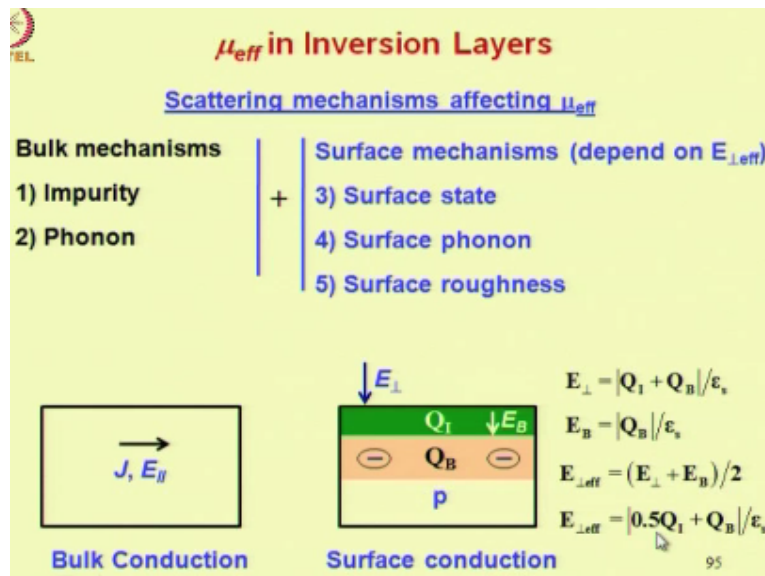


**Lecture-18**  
**Drift-Diffusion Transport Model: Equations, Boundary Conditions, Mobility and Generation / Recombination**

In the previous lecture we have started a discussion of the very important parameter drift diffusion model namely the mobility. We have pointed out that the question What is the mobility in silicon framed in this manner does not make sense because the mobility in any material depends on several factors. Whether the carrier is moving at the surface it is moving in the bulk. What is its doping level? what is the temperature?

And then you went beyond that where there is an electron or a hole and so on. So we have given an expression for mobility in the bulk. We began a discussion of mobility at the surface and we pointed out that mobility at the surface is less than mobility in the bulk because the electron is subjected to additional scattering mechanisms, we outline these mechanisms and explain how the mechanisms affect the mobility now let us proceed further along these lines.

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So we are talking about mobility in inversion layers these were the mechanisms that we had outlined which affect the mobility. Now. The effective ability. Is the mobility of carriers moving along the surface? For low values of parallel electric. Okay, so, in fact, this new effective is the new not or low field mobility of the bulk corresponding to the low parallel electric field affected by the presence of the perpendicular electric field and presence of charges and so on surface charges, surface states and also surface roughness.

So we said that this surface is not smooth it is rough, so presence of all these affect the mobility, now the important point to note is that. The surface scattering mechanisms depend

on an effective value of perpendicular electric field. So with refers to this diagram here. If we call perpendicular as a field just at the surface of the semiconductor. Then the effective value of electric field is less than this value.

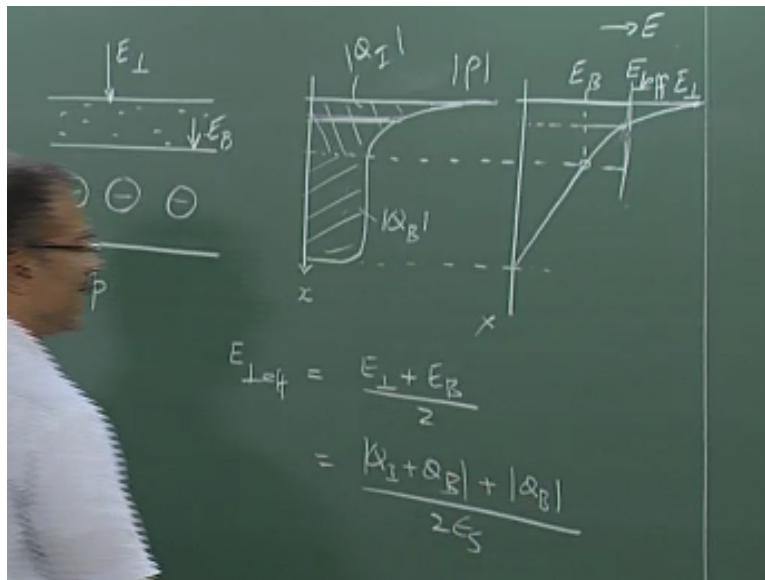
Because not all the carriers over the thickness of the inversion layer, so the green region here is the inversion layer, so not all carriers in the thickness of the inversion layer experience the same value of perpendicular electric field. And that is why one has to use an effective electric field or average electric field that is valid over the entire thickness of the carriers now let us look at a formula for this effective value of electric field.

So you identify the inversion layer charge and beneath that a depletion layer charge. Now this  $E_{\perp}$  is due to this total concentration because of inversion charge plus depletion charge so we can write this. In the following manner, so  $E_{\perp}$  is modeled as  $(Q_I + Q_B)/\epsilon_s$ . We are putting a model here because the  $Q_I$  and  $Q_B$  are negative we are looking at.

P type of straight an inversion layer is therefore made of electrons we can identify another perpendicular electric field which is at the boundary between the inversion charge and the depletion charge. And that we call as  $E_B$ . Again by Gauss's law this  $E_B$  can be written as  $Q_B/\epsilon_s$  where  $Q_B$  has a model assigned. Evidently the thickness of the electrons will experience an average value of the electric field given by  $E_{\perp}$  that is the field entering.

The surface or field inside the semiconductor at the surface plus the field at the edge of the. At the boundary between the inversion layer and the depletion layer.  $E/2$  so this average electric field can be assumed to be influencing the entire thickness of the inversion layer, now substituting  $E_{\perp}$  and  $E_B$ . In terms of charges using the Gauss law we get. The effective value of a perpendicular electric field as  $5 \text{ times } (Q_I + Q_B)/\epsilon_s$ .

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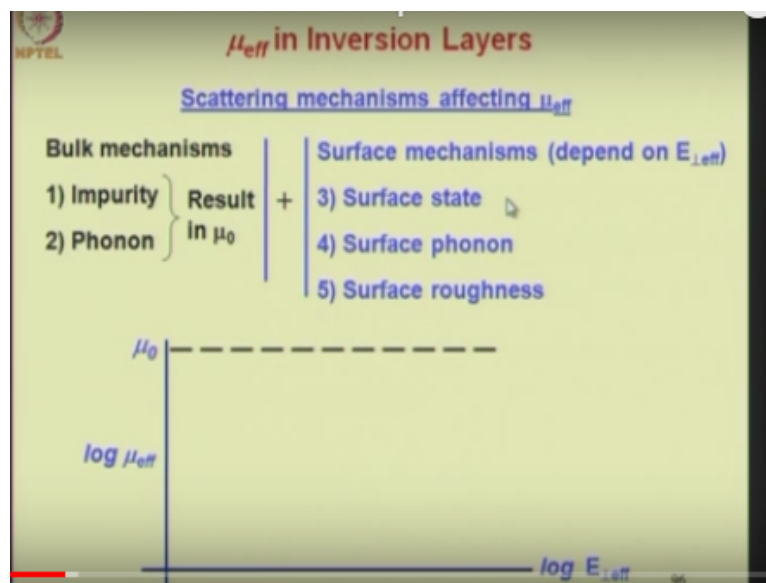
Let us work this out on the board so this is your inversion left. And let us say beneath this you have the depletion layer if I plot the charge picture the modeless of the charge right versus distance X. Then the picture would be something like this now I would like to emphasize that I am not drawing a diagram to scale so you really do not have such a thick inversion layer as compared to the thickness of the depletion layer put here right.

Okay this is a diagram is not to scale so this inversion of electrons. It is a P type of state and this is the perpendicular electric field. Now let us spot the electric field for this so your electric field will vary almost immediately in the depletion region and then it will increase something like this and this. And this is the perpendicular electric field at the surface according to our symbolism and this is the field EB that is the field here.

So this inversion charge experiences this electric field So for example at this point if you see if this is an inversion charge then it experiences so in fact I should really show it as a row and not as Q, models of row because the charge density and the Q. Is the integral under this right so for example this shaded area would be QB and roughly the shaded area. Would be Qi, okay this inversion charge here experiences the field of effective value.

But we are going to use an effective value so that is average of these 2 something like this right that is the average So we assume that all the charges are experiencing this field, so his is E perpendicular effective, so I have E perpendicular effective = E perpendicular + EB/2. Now putting in terms of the Gauss's law Qi +QB E perpendicular depends on this plus this depends on QB/epsilon S/2, here. So that is how you get half of QI and then 2 times QB/2, you get QB.

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Now using this effective electric field let us plot the behavior of mobility so again we are using a log plot. Log of effective mobility versus log of effective electric field because we shall find that the various scattering mechanisms. Because of them the mobility behavior will turn out to be a power log. Function of this electric field. That is why we are choosing log-log plot. Where the various segments would turn out to be straight lines now we start with the mobility in the bulk.

So this is resulting from impurity and phonon scattering in the bulk of the surface state scattering. This is called coulomb scattering because it is because of coulomb force between the charges and the carriers now this goes according to this formula so we are accepting this result from quantum mechanics clearly this TITA power 1 would be possible because a mobility would decrease as these charges increase.

On the other hand, the power of perpendicular electric field here gamma 1 is positive this means that mobility increases with this value of effective electric field. That is why you see the curve rising. As a function of log of effective perpendicular electric field. Now we have explained this in the previous lecture that this is because of the screening effect of carriers so in terms of our diagram here.

Which is this perpendicular electric field you also increase the effective electric field perpendicular electric field. More and more carriers gather together so. In an approximate manner we can imagine that the thickness of the inversion is going on increasing. So as a result what happens is supposing now you have here which is scattering. Now the electric field from these charges will terminate on electrons here.

So. Let us say, the part of the inversion layer this much terminates the electric field because of them. Then the inversion layer beneath has no effect of these charges have no effect on the

inversion layer beneath. Because the field from there has terminated so we say that this inversion layer this part of inversion layer has screened the charges which cause scattering.

Therefore, this thickness of the inversion layer is not affected by this scattering because of this and therefore this mobility will be higher so overall mobility of this inversion layer would be higher and higher because more and more part of inversion layer will be unaffected by this charge. When you increase this inversion charge increases when you increase this electric field and therefore effective mobility that you see increases.

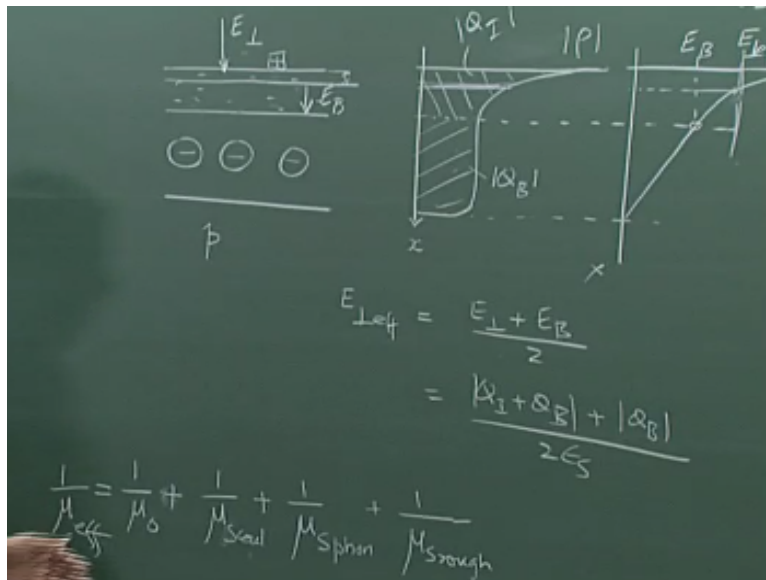
Now this line will shift parallelly down if your surface state increases, now because of phonons mobility falls as a function of the effective electric field this because the carriers are accelerated towards the surface by the perpendicular electric field we have explained and therefore there are more chances of scattering from the phonons at the surface. So more acceleration for a higher perpendicular electric field and higher scattering the mobility falls now as the lattice temperature increases. Then this line shifts parallel down.

This is like the bulk phonon scattering even there the mobility falls as temperature increases the surface roughness also increases the scattering and therefore the mobility falls according to this power law.  $\gamma_3$  is positive so you have the mobility falling here. Now this lines are all placed in such a way that you see that for low electric fields the coulomb scattering seems to be important.

Then for intermediate range here the coulomb scattering sorry the surface for non-scattering is important and then for higher electric fields or roughness scattering becomes important now we can put all these things together like we did in the case of the mobility behavior as a function of temperature even there we have a log log plot and we had 2 segments. And then the overall effect was a curve that assume to take to these 2 segments.

So we can adopt a similar strategy here and the overall curve would turn out to be dot sticked to this segment so it would look something like this. Now this is because we have the remark how mobility because of several scattering mechanisms can be expressed.

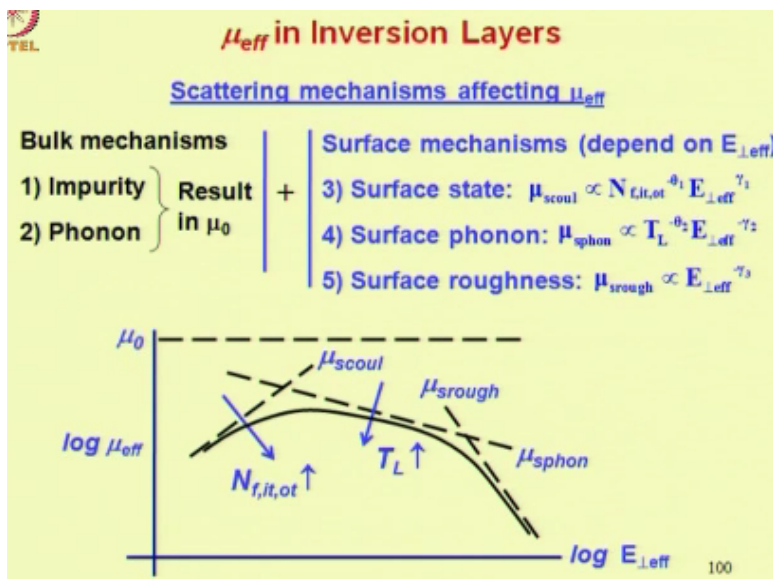
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So you recall we can use Matthiessens rule n. So 1/the mobility overall mobility because of several scattered mechanisms =  $1/\mu_1 + 1/\mu_2$ , so in this case we can put here. Whom surface coulomb for example and surface phonon +  $1/\mu$  surface roughness and of course. You have the mobility in the bulk so that we can write in the beginning in fact as  $1/\mu_0$ .

Because the mobility in the bulk is this and then mobility is falling because of other scattering mechanism. So we can write the overall mobility  $1/\mu$  at the surface. Is some total of all these. So this is the effective mobility right so because of this behavior because it is the reciprocal kind of formula right adding up the reciprocals that you can you find that your overall mobility curve  $1/\mu$  effective or rather the  $\mu$  effective it is asymptotic to the segments because of each of these individual things

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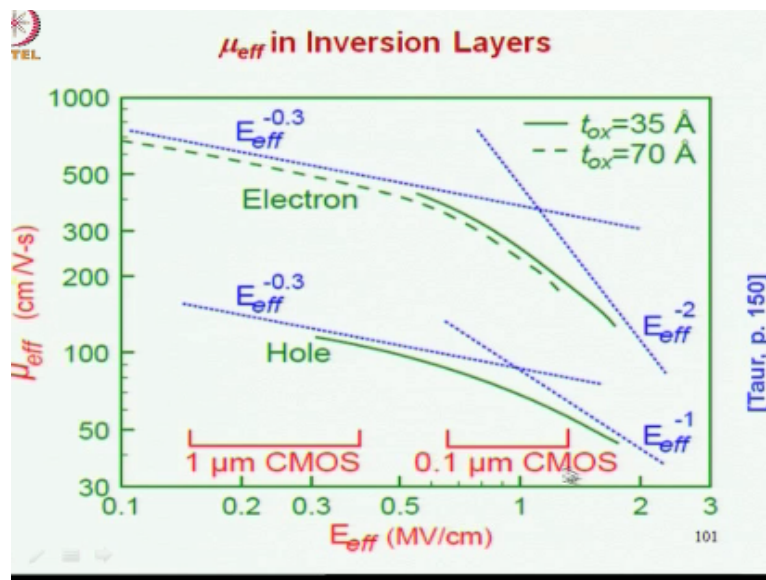


Let us look at some practical data to see where their devices operate on this effective mobility versus effective field graph so you find that 1 micron CMOS. Which is a thing of the past actually now. So for these devices the surface phonon behavior was important so his phonon.

You can see that surface coulomb scattering is not really important for these devices that come somewhere lower on this graph for lower effective values lower effective fields.

On the other hand, what could be called as modern CMOS is moving towards a right on this graph. So 1 micron CMOS you see you are at the borderline here of surface phonon scattering and surface roughness scattering and as you move towards the right. Another channel length of the CMOS devices decreases the surface roughness scattering is what becomes important? So really one needs to have a formula for this range of effective mobility versus effective electric field behavior.

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So we can develop a formula. For that range as follows. We neglect the surface coulomb scattering and the result we can write in the following way so this is the bulk mobility and the sum total of the scattering again is 1/Mu phonon and 1/scattering roughness both of these scattering mechanisms. The scattering increases with effective electric field and therefore they can be clubbed together. Okay and then there is a constant of proportionality coming here.

So what I can do is I can put the same 1/Mu<sub>0</sub>. Here in the multiplication factor an adjust this E<sub>0</sub> to take the effect of this Mu<sub>0</sub> into account and write the constant.

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$$\frac{1}{\mu_{\text{eff}}} = \frac{1}{\mu_0} + \frac{\text{const } \mu_0}{\mu_0 (E_{\perp})^{\gamma}}$$

Now let us look at this point this part so I have  $1/\mu_{\text{effective}} = 1/\mu_0 + 1$  by this sum total of this right now each of this mobility it falls with. Perpendicular effective value and I am using a power law, so I can say it is proportional to  $1 + \text{constant}/2$  to power gamma. That is the power that is used there. It falls so really it is minus gamma, okay because the mobility here falls with perpendicular level now how do I write this constant.

So what I am what we are doing is we write this constant multiply by  $\mu_0$  and this entire part there writing it as  $E_0$  to the power gamma. So that. I can take this  $1/\mu_0$  out and the inside part will become a simple relation like this. And this cannot be identified as a critical electric field beyond which this perpendicular effective electric field has influence and this electric field can be obtained from measurements.

So that is approach that is taken and that is how you get this formula yes. It should be  $E_0^{\text{power} - \gamma}$  because. This is a denominator and this is a numerator, but ultimately it should come in this form, so now this formula can be written. In this fashion for  $\mu_{\text{effective}}$  and  $E_{\perp}$  effect of electric field is expressed in terms of the charges in inverting charge and depletion charge.

Now this formula is very very important it is called the universal mobility formula because you can see that  $Q_I$  is a function of the gate voltage in a mass it can be it is a function of bulk to source bias. Similarly,  $Q_B$  is also a source from bulk to source bias and also it depends on doping in the subject. So you see both  $Q_I$  and  $Q_B$  overall charges can be said we are function of gate bias the bulk source  $x$  bias and the doping level and you see all this effects.

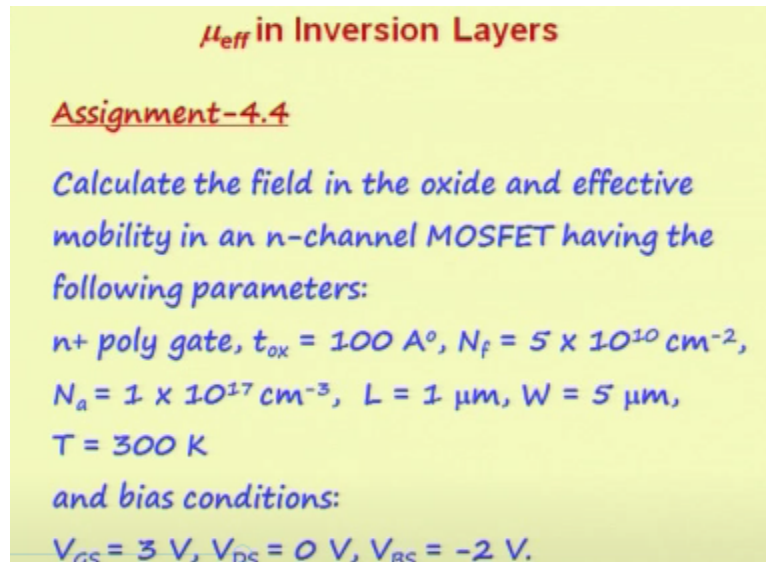
Effects of all these 3 parameters are nicely incorporated to this in this single formula so you do not write a mobility expression, okay, characterizing the mobility as a function of gate voltage as bulk to source voltage and then doping and so on right so all the effects have been



incorporated in this in a nice manner in this effective perpendicular electric field approach these are the values typical values of  $E_0$  and  $\gamma$ .

So you see that the critical electric field is of the order of mega volt per centimeter. So this means that once your perpendicular electric field =  $E_0$  you see mobility falls by a factor of 2 that is the significance of this  $E_0$ , the critical electric field.

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**$\mu_{eff}$  in Inversion Layers**

**Assignment-4.4**

Calculate the field in the oxide and effective mobility in an n-channel MOSFET having the following parameters:

n+ poly gate,  $t_{ox} = 100 \text{ \AA}$ ,  $N_f = 5 \times 10^{10} \text{ cm}^{-2}$ ,  
 $N_a = 1 \times 10^{17} \text{ cm}^{-3}$ ,  $L = 1 \text{ }\mu\text{m}$ ,  $W = 5 \text{ }\mu\text{m}$ ,  
 $T = 300 \text{ K}$

and bias conditions:  
 $V_{GS} = 3 \text{ V}$ ,  $V_{DS} = 0 \text{ V}$ ,  $V_{BS} = -2 \text{ V}$ .

Now, here is an assignment for you calculate the field in the oxide and effective mobility in n channel MOSFET having the following parameters. So the MOSFET has N+ polysilicon gate oxide thickness is 100 Angstrom the fixed charge is  $5 \times 10^{10}$  per centimeter square it is the number of fixed charges then substrate doping  $N_a$   $1 \times 10^{17}$  per centimeter cube, channel length is 1-micron channel width  $W$  is 5 microns and temperature is 300 K.

So biased conditions given are gate source voltage 3volts drain source voltage 0 volts, so the MOSFET is not really carrying a current So for this condition the expressions are somewhat simple so that is why  $V_{DS}$  has been set to 0 and bulk source bias is -2 volts. In the first level course. You have done. You have studied the MOSFET up to some level and that understanding is sufficient to solve this problem.

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$$\frac{1}{\mu_{eff}} = \frac{1}{\mu_0} + \frac{\text{const } \mu_0}{\mu_0 (E_{\text{eff}})^{\gamma}}$$

$$= \frac{1}{\mu_0} \left[ 1 + \left( \frac{E_{\text{eff}}}{E_0} \right)^{\gamma} \right]$$

Now let us look at the saturation with saturation velocity. Okay so you will recall that on the mobility versus electric field curve. The 2 important parts where the initial slope so this is mobility versus electric field here this is parallel electric field, so the initial slope, this is what we said is new effective. If you take into account, the perpendicular electric field and then the other limit that is important to model is the saturation velocity.

So this VSAT and this is drift velocity not mobility, so velocity will occur. So that is what we are doing now we are finished with Mu effective and we are now modeling VSAT and then we can get the complete curve joining these 2 ends.

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**$v_{sat}$  in Bulk and Surface Layers**

**Si**  $v_{sat} = \frac{2.4 \times 10^7}{1 + 0.8 \exp(T_L/600)} \text{ cm/s}$

**GaAs**  $v_{sat} = 1.2 \times 10^7 - (1.5 \times 10^4) T_L \text{ cm/s}$

In above relations,  $T_L$  is in K

$v_{nsat} \approx v_{psat}$  and primarily a function of  $T_L$ , because at such high carrier velocities, coulomb scattering, surface scattering and effective mass, all these become unimportant

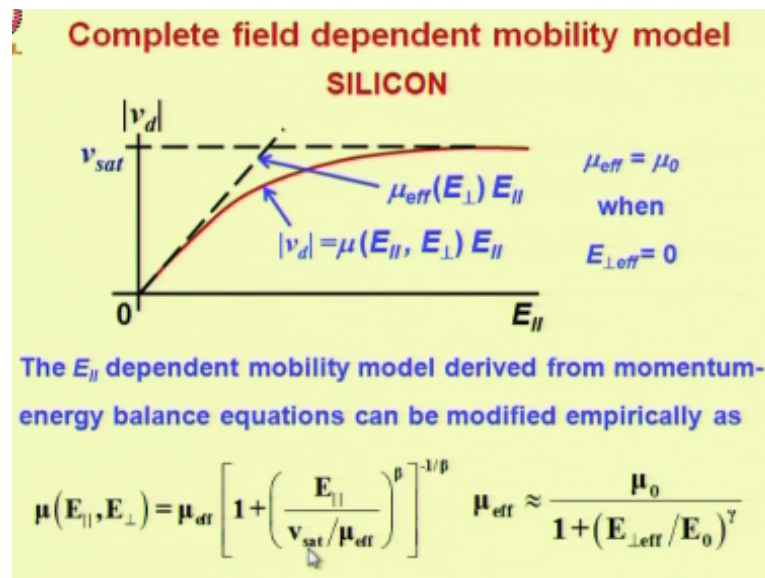
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So the saturation velocity formula there is an empirical relation. So this it falls with lattice temperature for silicon this is the relation for gallium arsenide also. The saturation velocity falls with temperature according to this formula now you see that this number 10 power 7 is the order of thermal velocity. Now you might wonder why other than temperature effective mass doping level and so on why they are not in the picture.

So it is found that the saturation velocity of electrons  $V$  and SAT and holes  $VP$  SAT are approximately the same and primary a function of lattice temperature because at such high carrier velocity coulomb scattering, surface scattering and effective mass all this become unimportant because the carrier velocity are high if there are any charges going to scatter the carrier does not spend much time near those charges right.

Because they are shooting past charges and therefore charges are not able to have much impact.

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Now let us get the complete field dependent mobility model For silicon we said that there is no velocity overshoot beyond the saturation value so the smooth curve joining these 2 ends the slope for low values of Parallel electric field is  $\mu$  effective it is a function of the perpendicular electric field so this line is a straight line  $\mu$  effective into  $e$  parallel. The complete drift velocity expression is mobility which is a function of both  $E$  parallel and  $E$  perpendicular multiplied by the parallel electric field.

So there is a so-called free dependent mobility which depends on both parallel and perpendicular electric fields  $\mu$  effective is  $\mu_0$  when perpendicular electric field effective value or perpendicular electric field is 0. The parallel dependent mobility model the  $E$  parallel. Mobility model derived from momentum energy balance equations can be modified empirically to express the results that we have got okay here so you will recall that.

When we derived the field different mobility model from the balance equations momentum balance and energy balance equations we did not take into account the perpendicular electric field and the scattering because of surface and so on so that is why we have to use some impression so we will use a form of the expression that we have got there but adjust the power and so on.

Now this is what is the resulting expression so the  $\mu$  take into account both parallel and perpendicular electric fields is given by the effective mobility. Multiplied by this term which consists of the parallel electric field divided by saturation velocity by  $\mu$  effective raised to the power of  $\beta$  and  $1 + \beta$  quantity the whole thing is addressed to the power  $-1/\beta$ . And the effective mobility is given by this formula.

This we just now saw a few minutes ago, there does, look at this equation okay. How do we get this now to get the but also the velocity field expression it is very useful to work with the reciprocals.

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$$\frac{1}{\mu_{eff}} = \frac{1}{\mu_0} + \frac{\text{const } \mu_0}{\mu_0 (E_{\perp eff})^\beta}$$

$$= \frac{1}{\mu_0} \left[ 1 + \left( \frac{E_{\perp eff}}{E_0} \right)^\beta \right]$$

$$|V_d| = \left[ \left( \frac{1}{\mu_{eff} E_{\parallel}} \right)^\beta + \left( \frac{1}{V_{sat}} \right)^\beta \right]^{1/\beta}$$

$$|V_d| = \mu_{eff} \left[ 1 + \left( \frac{E_{\parallel}}{E_0} \right)^\beta \right]^{1/\beta}$$

Because we have remarked supposing you look at this problem from curve fitting point of view. Then a curve which fits this straight line. For small values of electric field and this straight line horizontal straight line. For large bodies of electric field right can be easily written as follows.  $1/V_D = 1/\mu_{eff} E_{\parallel}$  by this initial segment for small values of electric field and that  $\mu$  effective into  $E$  parallel. So there is expression for this linear segment.  $+1$  by.

The expression for the other segment the other asymptote that is  $V_{SAT}$ . Notice why does this approach work because  $V_{SAT}$ . When your electric field is very very small then this quantity is large this dominates over this so this falls off and therefore you get  $V_D$  so this falls off and so  $V_D * \mu_{eff} = V_{\parallel}$ . On the other hand, when this  $E_{\parallel}$  becomes very large then this quantity falls off so you have  $V_D$  equal to approaching  $V_{SAT}$ .

And we say now that is a nice thing about this formulation of the equation. Okay, now this approach can be used to actually fit a curve if you know the asymptote, various asymptote denominator right and add up like this, now the issue is how smoothly the curve make a

transition between this straight line and that straight line so this corner here. Right that can be controlled by raising this each of the terms to a power.

So if the beta is high then the corner will be sharp, so for example if I draw another curve here if beta controls the corner. This is how you can approach this problem of getting forfeiting field curve free from an empirical forfeiting point of view physically we get this end and that end and in between however we use a curve fitting approach.

Now, the curve corresponding to the velocity filter that we did to derive using momentum and energy balance equations corresponds to using beta equal to 2. Okay whereas in practice we know that theory always does not fit into experiment perfectly. So we can adjust this values of beta to get a match between theories and experiment. Now you can easily see that this expression can be written as; so I will remove this beta and put it on this site as  $1/\beta$  and so I get.

The VD equal to so in fact, I can do it here. So if I put VD here all that I would do is put this as to the power  $-1/\beta$ . Now if I transform this further I take this  $\mu$  effective E parallel out I will get this is power beta and then you raise it to  $-1/\beta$ . So this comes to a numerator  $\mu$  effective E parallel into  $1 + \mu$  effective e parallel / VSAT power beta to the power  $-1/\beta$ .

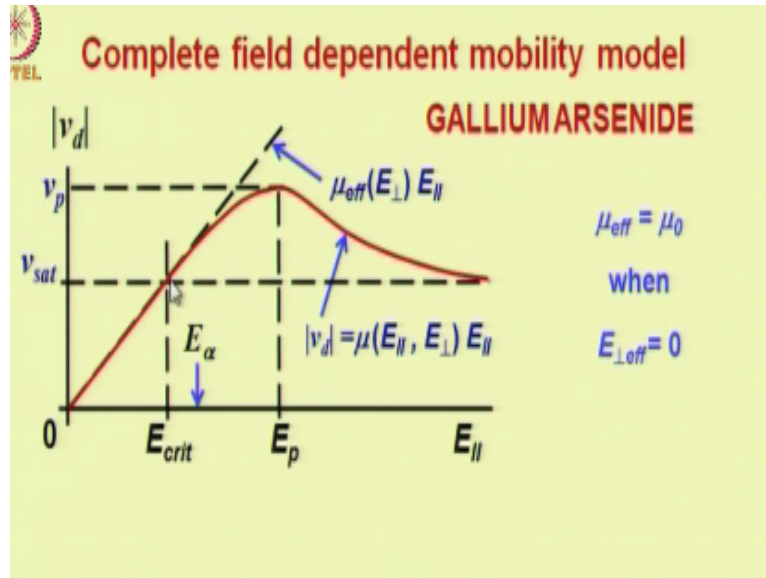
So now further here I could push this new effect in the denominator and write this as VSAT by  $\mu$  effective, the idea of writing like this is to recognize this has the dimensions of an electric field a critical electric now from here. I am getting an expression for velocity versus the parallel electric field the perpendicular electric field is contained in  $\mu$  effective and this is a so-called. critical electric field that one can use now.

If I push this to bring it in the format that we are familiar with if I push E parallel to this side. You can identify this whole quantity as the field dependent mobility right so this is the so-called mobility new function of e parallel and e perpendicular. Because your expression is drift velocity into mobility field dependent mobility into parallel electric field so that is the shape of this curve so this is how if you write develop the expression for drift velocity.

Using the reciprocal approach right so write  $1/VD$  equal to  $1/1$  asymptotic segment  $+1$  by the other asymptotic segment and then you can get the expression very easily so both expressions for mobility for number of scattering mechanisms and expression for drift velocity as a function of parallel electric field so both can be arrived at using this approach where you put the various terms in the reciprocal to start with this kind of form of an equation.

And then you manipulate to get the expression of your interest. So in silicon 300 k the parameters are beta N for electrons it is 2 so your equation developed from momentum balance equations seems to work well. However, for holes that the power of 2 does not work well and it has been found empirical adjustment to 1 is gives you a better fit with the measured data.

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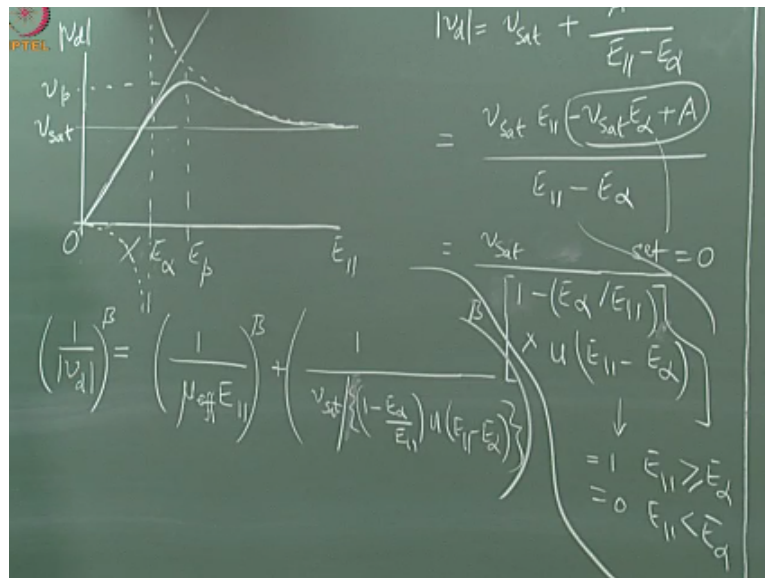
Now we can use the empirical curve fitting approach to develop a model for the velocity field characteristics in gallium arsenide. So we can show the velocity field characteristics of electrons because in gallium arsenide the whole mobility is very poor and so one really does not make devices with. Which depend on the whole current the features of the loss of the field are shown here. The curve rises linearly for small values of E parallel as in the case of silicon.

But it overshoots the saturation velocity and reaches a peak at EP electric field. And the value of the peak velocity is VP and thereafter for higher values of E parallel it decays to the saturation velocity the linear segment here is of the form the effective mobility multiplied by e parallel. Now we have introduced some characteristic field parameters here. E Alpha is the electric field until which the curve remains linear the velocity versus electric field curve remains linear.

The e critical is the field for which the velocity is equal to the saturation velocity now in this particular diagram e critical is less than e alpha and therefore e critical is really a point in the linear portion of the curve however it can happen in some situations that E critical is more E alpha, in this case E critical should be regarded as the point corresponding to an extrapolated linear segment where the extrapolated linear segment reaches saturation velocity.

Now let us see how in terms of the features shown in this diagram we can develop an equation. For the velocity field curve so we are looking for a function which interpolates the asymptotic limits  $V_D$  equal to  $\mu_{\text{effective}}$  parallel that is the linear segment here for  $E_{\text{parallel}}$  very small and  $V_D$  equal to  $V_{\text{SAT}}$ , the for  $E_{\text{parallel}}$  into infinity and importantly the deviation from silicon is that it must peak in-between. So let us see how we can develop an equation like that.

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So this is  $V_D$ , this is  $E_{\text{parallel}}$ , this is linear asymptote for small  $E_{\text{parallel}}$ , this is a saturation value. Now we want to achieve a peak we can choose a function. Which decreases like this. As an asymptote so the function we want to develop will start like this it will reach a peak and then if you follow this asymptote for a smaller value of  $e_{\text{parallel}}$ . So in terms of this linear asymptote.

And this curved asymptote we can use our reciprocal rule which we used in the context of silicon devices to do an empirical perforating. So  $1/V_D = 1$  by effective mobility into  $e_{\text{parallel}}$  that is this segment + 1 by now the expression for this curve segment now what kind of an expression can we choose there are many possibilities let us try a simple thing like this so you see that for  $E_{\text{parallel}}$  very small  $V_D$  goes to infinity for  $E_{\text{parallel}}$  large it saturates at  $V_{\text{SAT}}$ .

So this kind of an expression now there is a difficulty with this expression if you try it out you will find that this expression is not able to model the speak very well because it falls very rapidly as you increase  $E_{\text{parallel}}$  and therefore you do not get the sufficient amount of peak out of this expression even if you adjust the constant  $A$ . A variation of this would be you shift the curve in such a way to the right in such a way that it goes to infinity at some field  $E_{\text{alpha}}$  which is more than this  $E_{\text{parallel}}$  equal to 0.

So whereas this curve goes to infinity. When  $E_{\text{parallel}} = 0$ , I want to shift the curve so that it goes to infinity at  $E_{\alpha}$ . Now how do I do that so I will simply put  $-E_{\alpha}$  so clearly when  $E_{\text{parallel}}$  approaches  $E_{\alpha}$  this thing blows up okay, let us look at the form of this expression because now you have 2 constants can we reduce it to a single constant So let us manipulate this.

And we will get  $V_{\text{SAT}} e_{\text{parallel}} - V_{\text{SAT}} e_{\alpha} + A$  upon  $E_{\text{parallel}} - E_{\alpha}$ , now note that this whole quantity here is a constant. We can treat this whole thing as a constant right different from  $E_{\alpha}$ . We have already chosen a constant with zero and see whether I would be sufficiently good because we then have a free parameter  $E_{\alpha}$  that should be sufficient to simulate the peak. So if you do that, you said it equal to 0.

Then, your equation becomes  $V_{\text{SAT}} E_{\alpha}$  upon  $V_{\text{SAT}} e_{\text{parallel}}$  upon  $E_{\text{parallel}} - \alpha$  you divide the numerator and denominator by  $E_{\text{parallel}}$ . This is your expression so for  $E_{\text{parallel}}$  approaching  $E_{\alpha}$  from values higher than  $E_{\alpha}$  it will go to infinity. But there is a trouble with this equation when  $e_{\text{parallel}}$  is less than  $e_{\alpha}$  So what happens when  $e_{\text{parallel}}$  is less than  $e_{\alpha}$  then this quantity approaches 1 from more than 1, right.

So you will find far less than  $e_{\alpha}$  the curve is like this. Corresponding to this when  $e_{\alpha}$  is 0, this is infinity,  $V_{\text{SAT}}$  infinity is zero so this is what it is and then. When the  $E_{\text{parallel}}$  approaches  $E_{\alpha}$ . Then it goes to - infinity but we do not want this curve so how do we cut it off well we do a trick we multiply this denominator. By a step function the unit step function which is one for  $e_{\text{parallel}}$  more than  $e_{\alpha}$ .

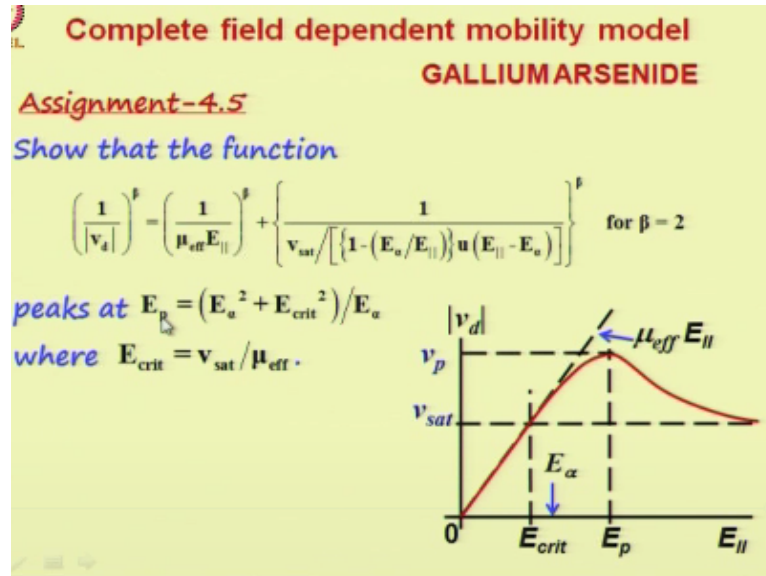
So this quantity = 1 for  $e_{\text{parallel}}$  more than or equal to  $e_{\alpha}$  for 0 otherwise so we are multiplying this here. With the denominator then this part will be cut off and we will have only this now this is a function that we put here so let us put it  $V_{\text{SAT}}$  divided by  $1 - E_{\alpha}/e_{\text{parallel}}$  into the unit step so now the VD will start off on the asymptote and for large volumes of the parallel it will go to this asymptote.

And then it will peak in between so this peak location is  $E_P$  and this peak value is what is  $E_P$ . now I am going to give you an assignment to find out what is the  $E_P$ . And what is a  $V_P$ . if we choose a certain  $E_{\alpha}$  okay, so that is left as an assignment no we need to do one more thing with this equation and that is I want to control the sharpness of this peak so I may get a broader peak I may get a sharper peak and so on right I want a freedom to realize whichever kind of peak.



I get so one more kind of parameter which I would like to introduce to get a better fit with measured data and that we know what we did in the case of silicon so we raise this to the power beta. Reached on this beta, if it is large the corner will be sharp, it will push this peak up and make it sharp, that is how at this particular form of the expression Now that is expression put down here in the slide.

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So the assignment for you is show that this function that we developed just now for beta equal to 2 peaks at EP given by this formula which is a function of E alpha square+e critical square we have already defined the E critical earlier it is the electric field at which the linear segment which leads to the velocity VSAT divided by e alpha/Mu effective and hence we show that you can choose. In terms of EP. And E Critical now what is the significance of this expression.

When you measure your data from the measured data you can get the location of the V.P. and you can also get this value E critical. Because you know what is V SAT and you know the mobility from your measurement therefore from the measured data you can substitute in this formula the values and then you will get the parameter E alpha which you will use in your model of the velocity field curve.

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## Complete field dependent mobility model

$$|v_d| = \mu(E_{||}, E_{\perp}) E_{||}$$

$$\mu(E_{||}, E_{\perp}) = \mu_{\text{eff}} \left[ 1 + \left( \frac{E_{||} - E_{\alpha}}{v_{\text{sat}}/\mu_{\text{eff}}} \right)^{\beta} u(E_{||} - E_{\alpha}) \right]^{-1/\beta}$$

$$\mu_{\text{eff}} = \frac{\mu_0}{1 + (E_{\perp, \text{eff}}/E_0)^{\gamma}}$$

**Bulk GaAs (300 K):**  $\beta_n = 2$

$$E_{\alpha} = \frac{1}{2} \left[ E_p + \sqrt{E_p^2 - 4(v_{\text{sat}}/\mu_{\text{eff}})^2} \right] \quad E_p = 3.2 \text{ kV} \cdot \text{cm}^{-1}$$

**Si:**  $E_{\alpha} = 0$

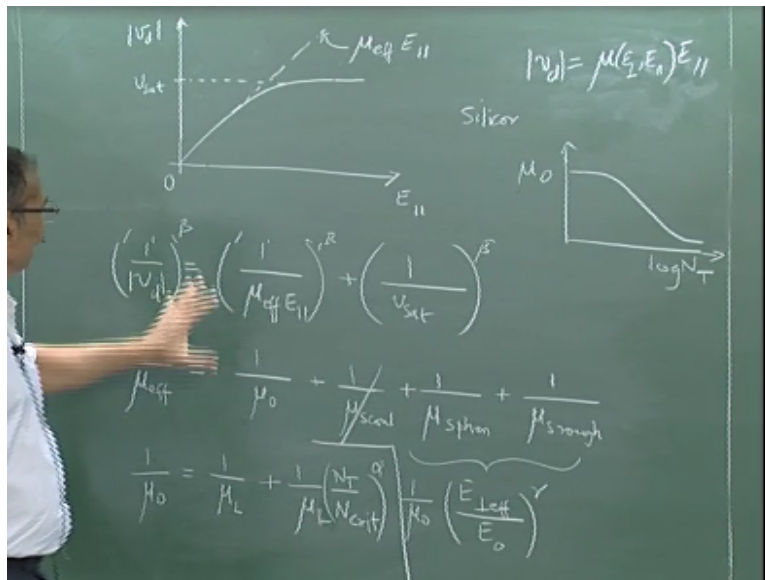
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So let us manipulate the model that we have just developed to write it in a form in which we can directly use it so the complete field dependent mobility model is  $v_d$  equal to the field mobility \*  $E_{\parallel}$  where the field dependent mobility is given by effective mobility multiplied by this term inside and the effective mobility itself is given by this formula. Now for bulk gallium arsenide 300K. For electrons  $\beta = 2$ .

And the  $E_{\alpha}$  is given by this equation which was there in the previous slide, it turns out that many measured data points lead to  $E_p$  of 3.2 per centimeter. Not what is interesting to note is that the same equation works for silicon also if you simply said  $E_{\alpha} = 0$ , if you said  $E_{\alpha} = 0$ , the simple step will be just unity and if you go away from the expression and this is  $E_{\alpha}$  is becoming 0, so you have simply  $E_{\parallel}/v_{\text{sat}} \mu_{\text{eff}}$  here.

We have come to a close of this lecture. A summary of the important points so in this lecture we completed the field dependent mobility model. By adding. The effective mobility model to the model of the mobility in the previous lecture so the overall complete model of the velocity field characteristics can be expressed as follows.

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Now let us first talk about silicon on which we concentrated. For most of the lecture, so velocity versus electric field characteristics so we said that you can write the expressions very conveniently using the reciprocal approach so we write. Reciprocal of the drift velocity is given by  $1/v_d$  by the asymptote for small electric field that is this  $+1/v_{sat}$  by asymptote for high electric field that is this constant line and a parameter to control the sharpness of the curve.

At the corner now the effort to mobility itself again can be written using this reciprocal approach.  $1/\mu_{eff}$  is by  $1/\mu_0 + 1/\mu_{scat} + 1/\mu_{sphn} + 1/\mu_{rough}$ . Here we neglected this scattering, here we wrote about combination of these in the form of the equation  $1/\mu_0 * a$  perpendicular effective electric field by a critical electric field whole power gamma.

So this was put in this form. Now the  $1/\mu_0$  so  $\mu_0$  itself was written as  $1/\mu_{not} = 1/\mu_{mobility to lattice scattering} + 1/\mu_{initium mobility scattering}$ , now this initium mobility scattering was itself, the lattice scattering mobility as  $1/\mu_{not} * N_T/N_{crit}$  whole power Alpha. We did. Discuss a. Small modification of this formula to predict the. Saturation of the mobility for high doping levels but this is ND on a large scale versus the mobility  $\mu_0$ .

Okay so to predict the saturation here we did modify this formula a little bit but essentially what you find is that if you use a reciprocal approach you can quickly write the entire complete field independent mobility model using this approach and then you can manipulate these equations and get your results in the form of this expression  $v_d$  is equal to a field dependent mobility \*  $E_{parallel}$ , where field dependent mobility is a function of both perpendicular and parallel electric fields you can always get in this form.

So writing in that form it comes out as  $v_d = E_{parallel} * \mu_{effective}$  upon  $1 + e_{parallel} / \mu_{effective} / v_{SAT}$ . Now whenever we write like this we might dimensionally check whether

this has a unit of the electric field so this is centimeter square per whole second and saturation velocity is centimeter per second so it comes out as centimeter per whole where it comes out as whole per centimeter, so there is a mistake there. Should be  $V_{SAT}/\mu_{eff}$ .

So that is the critical here, whole power  $\beta * 1/\beta$  where the  $\mu_{eff} = \mu_0$  divided by  $1 + \frac{E_{perp, eff}}{E_{crit}}$  whole power  $\Gamma$ , okay, so this is your final expression for the drift velocity versus the parallel electric field characteristics in terms of the effective perpendicular electric field that is given by 0.5 times the inversion charge plus the bulk charge by  $\epsilon S$ .

As with model sign perpendicularly effective electric field is interpreted as follows so this is your beta state and this is your inversion layer so here your field is  $E_{perp}$  at this point your field is  $E_B$ , this is the depletion charge so this field depends on inversion as well as depletion charge and this field depends on the pushing charge alone. And the effective electric field is average of these 2 so that is the field experienced by the inversion let on average  $\Delta_{inv}$ .

So that is the summary of the present lecture you must learn from this lecture how to write out the expression velocity versus field, when you use this form of writing quantity in terms of reciprocals that is one of the key things that you have to learn how you can use this approach to empirically fit the curve. Once you know that asymptotic linear segments how to fit a curve to asymptotic linear segment because we use this approach also to develop any equation for gallium arsenide velocity field curve.

Right using 2 linear segments one like this and another linear segment coming like this so using reciprocal approach to fit a curve and the approach that will start with this linear segment reach a peak and then approach this nonlinear segment right. So those are the things that we learned from this lecture.