Analog Integrated Circuit Design
Prof. Nagendra Krishnapura
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 57
Continuous-Time Active Filters

Hello and welcome lecture 57 of analog integrated circuit design we will continue our discussion on continuous time filters will start from second-order filter prototypes and see how to realize them with op-amps and trans-conductors.

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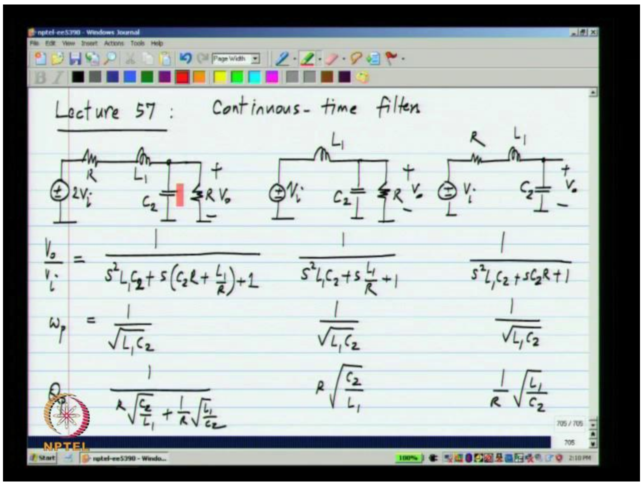

Now, we let us assume that we already have a prototype passive second-order filter and we start from there and synthesize the active counterpart. Now, the business of finding what transfer function is optimal what order we should use and the prototypes and. So, on can be found out from either simulators like mat lab or from sources like the book by zverer our only job is to translate the passive prototype into the active one, and which can be generally done for a any filter of any order.

So, will consider second-order low pass filters and a second-order low pass filter prototype can consist of many forms, which are sort of related this is what is known as w terminated, because there is a termination resistance at the input and termination resistance at the output, and you can also have it only on one side like this, this is an example of a singly terminated filter that may label is L1 and C2, Ll and C2 and this is an example of also a singly terminated the filter in this case the resistance is on the output side and in this case on the input side, and these are driven by the input voltage V i and in the w terminated case I will make the input signal two times Vi, it is just to take into account the attenuation at d c we have R and R. So, the d c gain will be half when the inductor is a short and capacitors open.

So, just to make all of them the same in terms of their output voltages V naught I will apply to V i. Now, the transfer functions V naught by V i can be easily derived for all of them i am not going to do it here, but they turn out to be ((No Audio: 02:55 to 03: 15)) in this case and we will have one by S square L1 C2 plus S L1 by R plus one in this case and similarly one by S square L1 C2 plus S C2 R plus one. Now, the natural frequency if all this filters is the same and that is one over square root of L1 C2 its the same for all of them. A second-order system is characterized why its a natural frequency omega p and the quality factor Q p.

And in this case the quality factor is one by R square root C2 by L1 plus one over R square root L1 by C2, and in this case it will be R square root C2 by L1((No Audio:04:19 to 04:37)), and in this case it is one over R square root L1 by C2. So, these are just three second-order low pass filter prototypes and will say how to turn this into active forms the components that result in filtering are the inductor and the capacitor and we have to synthesize the function of these things in some way without actually using inductors that the idea, because on chip inductors cannot be made of very large value they occupy a very large area. So, we will stick to using active elements and capacitors, if we manage to realize the node equations describing this circuits using only active elements and capacitor we will implement the same function for that we will first have to write down the node equations of these…

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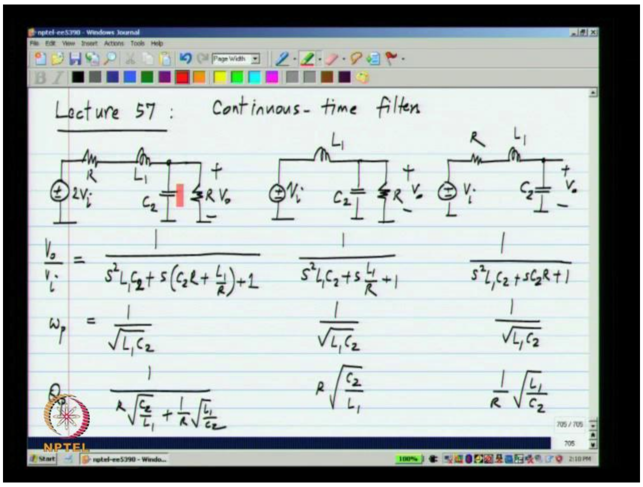

Now, the node equations are in written in terms of the state variables there are two state variables in each case the inductor current I1 and the capacitor current V2, and the output voltage is equal to the capacitor voltage in this particular filter. So, I have I1 and V2 and similarly I1 and call this V2. So, what I will do is relate the voltage across the inductor to its current that is the equation describing one of the state variables, and we have one for another state variables in general, if you have an n th out of filter you will have n such equations.

So, first of all the voltage across this inductor L1 will be equal to S L1 the impedance of the inductor times the current I1 and what is the voltage across this, it is this voltage minus the drop across the resistor 2Vi minus I1 R that is the voltage at this point and the voltage on the other side is by definition V2, and this will be equal to S L1 times I1 similarly the current through the capacitor C2 will be the voltage across the capacitor times S times C2, which is the admittance of the capacitor, and what is the current flowing through the capacitor it is I1 minus the current flowing through this resistor, which is V2 by R, and similarly I will write it for this the voltage across the inductor is Vi minus V2 and that will be equal to S L1 times I1, and a current through the inductor is I1 minus V2 by R, and that will be equal to S C2 V2 and for last one the voltage across inductor is V1 minus I1 R minus V2 that will be equal to S L1 times I1, and the current through the capacitor is nothing, but I1 itself.

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So, essentially we see that certain set of electrical quantities in this case some voltages are equal to this S L1 with this multiplication by S denotes a differential operator times on current, and what is usually done is we make all the state variables voltages, because in our realization of the continuous time filters will not have inductors in place of every inductor it will turn out we will have a capacitor an inductor takes a voltage, and integrates that to produce the current in the inductor a capacitor takes a current, and integrates that current to give a voltage across the capacitor.

So, the function is basically either integration or differentiation depending on the way you think about it, and it can be performed either with the capacitor or an inductor we will do all or operation using a capacitor. So, all the state variables we have in terms of currents that is inductor currents we will normalize them by some resistance are to result in voltages. So, I will define I1 are to be V1. So, that even the first state variable is in terms of a voltage, if I do that these can be written as 2Vi minus V1 minus V2 is equal to S L1 by R times V1 and similarly V1 minus V2 is S C2 times R times V2, and here its the same V i minus V2 will be S L1 by R times V1, and this will be V1 minus V2 will be S C2 R V2, and there V i minus V1 minus V2 will be S L1 by R times V1, and V1 here will be S C2 R timesV2. So, here all the state variables are voltages and this we can realize with a circuit that as only capacitor, where the state variables corresponds to voltage across capacitor.

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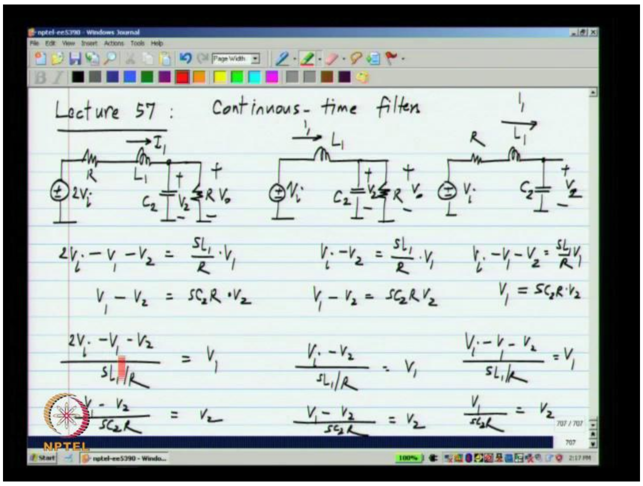

So, it looks like we need to make certain voltages related to derivatives of other voltages now, these equations ((No Audio:10:38 to10:58)) can be equivalently written as...((N o Audio:11:01 to 11:40)). So, these are written in a way that a certain state variable V1 equals some combination of other voltages, which could be state variables are the input voltage divided by S L1 by R that is these combination of voltages is integrated over time. Similarly, V2 is given by this, which is V1 minusV2 and integrated over time gives you V2 and. So, on in all the cases it is like that these equations are in a form, which can be realized as integrator S which integrate a combination of voltages.

So, it turns out that by implementing these equations will get exactly the same function as the original filter now, this is a very convenient form for us to implement as well, because we know that we can make integrators why using a voltage control current source converting the voltage to a current, and then passing the current through a capacitor. So, we have that is how we can implement integration and the output voltage will be the voltage across the capacitor, and that is exactly what we want from this equation. Now, an alternative way of implementing integrator could be using an op-amp in negative feedback. So, that is also possible will first look at how to implement these filters using voltage control current sources are trans-conductors and capacitors.

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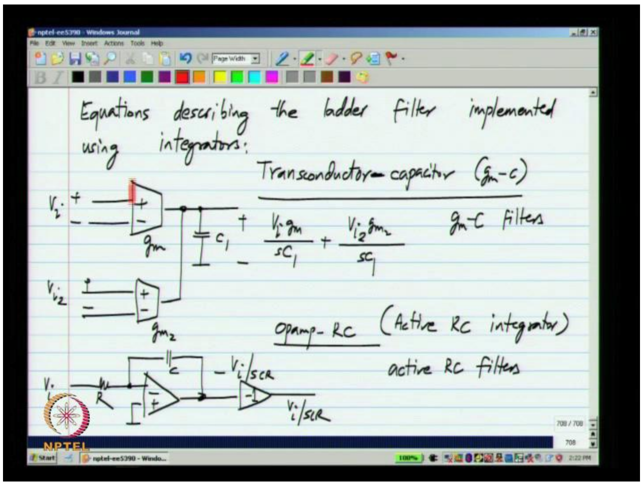

So, how do we make an integrator, if we take a trans conductor like this whose values g m and apply an input voltage V i at the output we will get a voltage V i g m by S C1 now, if we want to integrate a linear combination of voltages what we can do is I have multiple voltages input like this. So, if we have that then let us say we will call this g m2 the output voltage will be Vi2 gm2 by S C1. So, this one we can implement a linear combination plus integration. So, the first way of implementing integration is through trans-conductors and capacitors or what is known as g m-c technique and filters that use these are known as gm-c filters in alternative is to implement the integrator using an op-amp I want go through the details here, but if we make a trans-impedance amplifier, but replace the feedback impedance of the capacitor we know that we can implement an integration the output voltage here will be minus V i by S C R and I will show it along with a negative one gain. So, the output of that will get V i by S C R without minus sign.

So, this integrator is using op-amp R C and historically this kind of topology has been called active R C integrator and filter that use this integrator are known as active R C filters now, what I will do is I will synthesize all the filters using either this g m-c technique or the active R C and I will keep this minus one here. So, that finally, we get something without inversion, but this is that the block diagram level later we will see that we have to make fully differential implementations. So, this negative one block does not have to be explicitly implemented it can be implemented in simply by crossing wires also for g m-c filters I will not show the input with two terminals I will simply.

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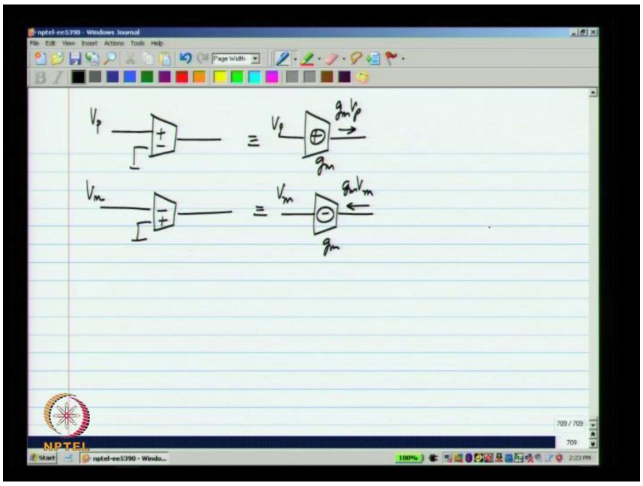

So, the trans-conductor let us say this to be connected like this V p. I will so, it as something with a single input and an output its assume that this plus inside means that the current is being pushed out and similarly, if I have to make something of this at V m then I show it again as a single input stuff, but with a minus inside the block all these are just to reduce the clatter in the diagrams that I am going to draw. So, this means that a current g m v m is drawn into the trans conductor again the implementation will be fully differential and we will connect the trans-conductor appropriately either positive or negative.

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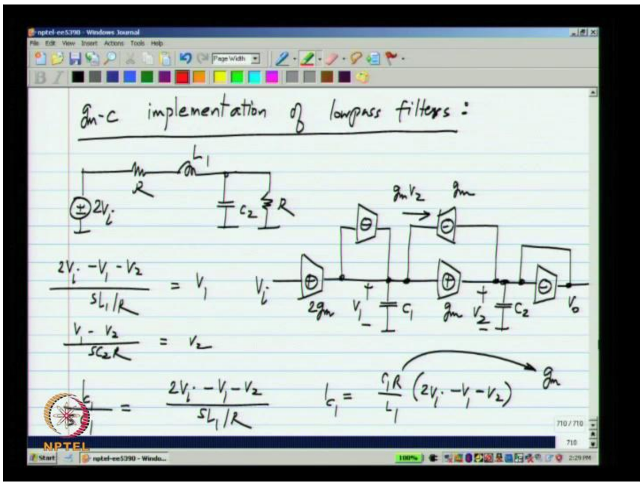

So, now, let us look at g m-c implementation of these low pass filters ((No Audio: 17:29 to 17:49)) will take the w terminated prototype((No Audio:17:55 to 18:07)) and we know that this circuit is described by these two equations. So, in our active counterpart will have two capacitors C1 and C2, but voltages V1 and V2 across them, and V1 on V2 and the input Vi are related by this relationship. So, this part seems.

So, simpler. So, will implement that first we will re write this as V1 minus V2 divided by R divided by S C2 this is equal to V2 now, V2 looks like some current divided by S times C2 and this is what you would expect, if the current through the capacitor C2 is what is shown in the numerator then the voltage across the capacitor C2 will be given by this equation. So, the current through the capacitor C2 must be equal to V1 minus V2 divided by R or in terms of trans-conductance's V1 times some g m, where g m is one over R minus V2 times the same g m.

So, we have V1 here we have V1-time plus g m. So, I will do that that gives me the first term and I have to get minus V2-time g m, and V2 is the voltage across itself , but we needed with a negative sign. So, we have that, and similarly we can implement the other equation the first one over here the voltage across C1, the voltage V1 will be whatever current is flowing through the capacitor divided by C1 and that should be equal to 2Vi minus V1 minus V2 divided by S L1 by R. So, what do we have the current through the capacitor must be equal to((No Audio: 21:07 to 21:23)) something like that, where we have some proportionality constant, and we have two-times the input voltage minus V1 minus V2 we assume that we have V i here, and we have to implement the fact of two in the circuit, and this quantity this as dimensions of trans-conductance, and I simply call that g m again.

So, what we need to do we need minus V1 minus V2 to implement minus V1, I drive this node with a trans-conductance, which is given from V2. So, get minus V2. So, this will give me, if this as value g m, g m times V2 and to get minus g m times V1 we have atom of the trans-conductor, and something other and finally to get two-times V i we have to have plus two-times g m, because the wait for this V i is twice that of the others we get the whole thing, this is V i and that will be V o and this will implement exactly the same transfer function as between this V i and V o, which is the voltage across C2.

So, by using trans-conductors and capacitors we can emulate the equations of the second-order filters and realize an active filters that has only active elements, which are trans conductors and capacitors will quickly do this for the other two-types the singly terminated types, and get likely different prototypes point that is probably what will use more often, but this serves as a introduction to how to synthesize a g m-c filter starting from w terminated L C ladder prototype.

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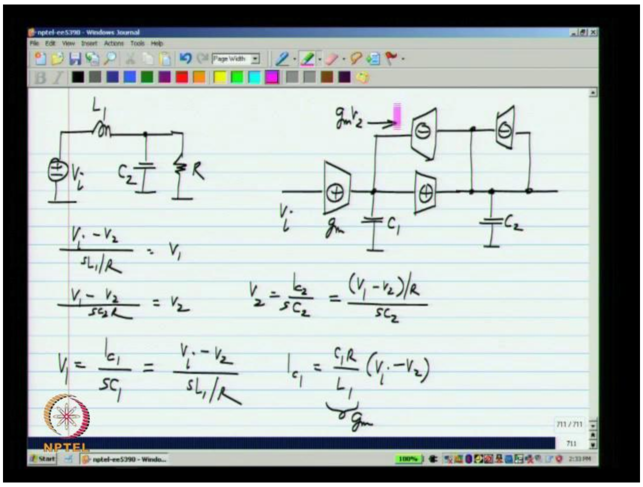

So, all I have to do is together singly terminated filter I omit either the termination at the source side or at the output side and V i is applied here, and the state equations are given by these as usual in our active circuit we only have capacitors C1 and C2. So, we can implement these two equations. So, V1 would be the current in the capacitor C1 divided by S C1 and that we would like to be equal to V i minus V2 divided by S L1 by R then what we do we just find the current in the capacitor, and that will be equal to C1 R by L1 times V1 minus V2. So, as usual as dimensions of conductance and I call that g m. So, I drive this with V i. So, that V i g m flows out and I have to drive another one trans-conductor with V2 and this should have a negative weight.

So, that g m V2 flows that way and similarly to implement the second one, V2 will be Ic2 divided by C2 and that is equal to V1 minus V2 I c2 by S C2, which is equal to V1 minus V2 divided by R by S C2. So, the current I c2 must be simply V1 minus V2 divided by R, which is V1 minus V2 times this g m so I have V1 times g m by using a trans-conductor with a positive weight and minus V2 times gm by using a trans-conductor with a negative weight.

So, this implements the same second-order equation and the difference is that difference between that and the w terminate prototype is this particular trans-conductor this is missing from the singly terminated prototype, but what is this element here this is nothing, but a negative trans-conductance connected in feedback, which is basically a conductance. So, we have the source side termination here that is missing from the singly terminated prototype. So, in the active circuit also we have a conductance here these conductance corresponds to the termination on the input side and this conductance, which is again negative g m in feedback corresponds to termination on the output side.

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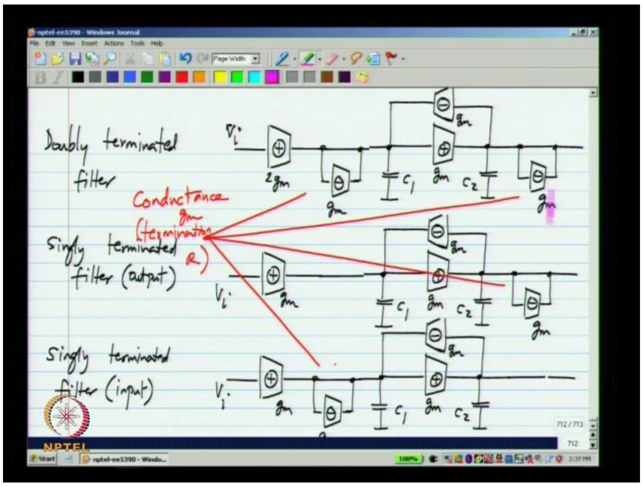

So, the termination on the input side is missing So, the positive trans-conductor that we connect here the positive conductance we connect here is missing and similarly, we can work out the prototype for the other singly terminated case I will just give the answer, and you can verify if you got the same answer as sighted this is 2 g m, I will draw it slightly differently here this is minus gm in feedback with essentially forms a conductance gm C2, and we have another termination another conductance trans-conductance corresponding termination on the other side this is g m, and we also have negative g m going the other way round, and for the singly terminated filter with a termination on the output side what we have is exactly this.

But without that one and if we consider a singly terminated filter ((No Audio: 28:52 to 29:03)) on the input side((No Audio: 29:04 to 29:35)) what we are missing is this particular termination, and you can see that all three topologies very similar to each other compared to the w terminated one the singly terminated ladders will be missing one or the other conductance all of these are basically conductance g m, which corresponds to the termination resistance. In these two singly terminated cases the trans-conductance value driven from the input voltage will be g m and in the w terminated case it is 2 g m that is a compensate for the attenuation of the w terminated ladder. So, that the d c gain all this filters equals one now, if we know how to make a trans conductor we will know how to make this active filter one of the interesting things to see that we had a inductor in the passive prototype now, you do not have a inductor any more, but we're realizing its operation using trans conductors.

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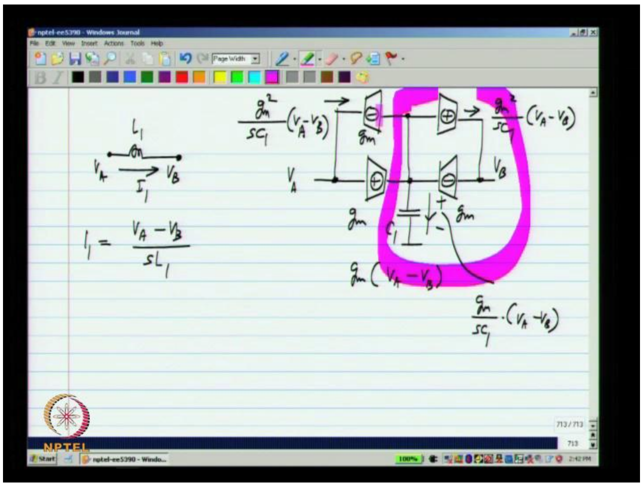

So, lets how that happens lecture we try too emulating inducted by itself using not an a inductor but using capacitors; that means, that have to implement the voltage current equation of the inductor, but without using an inductor how do I do that I know that I1 is VA minus VB divided by S L1. So, the operation is that of an integration, which I know can be implemented using a capacitor as well. So, if I am given a capacitors of C1 and I have these two voltages VA and VB, I have to draw this much current I1 from VA and then push it into VB, I have VA and VB, and that this is g m and g m a current flowing through the capacitors would be g m times VA minus VB, and the voltage it was capacitors would be g m by S C1 times VA minus VB.

Now, from VA, I was draw a current that is proportional to this from VB, I should draw a current proportional to this, but with negative polarity. So, if I do this what happens there is a current that this is drawn, which is proportional to which is equal to g m square by S C1 VA minus VB the value of this is also g m and similarly here I have to push the current out. So, i will use trans-conductor with the positive wire and the current being pushed out equals g m square by S C1 VA minus VB. So, this entire circuit emulates this inductor, but without using an inductor that is it emulates the voltage current equations of the inductor and this is the structure that you see repeatedly inactive filters.

So, this is known as a gyrator and gyrator in words the impedance, if you have a capacitors terminating a generator from the other side it looks like on inductor vice versa in case of active filter is usually used to emulate inductor, and in our circuit also you can see structure that is similar to this not the complete structure by something similar to this structure. So, if I highlight the this part of it you see that I have exactly that over here. So, this is our g m-c active filter, which is a second-order filter and we can realize higher order filters using a second-order filter.

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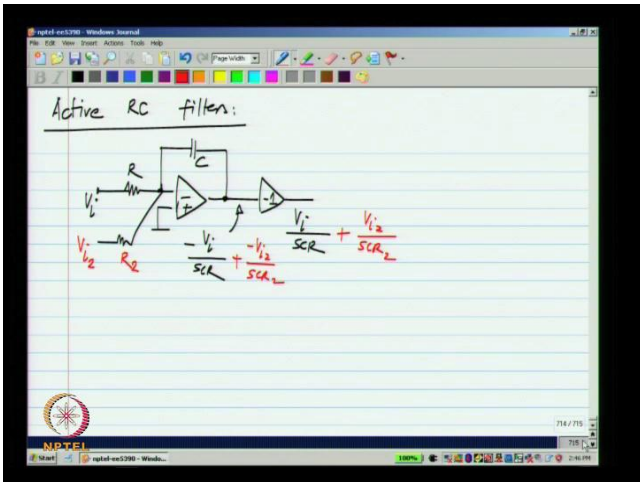

Now, similarly let us also do this with active RC circuits now, again I can do it for all three, but I do it draw only for one of them one of those three cases that I showed that is a w terminated and singly terminated with either termination at the source side or the output side, and we can take the other two as an exercise, and see what structures come out. So, as you said earlier we have to have active versus circular or op-amp circular which can performed the function of integration and integrator using a op-amp looks like this and I will keep this minus one blocks. So, that I do not have the polarity reversal, if I applied V i here what I get here will be V i divided by S C R and here I will have minus V i divided by S C R.

So, let us implement one of the singly terminated prototypes that is this particular one and this we know as to implement the equation V i minus I1 R minus V2 divided by S L1 equals I1, and if we translate everything in terms of voltages will have V i minus V1 minus V2 divided by S L1 by R will be equal to V,1 and similarly the other expression would be I1 divided by S C2 would be equal to V2, and by normalizing it with some resistance will get V1 divided by S C2 R equal to V2.

Now, this is the active RC integrator now, this is the active RC integrator, and this I want to make a linear combination of voltages, which as to be integrated all I can do is add multiple input registers here, and output voltages will be we also get plus V i 2 by S C R2 and here minus V i plus minus V i 2 by S C R. So, this is exactly what we went to have. So, we can make integrators with multiple inputs.

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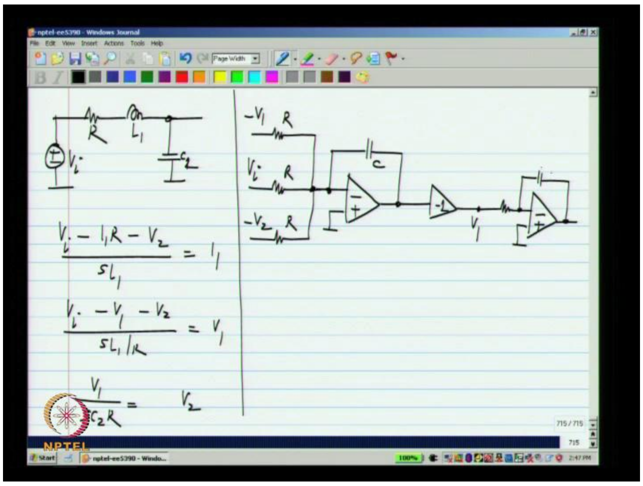

So, in this the first case we need to have an integrator with three inputs and this will be the output V1 under three inputs would be V i and minus V1 and minus V2, and the other integrator has also single input and that is equal to V1, I am running out of room here this would be V2 and I have to connect minus V2 over there, which appears over here.

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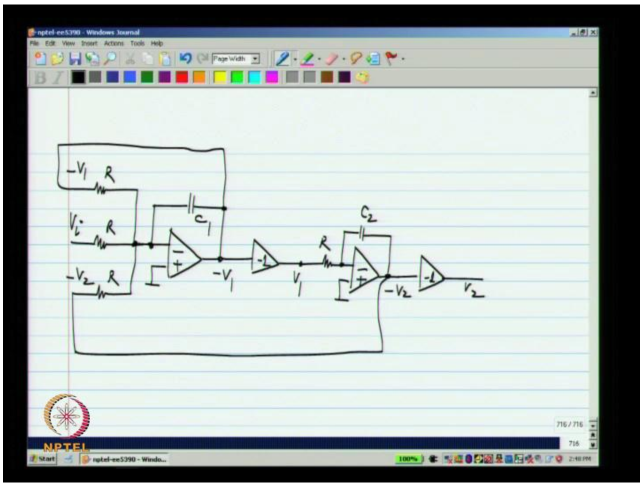

So, I can do this I have to connect minus V1 over there, which appears there. So, I can do and this will be some C1, and that will be some C2, and if I redraw it more neatly…

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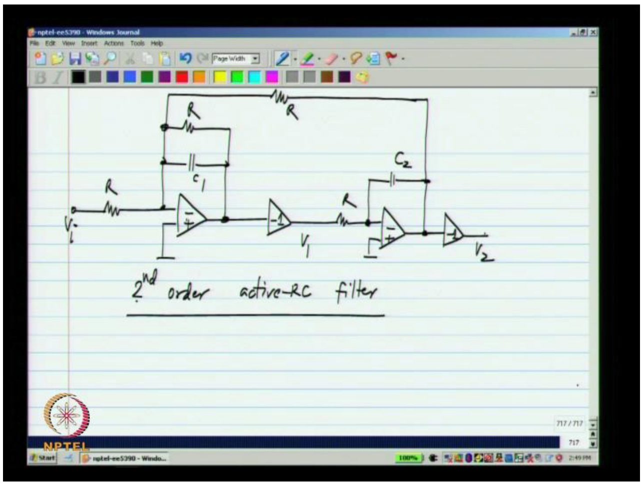

I will get topology, which is a classical active RC filter topology ((No Audio: 40:24 to 41:01)). So, the actual output must be inverted again to get rid of the polarity inversion, if I apply here I will get V1 and V2, where V1 is the scale version of the inductor current and V2 is the capacitor voltage in the prototype.

This is a second-order active RC filter and this procedure again can be extended to higher orders by writing down the expressions appropriately will see an example of that ladder, and then translating the equations into active RC integrator implementation ((No Audio:41:44 to 42:01)). So, now, we know how to translate the passive LC ladder prototype into g m c and active C filter implementations like I mentioned earlier all our implementations will be differential, because of the all the good things are differential operation offers, and same volts for the g m c and active RC filter as well I will quickly show how the differential implementation looks like although in all the future discussions I will show the singular encounter part that is, because the otherwise the diagram very messy, and this also get rid of the minus one blocks that we had in the active RC implementation that is simply corresponds to switching the wires.

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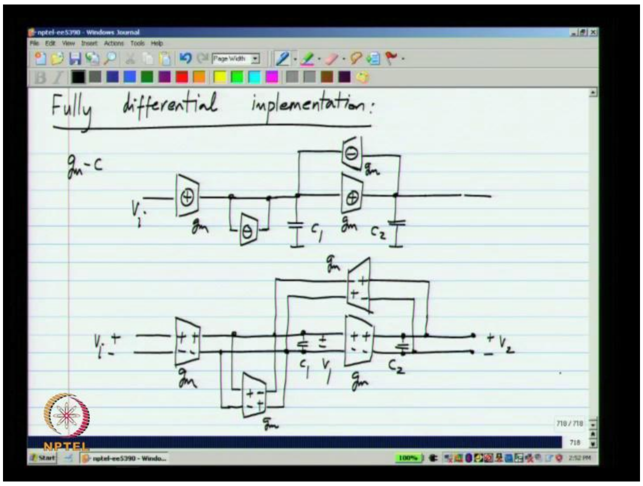

So, this is what we hacked and the fully differential implementation would be that these trans-conductor would be fully differential, and this negative sign means that if its plus minus on the side I reverse the polarity, and connected that is all is there to it similarly this is positive. So, we will have plus minus and plus minus and drive C1 here and C2 over their, and here I have plus minus, minus plus and drive the capacitor voltage hear will be V2 and the voltage here will be V1 voltage here is Vi.

So, this is the fully differential g m c second-order filter and will see more details of the implementation of such a filter and similarly with active RC what we will have is instead of this will have fully differential op-amp, which we know how to build with common mode feedback and all of that is of, and will have two capacitors

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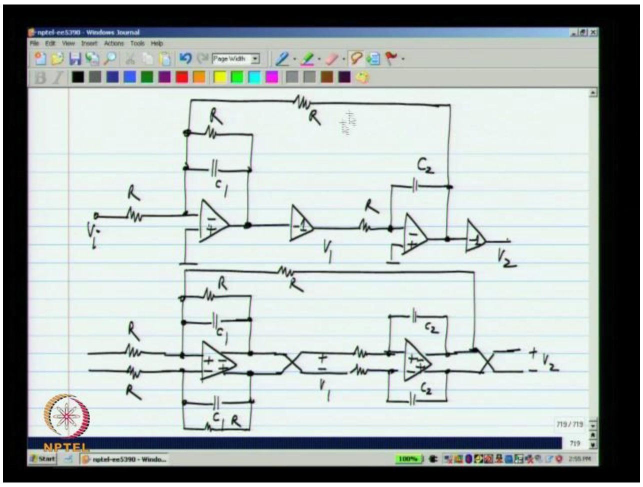

and feedback one for each side, and will have it to input resistors we are resistor cause each of these this minus one simply corresponds to crossing the wires like that even will have this capacitor and to get the actual output voltage V2 it will be simply to cross the wires, and V2 will be like that and V1 will be of that side finally, how to complete this feedback loop, which comes from minus V2 and the same on the other side.

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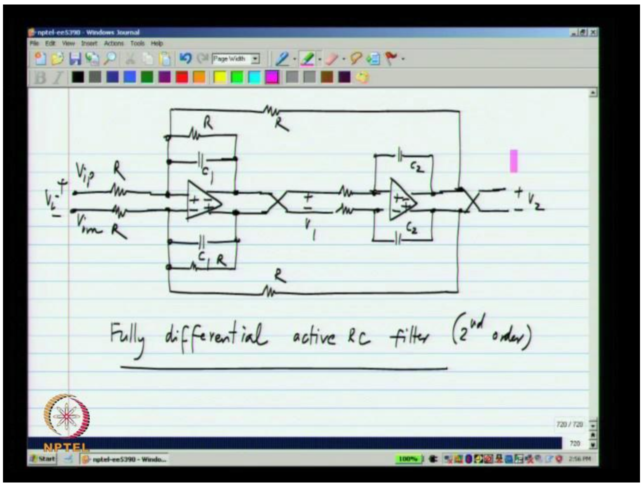

So, this is a fully differential active RC filter of second order and just as exercises can repeat this with the w terminate prototype, and the other variety of the singly terminate prototype, and it will be interesting for you to compare those topologies with what I shown here and the input voltage applied in between these two, which is really a difference between V i p and V i m.

So, the voltages that we see here V1 and V2 will be filtered versions of V i p and V i m the desired output the low pass filter output is V2. So, what we have done in this lecture is to synthesize g m c and active RC prototypes starting from ladder prototypes, which we tack as given and in the next lecture what we will do is extend this to higher order or use the second-order filters to make higher order filters, and discuss quickly some non ideal features of such filters and how to fix them.

Thank you I will see you in the next lecture.