

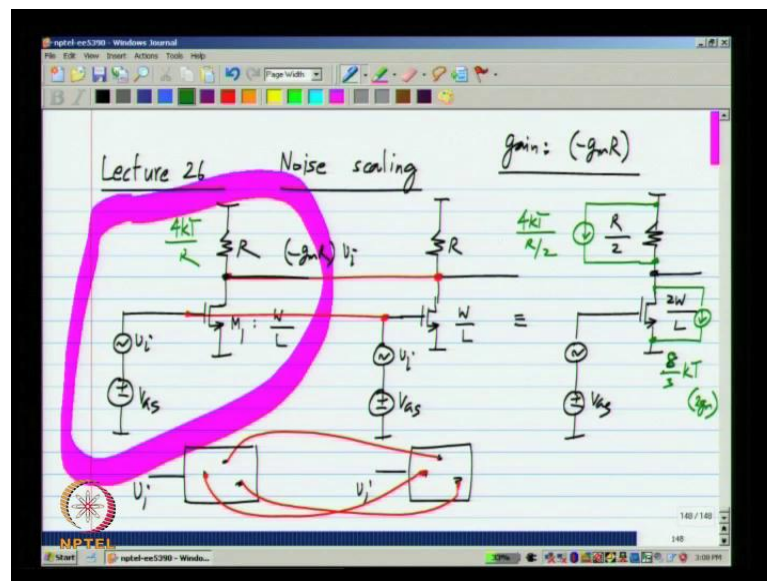
**Analog Integrated Circuit Design**  
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**Lecture No - 26**

**Noise Scaling; Basic Amplifier Stages- Common Source, Common Gate**

Hello and welcome to lecture number 26 Analog Integrated Circuit Design. In the previous 2 lectures, we looked at models for thermal noise and flicker noise and resistor and MOSfet. In this lecture what we will do is to see, how noise can scale in the circuit that is if you have a circuit with certain functionality. We will like to have a same functionality, but with a smaller noise and there are systematic ways of doing that that is what look at here.

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Let me take the example of the amplifier I had earlier, and I will use this simple bias scheme although it is not required, just for simplicity I assume that the transistor is biased with a fixed  $V_{GS}$  and an incremental  $v_i$  is applied over it. And we have a resistor in the drain, and this is the common source amplifier with a gain of  $-g_m R$ , let us say this MOS transistor has width  $W$  and length  $L$ .

Now, let me take an identical circuit with width  $W$  by length  $L$  and resistor  $R$ , and what I do is, I will connect the corresponding nodes of the two in parallel, now it is very easy to see that the functionality has not changed. The gain from the input to the output will still be exactly

minus  $g_m r$ , there are many ways of visualizing this, one way to think about it is that you imagine two identical circuits, with identical inputs everywhere,  $v_{gs}$  and  $v_i$ . If that is the case, let us say you have two identical circuits, here I will show you more abstractly with a number of nodes.

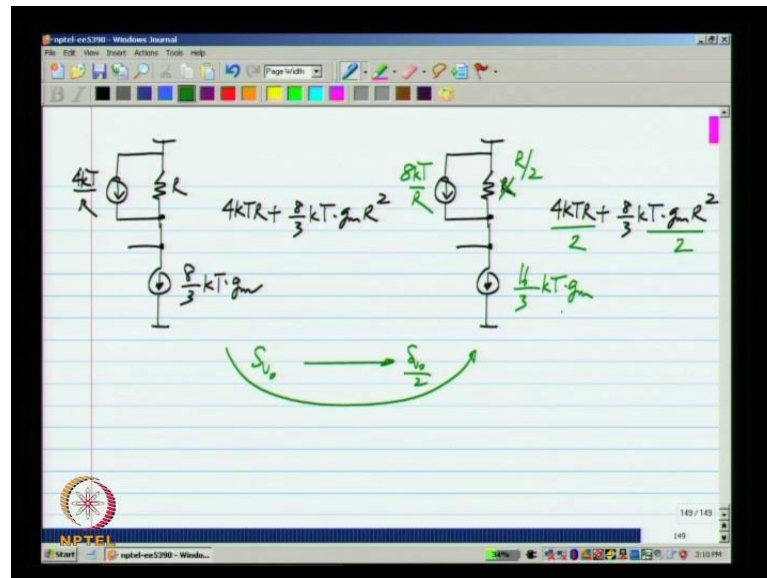
That two circuits are the same input in that case, the internal nodes will be exactly at the same voltages in the two circuits, this will be at the same voltages, that will also be that and that and so on. Similarly, here the gate voltage is the same the circuit has only two nodes, the input and output node, the output node voltage would be exactly same in the two circuits. So, we can connect the two circuits without affecting the operational circuit, because there are identical voltages, I can just connect them together I can do this and nothing will change in the circuit.

So, from this it is very obvious, how complicated the circuit is if I take two identical circuits and connect the corresponding nodes together, I will have the same functionality as before. So, this new amplifier also has a gain of minus  $g_m r$ , what does change though we can say that, this is equivalent to having a transistor of width  $5W$  and the length  $L$  and a resistance  $R$  by 2.

This circuit is very easy to see that, the noise current from the resistance has doubled, it is  $4kT$  divided by  $R$  by 2, instead of  $4kT$  divided by  $R$  in the original circuit, which is just that much. Similarly, the noise from the MOS transistor, what does happen to it, we see that the current in the MOS transistor has doubled, for which we have the same  $v_{gs}$  and twice the width. So, the current will double, and that will double the  $g_m$ , that will double the transconductance of the MOS transistor, because we have the same  $v_{gs}$  and we have twice the width, the transconductance is double.

So, this noise current which is  $\frac{8}{3}kTg_m$ , in the original case will become  $\frac{8}{3}kT \times 2g_m$ , when I say noise is double, I mean that the noise spectral density is double.

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So, originally we had a resistance  $R$ , and in the noise picture  $4kT$  by  $R$  and the noise of the MOS transistor,  $\frac{8}{3}kTg_m$  flowing into  $R$ , so output voltage noise spectral density was  $4kTR + \frac{8}{3}kTg_mR^2$ . What we have in the new circuit, we have  $4kT$  divided by  $R$  by  $2$ , so which is  $\frac{8kT}{R}$ , and similarly instead of  $\frac{8}{3}kTg_m$  we have  $\frac{8}{3}kTg_m$  and this resistance is not  $R$ , it is  $R$  by  $2$ .

So, if you look at the voltage noise, it will be  $\frac{4kTR}{2} + \frac{8}{3}kTg_mR^2$  also divided by  $2$ , so if the first circuit has a certain voltage noise  $S_{v_o}$ , that becomes  $\frac{S_{v_o}}{2}$ , that is the noise spectral density divides by a factor of  $2$ . When we double the circuit, when I say double the circuit, I take two identical copies of the circuits and joint the corresponding nodes together.

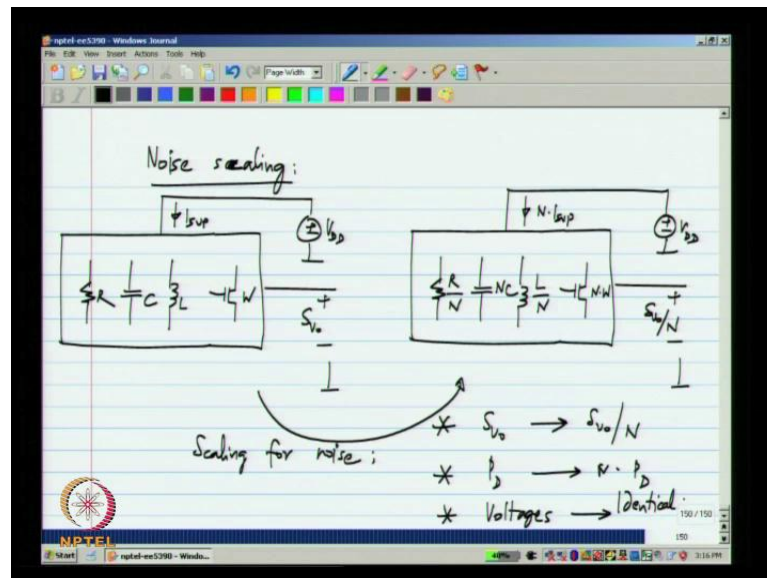
Now, I have taken a very simple example, but this turns out to be universally true again, it is very easy to imagine two identical circuits operating with two identical inputs and, in fact for noise calculation, we do not have to consider the inputs. Now, each of those components in each circuit can have noise, when you connect the corresponding nodes together, what happens is that let us say that we represent all the noises by equivalent current source in parallel.

The current source will appear in parallel, the noise current sources from the first circuit and the noise current source, in the second circuit will appear in parallel with the

corresponding mach's elements. So, the spectral density of the current noise source will double, because the noise in the first circuit and the noise, in the second circuit are uncorrelated from each other the spectral density will simply double. But, the impedance has become half, because with every element in the first circuit, there is another element from the second circuit in parallel, so whatever impedance each branch has it has become half of the original value.

Now, what is output voltage noise, spectral density it is equal to current noise spectral density times sum impedance square, so current noise has gone up by factor of 2, impedance has gone down by factor of 2, impedance square has gone down factor of 4 consequently. Therefore, the product goes down by factor of 2, if take two copies of an identical circuit and connect the corresponding nodes together, what will happen is that the voltage noise spectral density, at every node will reduce by a factor of 2. Now, there does not have to stop at factor of 2, or the factor does not even have to be an integer, so if you take 2 circuits, one of which is the scale version of the other, let us say scale by some factor n. The noise voltage spectral density, will also be scale down by the factor of n, and this is known as noise scaling.

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Let us say, we have one circuit with R C L and MOS transistor, let us say MOS transistor with I denoted by W, this is just an abstract representation of some circuit, and I consider the noise at some node and let us say that is  $S_{v_o}$ . Let us say instead of R what I do is I

connect  $n$  of them in parallel, this is way of thinking about it, but this works even if  $n$  is a non integer. So, this becomes  $R$  by  $n$  capacitances  $n$  of them, in parallel will be  $n$   $C$ , inductances  $n$  of them in parallel will be  $L$  by  $n$ , and MOS transistor  $n$  of them will parallel will have a width of  $n$  times,  $W$  it labels in  $m$  length that is a assume.

So, the noise of this will be  $S_v$  divided by  $n$ , so this is a systematic way of scaling the noise of a circuit, and as I have explained earlier, if I take these two circuits the noiseless version of them and operate them with exactly the same inputs. The node voltages will be exactly the same, in the two circuits this is MOST easily imagine by starting with the first circuit and then you take identical copy, and operate them with the same inputs.

Now, in the two copies the corresponding node voltages will be exactly identical, you connect the corresponding nodes in parallel, what you will get will be a scale circuit by a factor of 2. Now, because you are simply connecting nodes, which are of the same potential nothing will change in the circuit, and the functionality of the circuit will remain exactly the same.

When I illustrate that with an amplifier, I had a MOSfet of a width  $W$ , and the resistance  $R$ , I double the width, I double the resistance, and my  $f_m$  remains exactly the same, I was getting again the  $g_m r$  initially and I get the gain of minus  $g_m r$  from the now circuit as well. Now, what else does change in the circuit, from the power supply or in general the currents anywhere, I will consider the power supply, because that is what determines the power dissipation.

So, let us say this is  $V_{DD}$  and that draws a current  $I_{sup}$ , let us say what is the current drawn from this, it is easy to see that every branch connect, in the new circuit will  $n$  times, the branch connects in the original circuit, so the current here will be  $n$  times  $I_{sup}$ . So, scaling for noise, the noise voltage spectral density is  $1/n$  times the original, a power dissipation is  $n$  times the original, and the functionality that is the voltage in the circuit.

Let us assume that we are interested only in the voltage transfer function, when I say voltages, I do not mean the noise voltages, I mean the signal voltages will be identical in the two cases. So, this is a way of systematical taking some circuit and then reducing its noise without affecting the functionality, and you will encounter the scenario very frequently, in fact let us say you have some circuit you have designed, its functionality

is fine. It is gain let us say it is fine, but it has too much noise you have to scale up, the circuit in order to reduce the noise.

And this is the sort of dump scaling in that we are not doing anything intelligent with the circuit we are scaling the circuit, that we have to have less noise, and it also consumes correspondingly larger power. So, this is a given and sometimes, you may have to go other way around that is increase the noise, usually the objective will not be to increase the noise, what happens is you have some circuit.

And that is consuming too much power, and you have to reduce the power dissipation of that circuit, the way to do that is to scale up the impedance of every branch by a factor  $n$ , that is we go from right side to the left side in this picture. Then what will happen is the power dissipation will reduce by factor of  $n$ , and the voltage noise spectral density will also increase by a factor of  $n$ .

So, while saving power the noise will increase and let us say the noise of this particular circuit is not very significant, in that case you can leave with the increase noise, but you will be happy to take the reduce power dissipation. So, you will go both ways depending on whether rather power dissipation is important or the noise is important, so it is very important to know the systematic way of reducing noise.

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Thermal noise :  $\frac{4kT}{R}$  ;  $\frac{8}{3}kTg_m$

$\propto T$

$S_v \rightarrow \frac{S_{v_0}}{N}$

Carnot engine refrigerator

$T_2 > T_1$

\* input  $P_{mech}$

transfers  $P_{heat}$   $T_1 \rightarrow T_2$

$\frac{P_{mech}}{P_{mech} + P_D} = \frac{T - T/N}{T}$

$P_{mech} = (N-1) P_D$

$\frac{P_D}{N} T$   $T_2 = T$

$P_{heat} = P_D$

Now, we have looked at thermal noise in resistors and MOSfets, it is the thermal noise current spectral density  $4 k t$  by  $R$  in a resistor, and it thoughts  $k t g m$  in a MOS transistor, now the point is both of them are proportional to the absolute temperature  $t$ . So, another way of reducing noise could be to reduce temperature, so let us say I want I have some voltage noise spectral density  $S v o$ , and I want that to be  $S v o$  divided by  $m$ , I want to reduce it by a factor  $n$ .

So, what can I do I can reduce the absolute temperature by a factor  $n$  now this not any easy thing to do, you will have cool down the circuit, but the theoretically it is possible. Let us say you want to reduce the noise by a factor 5, now we are in 300 Kelvin at room temperature, if you want to reduce the noise spectral density by a factor 5, you have to cool it down to 60 Kelvin. If you do that then the noise spectral density will become, one fifth of before, now let us not worry about whether the circuit will really work at such a low temperature, and so on let us just assume that it does.

Now, there is an interesting calculation related to how much power this will consume, now how much power does it take to cool down a circuit of course, in practice it takes a lot of power it appears that, it will take a lot of power. What you will need is a refrigerator, but we will look at some fundamental refrigerator, I think all of your familiar with the Carnot engine from thermodynamics. And usually you think of it does something that takes thermal energy, and convert it to mechanical energy, and there is the very well known formula, for the efficiency of a Carnot engine.

It operates between 2 temperatures,  $t_2$  and  $t_1$ , and let us assume  $t_2$  is more than  $t_1$ , now what does it do, it produces some mechanical output power  $p_{mech}$ . And it wastes some power in the form of heat, let me call it  $p_{heat}$  and that goes from the higher to the lower temperature, and the classical formula tells you that the efficiency. The mechanical power to mechanical plus the wasted heat is  $t_2$  minus  $t_1$  by  $t_2$ , you know that only by cooling  $t_1$  down to absolute 0 will you get hundred percent efficiency.

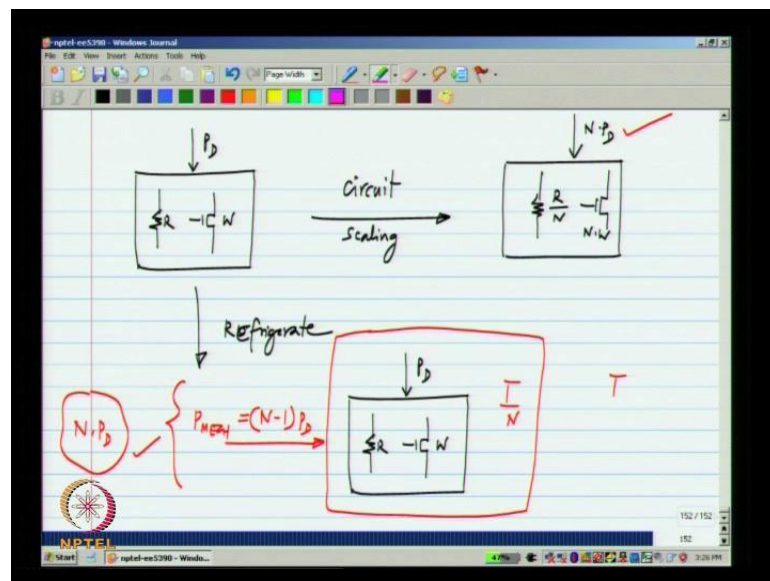
Now, what we want of course, is not a Carnot engine that produces mechanical output, but something that take mechanical power as input, and pushes the heat in the other direction. That is what the refrigerator does it pushes the heat from a lower temperature to a higher temperature, and that is why you have to put some work into it to be able to do that.

So, in this case the input  $p$  mechanical, and I will simply say that it transfer some thermal power  $p$  heat from  $t_1$  to  $t_2$ , so this is exactly the reverse of what a Carnot engine is right it turns out that the same formula applies even in reverse. Now, what are these quantities in our case, let us say I put some circuit which dissipation a power  $p_d$  inside the refrigerator, and inside this low temperature of  $t_1$ , and the outside temperature is  $t_2$ , I do this instead of putting this at outside the temperature of the  $t_2$ .

So,  $t_2$  is my  $t$  in my previous expressions, and if I want to reduce the thermal noise by a factor  $n$   $t_1$  should be  $t$  divided by  $n$ , and what is  $p$  heat, it is equal to  $p_d$  whatever power is being dissipated by the circuit has to be taken out. Exactly, that much has to taken out, that is why you can maintain at a constant temperature, if you take out more heat than the circuit is dissipating, then it will eventually cool down, and if you take less heat than the what does the circuit is dissipating it will keep heating up.

So, what need to be at a steady temperature, the amount of heat taken out of this system from  $t_1$  by  $n$  to  $t_2$  must be exactly equal to  $p_d$ , so this  $p$  heat is  $p_d$ , and  $p$  mechanical is the external power that I have to drive and that is why I have to find out. So, let me substitute in this formula,  $p_d$  equals to  $t_2$  minus  $t_1$ , that is  $t$  minus  $t$  by  $n$  divided by  $t$ . And I have to calculate  $p$  mech from this, if I do the calculations I will see that  $p$  mach is  $n$  minus 1 times  $p_d$  that just comes from simple algebra on this relationship.

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Now, this is very interesting, so what are the ways, so I have producing thermal noise, let us say that the circuit is consuming from  $p_d$ , and I have R and MOSfet, and also other components, I will show only two of these for simplicity. So, first thing I can do is circuit scaling, which says that I have to multiply every branch by a factor  $n$ , so R becomes  $R$  by  $n$  and  $W$  becomes  $N$  times  $W$ , and this will consume a power of  $N$  times  $p_d$ .

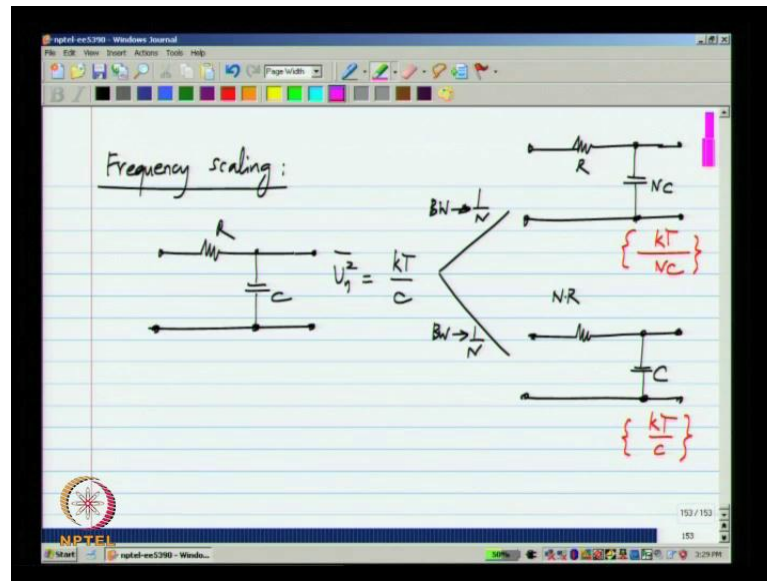
And what is the alternative I can choose two refrigerators, let me show the refrigerator outside this one, and inside this the temperature has to be  $t$  by  $n$ , and outside is  $t$  that is the assumption it is a same ambient temperature at which the original temperature circuits are operating. So, for me to sustain this condition I will have to have a mechanical power input of  $n$  minus 1 times  $p_d$ , this is an ideal refrigerator working according to the Carnot cycle, and what is the total power dissipated here.

The circuit consumes  $p_d$  and the mechanical power is  $n$  minus  $p_d$ , so the total power is  $n$  times  $p_d$ , we get exactly the same power dissipation, as when you do circuit scaling. So, this is just an interesting thing, normally you would not try to refrigerate circuits and in any case you cannot get this Carnot refrigerator with ideal efficiency, the efficiency have natural refrigerate will be much lower than this one.

But, it just illustrates the fundamental calculations, and shows you that the circuit scaling is also a fundamental thing. So, whether you try to reduce the noise by refrigerating it or you try to reduce the noise by scaling, it you will end up burning  $n$  times the power, when you want to scale the voltage power spectral density by a factor  $n$ . Now, I said that normally you would not refrigerate circuits, but in special cases that is the only way that you have to reduce the noises, let us say you done noise scaling and everything that could be possibly done to minimize the noise in your circuit.

Then you still want to reduce the noise further you have to cool it down, in some very special circumstances usually let us say some amplifiers were radio asponion and so on. You do have amplifiers, which are cool down you do have circuits, which are cool down, so that their thermal noise is reduced, but normally you would not think of doing something like that you would just do the noise scaling.

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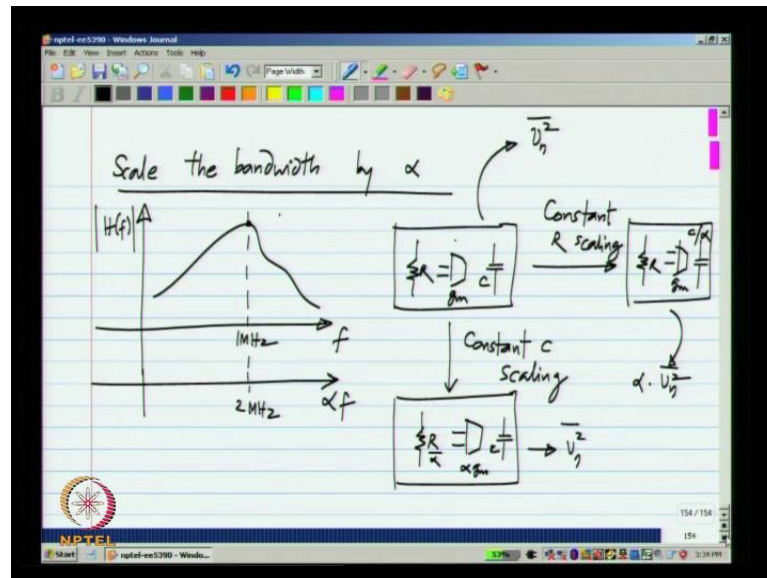
Now, around the topic of noise scaling we can also look at other scaling rules, let us say we look at frequency scaling, now again I will illustrate with we already analyzed the noise of this. We know that the output noise variance is  $k t$  divided by  $c$ , please keep in mind previously in the noise scaling part, we were talking about the noise spectral density and here we are talking about the variance of the noise or the means two noise.

Now, they two are related, but not exactly the same thing, so please be aware of what is being reduced, when we say that we reduce noise etcetera, etcetera, so the means square noise that we get is  $k t$  by  $c$ , and we can change the bandwidth of this circuit. So, let us say bandwidth goes to  $1$  over  $n$  times the original bandwidth, and similarly here I can reduce the bandwidth instead of modifying the capacitor, what I will do is I will modify the resistor  $N R$  and  $C$ .

Now, the voltage transfer function of these two circuits are exactly the same, they have a bandwidth of  $1$  over  $n$  times,  $1$  over  $R c$  radium's per second, but what is different between the two is noise. This circuit has a noise of  $k t$  divided by  $N C$ , and the lower circuit has a noise of  $k T$  divided by  $C$ , just a distinguish it from the previous scaling. Previously, we were trying to keep the functionality to same, but reduce the noise here we are trying to change the functionality, in that we are trying to reduce the bandwidth by a factor  $n$ .

And we want see what happens to noise or a different ways of scaling, the bandwidth, one is to play around the capacitances, one is to play around with the resistances. I would take in the example of a first order R C circuit, which has only one resistance and one capacitance, but this is the universally true rule, as you can see from this we can generalize.

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And say that, we can scale the bandwidth by a factor alpha, so what it means that if one of the circuit has some frequency response like this, and the remaining the other circuit will have exactly the same frequency response, where this is alpha f. So, let us have we take alpha equal to 2, what I mean is let us say this peak occurs at the peak of sound transfer function, it occurs at some frequency  $f$  naught, let us say 1 megahertz,  $f$  alpha equals to 2.

In the scale circuit, exactly the same shape will be preserve for the frequency response, and the peak will occur at 2 megahertz. So, as I said that there are two ways of achieving this, again I will take my original circuit, let us say it has some resistors, transconductors, and capacitors and I can make the frequency response scale by alpha, that is  $f$  goes to alpha  $f$ .

To do this I can have the resistance to be  $R$  the transconductance to be  $g_m$ , and the capacitor to be  $C$  by alpha, alternatively reduce the resistance by factor alpha, increase the transconductance by factor alpha, and keep the capacitance to be exactly the same.

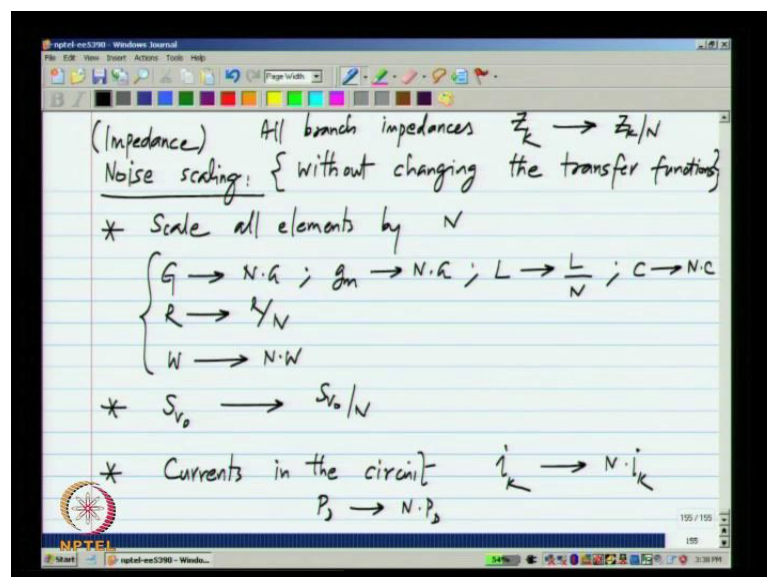
Now, this is known as a constant C scaling for obvious reasons like capacitances kept constant, and this is known as constant R or constant conductance scaling, now both of these will give you the exactly the same frequency response this.

And that will have alpha times a scale of frequency excess compare to that 1 if alpha is a factor of 2, so q means that whatever is happening in the original circuit at 1 by hertz will happen at 2 megahertz, in this circuit and that circuit, but the difference will be in the noise. So, let us say the integrated the noise between some nodes of this is  $b n^2$ , the integrated noise of this circuit will be alpha times  $v n^2$ , and the integrated noise of this circuit will be  $b n^2$  itself, now this is illustrated by the example.

So, this is constant c scaling and that is constant R scaling, so in constant R scaling, you keep all the conductance, and trans conductance to be the same. And you change the capacitances to modify the frequency response, and in constant c scaling you keep the capacitances to same, and modify all the conductance and trans conductance to scale the frequency response.

Now, these two again are very widely used, many times you want to have blocks like filters, which maintain the same frequency response, but different bandwidths. Let us say you want a butter worth filter at 1 megahertz or 2 megahertz or over the wide range of bandwidths, now you have to decide which kind of scaling you want to use based on what you want to happen to the output noise of the circuit.

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So, to summarize to do noise scaling scale all elements by a factor  $n$ , so this means that you will have  $n$  of them in parallel that is the easy way to visualizing it, but as I mentioned at couple of times earlier  $n$  need not to be an integer. For this means that a conductance will become  $n$  times a conductance, a trans conductance becomes  $n$  times the trans conductance resistance. Obviously, from this becomes one over  $n$  times the resistance, and an inductance becomes inductance divided by  $n$  capacitance becomes  $n$  times the capacitance and so on.

So, if you have a MOSfet of width  $W$ , it will have  $n$  times the width in the new circuit, if you do all of these the voltage noise spectral density, at any point in the circuit between any two nodes in the circuit will scale by a factor  $n$ . And currents everywhere in the circuit including the supply current, let us say  $I$  have some branch current,  $i_k$  this is not just a small signal current. It is the total current will become  $n$  times  $i_k$  and this; obviously, is true for the supply current as well, so the power dissipation becomes  $n$  times the power dissipation of the original circuit.

So, this is a way of noise scaling and scaling it without changing the transfer function in the circuit, you consider the any transfer function in the whole circuit in the new circuit, they will be exactly the same. This is again most easily imagine by imagining identical copies of the circuit, and connecting them node by node, if you have identical copies with the identical inputs. When you connect them node by node nothing changes, no voltages change, but the impedance would have become scaled, this by the way is also called impedance scaling, because all branch impedances  $Z_k$  now become  $Z_k$  divided by  $n$ .

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The image shows a digital whiteboard with handwritten notes on a grid background. At the top, it says "Bandwidth scaling:  $H(f) \rightarrow H(f/\alpha)$ ". Below this, there are two sections: "Constant C scaling" and "Constant conductance scaling".

**Constant C scaling:**

- \*  $C \rightarrow C$
- \*  $g \rightarrow \alpha \cdot g$
- \*  $g_m \rightarrow \alpha \cdot g_m$

Next to these are two graphs. The first graph shows a bell-shaped curve on a frequency axis labeled  $f$ . The second graph shows a similar bell-shaped curve, but it is wider and shorter, representing a scaled version of the original. Below the graphs, it says  $\bar{V}_n^2 \rightarrow \bar{V}_n^2$ .

**Constant conductance scaling:**

- \*  $C \rightarrow C/\alpha$
- \*  $g, g_m \rightarrow g, g_m$

Next to these is another graph showing a bell-shaped curve on a frequency axis labeled  $f$ . Below the graph, it says  $\bar{V}_n^2 \rightarrow \alpha \cdot \bar{V}_n^2$ .

The whiteboard interface includes a menu bar at the top with "File", "Edit", "View", "Insert", "Actions", "Tools", and "Help". There is also a "Page Width" dropdown and a toolbar with various drawing tools. At the bottom, there is a taskbar with a "Start" button and several application icons. The NPTEL logo is visible in the bottom left corner.

Now, we sometimes we have to do bandwidth scaling, what we mean by that is we preserve the transfer function should get scale to  $h$  of  $f$  by  $\alpha$ , so this means that if the original transfer function was something like this, on a scale  $f$ . The other one will be stretch out on the scale  $f$ , if I plotted on the same scale, that is what it will look like, but the shape will be preserve.

And in this we have different possibilities, constant capacitance scaling this means that all  $c$ 'es will be remain at  $c$ , and all conductance will be in  $\alpha$  times the conductance, and all trans conductance will also be  $\alpha$  times the transconductance. And in this case the noise transferred to be the same, if original noise was some value the noise means square value, the new means square value noise of the noise will be the same.

We have an alternative which is a constant conductance scaling, and in this case the radios the value of  $c$  by  $\alpha$ , but keep  $g$  and  $g_m$  conductance and transconductance is to be exactly the same. In this case what happen is that if the original circuit has a certain means square value of voltage noise, it will be  $\alpha$  times the whole value in the new circuit.

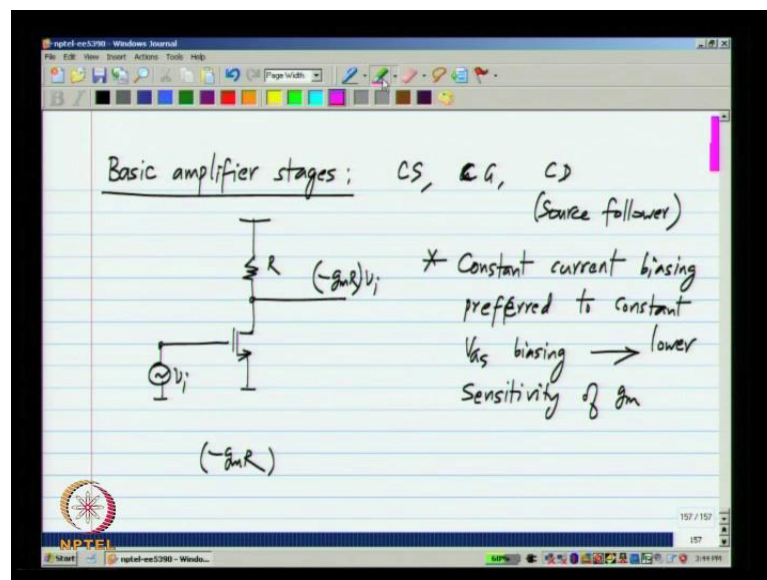
So, which one you choose depends on the constraints that you have, that is the summary of a systematically scaling noise in a given circuit, depending on whether you want to preserve the functionality, or you want to scale the bandwidth without the shape. And you will have do this very frequently when come to circuit design, specially impedance

scaling, because the first circuit that you realize may not be suitable for noise or power dissipation.

In either of these two cases you can alter the circuit without changing, it is functionality to either usually to reduce the power dissipation or to reduce the noise. Now, that we have looked at the noise of components, in the previous few classes we have looked at models of components, their large signal behavior, small signal behavior and noise mismatch and so on.

So, it is now time to go on design our op amps, design the circuits that we want using MOS transistors and other components, so before we do that we will do a quick review of the basic amplifier stages that you already know from undergraduate electronic courses. Now, the objective of this review will be first of all just a quick refresher, and also to look at effects, that you may not have looked at before as especially something like body effect with comes into the picture predominantly, only in integrated circuit design.

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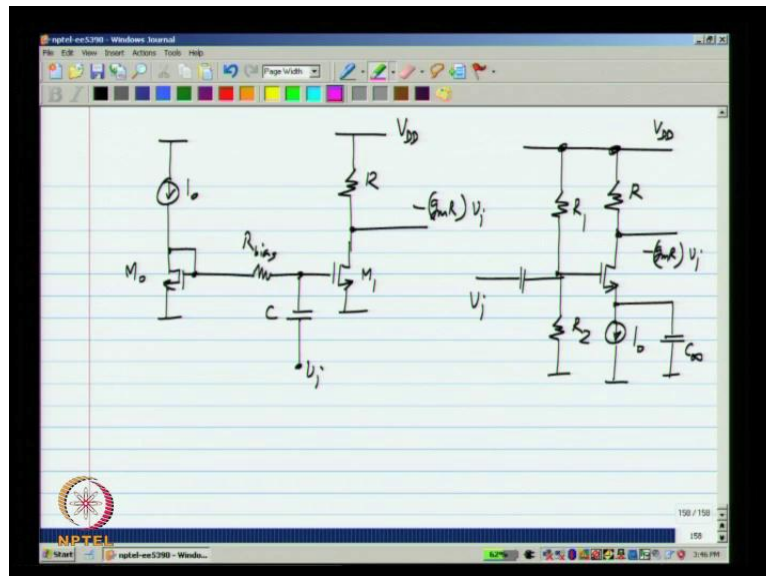
Now, what are the basic amplifier stages that we know, single transistor amplifiers source, a common source amplifier, the common gate amplifier or common drain amplifier, which is also called a source follower. A common source amplifier, as it is a small signal input between the gate and source, and the small signal output between the drain and source a small signal gain of this is minus  $g_m R$ .



So, the small signal picture looks something like this, I will use the symbol of the MOSfet for the small signal equivalent in this case, and this is  $R$  and output will be minus  $g_m R$  times  $v_i$ . Now, we can bias the transistor in various ways, and usually a constant current biasing is prefer to constant  $v_{gs}$  biasing, this is because of the sensitivity and when say sensitivity. We are talking about sensitivity to the parameters of the component or the MOSfet, and if you bias the transistor at a constant  $v_{gs}$ , and let us say that thresher voltage changes or mobility changes the  $g_m$  will change by a certain amount.

Now, let us you bias at a constant current and there are ways to do that, then if the thresher voltage or mobility changes, there can be a change in  $g_m$ , but it will be much smaller. Now, if you look at the ideal square law model, if you change the thresher voltage, but bias the transistor at a constant current, there will be no change in the transconductance or  $g_m$  of the transistor. And if you bias at a constant current on the mobility changes, there will be a change in  $g_m$ , but it will be much smaller than, when you bias the transistor with a constant  $v_{gs}$ .

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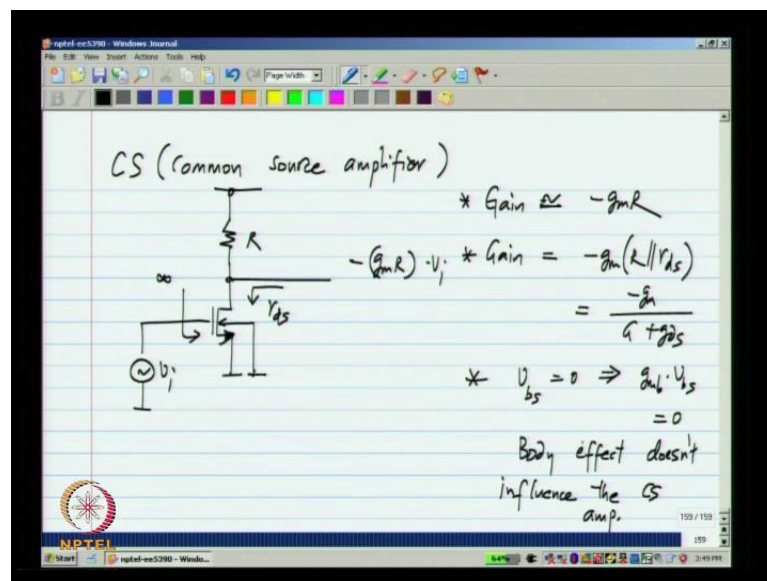
So, the constant current biasing is what is prefer I will quickly show an example, so one way of making a common source amplifier, so the current constant biasing is to use current mirror biasing. Let us say  $m_{naught}$  and  $m_1$  are identical, and this is some  $R$  bias this is very large and  $v_{gs}$  couple of signal to the gate of  $m_1$ , now this is bias at the



constant current my note. So, the gain minus  $g_m R$  from  $v_i$  to the output will not be affected too much by the changes in threshold voltage and mobility of  $m_1$ , and there are other ways of the biasing the transistor at the constant current.

Again, these you know from your basic circuit courses, let us say you connect constant current source at the source of the MOS transistor, and by pass it to the large capacitor for high frequencies. You bias the gate with the resistive divider and apply your input, this is the supply voltage, the output will be again minus  $g_m R$  times  $v_i$ , the signal is a c coupled to the gate. So, these are the biasing strategies, you can have your own other types of biasing, if you wish to have that, in this case I am only interested in the small signal performance, I just showed this to quickly illustrate how the transistor can be biased.

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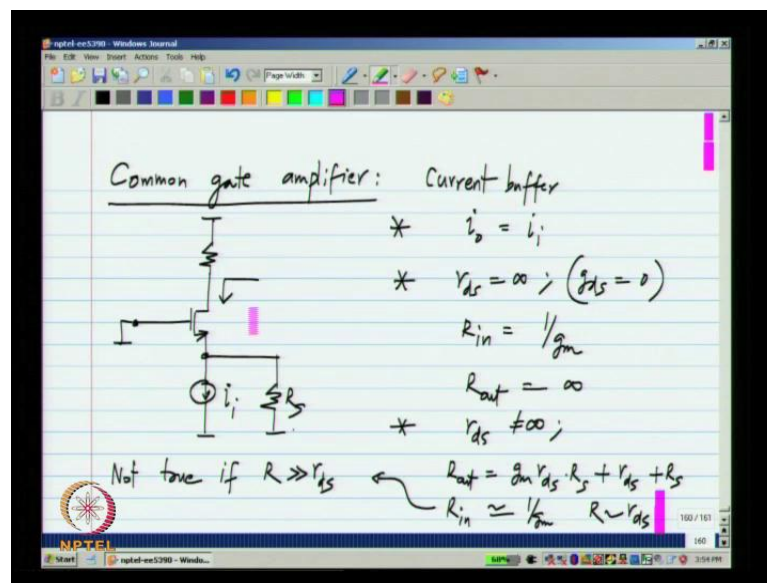


Here, again I show only the small signal picture, so what will be the output it is minus  $g_m R$  times  $v_i$ , so the gain is minus  $g_m R$ , now if you include the output conductance to the MOS transistor. It is really minus  $g_m R$  parallel  $R_{ds}$  or minus  $g_m$  divided by  $g$ , which is the conductance corresponding to this element plus  $g_{ds}$ , I have lot of times we would be operating in the regime where  $R$  is much smaller than  $R_{ds}$ .

So, the gain is approximately  $g_m R$  or the gain really is influence by  $R_{ds}$  as well, and that is something that has to be taken into account, so beyond that there is not much to the common source amplifier. The input resistance looking in here is infinity, if you have

some biasing network that may contribute to the input resistance, and the output resistance looking into the MOSfet is  $R_{ds}$ . And finally, what is the effect of body effect or the bulk transconductance  $g_{mb}$  in the circuit, I have shown the three terminal symbols here, but the source at the fixed potential. And the bulk will also be at a fixed potential, now it maybe at a different potential for d c, but small signal voice that is also at ground. So, the small signal  $v_{bs}$  equal 0 so; that means, that  $g_{mb}$  times  $v_{bs}$ , which is the effect of the bulk transconductance that is also 0, so simply the body effect does not influence the common source amplifier, so that is all.

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Now, let us go on to the common gate amplifier, what is the common gate amplifier, again I will ignore the biasing and look at only the small signal picture, the common gate amplifier has its gate connected to incremental ground. And it will have a load connected to the drain it could be a resistor, so let me just show a resistor for simplicity, and it will have an incremental current input  $i_i$ .

So, again there is some bias current this is only the increment, what is the functionality of the common gate amplifier, it is a current buffer with the output current  $i_o$  equals  $i_i$ , it is a current buffer with unity gain. Now, what is the use of such a thing what it does is to take a potential bad current source and termite into the better current source, so that we can look at by examining the input and output resistances.

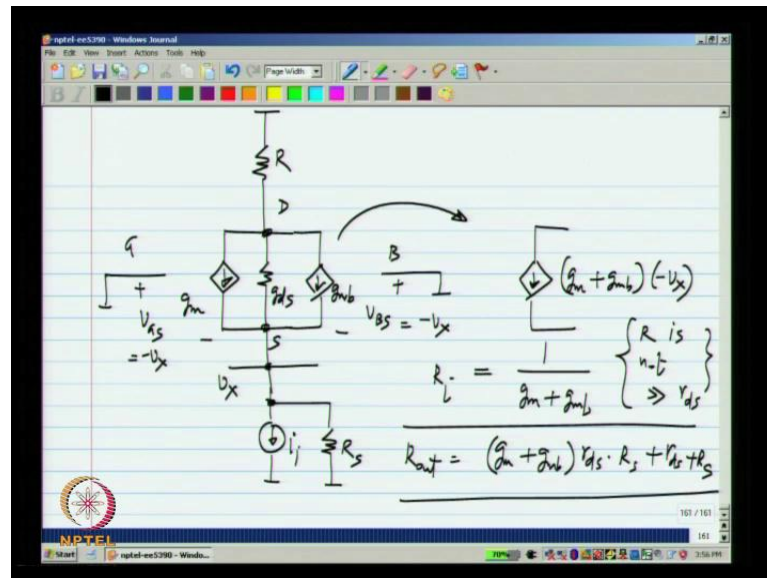
Now, first of all when  $R_{ds}$  equals infinity or the  $g_{ds}$  of the transistor equal 0, the resistance looking into the source of the transistor, the input resistance is just  $1/g_m$ , I would not include body effect, I will do that later and then see what the effect of that is. This is  $1/g_m$ , and the impedance looking from the drain that is the output resistance is infinity, because we have only current source here, and if you do not have any source on this side you will see an open circuit.

That is regardless of the value of the resistance of the input source  $R_s$ , now when  $R_{ds}$  is not equal to infinity  $R_{out}$  will be  $g_m R_{ds} R_s / (R_{ds} + R_s)$  plus  $R_{ds}$  this is something that we know from basic circuit analysis. Now, what is  $R_{in}$  is still approximately equal to  $1/g_m$ , but this is true as long as  $R_s$  is of the same order of  $R_{ds}$ , so this point this is not true if  $R_s$  happens to be much more than  $R_{ds}$ .

Now, this can be imagine very simply by corresponding  $R_{ds}$  to be an open circuit  $R_{ds}$  to be infinity, if  $R_{ds}$  is infinity we have an open circuit in a drain, so clearly any incremental current that you inject here cannot go anywhere. So, the incremental current going into the source has to be 0 so; that means, that  $R_{in}$  is infinity as well, when  $R_{ds}$  equals to infinity. So, we will not work out the exactly expression, you can take it up of an exercise and please do that, but result of that should be that when  $R_s$  is rather small, and when its comparable to  $R_{ds}$ . Then  $R_{out}$  will be  $1/g_m$ , and when  $R_s$  is much more than, that it will not be equal to  $1/g_m$  it will be some very high impedance, what we will assume this condition to be true.

And under those conditions it provides small input resistance, and a very high output resistance, which is exactly the quality you want from a current buffer, so if you have a current source of with an internal resistance of  $R_s$ , it will look like a current source of the same value, but much higher internal resistance. Now, the interesting thing is what happens to this circuit with body effect, now clearly here you can expect something interesting to happen, because the source is not at small signal ground, but the bulk will be at small signal ground.

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I will just see the  $g_m$ , but it means that its multiplying with  $v_{gs}$ , and similarly I also have  $g_{mb}$ , and bulk is at ground as well this is bulk, source, drain and gate, and normally I apply my input current here, it will have some internal resistance  $R_s$  and there will be drain resistance over there. So, without this we already determined the answers, that is the output resistance is high it is  $g_m R_{ds}$  times  $R_s$  plus  $R_{ds}$  plus  $R_s$  and so on.

What happens to the  $g_{mb}$ , now in this case the analysis is very simple, because this is  $v_{bs}$ , and let me call the voltage at this node  $v_x$ ,  $v_{gs}$  equals minus  $v_x$  and  $v_{bs}$  equals minus  $v_x$ . So, these two control sources are in parallel, and they are multiplying the same quantity minus  $v_x$ , so effectively what we have is a current source which is  $g_m$  plus  $g_{mb}$  times minus  $v_x$ , that is because in this circuit  $v_{gs}$  happens to be equal to  $v_{bs}$ . So, everywhere we have  $g_m$  in the original expressions, we have to substitute  $g_m$  plus  $g_{mb}$ , because originally we just have  $g_m$  times  $v_{gs}$   $v_{bs}$  is exactly to  $v_{gs}$ , so here we will have  $g_m$  plus  $g_{mb}$  times  $v_{gs}$  everywhere. So, what does this mean the input resistance is  $1$  over  $g_m$  plus  $g_{mb}$ , this again assumes that  $R$  is not much, much more than  $R_{ds}$  and  $R_{out}$ , the expression for that will be  $g_m$  plus  $g_{mb}$  times  $R_{ds}$  times  $R_s$  plus  $R_{ds}$  plus  $R_s$ .

So, you can see that in this particular case, the body effect  $g_{mb}$  is actually helping you  $g_{mb}$  is an additional control source in parallel with  $g_m$ , and it is simply adds up to  $g_m$

in case of a common gate amplifier. It further lowers the input resistance and further increases the output resistance, and as you know a current buffer is supposed to have a low input resistance and a high output resistance, and the bulk transconductance  $g_{mb}$  actually helps you in this direction.

Thank you, and in the next lecture we will look at the effect of body effect, on the common drain amplifier or the source follower, and also look at other transistor level circuits such as the differential pair, with which we can make op amps.