Analog Integrated Circuit Design

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Lecture No - 3
Step Response, Sinusoidal State Response

Hello everyone. This is the 3rd lecture of analog integrated circuit design. First, a quick recap of what we learnt so far. Basic principle of negative feedback was to sense the difference between the desired value and the actual value and to integrate the error. So, to drive the desired value in the correct direction, integrate the error or the difference and you drive the output from the integrator, such that the error reduces. We look that couple of analogies like adjusting the speed of an automobile or adjusting the volume control knob and this is what you do. There are many other situations that you can think of. Also, we made an amplifier using this principle.

We took the difference between v naught by k and the input v i and use them to integrate the difference and drive the output. In all these cases, we see that finally when studies state is reached here; it is assumed that the study state means the output is constant because the input is constant with time. When the steady state is reached, the output is the ideal value because the integrator will keep on changing its output if the input is not zero.

So, knowingly way for the integrators output to be constant is for the input to be 0 and that will happen only if the actual output is equal to the desired output. So, the key to proper operation of the negative feedback loop is sensing because you can imagine that. For instance, if your speed of meter has an error, then however you drive it, you will finally reach the wrong value.

So, what is the key to the negative feedback loop? It is the sensing and the sensing has to be accurate. This we will see repeatedly through the course. This is through an amplifier. This is true for amplifier as well because it does not matter exactly how you drive, whether you exhilarate fast or exhilarate slow, whether you integrate, whether you are integrating faster or slower, what matters is that you do it in the right direction and the right direction is given by the correct sensing. So, the sensing is the most crucial part of the negative feedback loop, and we will take lot of pain to make sure that the sensing is accurate when we build our circuits. So, using this basic principle of negative feedback loop, it derived our amplifier.

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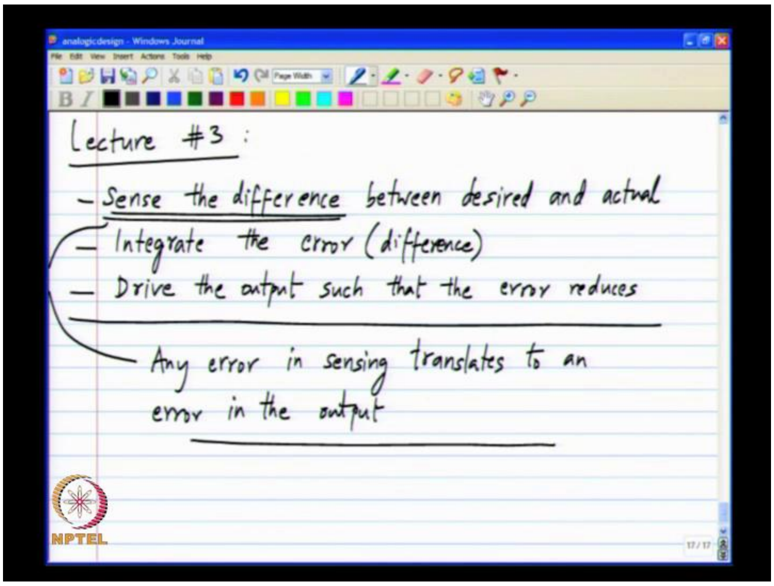

The amplifier looks like this. Omega u is the parameter of the integrator. It says how fast the integration is and we saw that for a constant v i. That is the only case we have considered so far. The output will reach k times v i. We also saw that the differential equation governing this system is, it is a first order differential equation and you can clearly see that the steady state solution that is from the derivative of v naught is 0. V naught equals k times v i and just to relate this to something, you already know the differential equation is the same as what you would see in a first order r c filter and the differential equation governing this system as you can see, the differential equations describing the amplifier that we design and the differential equation describing the r c filter are the same except for the constant k, which appears in the amplifier.

So, the solutions to them are same as well and the solutions are given by. So, the first part is the initial condition which decays with the curtain time constant, and the second part is related to the input v i and it grows with the same time constant. It is exactly the same behavior that is seen in a first order r c filter.

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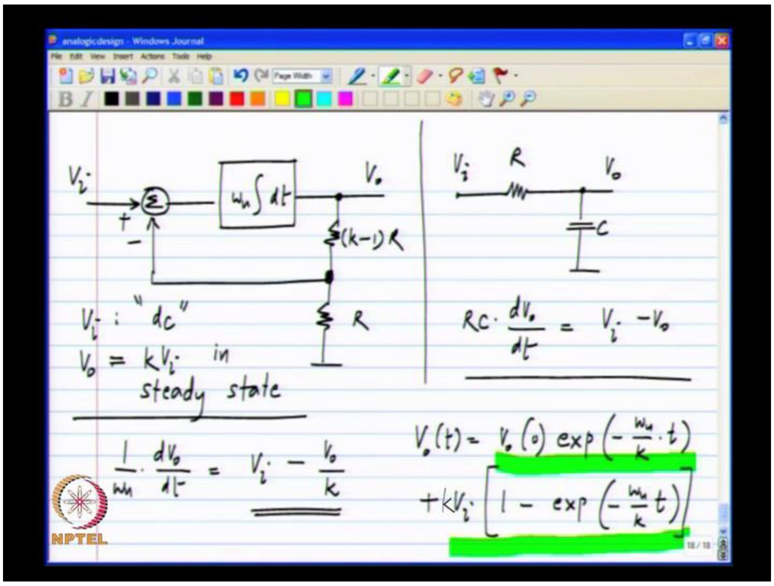

So far we have examined the negative feedback amplifier with a constant input, and we see that it behaves ideally. So, the output will be equal to k times v i if you wait for long enough time and we also looked that how to calculate the amount of time for which we have to wait. Now, what we would like to do is, we also have to see what happens when the input is changing because in a real life situation, the input will not always be constant. We have to also examine for changing inputs. So, we will consider two types of changing inputs. First is a step input that is the input goes from being one particular constant to another constant and secondly, a sinusoidal input which is very useful input analysis systems because any signal can be decomposed is the sum of sinusoids.

So, if you know the response of a system to a sinusoid, you can in theory compute the response to any other input signal. First, we will start with the step input. This case v i will go from constant to another constant and for simplicity, I will take the initial constant to be 0 and the final constant to be some v x. So, this is the nature of the input signal. Now, during this part when the input is constant, we assume that the system has reached steady state. So, v naught also will be equal to 0. This is known as the feedback voltage. The feedback voltage is v naught by k and that is also equal to 0 and the error naturally is also 0 and the steady state has reached.

Now, what happens when you apply a step? There is a step and the output of the integrator cannot change suddenly. So, even after the input step, the output will still be at

0. Then, what happens is because the feedback voltage is still 0, the error voltage will step up by v x. This is because the input has stepped the integrator, output has not changed. So, that means, the feedback voltage has not changed. So, the input to the integrator which is the error voltage steps up.

Now, because it steps up, the output voltage starts increasing. Let me show this with the blue pen. So, this part here, the output start increasing and the initial slope of this is simply equal to omega u times v x because the step v x appears directly at the input of the integrator. So, the slope of the output equals the constant of the integrator omega u times the input step v x.

Now, as the output increases in the feedback, voltage also increases which means that the error voltage decreases because we have a fixed input and the feedback voltage is increasing, and it is being subtracted from the input. So, the error voltage decreases. So, this means that the slope of the output reduces which reduces the slope of the feedback voltage, which further reduces the error.

So, we have already solved this, but now we are looking at it intuitively as what happens step-by-step. So, this goes on until the slope becomes almost 0, and the output reaches the new steady state and the error does something like this. So, this will be the nature of the error. So, it will show an initial step. Initially the error becomes very large because the integrator cannot respond instantaneously to its input and then, after the error gradually reduces, and then becomes close to 0 and then, the new steady state is reached and the new steady state value is when the output v naught equals k times v x.

So, this is what happens with a step input. We have already seen this from the solution of the differential equation. It is exactly the same thing that I have shown here. So, the output gradually approaches the ideal value and of some interest is this error will form. You can see that the error becomes very large equal to the input and then, comes down and gradually becomes 0.

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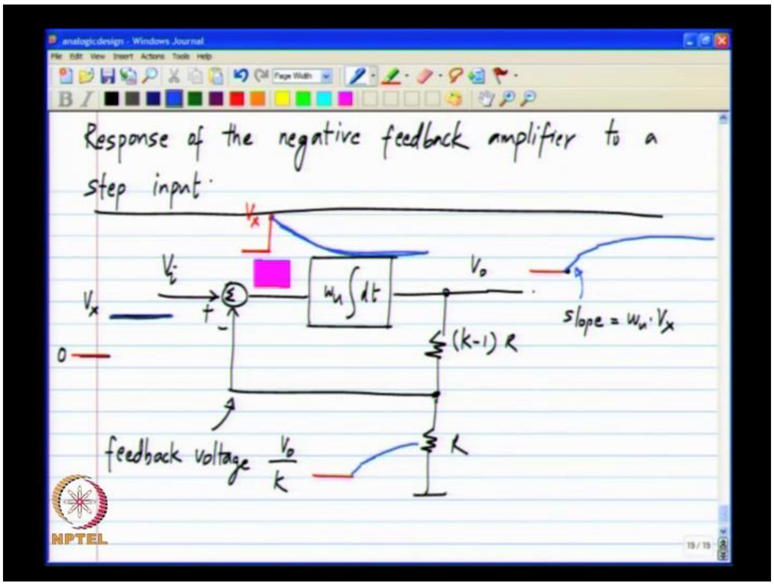

So, this is again through every negative feedback amplifier because the error cannot immediately become 0. The tracking cannot be immediate. Again, we can take the analogy of driving the automobile. Let us say you are driving at 40 kilometers an hour and then, I say 50, that is I tell you to go at 50 kilometers an hour. We cannot instantaneously change from 40 to 50. It will take some time for you. So, during sometime, there will be a speed error that is the desired speed will be 50 and the actual speed will be different from 50. This is exactly what happens.

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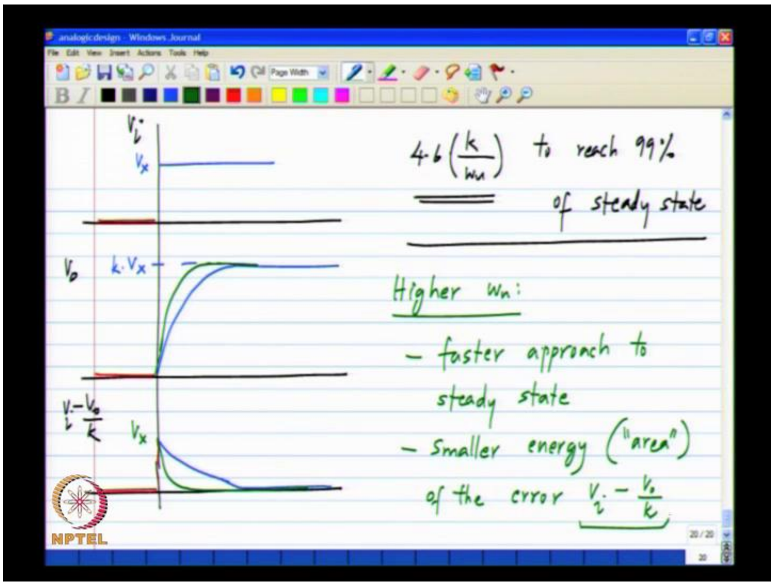

Just to look at this a little more that may draw the wave forms, the input waveform is like this and the output waveform should reach k times v x, but it does not do it instantaneously and I will also plot the error. This is the output v naught and this is the error v i minus v naught by k. So, that will be 0. Initially, it will also be 0 after steady state is reached, but in the middle, it jumps up and gradually comes down.

Similarly, k times v x will not be reached instantaneously. It does something all that shown. Now, we already saw how long it takes to reach a certain percent of the steady state. Yesterday we evaluated that it takes 4.6 k by omega u, this much time to reach 99 percent of steady state. So, we also see what happens if omega u increases. If omega u increases, then the amount of time taken to reach steady state is decreased.

Now, what is the case for a higher value of omega u? So, let me do it here. So, for higher value of omega u, this rises up faster and it goes that way and the corresponding error waveform will be like that one. So, the error will still jump from 0 to a value equal to v x, but when omega u is larger, it comes down faster. When omega u is small, it takes a long time and this is of some interest in practical applications as well. This we will see later.

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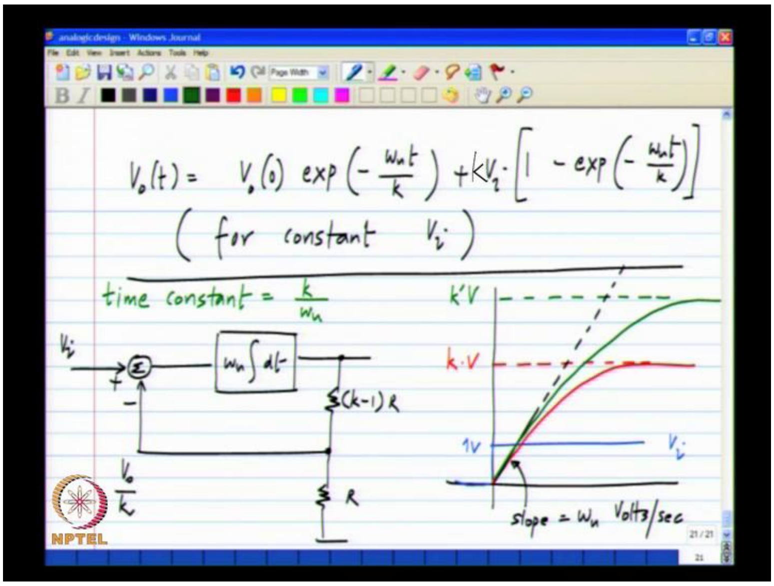

So, if you want to reduce the energy of the error, so to speak by energy I mean the area under this curve, you have to increase the value omega u. Higher value of omega u means that you have a faster approach to steady state. It also means that smaller energy by this. I simply mean the area under the error curve. So, to make the feedback system more accurate for step inputs, you have to increase the value of omega u. Now, the expression for the output is this. Once again let me emphasize that is for constant inputs v i.

Now, in this expression you see that the time constant depends on omega u as well as k, that is the time constant is what decides how fast the steady state has reached, and that depends on the speed of integration omega u as well as the gain that you are trying to realize. So, let us quickly a look at why this is slow. So, first of all let me draw a unit step. Let me assume that this is the input to the system.

So, there is one volt and this is v i. Now, what should happen if left side again is k? Then the output should be k volts and if the gain is some other k prime which is greater than k, it should be k prime volt. Now, the time constant of the system is k by omega u and you see that as you increase the value of k that is you try to make an amplifier with a higher gain, the time constant increases.

So, let us quickly see why this is the case. Let me redraw the amplifier. Let me again assume that the output is initially 0. So, what happens if the output is 0 and if the input is a step, the feedback voltage is also 0 that is v naught by k also equals 0 initially. So, that means that all of the input steps appear at the input of the integrator. So, as we discussed just a few minutes before the output starts ramping up and the output starts ramping up at a rate which is omega u times the size of the input step and the input step is one volt. So, the slope is simply omega u volts per second. So, that is always the starting slope.

Now, as the output increases, the value of the error decreases and the slope gradually reduces, but the initial slope is independent of the value of k. Let me draws this line all the way through. So, what happens is that for the k shown in the red that is for a gain of k is such with this slope and then, gradually reaches steady state and for the k shown in green, it again starts with the same slope and it has to reach a higher steady state. So, it has to increase to a larger value of the output. That is why it takes a longer time. That is the reason why the time constant is also proportional to the gain k.

So, for a higher value of k, you have to reach a higher voltage and you always start with the same slope. So, it takes a longer time to reach the higher voltage. So, that is an intuitive explanation for why the time constant is higher when you try to make an amplifier with a higher gain. So, that is what happens when you excide the negative feedback amplifier with a step input that is we know two things. Now, first of all when the steady state, the output is ideal that is in steady state, the output voltage is k times the input voltage where k is the gain of the amplifier because the integrator output reaches steady state only when the input to the integrator is 0, that is when v i minus v naught by k equal 0.

So, in steady state we have ideal behavior. Now, when you apply a step, what happens is that the integrator cannot respond instantaneously to a step. So, initially the integrator does not respond at all. The error will be quite large and as the integrator output ramps up, the error reduces and the error gradually reduces to 0, and the output gradually approaches steady state, and we also know how to calculate the amount of time during which it reaches a certain percentage of steady state, and we see that if you increase the value of omega u that is if you increase the speed of integration, you can reach steady state faster. This is intuitively obvious. If you ramp up faster, then you will reduce the error faster and you will reach the steady state quicker. We also see that the energy or the area under the error curve will be smaller if omega u is higher.

Now, we also see that from the expression, the time constant depends not only on the speed of integration omega u, but also on the gain k. Now, this can be intuitively explained by looking at what happens when you apply a step input. When you apply a step input, the input to the integrator is the step itself initially because the feedback value does not change instantaneously.

So, this means that when you apply a step, the output of the integrator does not change and then, it starts ramping up at a given slope and that slope is given by omega u. It is independent of k. Now, if the value of the gain k is larger, you have to reach a higher steady state because the output is k times the input and it takes a longer time. That is the reason why the time constant for a higher gain amplifier with the same integrator will be more than the time constant for a lower gain amplifier.

We will now look at another type of time wearing input which is a sinusoidal input. As you know sinusoidal inputs are very convenient for analysis of linear system because when you apply a sinusoidal input, the output consists of a sinusoid of the same frequency and it modifies an amplitude and phase. So, this is described by the transfer function. So, it is very easy to evaluate the output of a linear system for a sinusoidal input. We also know that from Fourier series and Fourier transform, any input can be decomposed into sinusoids.

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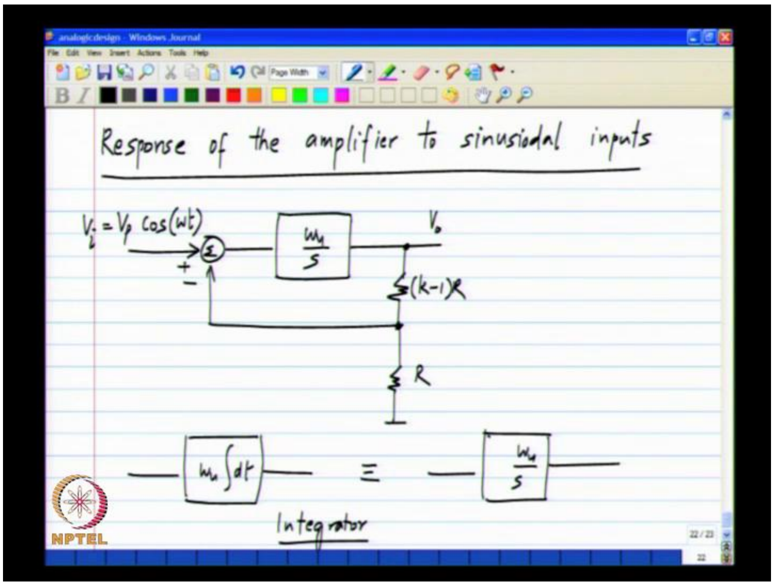

So, combining these two, we know that for any input we can find out the steady state of output easily by evaluating the response of the system to sinusoids. So, what we will be evaluating will be steady state response to sinusoids. We will ignore the transients. What we will be evaluating will be steady state output in response to sinusoidal inputs. We will be ignoring the transients, that is when you have a 0 input and then, you change to a sinusoidal input. The output will go through some transients and finally, reach sinusoidal steady state. What we will be evaluating is only the sinusoidal steady state will ignore the transients.

Now, from the analysis of step inputs, we know how long it takes for transients to die out and when you have a sinusoidal input, it is about the same time that takes for the transients to die out. So, we will not analyze this. Further, we will only look at sinusoidal steady state. So, in this case we assume that the input as sinusoidal of frequency omega. So, this is the system we want to analyze, and we want to see what the output is in steady state and as we know, this is very convenient to analyze not in the time domain, but in the Laplace domain where we replace the integrator with its Laplace transfer equivalent.

So, what is an integrator? In the time domain is the same as this algebraic operation in the Laplace domain that is omega u integral of time is the same as omega u by s is the same as multiplying the input by omega u by s, and you also see the dimensional consistency. Here, omega u has dimensions of frequency and s also has dimensions of frequency. So, the transfer function is dimensional less. If you apply the voltage, you get a voltage out.

So, to analyze the sinusoidal steady state response of our amplifier, what we need to do is to replace this by h Laplace transform equal. Now, as I had mentioned in the introduction, the advantage of Laplace transform analysis is that instead of solving the differential equations, we will solve algebraic equation and from the algebraic equation, it is very easy to get the solution by using the inverse Laplace transform and we are looking at sinusoidal steady state. So, it is even easier. We substitute s equal to j omega and calculate the magnitude and phase of the output sinusoid. Now, before we do that let us just examine the integrator by itself to see what it behaves like. We already examined integrator in the time domain and I have seen the implication of the constant omega u.

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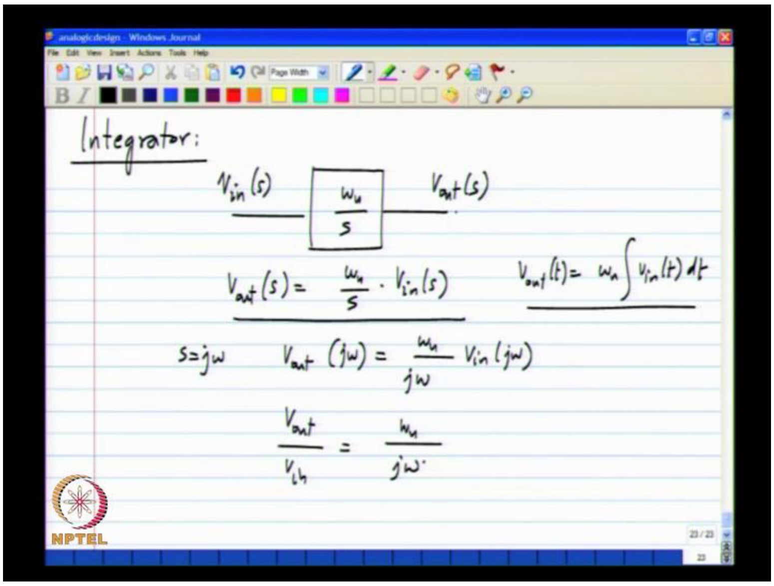

Now, we will do the same thing in the frequency domain or we will be different symbol for the input. Let me call it v in of s and v out of s and in the Laplace domain, v out of s is simply omega u by s times v in of s and in the time domain, we know that v out of t is omega u integral of v in of t d t. Now, the sinusoidal steady state response of this can be very easily seen from in the bode plot. It can also be easily seen from the transfer function. What we will be using is bode plot. So, it is a good practice to use, bode plot for a simple system such as this one. So, for sinusoidal steady state, what we need to do is to simply substitute s equals j omega and this uses v out of j omega to be omega u divided by j omega v in of j omega or the transfer function v out by v in is given by omega u divided by j omega.

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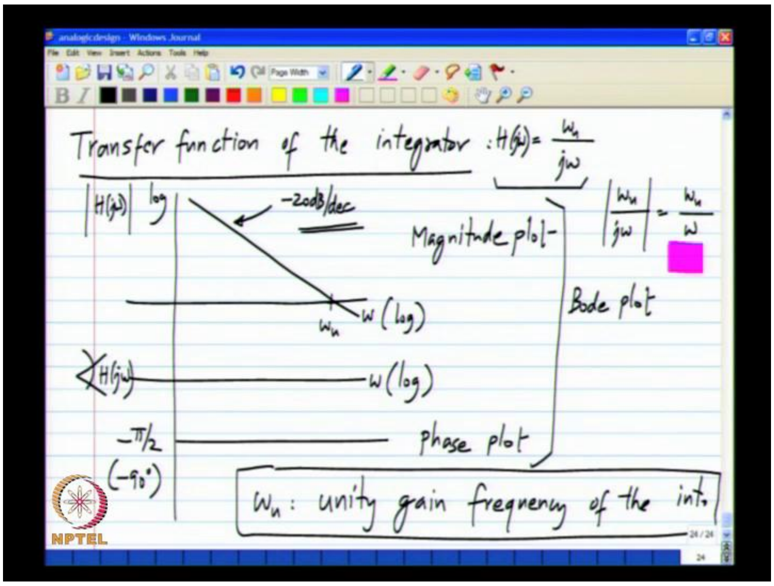

So, we will just draw the bode plot of this, and see how it behaves and the bode plot consists of the magnitude plot and the phase plot, that is real plot. I will denote the transfer function by h, the magnitude of h or j omega versus omega on a log scale. This is what is plotted for a bode plot and similarly, we plot the angle of h or j omega versus omega on a log scale and the combination of these two is what is referred to as bode plot, ok.

So, now, the magnitude is given by magnitude of omega u by j omega which is simply omega u divided by omega. So, the bode plot of this is very simple on a log scale, where you plot the magnitude on a log scale as well as the frequency on a log scale. This is the straight line with a negative slope and the slope is given by minus 20 db per decade that is if you take it in the decibel units. If you take 20 logs to the best of the magnitude drops with 20 db per every factor of an increasing frequency that is for every decade changes in frequency.

That is what the meaning of this one is. Where does it reach unity? It is very obvious. This function reaches unity when omega equals omega u. So, the magnitude of the integrators transfer function reaches unity when omega equals omega u. In fact, it is for this reason that the particular symbol has chosen omega u as the unity gain frequency of the integrator.

Now, for the phase of the integrators transfer function, this is even easier because we have a single j in the denominator. The phase of this, the transfer function is always equal to minus pi by 2 or minus 90 degree. So, the integrators transfer function as a magnitude that is decreasing with frequency at 20 degree per decade slope and it has a phase which is constantly minus 90 degree for all frequencies.

Note that in the bode plot, you do not have a representation for dc because the x axis is the log scale and you cannot represent zero frequency on a log scale, but you can extend it down to as lower frequency as you want, and for an integrator, the magnitude will keep on increasing as you go to lower and lower frequencies. So, this is what the transfer function of the integrator itself looks like, and like a mentioned omega u is the unity gain frequency of the integrator.

So, this will be a crucial parameter that will govern the behavior of negative feedback system that we build using this integrator. So, you choose an integrator which has a particular omega u. We have a time domain interpretation for it that relates to how pass the integration happens. If you apply a step, the output slope will be omega u times the input step. Now, we also have a frequency domain interpretation for it. So, omega u is the frequency at which the magnitude response of the integrator is unity that is if you apply a sinusoid at this frequency in steady state, the output sinusoid will have the same amplitude as the input sinusoid.

So, with this background, we can go back and analyze the transfer function of the entire negative feedback amplifier and then, relate it to the transfer function of the integrator by itself. So, this is the system we have and because it is an algebraic, because the integrator can be represented by an algebraic relationship, the analysis is very easy. So, here we have v naught by k and here we have v i minus v naught by k. So, all we have to do is to recognize that the input of the integrator is v i minus v naught by k, and that times v omega u by s is v naught.

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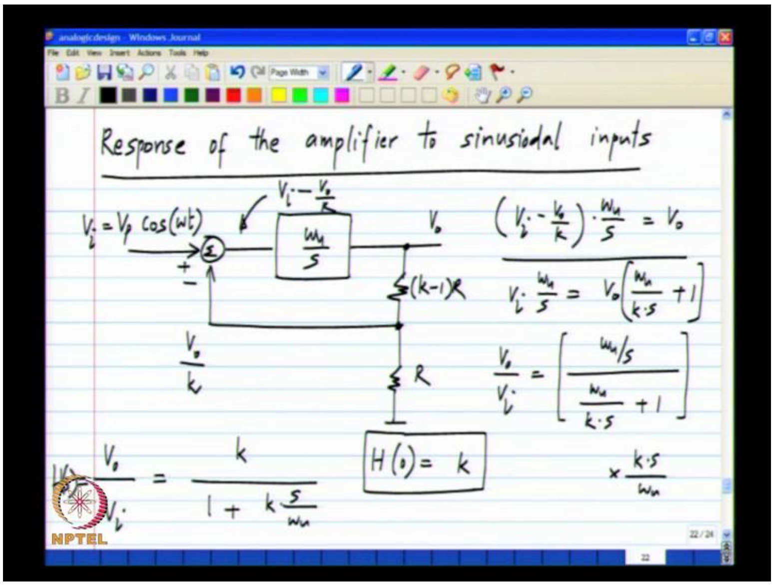

So, this can be rewritten as the transfer function v naught by v i is given by omega u by s divided by omega u by k times s plus 1. Of course, normally transfer function is not represented in this form. They are represented as a ratio of polynomials in s. So, we multiply both the numerator and denominator by k times s by omega u. So, when we do that v naught by v i become k divided by 1 plus k times s by omega u. So, what is this saying? Now, first of all this is the transfer function of our amplifier h of s and the value of h at 0 frequency is k.

So, what does this mean? Now, s is equal to 0 corresponds to dc. So, what this expression is saying is that the dc gain of the amplifier is k. Now, this we already knew from the previous analysis and it is good that these analysis is consistent with it because it was not we would be in trouble, but previously we calculated that the steady state solution to our amplifier is that the output will be k times the input when the input is constant. Now, when the input is constant, then it is the same as saying the input is dc and the analysis confirms that the gain of the amplifier equals k when the input is cd, and we will examine this transfer function little further. The transfer function v naught by v i of our negative feedback amplifier is this much.

Now, for this the dc gain that is h of 0 equals k and this is consistent with our time domain analysis. Now, it also has a pole at minus omega u divided by k. The pole is the value of s at which the denominator becomes equal to 0. So, you can easily see that for s equal to minus omega u by k and the denominator becomes equal to 0. So, this is a first order system and we have a single pole at minus omega u by k. So, if I draw it on and explain here, I show the real and imaginary part of s. There is a single pole on the negative real axis and it is at minus omega u divided by k. Of course, we know that for stable systems, the poles have to be in the left half plane and in this case, the pole is the left half plane.

So, that means, that this is stable. We already knew that it was stable from the time domain solution to the step input because the exponential had a negative argument and the exponential would always lie down. So, this means that the system was stable and it is again confirmed by our analysis in the frequency domain, and this is one thing which is very important, this type of comparing analysis because you cannot find the single analysis that will reveal all the aspects of the problem. So, you will always for any problem, you will go through multiple types of analysis. Sometimes you analyze it only for dc, sometimes for higher frequencies, sometimes in the time to time and frequency domain.

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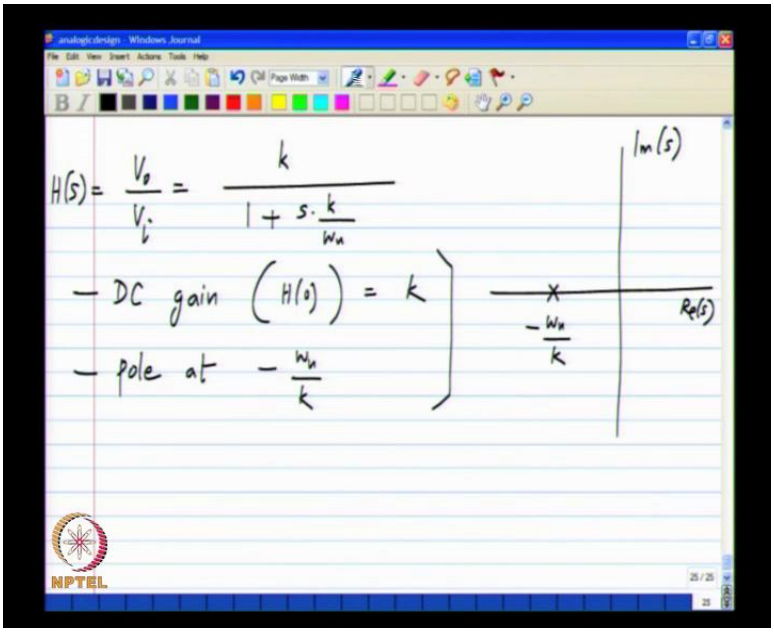

It is extremely important for you to reconcile one type of analysis with another because there are areas so overlap. For instance, we evaluated the steady state solution time to time. We have evaluated the dc gain in the frequency domain and these two have two matched up. Otherwise, there is something wrong with the analysis, either one of them.

So, this is something that you have to do and this also gives you multiple points of view to look at the same problem.

So, this summarizes that we have a dc gain of k and a single pole at minus omega u divided by k. Now, we would also like to look at the magnitude and phase plots of this transfer function. So, finally what we started off was in that we will look at the sinusoidal steady state response that is we will see what happens when you apply sinusoids to the output. We expect a sinusoid, but we want to see what its amplitude and phases. So, we are going to do that by evaluating bode plot or the magnitude and phase of this particular transfer function.

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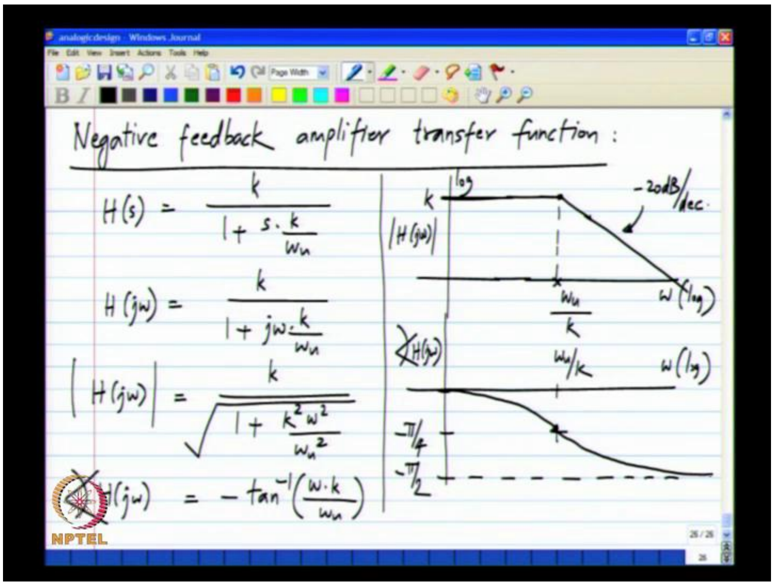

In the second expression here, I simply replaced s by j omega which is what I want to evaluate. The sinusoidal steady state and the magnitude of h is given by this. Simply take the magnitude of the complex denominator and the angle of h is given by minus tan inverse and from the second draw, the bode plot and if you are fluent enough with bode plots, you do not have to evaluate these things explicitly. Simply by looking at the transfer function and remembering the rules for bode plots, you can draw them.

So, first we will plot the magnitude as usual on a log scale versus omega also on a log scale. So, for very low frequencies, the second term in the denominator k square omega square by omega u square is much smaller compared to 1 and therefore, it can be neglected in comparison to 1. So, the magnitude of h will be very close to k. So, the magnitude of h will be very close to k and for very large values of omega, what happens is that the second term in the denominator is going to be much greater than 1 and 1 can be neglected in comparison to k square omega square by omega u square. So, in this condition, we can approximate the magnitude by omega u divided by omega.

So, What I have done here is to simply neglect the one here. So, in the denominator I get square root of k square omega square by omega u square which is nothing, but k times omega by omega u and k cancels out, and you simply get h of j omega approximately equal to omega u divided by omega and this is an approximation that is valued for omega much more than omega u divided by k, and we have already seen that h of j omega approximately equals k for omega, much smaller than omega u divided by k.

So, for very high frequencies, the magnitude response is inversely proportional to omega which means that it simply rolls of at minus 20 db per decade, and you also see from this particular expression that at omega equal to omega u, it approximately reaches unity. So, the frequency at which the transition occurs is nothing, but the pole frequency of the amplifier and it is equal to omega u divided by k. So, the magnitude will be close to the ideal value of k. We would like to have a gain of k. So, the magnitude will be close to the ideal value of k for low frequencies, where low frequencies are defined as frequencies much lower than omega u by k and for high frequencies, it also of at 20 db per decade.

So, this is the story of the magnitude response of the amplifier. So, now it is for the phase response. So, the phase is nothing, but minus tan inverse omega times k by omega u. Now, clearly for very small values of omega, we have almost zero phases. So, it starts with zero phases for omega equals omega u by k. The argument of tan inverse will be unity and we get the phase of minus pi by 4 and for very high values of omega, the argument of tan inverse will be very large and we get an angle of minus pi by 2. So, the phase changes from 0 to minus pi by 4 at the pole frequency and becomes minus pi by 2 at very high frequencies. So, this gives the magnitude and phase plots of our transfer function.

The summary to take home from this is that the dc gain of the transfer function is k. As we expect, it has a single pole at omega u divided by k and if you look at the magnitude plot, the magnitude remains at k for frequencies less than omega u by k and after that it roles of at first order meaning minus 20 db per decade. If you look at the phase response, the phase log is very small at very low frequencies and the phase log is minus pi by 4 or 45 degrees at the pole frequency, and it becomes minus pi by 2 at very high frequencies. Now, if you know the rules of drawing the bode plot, you do not have to evaluate the magnitude and the phase explicitly. The bode plot says that there will be break points at every real pole and 0 at every pole.

By appropriately evaluating the breakpoints, you can draw the entire bode plot without actually evaluating the magnitude and phase response, but maybe in the beginning you experience, you can evaluate it and see how it behaves. So, the next thing we have to do what we have started of it doing was to try and evaluate the sinusoidal steady state response, and what we have to do is to use this magnitude and phase functions and evaluate the sinusoidal steady state that we will do in the following class.