Analog Integrated Circuit Design
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Lecture No - 9
Negative Feedback Amplifier with Parasitic Poles and Zeros; Nyquist Criterian

Hello and welcome to another lecture of analog integrated circuit design, so for weight we looking at what happens if you have a delay in a negative feedback amplifier, and where those delays could come from? The delays come from parasitic poles in the system where also saw that pole acts like a delay for the integrated. So, today what we will do is elaborate a little bit on these things and then see how to conveniently judge the stability of an amplifier and how to design it. So, that it is well-behaved.

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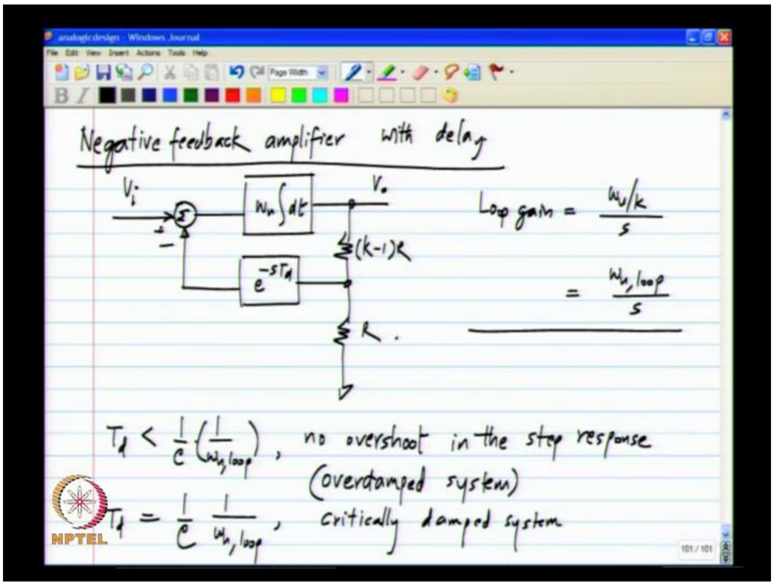

So, this is have a negative feedback amplifier and it could have a delay T d in the feedback path or anywhere in the system. While a quantifying amount of the negative feedback we have a computed the loop gain, and the loop gain of the particular system without, the delay is omega u by k divided by s which can be defined to be omega u loop divided by s. Here omega u loop there is a unity loop gain frequency and its given by omega u by k where omega u is the unity gain frequency of the integrator you to realize the negative feedback system.

Now, how much delay can be tolerated depends on the value of omega u loop. So, previous analysis shows that for T d less than or equal to 1 by e times 1 by omega u loop that is 1 by 2.718 times their time constant of the integrator, we get no overshoot in the step response and essentially behaves like an over damped system. That is for the T d less than this number which is equivalent to an over damped system for T d equal to 1 by e times the time constant of the loop gain, then there is again no overshoot and this is the critically damped system. So, this corresponds to the higher delays that you can have without having a overshoot in the step response, it also corresponds to the fastest step response that you can have without overshoot for that is why it is the critically damped response critically damped system.

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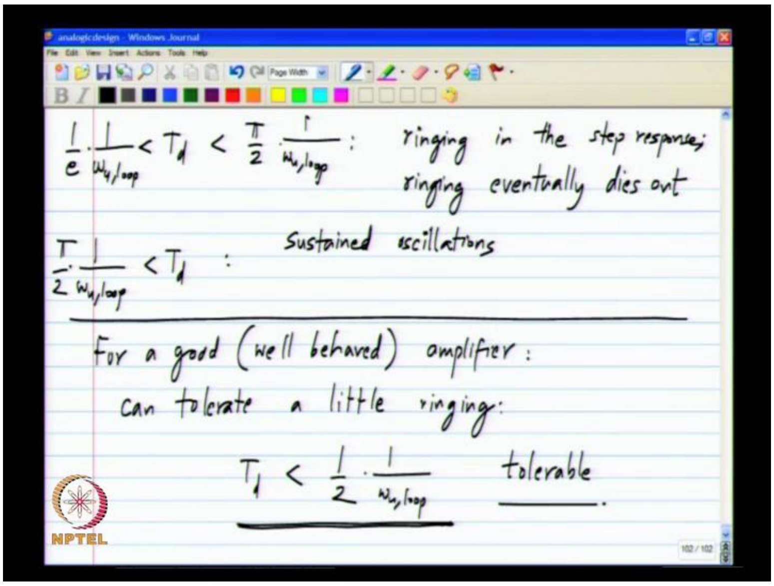

Now, when T d is when T d is between 1 by e times the one over omega loop and between pi by 2 on the omega loop, what happens is there is ringing in the step response, but the ringing eventually die's out. So, the system is stable in that if your apply a step there will be some remain that thing eventually dies out, and the same thing if you we have initial conditions in the system there will eventually die out, but there can be a lot of ringing. You can tolerate a little bit of a ringing, but these you see a lot of ringing and T d more than omega over your loop what happens there are sustained oscillations.

So that means, that if you apply a step then there is ringing and it blows up also if you have some initial conditions they want die out they have eventually a blowup and leave a response even without having a input. So, this is an oscillatorary condition and this is circling something that you do not want to have. And for a good amplifier, that means well-behaved amplifier you can tolerate a little ringing, it also depends a little on the application that a looking at you can tolerate a little ringing.

So, typically T d less than about to half of the on the omega over loop is tolerable in if your applications says that there has to be absolutely no ringing at all when you have to make T d to be less than 1 by e times 1 by omega u loop. So, this is what is tolerable and if you have T d beyond the certain start ringing more and more, and if you have T d more than pi by 2 times 1 by omega u loop you will have sustained oscillation and certainly you do not want to have that. So, now, let us see what happened when there is a extra pole and related to this one.

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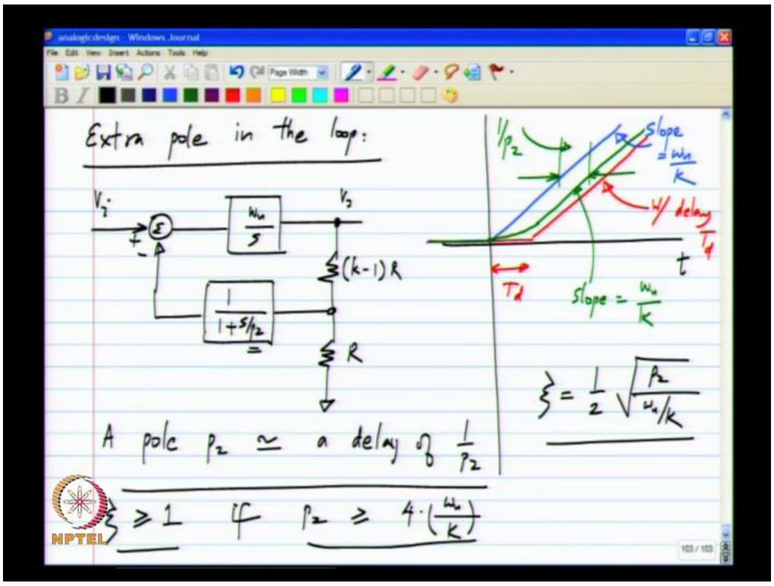

What we said was there was extra pole in the loop, somewhere we have not at actually looked at exactly where it can be and it can come from parasitic capacitance at various points in the circuit. So, for no we will simply modulate us where usual system and integrator with a unity gain frequency omega u, the a voltage reorder fraction 1 by k. And here I will simply have an ideal block which models the pole just like the delay it does not matter where in the loop this pole is. So, has long as this in the loop it will affect the response of the system.

So, the first thing is we see that where step response in the loop gain without the extra pole will be a ramp will slope is omega u by k or omega u loop. If you have an ideal delay of T d then the step response will are the same slope, but is delayed by an amount T d this corresponds to delay of T d. And finally, if you have a pole, if you have extra pole p 2 what happens is that you will also have something like this and then response slowly rises. But eventually it will settle to a slope which is the same as for that other cases, it is equal to omega u by k.

The slope is the omega u by k and it is shifted horizontally by an amount of depends on the pole. And In fact, it is exactly equal to 1 by p 2. So, a pole p 2 is approximately the same as a delay of 1 by p 2 we can expect a similar behavior that we know that if the delay is very small, there will be no overshoot in the response. As the delay becomes larger and larger there will be overshoot similarly if the pole p 2 is very large which corresponds to small delay be no overshoot.

And if a the pole p 2 becomes very, very small then you will start having overshoot and. In fact, we will already analyze this through the damping factor and there are seen that for this particular system, the damping factor will be half a square root of the unity loop gain frequency divided by the parasitic pole p 2. Now, for no overshoot we need to have the damping factor equal to one or more. So; that means, that the pole p 2 as to be at least four times greater than the unity loop gain frequency, I made a mistake in this expression this as to be p 2 divided by omega u by k.

So, the damping factor will be more than or equal to 1 if p 2 is greater than equal to four times omega u by k and this is consistent when the notion there a pole at similar to a delay a pole at p 2 corresponds to delay of one over p 2. So, if the pole is a very high-frequency the delays very small and there is no effect from the step response, if the poles comes lower and lower frequency, then there will be an effect on the step response in that we start ringing. And the critically damped case zeta equal to one corresponds to p two equal to four times omega u by k and you do not want to have a pole that is substantially lower this lower than this in frequency.

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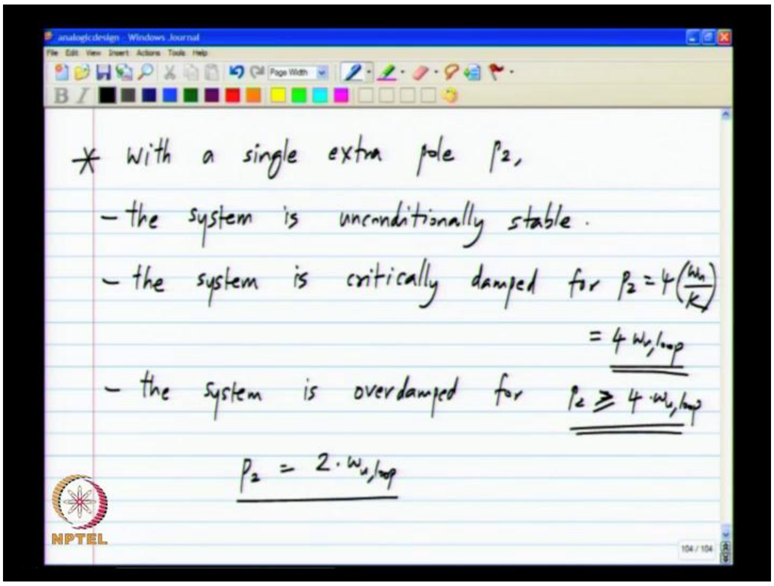

Now, one thing about the system with the pole that is different from the system with the delay is with single extra pole p 2, the system is unconditionally stable meaning they can be ringing in the system, but there never be sustained oscillation this is because its second-order system and that is what it turns out to be, will see that when you have multiple extra poles this is no longer the case. Firstly, the system is unconditionally stable the system is critically damped for p 2 equal to four times the unity loop gain frequency. And the system is over damped for p 2 equals p 2 more than four times unity loop gain frequency.

So, now again we can tolerate this condition we can violated this condition a little bit, but not by too much it is to have let us say p 2 is two times omega u loop and this case we will get a little bit ringing, but certainly we do not want make p 2 at a lower frequency than this. In that case you will have a lot more ringing and it will not be acceptable in an amplifier. Now, this shows that with an extra pole the system behaves as though there is an extra delay and the conclusions that we drew from the original analysis with the delay holds could even now just reinforces notion and into connect to the analysis. Well we are going to do later, which will make this solution to the whole problem easier we will do it for two parasitic poles.

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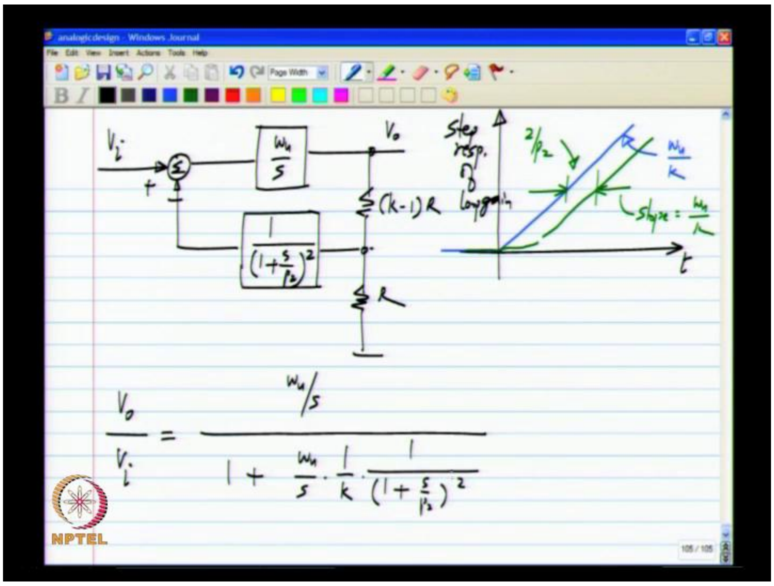

We will assume that the negative feedback amplifier as I assumed two extra pole is somewhere in the loop. So, what happens in this cases the step response without the parasitic pole of the loop gain will be a ramp to slope is omega u by k, and with this parasitic delay it will initially do something, but finally settled to a ramp with the same slope and the delay of 2 over p 2. In fact, if you have multiple poles in the system each poles can the thought of us contributing a delay of one over that pole. So, this is the units of step response of the loop gain and it again acts as a delay.

So, for this particular case we will try to solve for the condition, when there will be sustained oscillations just we see what happens when you go to higher and higher order systems. So, V naught by V i will be equal to the gain of the forward path plus divided by 1 plus the gain of the forward path times the gain of the feedback path, this you can work out for yourself. Even if you do not know the formula by writing of the variables around the slope you will be able to work out this.

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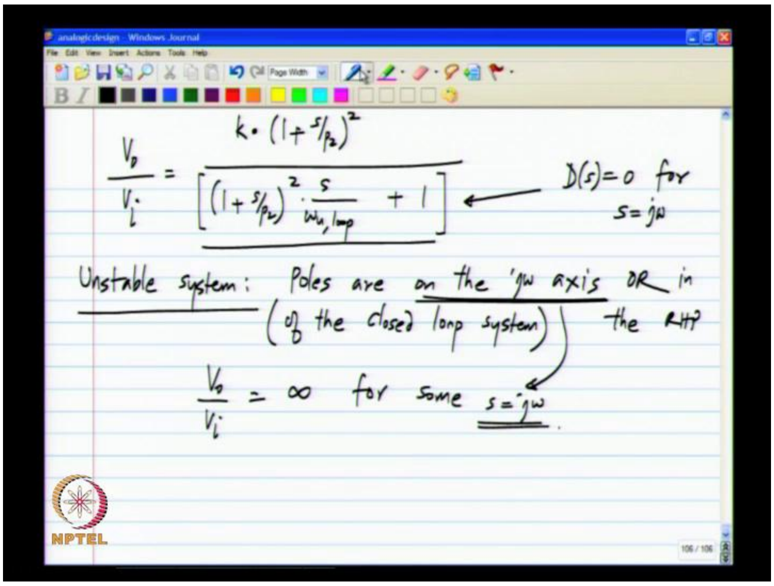

And this can be re re-written in the standard form by multiplying by 1 plus s by p 2 square times s by omega u by k. So, we will get this is what we will get and I will for convenience, I will replace this file variable omega u loop. And now you can clearly see that as we wrote to higher and higher order system, evaluating the step response in a closed form and finding out whether there is ringing or not becomes very difficult, which is by will low later go to the more convenient condition in which we do not have to solve for it analytically.

So, but we will look at we will look at the conditions for which the system is unstable. An unstable system doing the poles are on the j omega axis here one and when I say poles I am talking about poles of the close loop system around the j omega axis, or in the right half plane. And it turns out that it is rather easy to solve for the condition where the poles are an the j omega axis and that is what we will do.

When the poles are have the j omega axis what happens is for a certain frequency s equals j omega, this expression V naught by V i becomes a infinity. For this condition V naught by V i is infinity for some s equals j omega and if you have the single extra pole p 2 that is what never happen, but for more extra poles this can very easily happen. So, do be solved for a we will set the denominator to be equal to 0, for some s equals j omega. So, what I have to do is I have to substitute s the j omega in this expression and set the result to 0. So, as usual as encourage you to try this out before we look at my solution and find the condition for which it happens.

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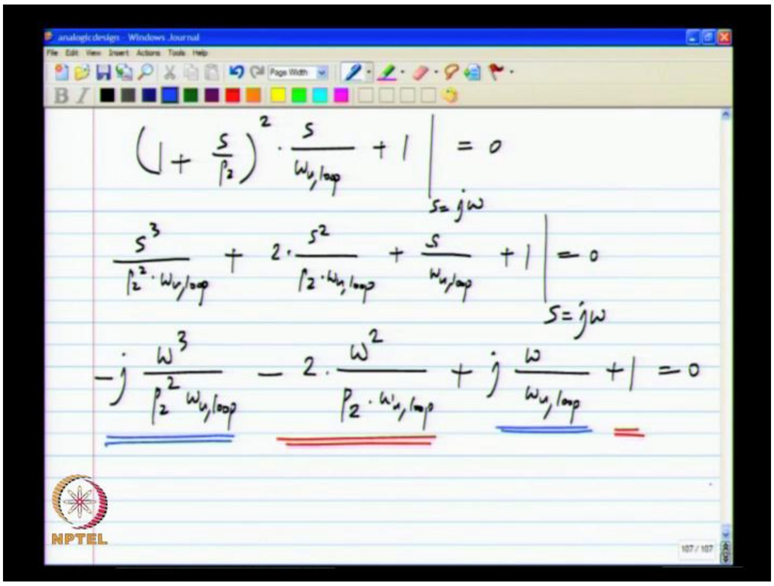

This is 0 for some s equals j omega and this is nothing but again for s to be j omega. So, with this means that minus 2 omega square p 2 omega u loop plus j omega by omega u loop plus 1 equals 0. So, as you can see there are two variables here the value of omega for which this happens and the condition one p 2 at which this happens. And there are 2 equations and there are two equations because there is the real part of this equation and there is also there imaginary part of this equation. And each of them as to be equal to 0 by equating them to 0 you will get the two conditions. Firstly, if you equate the real part to 0 you will get.

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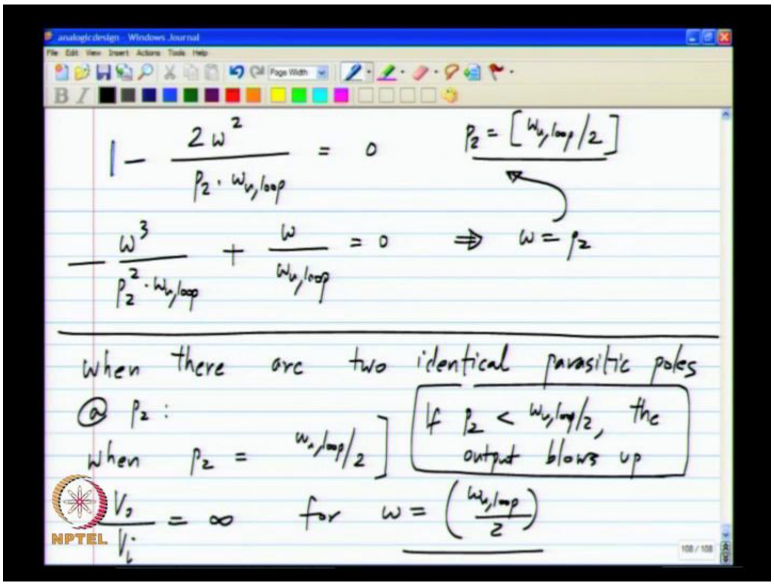

1 minus 2 omega square divided by p 2 omega over u loop equals 0 and if you equate the imaginary part to 0, you will get minus omega cube by p 2 square omega over u loop plus this part omega by omega u loop equals 0. And this gives you omega to be equal to p 2 and if you substituted here, you will get substituting in this first equation you will get p 2 to be omega u loop divided by 2 for the condition to hold. Just us to summarize what we have done is we have evaluated the close form transfer function and we are found out the condition, under which this becomes infinity. It becomes infinity when the denominator becomes 0 and we have evaluated some s equals j omega for which this can happen.

So, what it says is when we have two parasitic poles at the same frequency mind you in this case taken both the parasitic poles at the same frequency, this is for convenience of analysis is does not mean the reality is as to be always that way, but the conclusions we draw from this will hold from the general case. When there are two identical parasitic poles or p 2 and then when p 2 happens to be half of the unity loop gain frequency, the gain will be infinity for omega equals omega u loop by 2.

Now, it actually does not matter for what frequency the gain becomes infinity because once the gain becomes infinity at certain frequency at that frequency even without an input you will have an output. So, the amplifier becomes completely unusable because an amplifier is supposed to give you only the input times the gain it should not have anything and the related to the input. But the system is unstable it will have something that these and related to the input and the amplifier is useless.

So, this is the condition and it can also be shown that a if p 2 is smaller than this, the output blows up, if p 2 equal this you will have sinusoidal and the output even without an input. And it will be constant on amplitude and the p 2 smaller than this the ringing on the sinusoid keep growing and amplitude it will never die down.

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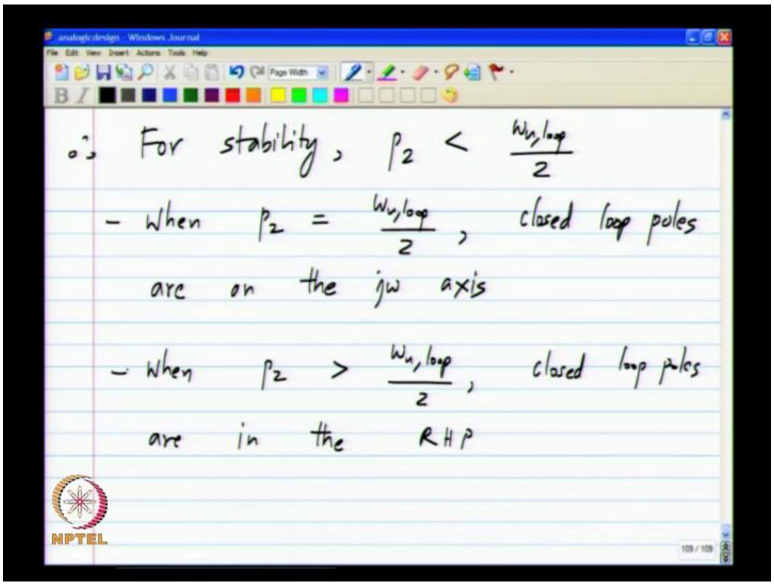

So, for stability p 2 should be less than half the unity loop gain frequency of and when p 2 equals the unity loop gain frequency by 2, the poles are the closed loop system are on the j omega axis, and when p 2 is more than omega u loop by 2 close loop poles or in the right of plane, which also means more unstable system. So, just before when p 2 becomes very small the equivalent delay becomes very large and this is becomes unstable.

So, we can carry this exercise for three extra poles and four extra poles and so on and it is most convenient, if you assume that all the parasitic poles are in the same location. But the conclusion that you draw from this is general, even if you have poles or different locations the total delay contribution due to all the poles should be limited to certain value. If the delay becomes too much the system will become unstable, I will quickly tabulate the results.

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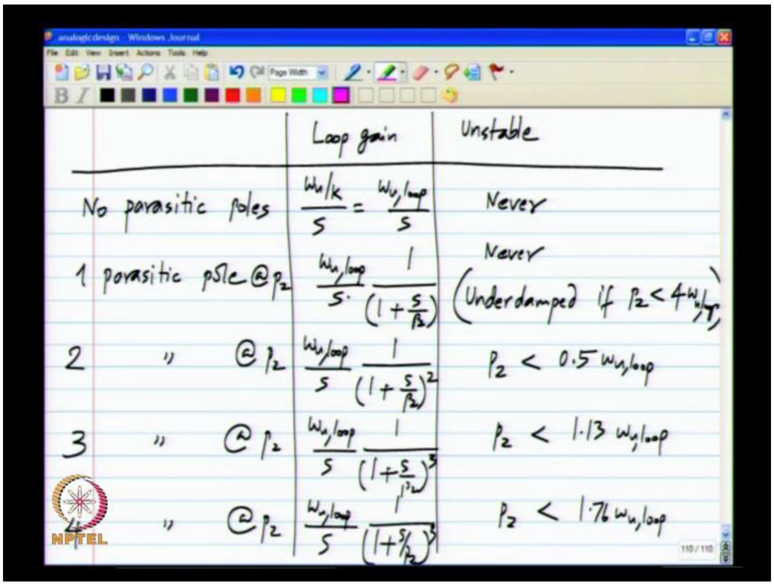

So, let us say we are the case where there they are no extra poles from, this is the ideal case that we consider and in this case the loop gain is omega u loop by s or omega u by k divided by s. Let me write it more clearly if you use an integrator of unity gain frequency omega u and make a make an amplifier with gain of k with it, it will have a loop gain like this which corresponds to omega u loop by s. And this will actually never ever become unstable it is unconditionally stable, it is the first order system with a pole at omega u by k and it is unconditionally stable.

So, let us say you have one parasitic pole at certain frequency p 2 the loop gain function will be omega u loop by s 1 by 1 plus s by p 2. And actually even this never becomes unstable, but it does get under damped or shows ringing, if p 2 is less than four times the unity loop gain frequency. And similarly you can have the two parasitic poles both identical at p 2 in this case the loop gain function is this and you can calculate when it becomes unstable. And it becomes unstable for p 2 less than 0.5 omega u, and you can do it for three parasitic poles at p 2 and this is the function and this becomes unstable for p 2 less than 1.16 omega u.

Sorry 1.13 omega u and finally, you can also try a 4 and 5, and how many were you wish to have. And here also the system can become unstable, this is the function and this becomes unstable for p 2 less than 1.76 omega u. So, to summarize an extra pole is like a delay and as the delay increases, we will tend to have ringing in the system and when the delay increases a lot you can even have instability. That means, the system can give you an output even without an input, you can think of it as a again becoming infinity the some particular frequency or the poles being in the right half plane.

Now, it turns outer for the special case of a single extra parasitic pole, the system is never unstable it can ring, but it never unstable. So, you still have to make sure that the parasitic pole is that high enough frequency to ensure that the ringing is limited and that limit is usually two times the unity loop gain frequency. And if you do not want any ringing at all the parasitic pole as to be at least four times the unity loop gain frequency. Now, you can have 2 or 3 or 4 or parasitic pole extra parasitic poles and they can be at a various locations, but for analysis easier if you assume that all of them are at the same location. And if have two identical parasitic poles are p 2 what happens is the system can become stable. And it happens for p 2 lesser than 0.5 omega u loop.

So, all of these have written here should be omega u loop, 0.5 omega u loop and for three extra parasitic poles it is 1.113 omega u loop's and four of them it is 1.76 omega u loop. Now, in the other thing is also notice is that when you increase the number of poles, the permissible location for the pole is that higher frequency. For instance with to parasitic poles, which are half are omega u loop and with fore parasitic poles it is at 1.76 omega u loop. This is because each pole contributes a certain amount of delay and when you have multiple poles delays due to each other poles and some. So, it effectively constitutes a greater delay.

So, the delay due to each poles should be smaller, if you have to ensure stability. And this can be easily related to the first result that we have obtain that if you have an ideal delay. And it delay becomes too much that is 1.5 times 1 by omega u loop the system becomes unstable it is exactly it is 5 by 2 times 1 by omega u loop is which can be approximated to 1.5 times 1 by omega u loop.

So, the greater the number of poles the higher frequency they have to be if they come to lower frequency you will risk instability. Now, what we are interested in amplifier designers not nearly avoiding instability because it can be ringing a lot, which dies down ringing dies down and it is really a stable system that is also not what we are interested in. What we are interested in is also well-behaved system where it does not ring too much.

Now, what happens is in his when you have multiple poles a evaluation of step response becomes difficult when we had only one extra pole it is very easy, we know the step response poles for the second-order system. And then classified into critically damped over damped under damped and we can easily identify the condition for which there is no ringing that is it as to be critically damped or under damped. Now, when you go to higher and higher order systems in is not the case, the analytical expression become too complicated and little becomes simply difficult to judge even stability, or it is even more difficult to judge in the system is well-behaved.

So, what we will do is we will not a evaluate the transfer function and work with the polynomials which is very difficult, what we do is taking our from the ideal delay case for which we have the solution. And the second-order case for which we also having a solution from their where the extra polite some conditions for higher order systems and these conditions will not be based on calculating polynomial, and the roots and etcetera. Because that is what is very complicated it will be based are have evaluating the polynomial for some sinusoidal frequency. That is the evaluating the sinusoidal frequency sinusoidal response of the loop gain, because of that it is much easier, and that is the preferred way of analyzing negative feedback systems.

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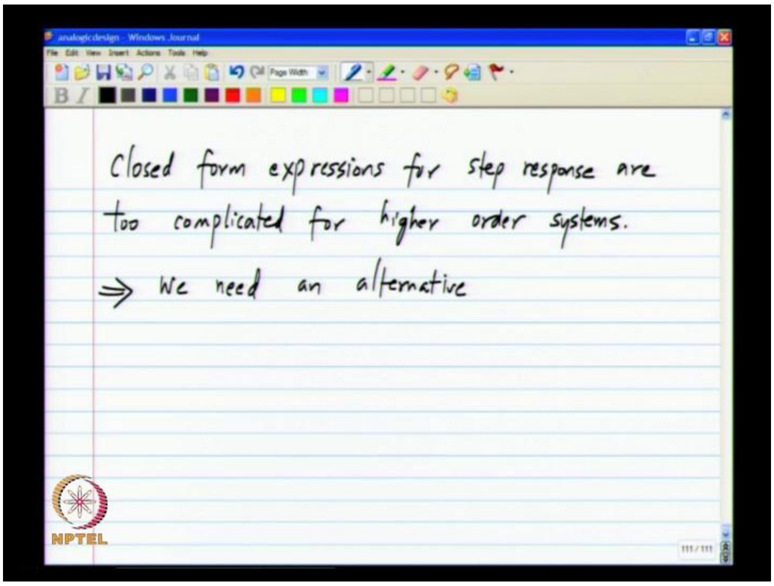

Where close form expression for the step response for too complicated for higher order systems, so which basically means that we need an alternative methods for analysis. And there is an alternative method, but does not depend on calculating the denominator polynomial and its roots and zones, and that is based on the Nyquist criterion. So, the Nyquist criterion would be familiar to the from courses on control systems, in this course not going to deal with the loop of the Nyquist criterion response. But we will just a state the Nyquist criteria with our purpose which is able to design well-behaved amplifier.

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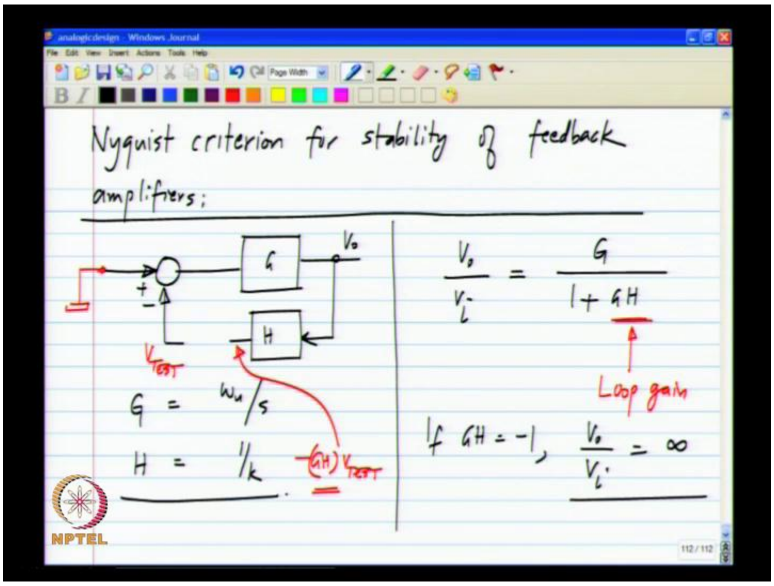

So, just a start of this if you take the classical representation of the feedback amplifier which also includes, the amplifier that we have derived and if you look at the control system textbooks, this is how it as written the forward path is denoted by G in the feedback path denoted by H. And for our proto type case G is nothing but the integrator and H is nothing but 1 by k ideally. And in reality there will be parasitic poles and either G r H or both there will be extra poles in the system. So, thus V naught by V i as a we know as expression G by 1 plus G H where the G H is nothing but the loop gain, as we are seen the loop gain as something that we used to quantify the amount of negative feedback.

So, the where we do it is by breaking the loop somewhere applying a signal in the forward direction of the loop and see what comes back. For instance in this particular case the we can break it here, we set the input is 0 for a evaluation of the loop gain because we are not interested in any particular input, but we are only interested in what comes back around the loop. So, we can apply some V test here and what comes back here we know is minus G H times V times. So, this G H is defined through the loop gain the minus sign is omitted because we assumed that you are making a negative feedback amplifier. So, we expect a minus sign. So, that is omitted and G H is the loop gain and as you can see from this expression the gain will become infinity if G H becomes minus 1 if the loop gain becomes minus 1.

So, the stability criteria usually revolver of the loop gain and the value of minus 1, that is you should not have loop came to be equal to minus 1. So, how to avoid it and the, what to do with the loop gain that is what the stability criteria work with. So, in a very crude way we can say that stability criteria is about avoiding G H being minus 1, there is a little more to it than that, but that explains the significance of minus 1 in under the loop gain.

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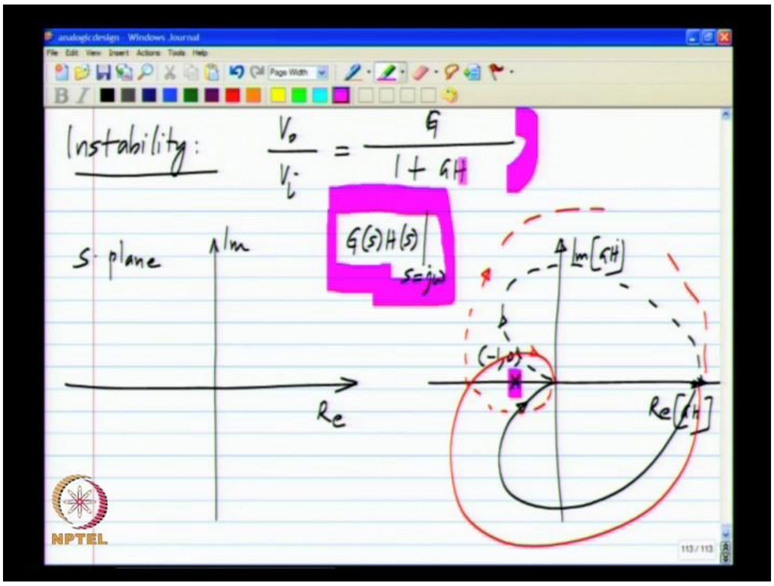

Now, what is the actual definition of stability or instability rather instability means, that transfer function V naught by V i which is G by 1 plus G H has some poles in the right half plane. So, if this is the s plane instability means, there are some poles in this half of the plane in their right half plane or on the imaginary axis. So, this includes the imaginary axis. Now, it turns out that for this to work out this thing you need to obviously, calculate the V naught by V i and there is the some denominator polynomial. You have to find out the root of the denominator polynomial and see whether they have positive real parts are not, if they have positive real parts well-being the right half plane and the system is unstable. But that is what you said was very difficult that is very difficult because you will end of with very higher polynomial and for which the roots are not very easy to determine.

So, the alternative is it use the Nyquist criterion which states the following. So, what you can do is instead of evaluating G by 1 plus G H and working with the polynomials you can simply evaluate G H. And that to not everywhere, but on their imaginary axis s equals j omega is essentially means that what you are evaluating is the sinusoidal study state response of the loop gain function s equals j omega corresponds to a sinusoidal frequency omega. And G of s H of s at s equals j omega corresponds to the loop gain for a sinusoidal frequency of omega. So, you can evaluate G of s H of s and plot its imaginary part versus the part. So, instead of calculating the polynomial we simply calculate G times H for sinusoidal frequencies s equals j omega and then plot by imaginary axis real.

We later going to how this plot exactly look like, like I said and rather likely that stability criteria involved avoiding the loop gain being minus 1. So, this minus 1 0 is a critical point in this whole technique, and what the Nyquist criterion states is that the plot of imaginary part of the loop gain verses the real part of the loop gain should not encircle minus 1 comma 0. That is the plot can be something like this and if it is like this for the negative frequencies, it will be the mirror image. So, in this case it is not enclosing minus 1 0 well make more concrete the definition of what enclosing means, but in this case you can clearly see that minus 1 0 is not inside this picture.

On the other hand if the picture looks something like this then it looked likes minus 1 0 is enclosed by this plot of imaginary part of the loop gain verses real part of the loop gain which is evaluated on the j omega axis. So, in this case the system will be unstable. So, I still made some make statements, you had to make many more things more precise which I will do, but what I like you to do is to first appreciate the advantage of this particular technique.

So, before if we had to ascertain stability you have to evaluate this function and evaluate the denominator polynomial of that and find the roots, see if the roots are positive real parts. That is a very difficult thing to do first of all the expressions become very combustion even for the baby cases that we took that is you have two identical poles, you have to do a little bit of work. Now, when you have multiple poles and that are not identical to each other it becomes very, very difficult. Instead what is done it is evaluate only the loop gain and that do not everywhere not on a general form, but on the imaginary axis that is for sinusoidal input.

So, as you know it is easy to calculate as well as easy to measure if you are only interested in the sinusoidal study state response. When you have a system you can apply a sinusoid see what comes out and evaluate the magnitude and phase shift of what comes out compared to what goes out. So, this Nyquist stability criteria is a way of ascertaining stability that is evaluating whether the poles in the right half plane by looking at only the loop gain and this is a tremendous advantages. And that’s why this the techniques that are sort of related to this are very widely used.

Roughly speaking it involves the sinusoidal study state response of the loop gain and the critical point of minus 1, and the intuitive reason for minus 1 being there is that if the loop gain exactly equals minus 1, you will have the for sure because V naught by V i becomes infinity. So, this Nyquist criteria says something about the plot of the imaginary part of G s and verses real part of G H and the make some statement which is exactly equivalent to the poles being in the right half plane. Exactly what this statement is will the elaborate in the next class and see with all the details.

Thank you.