Analog Integrated Circuit Design
Prof. Nagendra Krishnapura
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 51
Oscillator Phase Noise

Hello and welcome to lecture 51 of Analog Integrated Circuit Design. In the past few lectures, we have discuss the type 2 phase lock loop and also we discussed the effect of random noise from the charge pump and the loop filter on the output phase noise of the phase lock loop. Now, there are two important sources remaining that is, the phase noise of the oscillator itself and it is contribution on the phase noise of the phase lock loop, also the phase noise of the reference oscillator.

The reference is the periodic source and it will also have it is own phase noise, now what we will do here, is to first do some analysis based on an idealize model of an oscillator, to get a field for the general nature of phase noise in oscillator. We will not try to exactly calculate the phase noise from the device noise sources that is, rather to complicated and simulator is usually used for that. We will just see, why the phase noise of the oscillator has a certain shape that is commonly assumed.

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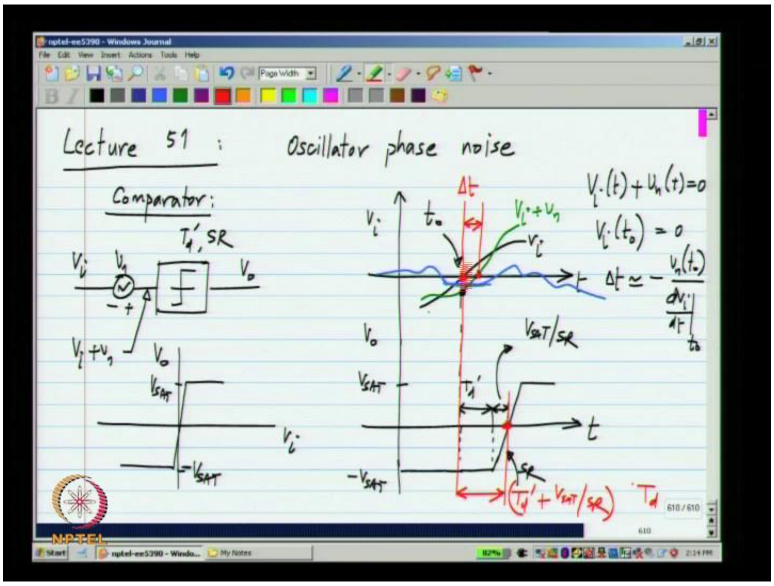

Now, what is an oscillator, oscillator there may be models that we can consider such as, resonance circuit with a negative resistance to cancel it is loss or a comparator in the negative feedback and so on. We will use the comparator in negative feedback that model, because it turn out to be easy for this particular aspect that is, to determine the nature of phase noise. And what is the kind of comparator that we will assume, the comparator can be thought of us nothing but, a high gain amplifier with some saturation levels.

Well sure does this and we would expect that, the characteristic V naught verses V i would be something rather very high gain and saturation. We will approximated by an infinite gain and let us assume symmetrical saturation levels of V SAT and minus V SAT and no comparative will make a discussion instantly. We will see that, it has a certain delay T d prime and also the output, if the input crosses 0, the output does not change instantaneously and let us say, it has certain slew rate.

Now, what happens, let us say that, the input verses time it is changing in some manner like this and here is where the input crosses 0. And what happens to the output, before this instant, the input was negative, the output would be at minus V SAT. And at this point, input crosses 0, but let us say that, the output starts changing after the delay of T d prime. This is because of the delay in the comparator and then, it starts rising at a certain a slew rate S R and finally, we will reach plus V SAT.

And it takes some time to rise from this minus V SAT to 0 and that amount of time is nothing but, V SAT divided by S R. So, effectively there is a delay of T d prime plus V SAT by S R, between the zero crossing of the input and zero crossing of the comparator. So, this is one thing to keep in mind and we cannot denote this entire thing by delay T d that is, the delay between zero crossings. The way you have initially modeled it is that, the comparator takes some time to start responding and after that, it also take some time to rise up, so the net effect is capture in this parameter T d.

Now, in addition to these things, we will also have noise and like with the most amplifiers, you can represent the noise of a comparator with some input refer noise source V n. So, what happens is that, the actual input of the comparator is V i plus V m, so let me reverse the polarity here. So, let us say the noise is of this value at this particular point around this point, it will also changing in some random way. The noise could be doing something other shot, but I was assume that, it is of this value over this into all.

So, V i plus V n will be this thing with some shift, it looks something, this is V i plus V n, whereas this is V i. Now, you see that, the zero crossing of V i plus V n is obviously different from the zero crossing of V i. And let say that, there is a time shift delta t between the zero crossing when noise is present and the zero crossing when the noise is not present. Now, how much is this delta t, we have to calculate V i plus V n and find it is zero crossing.

Now, that is generally difficult thing to do, but if we assume that, the noise is not changing much over this interval delta t. We can see that, noise can assume to be constant, so V i has been simply shifted down by amount of noise and amount of noise can be computed at the original zero crossing that is, what we want to find out is, where V i of t plus V n of t equals 0. And if I call this time instant, the original zero crossing p naught, I know that V i of T naught is 0. Now, the shift delta t can be calculated approximately as the negative of V n at T naught divided by the rate of change of the input.

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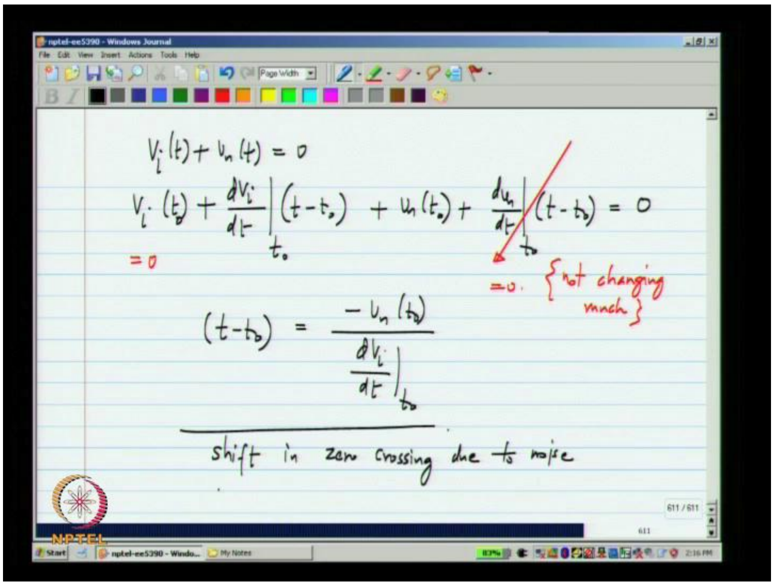

So, simple algebra shows that this is the case, if you want, you can expand it in Taylor series, V i of t plus V n of t must be 0. If we expand it in Taylor series, can retain only the first order term V L of T naught plus d V i by d t evaluated at T naught times t minus T naught and similarly for noise, V n of T naught plus d V n by d t evaluated at t naught, t minus T naught must be 0. What we assume is that, the noise is not changing much around the zero crossing, which is a reasonable assumption, this is 0 and this we know is 0, V of T naught that is the definition of t naught.

So, the shift in the zero crossing is nothing but, the sample of the noise at the original zero crossing divided by the slope of the signal at the zero crossing, so this is the shift in the zero crossing due to noise. To summarize, we have a comparator and if the input crosses 0, the output will cross 0 after a certain delay T d and that T d is composed of the starting delay of the comparator and something related to the slew rate of the comparator.

And in addition to this, if the comparator has an input referred noise as most real comparator do then, the output zero crossing will be shifted from the ideal zero crossing by certain amount. And the amount is the sample of the noise at the original zero crossing divided by slew rate of the input signal, also evaluated at the zero crossing, keeping these things in mind, we can analyses the oscillator and it is noise.

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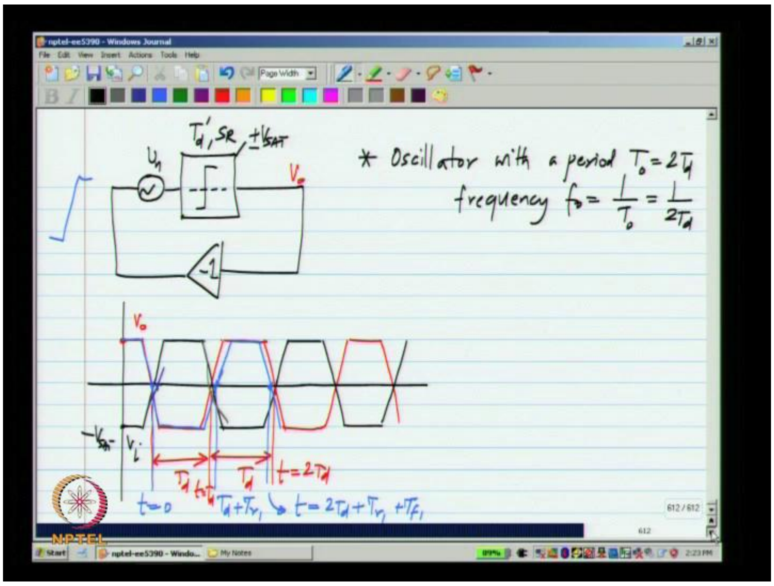

The comparator is characterized by delay slew rate and the saturation voltages and to make an oscillator all I do is, put it in negative feedback. Now, what happens in this case, let us say initially that, the output is at positive saturation level, that is V o, this is V o. So that means, that the input will be at the negative saturation level, minus V SAT and this is V i. What happen is, if the input is V i, after the certain delay, the output will changeover also to minus V i.

We do not know what the initial condition was, so we will say that at this certain point, it will cross over to minus V i. Obviously, the input will do the opposite and it will do that and the output will again switch to the positive value after delay of T d this is something we have seen. At this point, the input is changing to positive value, so the output will change to a positive value after a duration of T d, which is T d prime plus V SAT by the slew rate.

And now of course, input also switch sign and the output will change it is sign after the delay of another T d and this will gone repeating. So, this will keep happening and you see that, the output is periodic with a period T naught of 2 T d or an oscillation frequency f naught is 1 over T naught, is 1 over 2 times T d. So, this is what going to happen, now the question is, what happens in presence of an input referred noise. And presence of an input refer noise, let us first assume that, the first zero crossing appears over there, whatever this time is that, is time of first zero crossing and will call that, t equals 0.

Now, what happens is that, the input changes from minus V SAT to V SAT at that instant and the output must follow it, after the delay of T d. So, the input change is like this and the output must follow it after delay of T d, but what happens is, because of this noise, the input here does not change it is sign at this particular instant. So, what it will do is, the input may change it is sign at a slightly different instant. So, the output will change it is sign at that instant and let me call that, if this was t equal to 0, originally this would have been t equals T d that is, the time of the first zero crossing.

Now, it is something else and I will call that T d plus tau r 1 and r refers to the rising part of it, we will remove all these intermediate variables later, but that is what it is for. And then, what happens, the output will of course change to positive saturation value. And the input should have done that, again because of the noise, it will not change it is sign exactly at that instant, so it may do something like that, it could be even earlier. So, what the output is do is, it will also go down a little earlier compare to before.

So, again this would have been t equals 2 times T d and in the presence of noise, it will be t equals 2 T d plus tau f 1, where t au f 1 is this time shift and tau r 1 is the time shift between the red and blue curves over there. It is very easy to see that, these time shifts go on accumulating, this would have been 2 T d plus tau r 1 plus tau f 1, because you are starting from T d plus tau r 1. And then, from there, it would go to 0 at distance of T d, now it does it at T d plus 2 f 1, it is not this shift that is 2 f 1.

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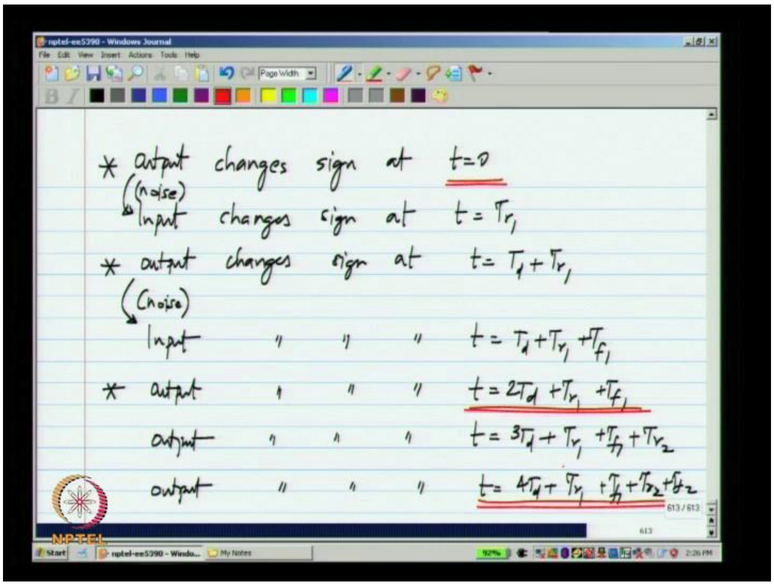

So, the cycle is as follows, let us say the output V changes sign at t equal to 0, this is the definition of t equals to 0. Now, after this, the input which is inverted, changes sign at t equals tau r 1 let us say in presence of noise. Now, because of this, output changes sign at t equals T d plus tau r 1, now input should change sign at the same instant. But, again because of noise, input changes a sign at this plus T d, which is the nominal value plus some number tau f 1.

So, it should have been at 2 T d, so it will be 2 T d plus tau r 1 plus tau f 1, so because of this, the input change sign at T d plus tau r 1 plus tau f 1 and output change a sign at a distance T d from this one, which is 2 T d plus tau r 1 plus tau f 1 and it will go on accumulating. So, the next time the output changes sign will be 3 T d plus tau r 1 plus tau f 1 plus tau r 2 and so on and the following term will be, so it will go on doing that.

Now, if I consider only the completes periods, so this is the starting point 0 and after one period, it will be this one, 2 T d is the ideal period, but you have tau r 1 plus tau f 1 and 4 T d is two periods, but we also have these two additive quantities. So, what happens is, each cycle is different from what it is, what it should have been, because of noise added during that cycle. I will say, noise added during that cycle, it really is the noise sample at the beginning of that cycle.

And also in the middle, because the noise is added during both the positive and the negative 0 crossings. But, it is very easy to visualize that, every period will be different from, what should have been. Ideally it should be 2 times T d, but the input crosses 0 at twice and each of those instances, some noise is added which shift the zero crossing, so the period will be T d plus tau r for that period plus tau f for that period.

Now, if you look at the deviations of the zero crossing from the ideal values, what happens is, let say the first zero crossing is defined to have happened at t equal to 0. The next one will have this error added in the first cycle, the next one will have the error added in the first two cycles and so on. So, it is cumulative, so if you look after n periods, you will have errors added in the first n periods.

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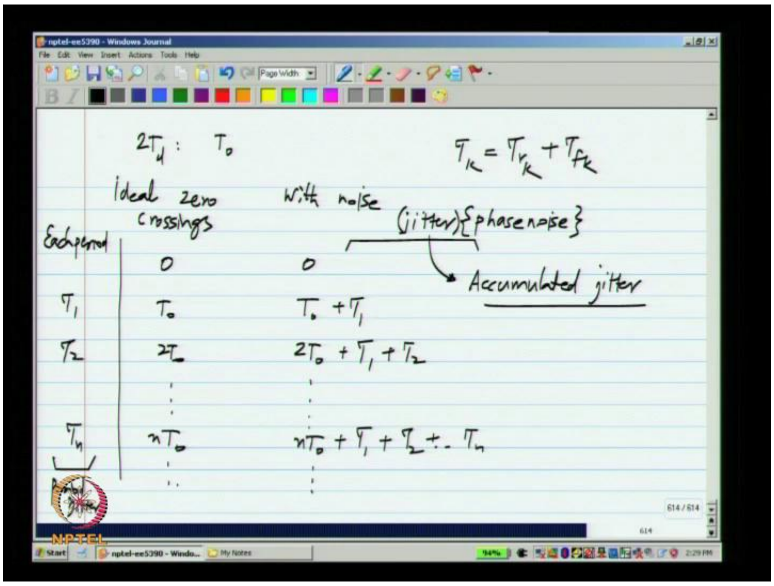

I will define 2 T d to be a period T naught, so the zero crossings, so it would ideally occurred at T naught, 2 T naught. First one is at 0 then, T naught, 2 T naught and so on and with noise what happen is, I have 0 T naught plus tau 1, where tau is basically the sum of the shift in the rise and fall adjust in that particular period. So, it will be 2 T naught plus tau 1 plus tau 2 and so on and T naught plus tau 1 plus tau 2, all the way upto tau n.

So, this will go on, so this part here, this is the jitter and represented in a different way, it will be the phase noise. So, these are the errors in the positions of the zero crossings, correspondingly you can think of the mesh errors in the phase of the signal. So, what will be the phase noise spectrum look like, by the way this jitter which is the total jitter that is, you assume that, you have an ideal oscillator and a noisy oscillator and both of them have their first zero crossing at t equal to 0.

Then, you measure the differences between corresponding zero crossings in the following cycles, the resulting difference is known as jitter, it is one particular kind of jitter, it is the accumulated jitter. Now, if you look at each period, tau 1 is added during the first period, tau 2 is added during the second period and so on and tau n is added during the n th period. These quantities taus are known as period jitter, so accumulator jitter is nothing but, cumulative sum of the period jitter values. This makes sense very easily, the oscillator is an autonomous system, there is no input, it will start at some point. And then, at every period, it accumulate some error, some timing error because of noise in the circuit and this will go on accumulating.

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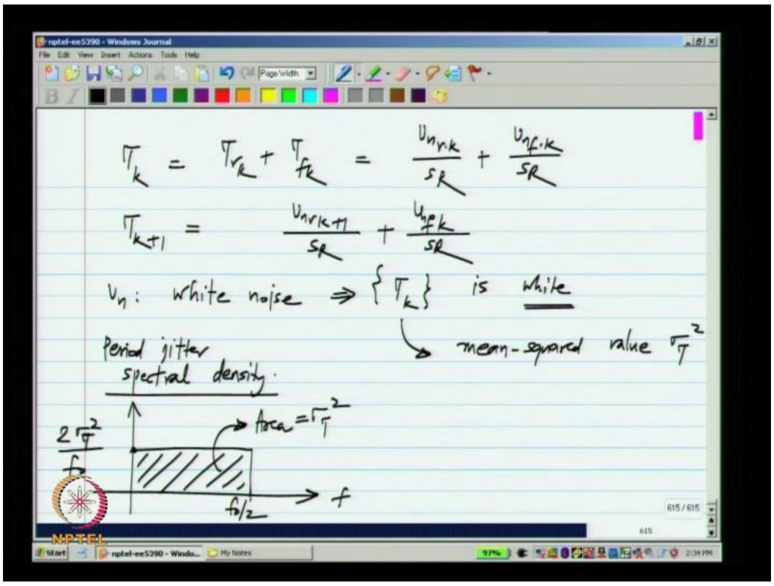

Now, first of all what is the value of tau k that is, sum of rising and fall jitters and that is nothing but, value of V n at the rising edge divided by this slew rate plus the value of V n at the falling edge divided by the slew rate, in fact in the k th cycle also. We would not worry about the details of these things except to say that, it is inversely proportional to slew rate. The values of, if you look at the k plus 1 th period, this will be related to the rise and fall noise values, the noise values are added during the rise and fall times divided by the slew rate.

Now, what is the kind of noise that will be this V n, most frequently this V n would be white noise. It could have other types of components, but we see that, the white noise is fundamental property of the MOSFET and every amplifier including comparator will have some input referred white noise. It may also have other kinds of noise like the 1 over f noise and maybe some other frequency depended noise, because of the frequency dependence inside the circuit, but there will be white noise.

Now, if this V n is white, we can easily see that, the period jitter values tau 1, tau 2, etcetera also form a white sequence. Because, from these expressions you can see that, the value of jitter added in the k th cycles is uncorrelated from value of jitter added in the k plus 1 th cycle. This is because, V n itself is white and the noise added in one cycle and the noise added in the other cycle are uncorrelated, so this means that, the periodic jitter sequence tau k is white.

And let us assume that, it has some mean square value or the variance, sigma tau square, all that we are saying here is that, the period jitter values are white that is, noise added in each period is uncorrelated from noise added in other periods. This is sort of easy to understand, we have encountered white noise earlier and this also comes from white noise. Now, what we are interested in for phase noise is the accumulated jitter, the accumulator sum of taus.

So, first of all, we will evaluate the spectral density of this tau k with the spectral density of period jitter. So, note that, the period jitter is a set of samples, sampled at frequency f naught that is, the frequency of the oscillator of itself. So, it will have energy upto f naught by 2, the Nyquist frequency and also the area under this spectral density has to be equal to the mean square value of variance, which is sigma tau square.

So, it is easy to see the level of the white spectral density that is, simply 2 times sigma tau square divided by f naught. So, that is the spectral density of the period jitter, now how is the phase noise related to the period jitter. Phase noise is really the phase difference between the ideal signal and the signal with noise. Now, here we have a time differences and to convert time differences to phase differences, we have to multiply by 2 pi divided by the period.

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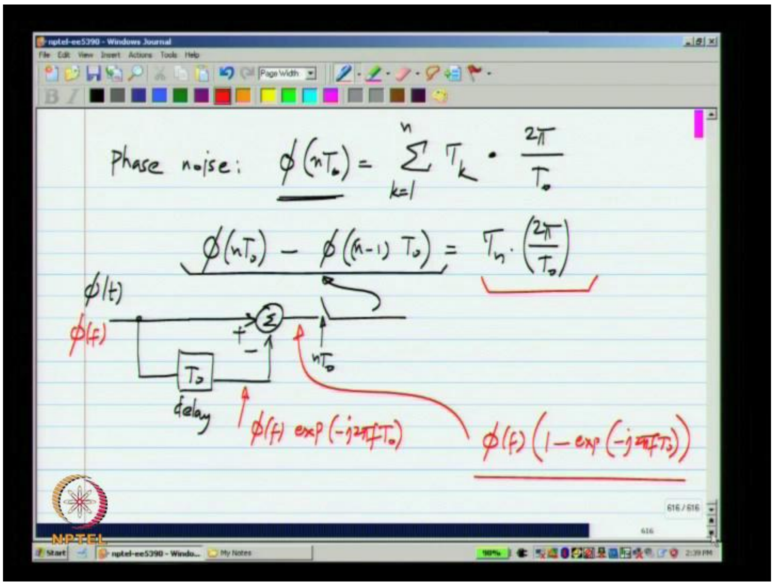

The samples of the phase noise at integer multiples of T naught will be summation from 1 to n tau k and the whole thing multiplied by divided by 2 pi divided by T naught. To calculate the spectral density of phi of t, what we do is, first we take the positive difference of this phi of n T naught minus pi of n minus 1 T naught that is, the difference between the summation for n and n minus 1, so that leave only the term for n, this tau n times 2 pi divided by T naught.

Now, what is phi of n T naught minus pi of n minus 1 T naught, you have this continuous process phi of t, you delayed by a delay T naught and sample the output at n T naught. The resulting samples will be exactly this, it will be phi of n T naught minus phi of n minus 1 T naught and also assuming that, there is an significant earlier sign and so on. The spectral density here and there that is, before and after sampling will be exactly the same, there will be different if there is the earlier sign, which we have assume is not there.

The signal here is phi of t minus the same signal delayed by T naught and in the spectral domain, if I represent these, if this is phi of f, this will be phi of f exponential minus j 2 pi f T naught. And the spectral density of that is the spectral density of this quantity, scaled version of tau n. So, what is that, the signal here is phi of f 1 minus exponential minus j 2 pi f T naught.

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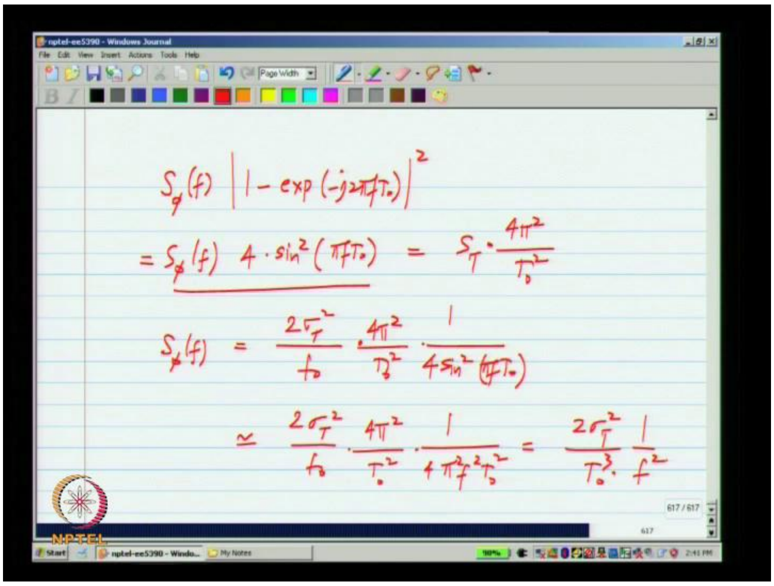

And that spectral density would be S phi of f times the magnitude of 1 minus exponential minus j 2 pi f T naught square, which is given by and this will be equal to the spectral density of the period jitter times 4 pi square by T naught square. So, here we have 2 pi by T naught, we will have spectral density of this multiplied by 4 pi square by T naught square. So, what does this mean, S phi of f is S tau already calculated that is, 2 sigma tau square divided by f naught times 4 phi square by T naught square and divided 4 sin square pi f T naught.

You see that, this spectral density is significant at lower frequencies when the sign function goes towards 0 and that is where, we want to estimate the phase noise. So, I will approximate the sign by the argument and get 2 sigma tau square divided by, I have T naught to the 4 and f naught, so T naught cube, f square let me separate out the 1 over f square dependence. So, the main lesson to take on from this is that, the phase noise of an oscillator has 1 over f square dependence.

The other way of driving this, I feel that this is an easy way of looking at it, basically the principle is that, in every period some jitter is added. And if the noise of the amplifier or whatever you device use for the oscillator is white then, the jitter added in each period will be uncorrelated from jitter added in other periods. And the total jitter is simply the cumulative version of this jitter, the jitter added in the second period, the jitter after the second period is the jitter added in the first two periods and so on.

So, after n periods, it will be the jitter added in the first n periods, because of this we get a cumulative sum and the spectral density of that reduces with frequency and for low frequencies, it reduces as 1 over f square. So, this is an approximate way of appreciating all oscillators have a spectral density of 1 over f squared. Now, the same results can also be derived most famously by what is known as Le Chant model which assumes that, you have an L C resonate network and you have a negative resistance to cancel it.

And then, that is done in the frequency domain, but this time domain accumulation is probably easier to understand. Now, the comparatives noise always will have the white component, but it may have other frequency dependences such as, the 1 over f noise. And it turns out that, because white noise gives you 1 over f square dependence, the 1 over f noise will give you 1 over f cube type of dependence. Now, there are lot of satellites involved here, which have glossed over such as, a sampling the noise and so on. I will not deal with them, but all oscillators you will find that, you have 1 over f square region that is, the region of frequencies, where the phase noise goes as 1 over f square. And also in many oscillators, you may have find significant region, where it goes as 1 over f cube, this is because of 1 over f noise of MOS transistors.

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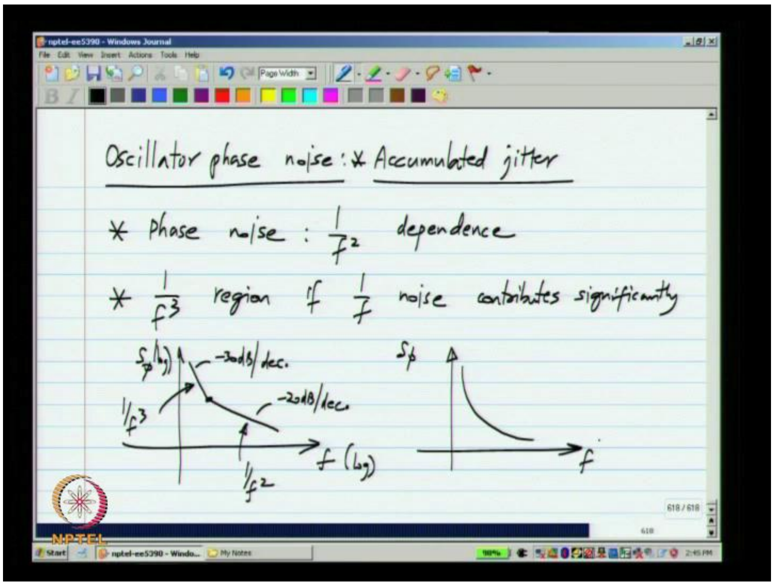

The phase noise will have 1 over f square dependence and 1 over f cube region, if 1 over f noise contributes significantly, so if you plot S phi verses f, both on a log scale, you will see a minus 20 degree per decade. This is the spectral density, which has a 1 over f square dependence, so it is a minus 20 dB per decade slope and there is also a minus 30 dB per decade slope and if you plot it on a linear Y axis, it looks something like that. Now, what does it mean, to have this particular phase noise spectral density, this is the spectral density of phi of t, the term phase noise means something slightly different from this, which I will show shortly.

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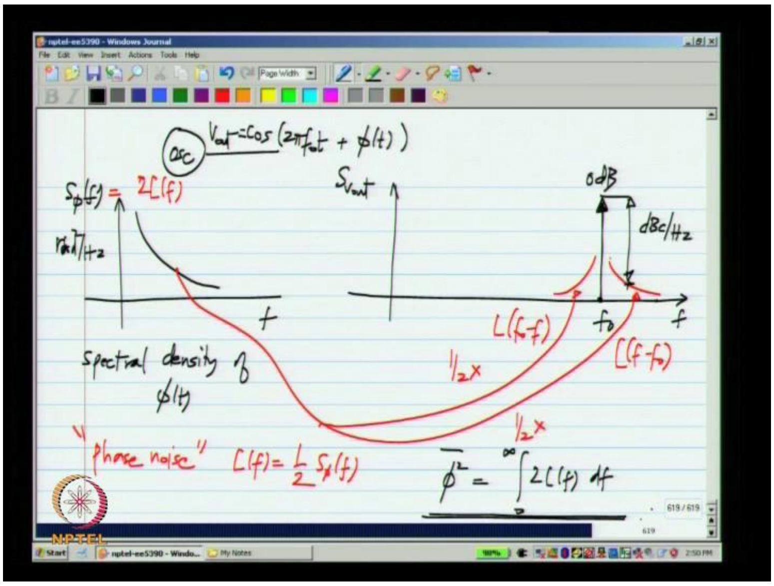

So, if we have an oscillator whose output is, let say cos 2 pi f naught t plus phi of t, as I said many times earlier, this does not have to be across, it can be any other pulse shape. And let say, this spectral density of phi of t that is, S phi of f verses frequency is like that then, the output signal, let me call it V out, verses frequency will have something which can be approximately thought of as an impulse at f naught. And this goes into a two side banks, above and below f naught.

We will have that and that also goes over there and each of them will contain half the power of this S phi of f. So, the phase noise, the traditional definition of phase noise is L of f, which is half of S phi by of f. Then, the actual oscillator will have the upper side band, which is L of f minus f naught and the lower one, which is L of f naught minus f. And because of used unit amplitude here, this has unit power 0 dB and this S phi is an radian square per hertz, basically some dimensionless this quantity divided by hertz.

So, here it is measured as the difference between 0 dB and this value in hertz and that usually referred to as d B c per hertz that is, d B relative to the carrier level. This represent, this impulse here represents the power of the carrier, again I have to say that, this is only an approximate representation. In reality, if you have an ideal periodic signal you will see an impulse, if you have a phase noise you will not see an impulse, you will see something that is neared out.

But, for small values of phase noise, which is of course always the case, you can think of it as this impulse plus some additional noise side bands. And a noise side bands have dimensions of 1 over hertz and they are measured relative to the carrier and hence, this phase noise L of f is measured in d B c per hertz. And this L of f is nothing but, half of the spectral density of a phi of f and sometimes you may have to calculate the variance of this phase or maybe the variance of the jitter.

Variance of the jitter is simply T naught square by 4 pi square times variance in the phase. If you want to do that, you have to double phase noise that is, take 2 times L of f to get S phi of f and integrate it from 0 to infinity to get the variance. The variance of the mean square value is given by integral 2 times L of f 0 to infinity and any time you have to calculate the variance or the variance of the phase or jitter, this is what you will have to do. And this turns out is true for every oscillator, it will have a 1 over f square type of a behavior in it is phase noise. This is basically, because the oscillator is autonomous system and each cycles starts from the end of the previous cycle. So, the jitter gets accumulated over all the cycles.

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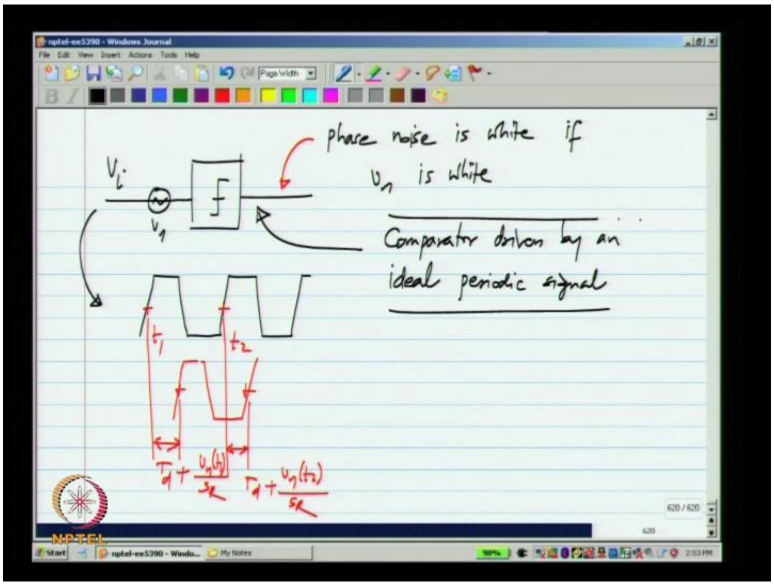

Now, just to contrast this, let us say we have a same comparator and it has noise, let say it is driven by an ideal periodic signal. Now, what happens, the output will have jitter in fact, this is one of the problems that you will encounter, when realizing buffers and so on for the periodic signals. You put a periodic signals through a string of buffers, the buffers themselves will add noise and you have to make sure that, the noise of the buffers is not significant.

So, the output will be something, it should be delayed, the first zero crossing should be delayed by nominally, the value T d. But, in reality, it will be T d plus the noise value at this zero crossing divided by the slew rate of the input signal and so on. And this will be, also T d plus noise value, let me call this t 1 and t 2, V n of t 1 and V n of t 2 divided by this slew rate. So, the point here is, the input is appearing exactly periodically and so, there is no accumulation of jitter.

Now, in this case, if you evaluate the phase noise of the signal, phase noise also will be white if V n is white. And this is for the case of a driven system that is, a comparator driven by an ideal periodic signal. So, if you have driven systems, white noise sources usually result in white noise and if you have a autonomous system like an oscillator, white noise will give you 1 over f square noise.

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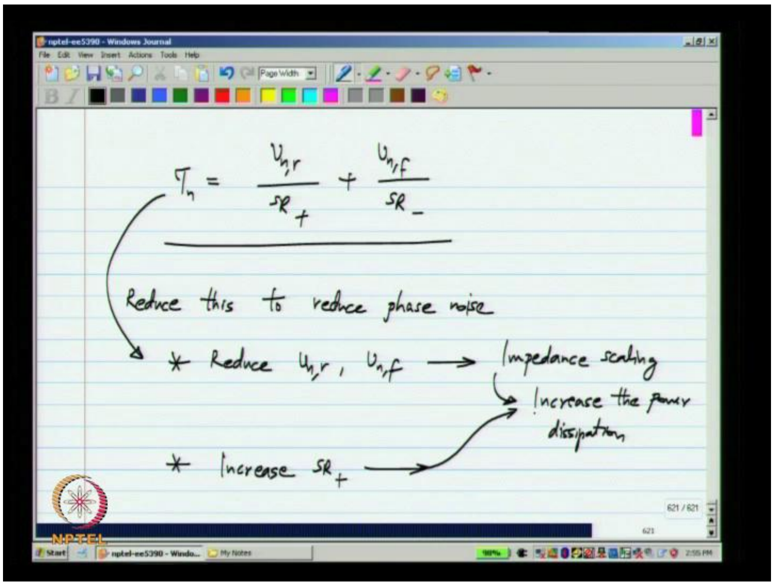

Now, we will look at the oscillator topologies in brief, we see that the jitter added in the n th period is the noise added during the rising edge divided by the rising slew rate plus noise added during the falling edge divided by the falling slew rate. So, how do you reduce this, to reduce phase noise this is what has to be reduced, so if there is a lot of phase noise that means, that there is the lot of deviation from periodicity and you may have to reduce this value.

So, what are the two things you have to do, first of all reduce the noise values that is, you have to basically reduce the input refer noise of this comparator in some way. And this of course, the standard way to do this is, if you are able to find a different circuit that has less noise, that is good. But, for a given circuit, you have to do impedance scaling and if you do this, you will automatically increase the power dissipation.

And we can also reduce the jitter by increasing the slew rate that is, increasing the slopes of signals in your circuit, this also can be done and this also usually involves increasing the power dissipation. So, whichever way you try to compact phase noise, either by reducing the added noise sources in the circuit or by making the circuits faster and faster, you will end up with a higher power dissipation. And there is consistent with everything else that, we know more about noise in circuits also to increase the signal to noise ratio, we have to increase the power dissipation.

Now finally, before we move on to oscillator topologies, why is the phase noise important at all. Now, a jitter is important in many context, you can have digitals circuits, there also jitter can be important, because it basically takes away from the period available for our signal processing. So, jitter is important in sampling, if you use a jittery clock to sample a signal, you will effectively add noise to the signal. So, the higher the precisions or the faster the sampling, you have to have lower jitter. Also there is another communications application, in which periodic signals are used very widely that is, to mix a signal.

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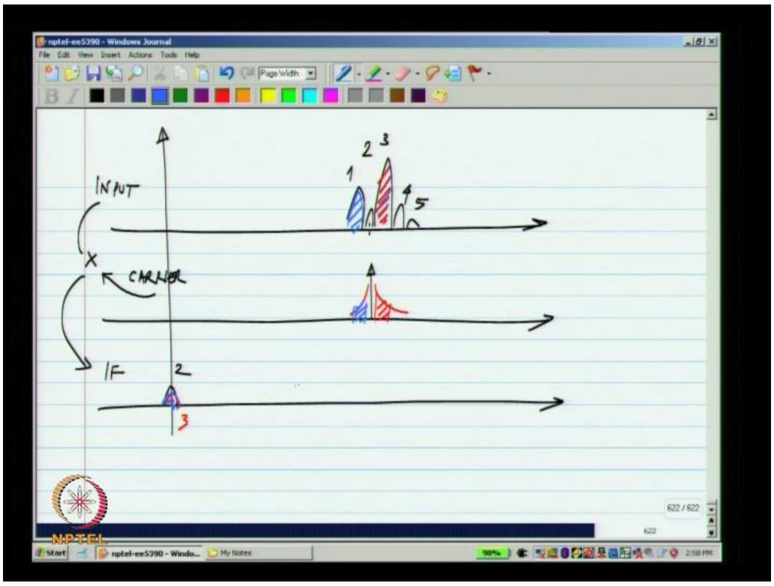

Normally what happens is, the radio spectrum consist of a number of signals, which are separated by a small frequencies and the standard architecture in a radio is to use a periodic signal to translate one of these signals. Let me level these channels 1 2 3 4 5, etcetera to convert one of these signals to a desired intermediate frequency. For simplicity let me assume that, I will use a carrier here to select this channel number 2 and move it to base band.

This is multiplied by this carrier to give you the IF and this channel number 2 must appear over there. But, let say you have these noise side bands with a carrier, what happens is that, this carrier does not have components only at this particular frequency, it has at all this other frequencies as well. Now, this strength here is smaller than that of the impulse, but it is not 0. So, what happens is that, the portion of a this channel number 3 can appear over there that is, from 3 and from 1 also could appear over there.

What happens is, this 3 will get mixed by this part and appear there and 1 will get mixed with this part and appear there and so on. So, it affects the phase noise to increase the noise and reduce the signal to noise ratio of channel number 2 or the desired signal and also the desired signal can be much smaller than the undesired signals, so this effect can be very strong. So, this is why, there is usually a specification on phase noise in oscillator that are used in a radio receivers.

Thank you, I will see you in the next lecture.