Analog Integrated Circuit Design

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Lecture - 47
Type I PLL Loop Gain and Transfer Function Reference feed Through

Hello and welcome to lecture forty seven of analog integrated circuit design. In the previous lecture, we looked at how to implement a phase detector, and the kind of non ideal characteristics that appear, that is there will be a periodic error in addition to the measurement of phase that is why in this lecture what we will do just to evaluate the effect of that that will also give us some practice in the kind of analysis that is relevant for a phase lock cube that is to measure the incremental phase error at the output and see what its effect is and then we will see that there are some shortcomings of type one phase lock cube and then we will see how to improve on the structure of the phase lock cube.

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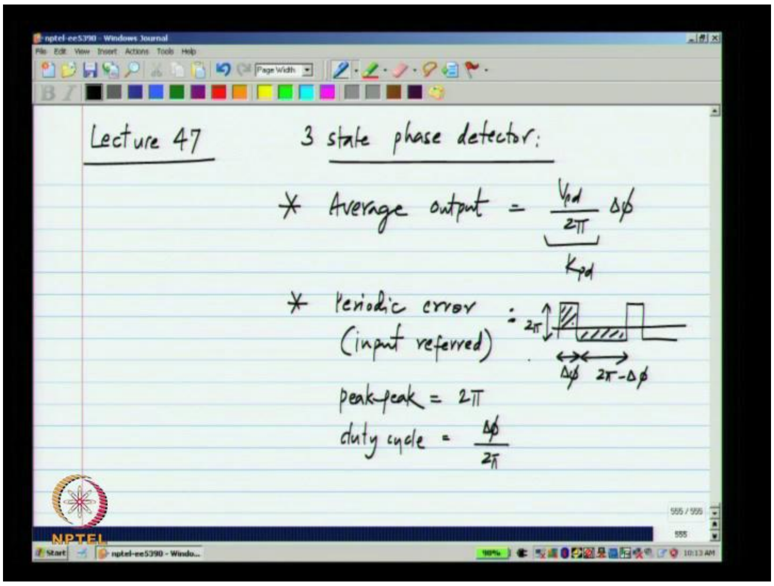

We saw that, the phase detector that we implemented. It has an average output which is, proportional to delta five or the difference in the phase between its two inputs in this V p d by 2 pi is nothing but, the gain of the phase detector, but there is also a periodic error, which when referred to the input will have a peak to peak value of 2 pi, and it will have an average value of 0 that is the positive and negative areas are equal, and the duty cycle would be delta five divided by 2 pi. So, this is the periodic error that gets added to, the input of the phase detector just because of the way, the phase detector is implemented.

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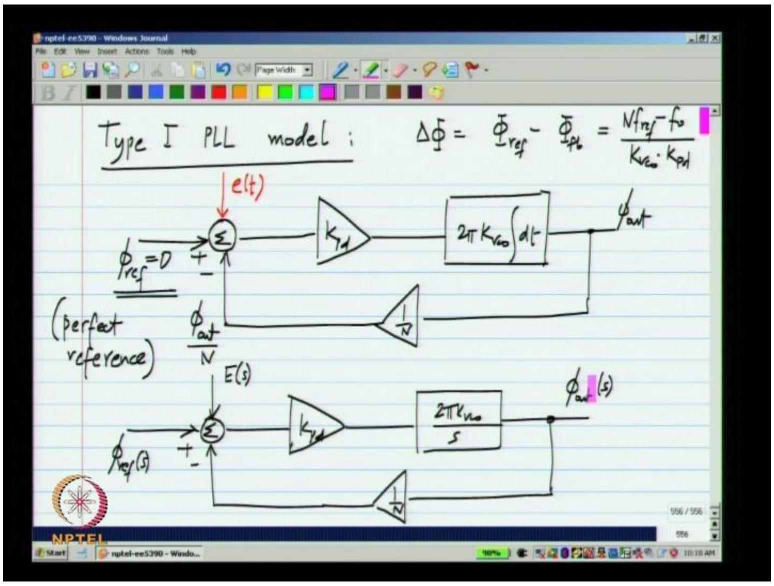

Now, let us put this in the incremental model of the phase lock loop. We have evaluated this, any input phase increment is phi ref, and here we have a gain of K p d and the V c o, which basically integrates the control voltage to result in the output phase, and the feedback divider is simply an attenuation of 1 by n. Now the additional thing we have because of the way the phase detector is implemented is a periodic error e of t, which I described as earlier, it has zero average and a duty cycle of delta phi by 2 phi, where delta phi is the operating point.

Phase difference it is not in the incremental domain there is operating point phase difference between the input and the fight back phases, and that will be equal to the difference between the output frequency and the free running frequency of the V c o divided by the gain of the V c o, and the gain of the phase detector. So, now, what do we do with model such as this we have a periodic error e of t, and what this says is that the output incremental phase phi out will not be 0 even if phi ref is 0 now, what is the meaning of phi ref being 0 phi ref this phi ref here the lower case phi ref refers to the phase error compared to the ideal ramp.

As you know, the phase ref of the periodic signal will be an ideal ramp, if it is perfectly periodic, but in general it will have some error, but in this case if we set phi ref is equal to 0, it means that, we are operating using a perfectly periodic input, but phi out will not be 0 because e of t is not 0. So, what does this mean even with a perfectly. Periodic input the output of the phase lock loop will not be perfectly periodic. So, we have initially done the analysis with this e of t set to 0. So, in that case the output will be periodic at n times f rep now, we will insert e of t and assuming that errors are small.

We can calculate the error from this and add it to our previous result. So, the thing is to calculate phi out from v of t and phi ref will be set to zero. So, this as I said means a perfect reference. Now, that we have the system diagram, we can calculate the output for any input, but we can do it in a simple way while making some assumptions about e of t. So, if I put down the Laplace domain model, everything will be exactly the same except that the integrator will be replaced by 2 pi K v c o by s, and this could be some input and this could be some input, and that will be the output. So, we can calculate the transfer function from here to there, and it is exactly the same as the transfer function from phi ref to phi out.

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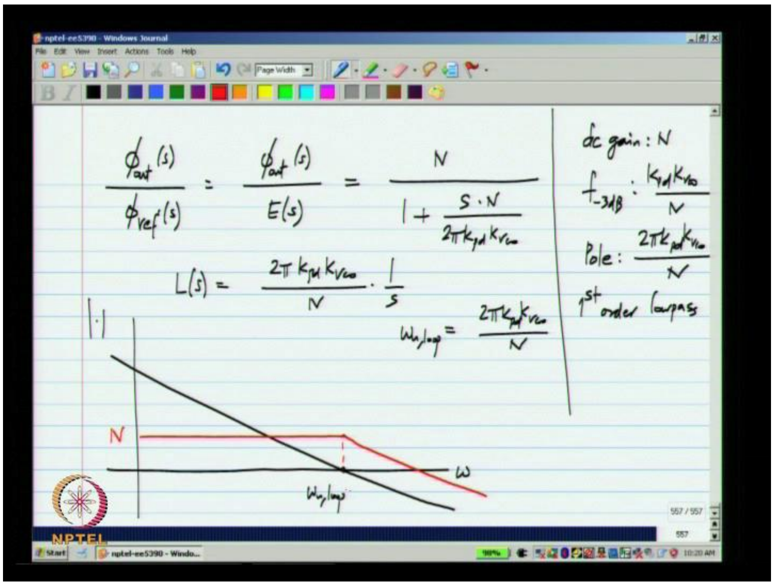

This is of course, assuming that each of this is acting one at a time and that will be equal to, it will be low pass transfer function with a d c gain of n and a three d b frequency in hertz of K p d K v c o divided by n. So, the pole is at 2 pi K p d K v c o by n. So, it is a first order low pass transfer function, and the loop gain of this feedback loop, we have discussed loop gain extensively while talking about op amps and other feedback circuits. So, I will simply assume that you know the results.

So, the loop gain is you break the loop and you go around and see what is the gain, and that will be 2 pi K p d K v c o by n 1 by s. So, if I now plot the magnitude response versus omega on a log scale, the magnitude response of the loop gain will be something like this, where this is the unity loop gain frequency, and if I plot pi out by phi ref it will have a d c gain of n, and after that twenty degree per decade roll off, and the unity loop gain frequency will be the three d b bandwidth of the system.

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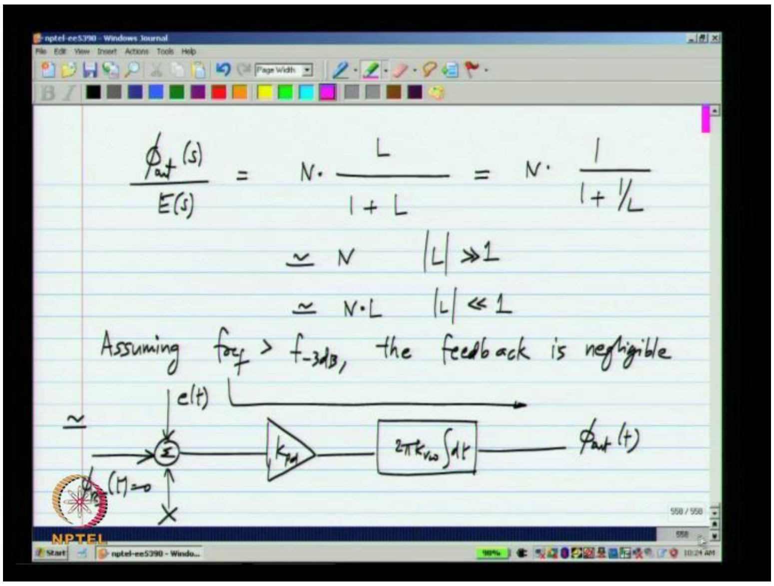

Now, we have seen earlier that the close loop gain of a feedback system can be expressed as in this particular case n times loop gain by one 1 loop gain or in other words also n times one by 1 plus 1 over L, and we know that, this is approximately equal to n, where the magnitude of the loop gain is very large, and approximately equal to n times L, where the magnitude of the loop gain is very small. We have seen all of these things, what does it mean, if the magnitude of the loop gain is very small, we can simply neglect the feedback for the fit.

We assume that nothing is coming back, and the gain from here to there will be simply the gain of the forward path. It will be the product of E of s K p d 2 pi K v c o by s that will be the output. So, now, this E of s itself, it is a rectangular wave at a frequency of f ref. So, that is what E of t is, now, what does this mean, it will have f ref and its harmonics.

So, now, the calculation becomes very easy, if we assume that f ref is well beyond this unity loop gain frequency or the three d b bandwidth of the system, in that case the feedback is negligible, what I am trying to say here is that E of t is a periodic signal in the frequency of f ref.

So, that means that it will have f ref in its harmonics, and if f ref is higher than the three d b bandwidth of the system or the unity loop gain frequency of the system, then all the frequency components of the system for E of t will be beyond the unity loop gain frequency; that means, that for these frequencies there will be negligible feedback, and we can compute the output by simply considering the forward path and completely neglecting the feedback. So, that makes the things a lot easier and that is what we are going to do.

So, approximately, we can calculate the effect of E of t, I am assuming phi ref is 0 that is a perfect reference by considering only the forward path. So, this part I will show just to show that I have neglected the feedback. So, I have K p d and 2 pi K v c o integral versus time. So, all that I need to do is to calculate phi out as a result of this path, now, e of t is a rectangular web. So, all I have is the integral of the rectangular web with some proportionality constant, which can be very easily evaluated.

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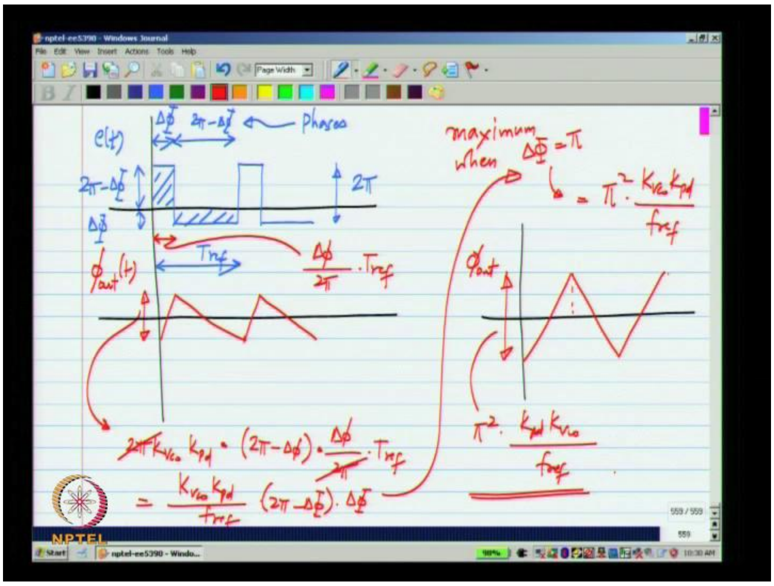

As usual, I suggest that you take it as an exercise and do it yourself, pause the video at this point, and then compare it to the result that I derived. So, here I have shown two cycles of e of t. Now this is going through an integrator, what does it mean, the output will rise during this part when e of t is positive, and it will fall during that part, when e of t is negative, and also, we know that, this area equals that area. So, the amount of rise equals the amount of fall, this would have to be the case anyway because the system is in study state, otherwise this quantity phi out will be continuously increasing or decreasing.

So, this is what it looks like, this is phi out of t and it will be some sort of triangular wave whose peak to peak value equals the area under the positive part of e of t times the proportionality constant 2 pi K v c o times K p d. Now, for e of t, we know that, the peak to peak value is 2 pi and the duty cycle is delta phi divided by 2 pi, and you can calculate it very simply to show that, this has to be 2 pi minus delta phi and that has to be delta phi.

So, here again delta phi refers to the phase difference between the reference and feedback at the operating point. So, you can clearly see that, 2 pi minus delta pi times delta pi that is the positive area equals delta phi 2 times minus delta phi, which is the negative area. So, the peak to peak value here will be simply 2 pi K v c o K p d, which is the proportionally constant times this height, which is 2 pi minus delta phi times this width by the way what I have shown here is in terms of phases, but what I need here is width in terms of time, and we know that phase shifted 2 pi corresponds to t ref.

So, that is t ref. So, this duration is nothing but, delta phi divided by 2 pi times t ref. So, what I have here will be delta phi divided by 2 pi times t ref. So, this can in turn be written as this goes away and t ref is one over f ref. So, the phase error and the output due to the periodic error in the phase detector is a triangular wave whose peak to peak value is given by this expression K p d times K v c o divided by f ref times 2 pi minus delta pi times delta phi. So, this clearly depends on delta phi but, we can easily see that, this is maximum, when delta phi equals pi.

So, this term is increasing as delta phi decreases, and this term is increasing as delta phi increases, the two will be equal, when delta phi equals pi, and at that point for this value this will be equal to pi square K v c o K p d times f ref. So, in that case also the duty cycle of this will be fifty percent, and this phi out of t will be a triangular wave, which is symmetrical.

So, now, that we have calculated phi out of t then what, what we need to do is to look at what the output should be in the ideal periodic case, and look at what it will be with the addition of phase error phi out, and see what the effect is. In fact, this kind of calculation will appear repeatedly in phase lock loops and in general wherever you are looking for periodic signals.

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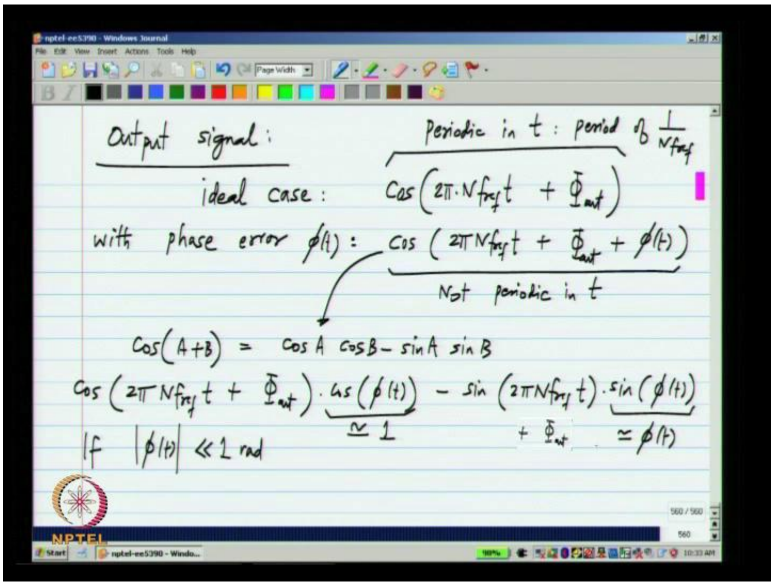

What is the output signal in the ideal case, it is just a periodic signal, again I will show it as co sine, but I will emphasize that the pulse shape can be anything, the wave shape can be anything, co sine plus some arbitrary phase phi out. Now, with the phase error phi of t what you get is. Now, clearly this function is periodic in t with a period of 1 over n f ref, and this is in general not periodic in t, if phi of t is not zero.

So, what we will think of it as an ideal periodic signal plus some error, and that can be got by expanding this, we know that cos a plus b is cos a cos b minus sin a sin b. So, this whole thing will be, and if the magnitude of phi of t is very small compared to 1 radiant, what happens for very small values phi of t, this part is approximately equal to 1, and this part is approximately equal to phi of t.

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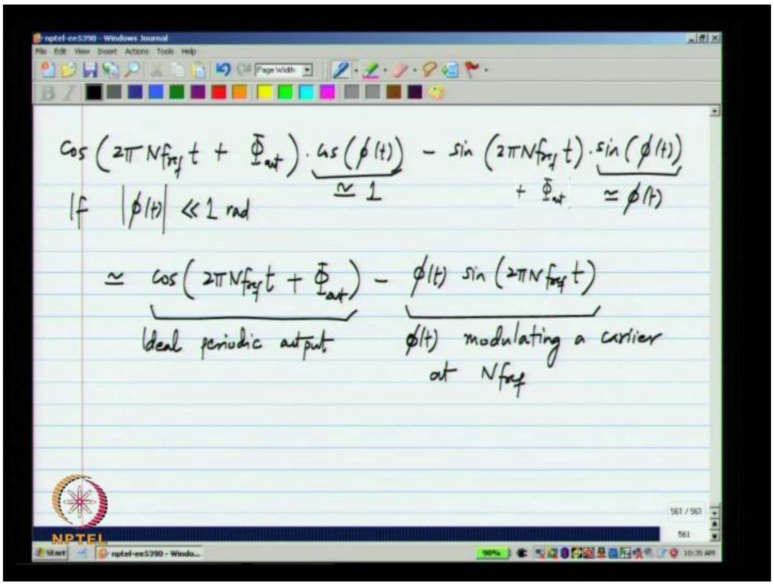

So, what we finally get, in presence of a phase error is this, we will get this to be this is the ideal periodic signal that we would have got plus an error, and what is this error this is a signal which is again periodic at frequency n times f ref, and this phi of t this phase error modulating that carrier. Now, there are two ways to characterize a signal that is almost periodic, but not exactly. So, and this is just like any other signal, we can do it in time domain or in the frequency domain. So, in the time domain it is characterized simply by this phase error phi out of t, ideally this phi out of t is 0 in reality, it is not zero, and this leads to the description of aperiodicity in terms of what is known as jitter.

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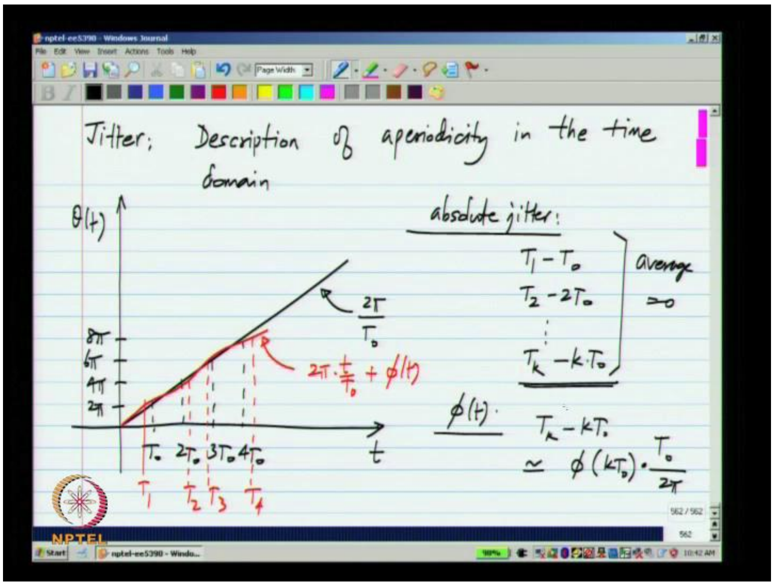

So, let me draw the phase of an ideal periodic signal it will be a ramp, and it will cross integer multiples of 2 pi at integer multiples of some interval T naught, which is the period of the periodic wave form. This is phase versus time and this will be a ramp with a slope of two pi by t naught. I mean reality, there will be some phase error and the phase may do something of this sort. What happens then, this will cross 2 pi at sometime T 1, its crossing 4 pi at sometime T 2, and its crossing 6 pi at sometime T 3, and 8 pi at some T 4 and so on.

And now, ideally there is T 1 T 2 T 3 and T 4 will be integer multiples of t naught. In reality they will be different. The absolute jitter is nothing, but the difference between the ideal crossings of 2 pi and actual crossings of 2 pi. So, in general T k minus K times T naught, and the average value of this will be 0, because the average slope of this red curve and the black curve are the same. So, the average value of the jitter numbers will be 0, sometimes this jitter will be negative and sometimes it will be positive.

Now, this jitter is directly related to the phase phi of t, now, what is this red curve after all, it is 2 pi times T by T naught 2 pi times the frequency of the signal times T plus this phi of t, and for small values of this error that is small values of jitter here to make it visible I have shown it as exaggerated number, but we can assume this to be relatively small compared to one period.

So, in that case these deviations can be approximated by the value of the phase at K T naught times T naught divided by 2 pi. So, this phi is in terms of phase in radiance now, we have to translate it to times. So, if you want to do that then the scaling factor of T naught by 2 pi will appear. So, the jitter values will be approximately phi of K T naught times T naught divided by 2 pi there will be a minus sign over here.

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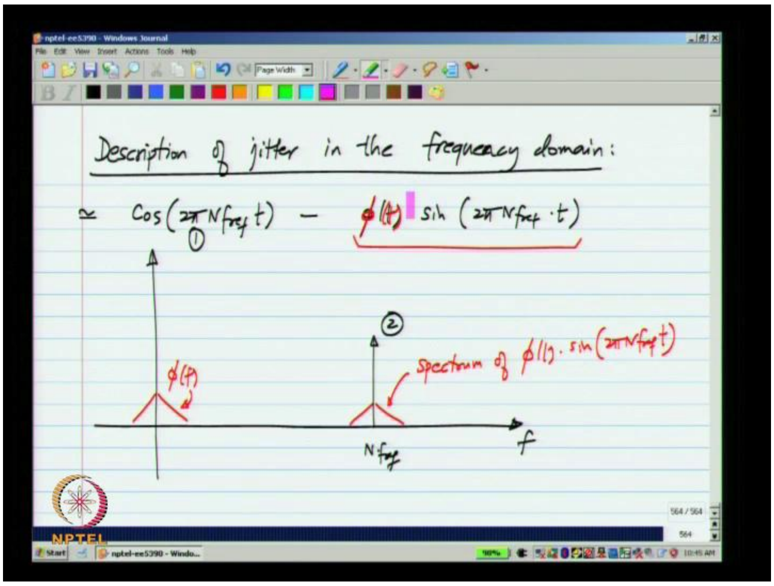

In terms of time, it will be, where this phi is basically the phase error that appears and this is the description of aperiodicity in the time domain its commonly used in certain kinds of applications, but in communications applications phase lock loops are used very it is also common to use jitter in the frequency domain, and now in our case we have evaluated the output to be cos 2 pi and f ref t minus phi of t sin 2 pi N f ref times t, when phi of t is very small.

Now, what is this in the frequency domain, the first cos 2 pi N f ref t corresponds to an impulse at n times f ref. So, this is the first part of it, and the second part it is phi of t modulated, but sin 2 pi f in ref times t. Now, let me say that, phi of t has a spectrum of the sort that is phi of f. Now, phi f t times sin 2 pi N f ref t will have a spectrum that is centered around n times f ref, because this phi of t here is modulated to n times f ref.

So, what is supposed to have in an impulse as an impulse and something on the side, now exactly what is on the side depends on the spectrum of phi f t, now, in our particular case, we have evaluated the effect of e of t to be a periodic phi of t it is a triangular wave. So, if phi of t is periodic; that means, the spectrum of phi of t will consist of impulses and those impulses will also get modulated to n times N f ref and now later we will see different kinds of phi of t.

Let us say, there will be something called the v c o phase noise in which case the spectrum of phi of t will not be a set of impulses, but some continuous function and that continuous stuff will also get translated to n times f ref. Now, this is how we interpret the result from the model of the phase lock loop. The results that you get are in terms of phase, and then you either interpret that in the time domain of the jitter or in the frequency domain as this sidebands or phase noise as the case may be.

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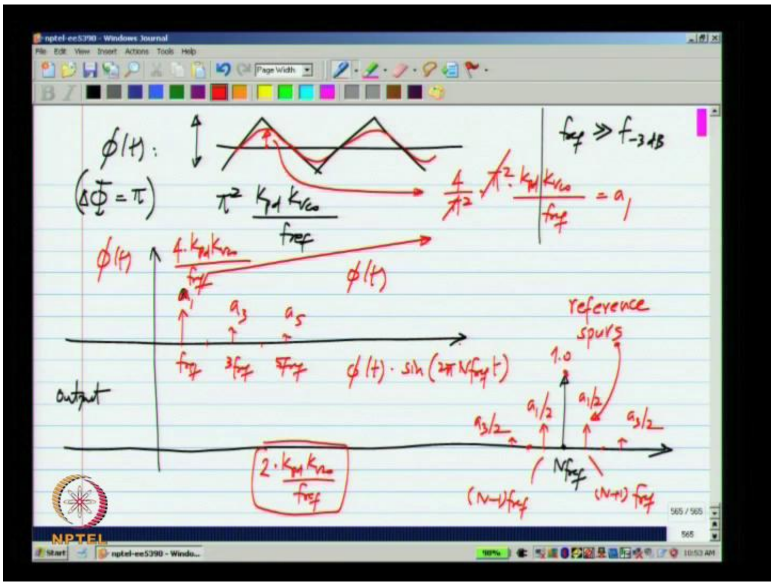

So, in our case what is phi of t it is a triangular wave I will take 1 with the highest peak to peak value this is for delta phi equals to pi, and we know that this peak to peak value happens to be pi square K p d K v c o divided by f ref. By the way all of these things are under assumption that f ref is much more than the bandwidth and this is a very reasonable assumption as we will see soon.

Now, what we will be the spectrum of the phi f t, it will consist of impulses at f ref 3 f ref 5 f ref and so on. Now, if the triangular wave is asymmetric, we will also have a component set 2 f ref and 4 f ref etc. We have considered the symmetric case or delta phi equal to pi it will be symmetrical. Now, what will be the spectrum of the output as I said earlier, it will consist of an impulse corresponding to the cos 2 pi N f ref t and all these impulses of phi f ref will also get modulated to n f rest.

So, we will have and so on. This will be at n plus 1 times f ref, and this will be at n minus 1 times f ref and so on. So, in general there will be at spacing of integer multiples of f ref from n f ref, which is the desired output frequency. So, what happens is if you have a periodic error the output will consist of these other sidebands in the frequency domain, and the strength of the sideband should be limited to some value. So, that we can still think of the output as sufficiently good periodic signal, and the sidebands are known as reference spurs.

Spurs is a shorthand for spurious components, and these are the spurious components in the output, this is what should have been there, but we also get these things, but if these things are sufficiently small, the output can still be thought of as periodic. So, now, what we should do is calculate the strength of these things, and from there calculate the difference between the strength of the main output, which is supposed to be there and the strength of these spurs which is not supposed to be there, and make them sufficiently large.

And, this is something like making sure that the signal noise ratio of a signal is sufficiently large. So, that is what we have to do and how do we go about doing that we have to calculate the Fourier component triangular wave, and when you modulate it by sin 2 pi n ref t, what happens is half of it goes to the positive side, and half of it goes to the negative side, what I mean is let us say, this is a 1 a 3 a 5 and so on.

And, when you multiply this is the spectrum of phi t, and when you take phi of t sin 2 pi N f ref t, what happens is you will have a 1 by a 2 here a 3 by 2 there, and also a 1 by 2 there and a 3 by 2 there and so on. And, In general for any signal that you have here, you can calculate the spectrum of the modulated signal you would be very familiar with this from communication systems and so on. So, here what I will do is not calculate it exactly for all the components, but do it only for the fundamental components that is the component at f ref, and you know that in general for most signals these components tend to get smaller as you take the higher order harmonics. So, that is not universally true, but in this triangular wave that is true.

Now, what is the value of a 1 you can consult some standard text books on fourier series. So, now, it turns out that the fundamental component of the triangular wave, will have a peak that is slightly less than the peak value of the triangular wave. So, it turns out that the peak value of the fundamental component of the wave will be 4 by pi square times the peak to peak value of the triangular wave.

So, that is the peak value that is the 1. So, this you can get from some standard text book on communications or Fourier series. So, this a 1 will be 4 times K p d K v c o divided by f ref as a 1 by 2 will be naturally 2 times K p d K v c o divided by f ref. Now, we want this to be sufficiently small compared to the main one which has a strength of 1, this has a peak value 1, and this will have a peak value that I just now calculated a 1 by 2, and we will look for a relatively modest requirement.

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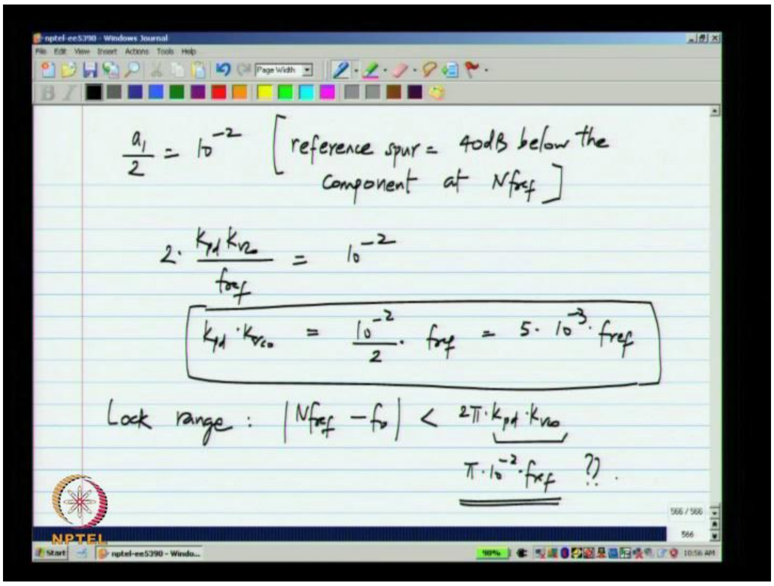

We will say that a 1 by 2 is 10 to the minus 2, what this means is the largest reference spur is forty d b below the component at N times f ref. Now, this is a relatively modest requirement usually you like this to be sixty d b or eighty d b below the component at N times f ref. Then all we have to do is plug in the numbers 2 times K p d times K v c o divided by f ref should be 10 to the minus 2 or K p d minus K v c o is 10 to the minus 2 divided by 2 times f ref or 5 times 10 to the minus 3 times f ref.

So, this is the constraints that we get from this phenomenon, which is also known as reference speed through the output will have reference spurs to make the reference spurs sufficiently small, we need to make sure that this number is smaller than some value, and also you see that the value a 1 should be smaller than some number; that means, that the value of K p v times K v c o should be smaller than a certain number. Now, let us go back to what we had for the lock range, and the lock range was related to the product of K p g times K v c o.

And now, what does this mean what was the lock range that is the difference between the output and the free running frequency was less than 2 pi times K p d times K v c o. Now

I will plug the value of K p d K v c o from here, and that comes out to be pi times 10 to minus 2 times f ref.

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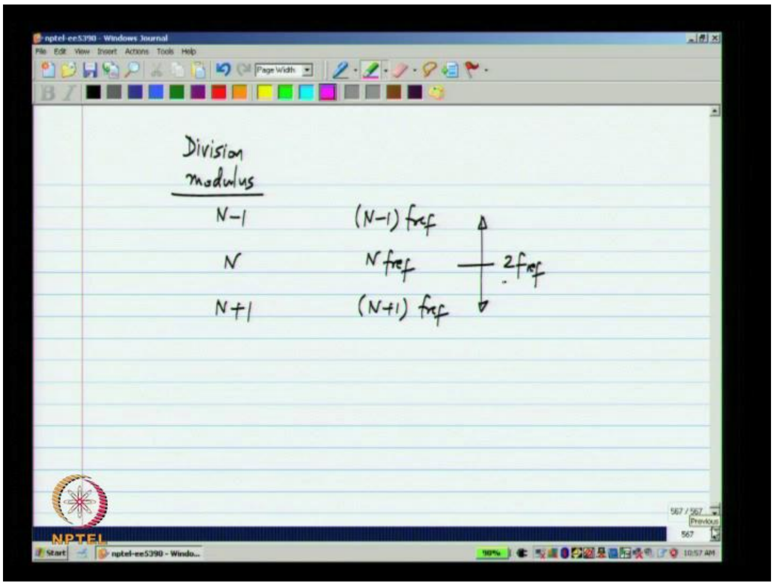

Now, what does this mean one of the things I said was that we would like to change the division module as to change the output frequency. So, if it is N, the output will be N times f ref, and if the module is N plus 1 the output is N plus 1 times f ref, and if this is n minus one it is n minus one times f ref and so on. So, what does it mean let us say I want to move the division modulus by one in each direction.

So, the range has to be at least 2 f ref. So, between these two we have a range of 2 times f ref. now the range that I am getting is much smaller than f ref. So, what this means is I cannot change the value of n at all. So, let us say that for some particular value of N this N f ref happens to be equal to if not I will not be able to change the value of N because I will not be able to deviate more than a very small frequency so that means, that I cannot really make my frequency synthesizer.

So, I hope the result of the calculation is clear, we went through some detailed calculation where do not get distracted by the details, but you need to do this in order to practice calculations involving phase lock loop in the phase domain in the frequency domain and the time domain and so on. But, what is the summary, we have a lock range that is limited by the periodicity of the phase detector to move the v c o from the pre-running frequency.

We have to apply a non-zero control voltage and to have a non-zero control voltage, there has to be some phase difference between the input and the feedback quantities, and that phase difference can only be a maximum of 2 pi in either direction, this is because of the periodicity of the phase detector. So, the lock range is limited to two pi time k p d times K v c o.

On the other hand, the phase detector implementation that we had, has periodic errors and this periodic errors causes periodic fluctuations in the output phase even, when the input is perfectly periodic. So, this means that the output is not exactly periodic what we did was to analyze its spectrum in the frequency domain, and we see that in addition to the components at N times f ref. We also have additional components at integer multiples of f ref away from it.

This phenomenon is known as reference speed through because the references at f ref components related to reference will appear at the output. And the spurious components are reference spurs, and to have a reference spur that is hundred time slower than the fundamental that we want, we find that the lock range is much much smaller than f ref.

So, now, what does it mean for the lock range to be smaller than f ref; that means, that I cannot build a frequency synthesizer, what is a frequency synthesizer I changed the division module as an and I changed the output frequency in steps of f ref. Now, with my lock range limited to less than f ref, I will not be able to change the value of N at all. In fact, at best I will be able to build this phase lock loop for a particular value of N which makes N times f ref f naught, and that is about it, and this is clearly not a satisfactory situation, and we need to find a way around it, and what is the way around it, the key here is that the periodic error that is injected is proportional to delta phi, this we have seen.

So, the periodic error injected will be proportional to delta phi, and delta phi, if it happens to be 0, there will be no periodic error at all. And now, this makes sense because, if the input and feedback pulses are aligned exactly the output of the phase detector will be continuously 0. It is supposed be 0 and there is no periodic error at all. In that case, there will be no periodic modulation of the v c o. So, we will try to exploit this to come up with a better phase lock loop in which case, this tradeoff between the lock range and the reference speed through is broken in summary.

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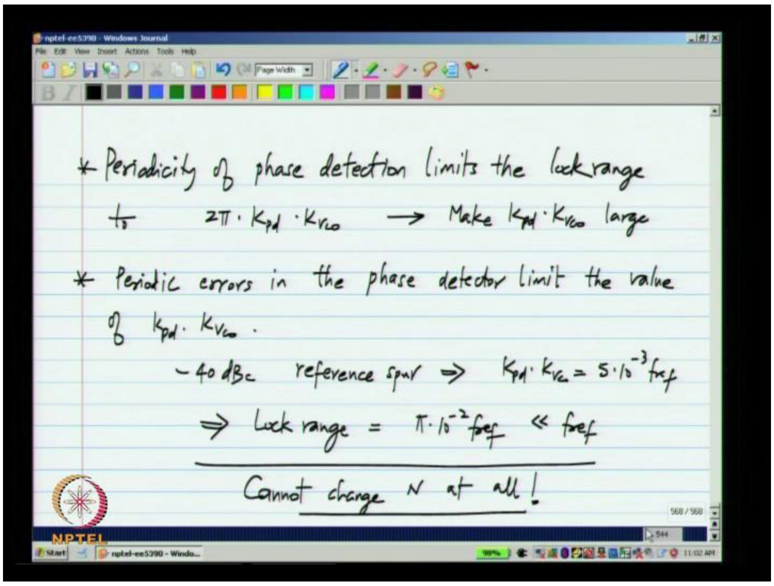

So, to have minus 40 d b c this is a unit that is used to show that it is relative to the main component. This means that lock range, which is this basically means that you cannot change N at all. In fact, you may not be able to build the p l l for any value of N. Now, if the free running frequency of the v c o happens to be equal N f ref for some value of N you will be able to build it for that, but then you will not be able to change the N for that in the next lecture, we will see how to get around the state of by introducing some refinements of phase lock loop. Thank you.