise out of it which in fact, I can take this number out, and this is the expression I will be left with, now we know that this numbers comes from to more transistor numbers comes from the resistor. So, what do you think of the relative contribution to the MOS transistor of the resistor to the noise, will see this also with the integrated noise. But, what do you think with the related contribution, we see that the contribution of MOS as number 1 multiplied by some 8 3rd K T g m.

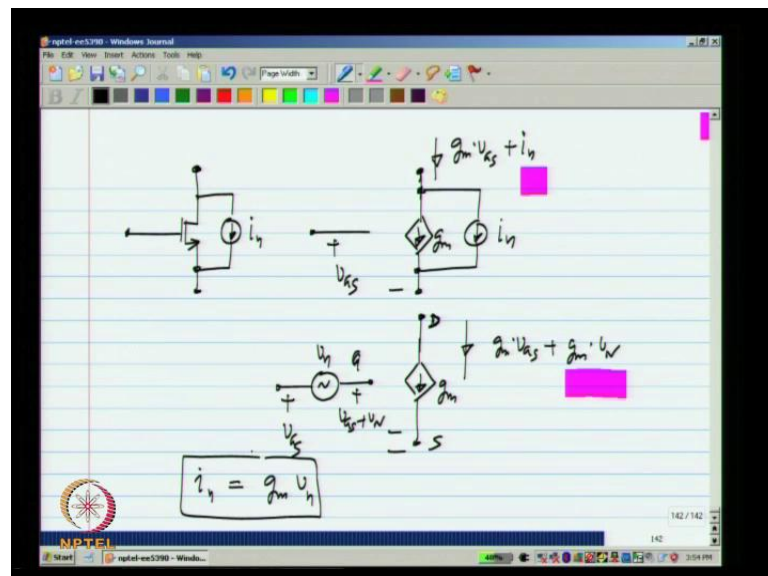
And the contribution resistor as 3 by 2 divided by g m R, now given that amplifier we expect that g m R is the number that is significant may be higher than 1, they could also be some modest number like 2, but if you do have the amplifier where g m R is much higher than 1. It is very easy to see that most of the noise contributed by the MOS

transistor, and not by the resistor, we will interpret even more later, but it is very obvious from the formula.

Now, to get the mean square noise we have to integrate S_v naught from 0 to infinity frequency and that will give me, that is what the result is going to be. And we will have some expression of this it sort, it as $K T$ by C multiplied by $g_m R$, which happens to be gain and then, inside this we have the contribution due to the MOS transistor, and the contribution due to the resistor. Now, one quick ((Refer Time: 32:26)) check result, we assume that the MOS transfer is not there and then, will reduce to the $r c$ circuit that we analyses yesterday.

And so we will have be left with $K T$ by C , there are also we can see that if we take only second term parenthesis, we will be left with $K T$ by C , so at least it passes the synaptic check. Now, the actual output noise turns out to be $K T$ by C times sum number that is related to the gain of the amplifier, this is again the general property, if you try to make large gain and the output you will also see large amounts of noise. And also as I mentioned earlier, they contribution due to the Mosfet is much more than the contribution that due to the resister, now this again turns out be generally true.

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In any circuit that is why chain of the amplifier with the gain more than 1, now let us say each one contributes some noise. Now, you expect that the noise from the first stage will matters most, and noise from the next stage matter less, and the noise from the next stage

matter even less. Earlier I said that we can model the noise of the MOS transistor, using a current source in parallel, and the spectral density of the current source is given by $\frac{K T}{g_m}$.

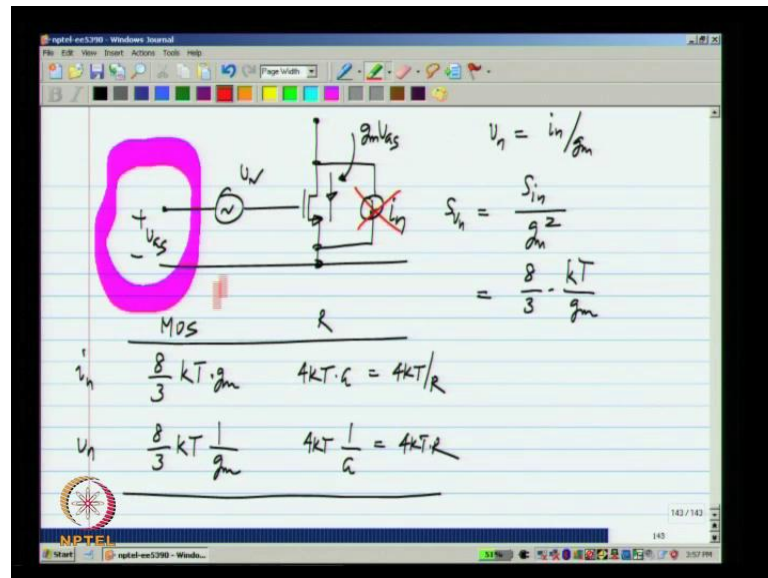
And the same thing can be equivalently modeled by the voltage source, and the spectral density of the voltage source is given by $\frac{K T}{g_m}$ divided by g_m , these are just two equivalent ways of specifying the same noise. The noise in a MOS transistor we have a $\frac{K T}{g_m}$ as the noise current or $\frac{K T}{g_m}$ divided by g_m as the noise voltage. Now, the model that we use for the transistor noise is a MOS transistor with the current source in parallel, now this adds to the drain current.

Now, in the small signal case, we have the current source $g_m v_{gs}$, there are also the conductance times and the current source $g_m v_{gs}$ times the by $d s$, but I ignored those things for now. In addition to the current contributed by the control current source, I have the noise current i_n , so the total current here is $g_m v_{gs}$ plus i_n . Now, I can think of this as not a change in the current $g_m v_{gs}$, that is why we have the noise in series with the gate source.

Let us just assume that this is the case, and there is no noise in parallel with the drain source in that case, let say up wise some v_{gs} here, and what appears between these two the gate and sources of the actual transistor is v_{gs} , I am assuming that this is the drain, this is the gate and that is the source. So, what will be the current here it will be $g_m v_{gs}$ plus $g_m v_n$, now this noise current here can be equivalently represents here by $g_m v_n$, if the two are the same.

So, the same noise the noise of MOS transistor can be represented by a noise parallel with the Mosfet or the noise v_n in series with the gate of the Mosfet as long as the two are related by this expression. Now, recall that for the resistor we represented it either by the current source in parallel with the resistor or a voltage source series with the resistor, and the two are related by the resistance value R . In this case two are related by the trans conductance g_m .

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So, the noise of the MOS resistance very well can be represented by input noise source something like that, and the spectral density of this given that V_n is related to i_n by i_n divided by g_m . The spectral density of V_n will be the spectral density i_n divided by g_m , again the spectral density are related to squared voltages and currents, so this will turn out to be 8 by 3 $K T$ divided by g_m . So, just to make the association clear, the MOS transistor current noise as spectral density $8/3$ rd $K T g_m$.

And the voltage noise and in the series with the gate as the spectral density $8/3$ rd $K T$ divided by g_m , have to compare to the resistor, the resistor current noise as the spectral density $4 K T$ times the conductance, which is $4 K T$ divided by R . And the voltage noise will be $4 K T$ times 1 over g which of course, $4 K T$ times R , now you can see that the expression are analyses, the only difference is that the MOS transistor as three terminals. So, the current source that appears is between the drain and source, voltage noise is something that the series with the gate, so that it add to the V_{GS} of the MOS transistor.

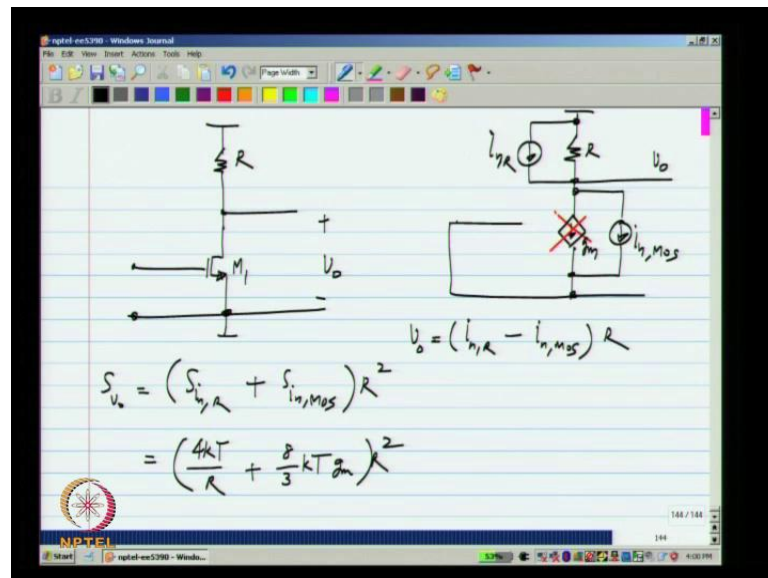
Now, this description is many times very convenient will say exactly why, the reason is that, we apply an input to the gate source. Now, most of the times what we are interested is doing is comparing the amount of noise to the amount of signal that we have, in this case V_{GS} is signal. Let us say the module the that is why I ordered the noise of the MOS transistor, using a the current source, now to compare to the signal noise, what I

have to do is to calculate the signal that appears in the MOS transistor due to V_{GS} , that happens to be g_m times V_{GS} and compare that to i_n .

When I say compare it to i_n , compare it with some matrix of i_n , such as its mean square value, now in this case it is quite simple, but it is even simpler, if let us say the noise is represented by i_n , but the series voltage source V_n . So, since V_n is simply added to V_{GS} by comparing to the magnitude some matrix of v_n the size of V_{GS} , which could be amplitude value I can say something about the signal to the noise ratio.

So, referring all the noise to the gate side makes it possible to have the easy comparison with the input which is V_{GS} for a MOS transistor. In a single MOS transistor it is very easy and whether the noise is represented by current source or voltage source, we can do the calculation easily, but this concept can be extended to more complicated circuits, that is why we take the same circuit again.

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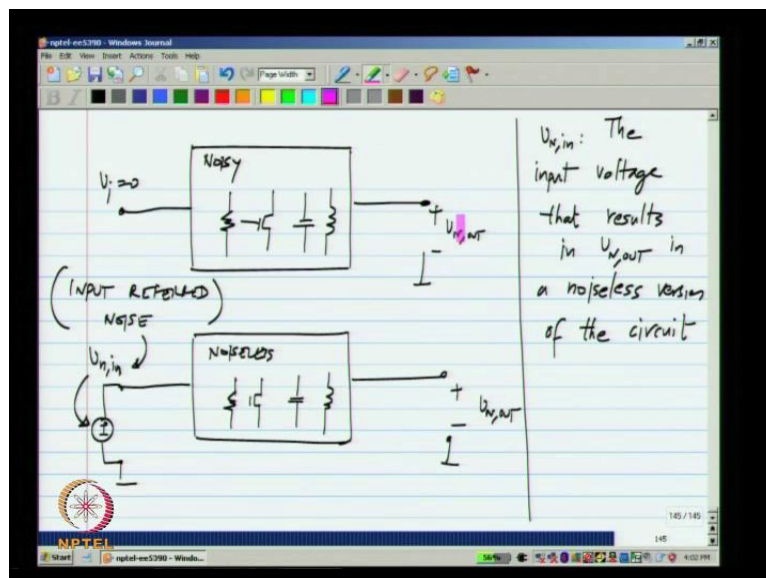


And this time we do not use the capacitor, so because of that the noise will not be band limited, so I cannot calculate the variance, but I will be satisfied with calculating only the spectral density, so let me just do that. This can be thought of another practice calculation in and noise, I want to find the spectral density of V_{naught} , so what do I do, I will again consider the case where V_{GS} is 0, because I calculate the noise without the input signal I have $g_m v_s$, but this is inactive.

Now, in addition to that, we have the noise connect i_n due to the MOS transistor and noise connect i_n due to the resistor, $i_n R$ and this is i_n MOS, now previously the sum of these two $i_n R$ minus i_n flowing through the parallel combination r_c , now the flow only through the resistor. So, the output voltage here is nothing but, $i_n R$ minus i_n MOS times R , so the spectral density is nothing but, $S_{i_n R}$ plus $S_{i_n MOS}$.

As I said these are un correlated quantities and the spectral density simply add up times R square; which gives me $4 K T$ by R plus $8/3rd K T g_m$ times R square, so that is the spectral density of g_m output noise. Now, that is why I have the amplifier with a number of devices, and I can calculate each device from output and finally, calculate the output noise, now there is the alternative representation of noise that is many times useful.

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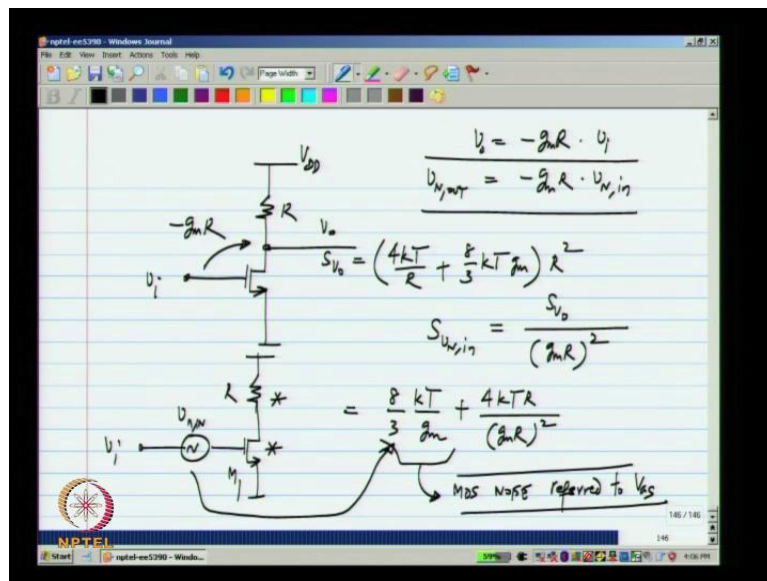
So, let us say have the noise is circuit with the number of components, there could be number of resistor Mosfet, and also the capacitor in inductors, which do not create noise, but influence the noise spectral density. Now, let us say with the zero input I calculate the output noise, I can take the noiseless version of the same circuit, apply some input $V_{i, in}$ and $V_{i, in}$ such that, the output of the noiseless circuit is $V_{o, out}$.

This is the input voltage that will give you the same noise as the actual circuit when applied by input, now this is not the voltage that will exist in circuit, this is something that is use for calculation convenience. And this is known as the input refer noise, this $V_{i, in}$ is known as the input refer noise and the way to calculate it is, first you calculate the

output that is you calculate the output function from the noise of each of this components to the output, that will give you the output noise voltage. And you divide that by the transfer function between the input and output, whatever you identify the input of the circuit.

Now, that will give you the input referred noise spectral density, and you can calculate the input noise spectral density by spectral density just like we do for any other calculation. The reason that is convince is let us we apply the input voltage source as the signal, we can immediately compare that with the input referred noise, and see whether the signal to noise ratio sufficient or not. As I mentioned earlier the input refer noise is not the noise voltage that exist in the circuit, it is an equivalent source that will give you the same output in a noise less circuit as the actual noise in a noise circuit the real circuit.

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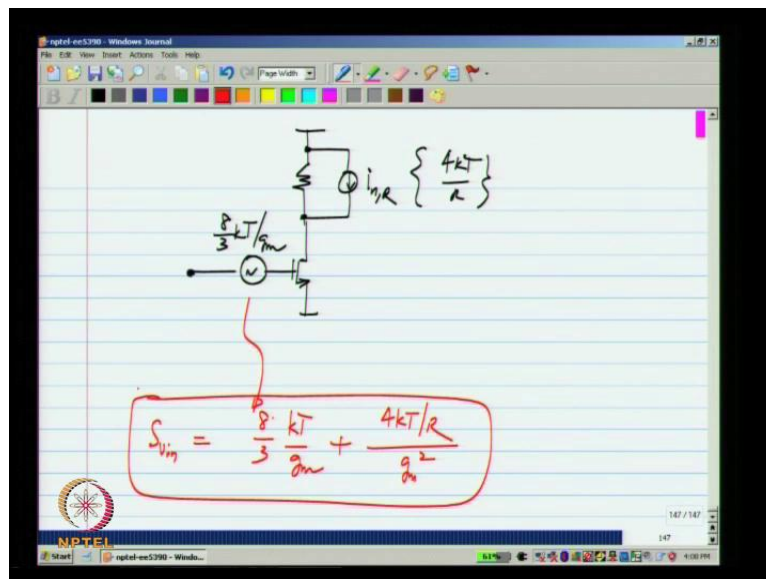


So, we will do this calculation for our example, we know that the output noise spectral density $4 K T$ by R plus $8/3 K T g m$ times R square this is of course, calculated from the small signal model, but show it here I will just show it here. Now, we also know that the transfer function this point to that point is the gain of common source amplifier, from V_i to V_o , the gain is minus $g m R$, V_o is minus $g m R$ times V_i . So, that means the actual output noise is related to the input to the noise minus $g m R$ times $V_{N,in}$, where $V_{N,N}$ is the input referred noise.

Now, of course we cannot calculate the noise voltage itself, but we calculate the spectral density, so S_{VN} in the spectral density of the input referred noise is S_V or the spectral density of the output noise is divided by $g_m R$ square. So, what we get if we do that that, will be equal to $\frac{8}{3} kT$ divided by g_m plus $\frac{4kT}{R}$ divided by $g_m R$ square, now what this means is a noiseless version of the circuit, I will just put the star next to it, to denote it is noiseless.

This resistance and this Mosfet do not have any noise, I can represent the noise by the input referred noise, like this one and this will have a spectral density equal to that. And if you observe here the first term is nothing but, the input referred noise of the Mosfet, the noise of the Mosfet is referred to a gate source voltage, so if we had use the alternative model of the Mosfet.

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If we use the model where I have $\frac{8}{3} kT$ divided by g_m , and let us say the resistor noise is modeled by this $i_n R$ spectral density of $\frac{4kT}{R}$, so that is why I use this. Then it would be immediately clear, but the input referred noise will be $\frac{8}{3} kT$ divided by g_m plus something, that is because this appears at the input of the Mosfet. So, that is why it is many times better the model noise of the Mosfet, as input referred noise, referred to V_{GS} instead of and output.

And if you look at the other term, what we could have I can rewrite it as, divide that as $\frac{4kT}{R}$ divided by R divided by g_m square, now what is $\frac{4kT}{R}$ that is spectral

density of the noise current that is added here. And the transfer function from the input to the drain current is g_m , so the current that is added here divided by g_m will get added to the input. So, in general it is true that, if you have the number of amplifiers, the noise added at the output of something will get divided by the square of the gain, when refer to the input.

In this case the noise current added by the resistor is divided by the gain, where gain refer to the gain to the input voltage to the drain current, as the square of that and get added to the input. So, by modeling the noise in different ways, there are many cases where we can simply write down the noise spectral density by inspection, first we can write this down. So, the input refer noise spectral density is $8kT$ divide by g_m plus the noise added by the resistor, which is having the spectral density $4kT$ divided by R divided by the gain of the Mosfet from V_{GS} to the output current, which is g_m square, so this is the alternative way of calculation.

Now, we can calculate the noise in any which way you want, as long as you get you the right results in fact, initially I suggests that you do in both ways, you calculate the output noise by proper calculation. And you also try to do by inspection just for practice, and make sure that the two match. And after enough practice there will be many circuit for which you will able to simply look at the circuit and write down the expression for input noise.

Thank you I will see you in the next lecture.