

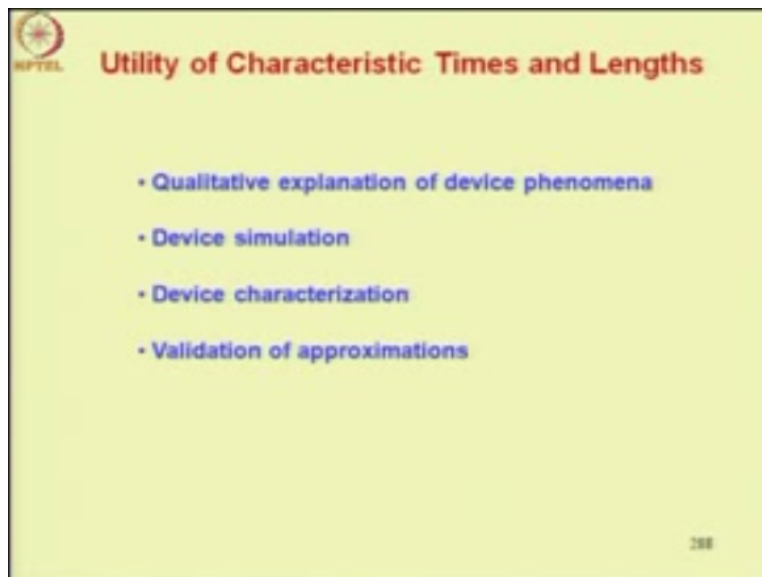
Semiconductor Device Modeling
Prof. Shreepad Karmalkar
Department of Electrical Engineering
Indian Institute of Technology- Madras

Lecture - 25
Characteristic Times and Lengths

So far in the module, we have derived and discussed various characteristic times and lengths. For example, we have discussed minority carrier lifetime, dielectric relaxation time, momentum relaxation time, energy relaxation time, transit time, mean free time between collisions and lengths such as de Broglie wavelength of an average thermal carrier, diffusion length, Debye length and so on.

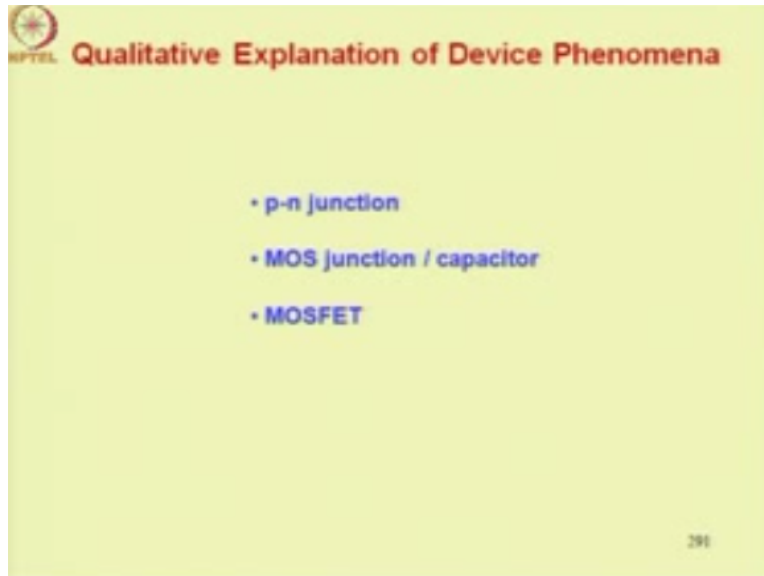
So now we need to discuss what are utilities of these characteristic times and lengths. Now that is a topic of this lecture today.

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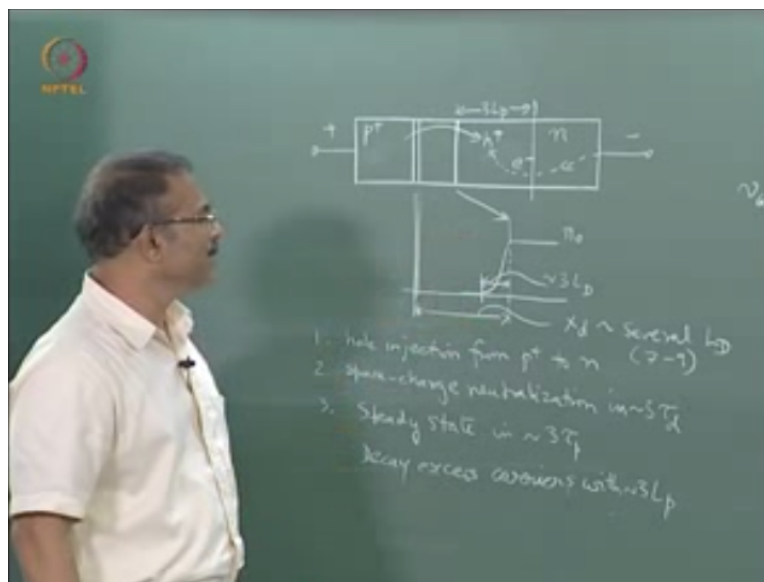
So we shall consider the following issues: Qualitative explanation of device phenomena, how the characteristic times and lengths are useful for this purpose, then how they are useful in device simulation and characterization and finally how they help us validate various approximations.

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Let us begin with qualitative explanation of device phenomena, let us look at the p-n junction, okay.

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Now suppose we are talking of operation of a p-n junction right and we want to explain what happens if you suddenly apply a forward bias. What are the sequences of events, how the conditions in the device evolve? So now in terms of a knowledge of various characteristic times, we could discuss or describe the evolution as follows: Moment you apply a forward bias the P region injects holes into N region.

Now once the holes are injected there will be a space charge and now this space charge will be neutralized in a dielectric relaxation time because of electrons which are drawn in from the contact, so we can say space charge neutralization. So before that we can start with whole injection from P+ to N. Now there will be electron injection also from into P, we are not talking much about it because you know that the electron injection in P+ region is really very small.

So let us look at just one aspect and we can also you know then extend this explanation to injection of electrons also from N to P. So the first stage is whole injection from P to N, the second stage is space charge neutralization okay by drawing in electrons. Now this happens in about 3 times the dielectric relaxation time of this region. Thereafter once you have access electrons and holes, they will start recombining and steady state will be reached in about 3 lifetimes.

So steady state, the third stage is actually meant of steady state in about 3 times the lifetime of holes in N type semiconductor. Now once the steady state is reached, you will find that excess electrons and holes, the decay over length and you know that this length is about 3 times the diffusion length of holes. So beyond 3 times the diffusion length of holes, you have no excess carriers and conditions here will be near equilibrium.

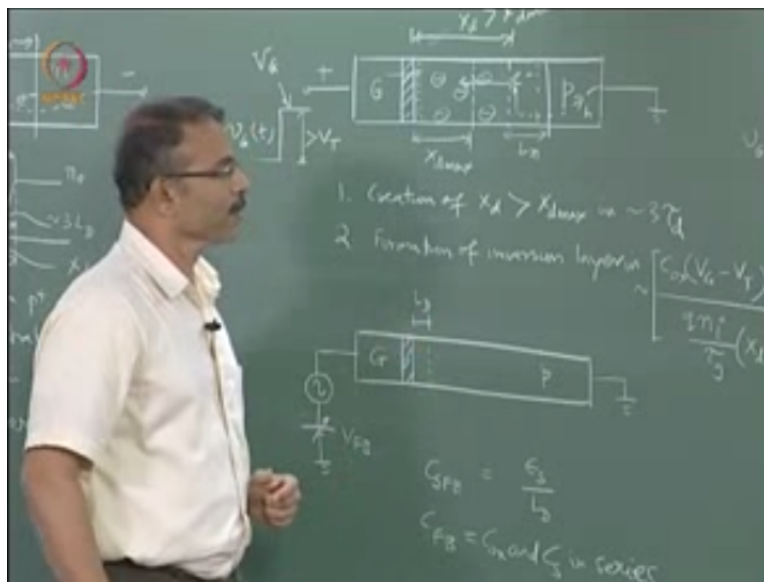
So as a part of the steady state, you will have decay of excess carriers within say 3 times the diffusion length, okay. Now this is how you can describe the various things happening in P-N junction, right when you apply forward bias. Now let us concentrate on the space charge region. Within the space charge region if you plot the electron concentration, it would look something like this. From the edge of the space charge region the electron concentration decays rapidly.

Now what is the width of the space charge region, according to the problem that you have done in assignment, this depletion region we are assuming that the depletion region on this side is really very small. If you want to include that also so the space charge region width will be several Debye lengths right of this likely doped region and within this the extent in which this fall will occur and assignment was given to you to find out how much distance does it take for the electron concentration to fall to one-tenth of its value right.

So it will fall to about 5% of its value and this will also be of the order of Debye lengths except that this would be about 3 times Debye length whereas the X_d would be you know a 7, 8, 9 times Debye length okay about 7 to 9. Now from here you can figure out that the depletion approximation will be valid for the depletion region if this distance is small compared to this distance, okay.

In other words, if your depletion width turns out to be about 3 times Debye length or smaller than your depletion approximation will not be valid. So this is how you can describe the various conditions in a device based on the characteristic times and lengths. Now we have considered the example of a p-n junction. Next, let us take up a MOS junction or a capacitor.

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Now this is a MOS junction. Let us say we apply a step voltage to this. We suddenly apply a step voltage. A forward voltage in this case or positive voltage so that I want to create an inversion layer on this side. I am applying a gate voltage that is more than the threshold voltage. Now how will the conditions in this device evolve to create the inversion layer.

Now what will happen is the first stage would be creation of a depletion layer right because the majority carriers holes can be quickly removed from here in dielectric relaxation time. So creation of a space charge in dielectric relaxation time from majority carriers, so let us say this is

the depletion region edge and this is the space charge that you have created because you applied a forward voltage, it immediately needs a charge.

So the field that you have created needs a charge here to terminate. Now this X_D will be more than depletion width under inversion okay under steady state conditions when inversion happens. Because there is no inversion layer here the entire electric field has to terminate on depletion charge so this will happen in about 3 times a dielectric relaxation time, so creation of $X_D > X_{D\max}$ okay.

Because your gate voltage is more than V_T in about 3 times dielectric relaxation time. Now once this has happened you have created a depletion layer like this. Now inside this the P-N product will become $< n_i^2$ and electrons and hole pairs will be generated. Similarly, electron on hole pairs generated here, part of those some electrons will get attracted to the surface because the field is in this direction it tends to attract electrons.

So progressively what will happen is that electrons generated here within a diffusion length from the depletion edge and electrons generated within the depletion layer, okay they will all start migrating towards the interface to create the inversion layer. And as inversion layer is built up, the depletion layer will progressively collapse, reduce and ultimately reach the $X_{D\max}$ which corresponds to the steady state value and at that point inversion layer will be formed.

So the second stage is formation of inversion layer and how much time will it take, well we can write a simple formula as follows: you know that under steady state, the inversion layer charge would be $C_{\text{aux}}(V_G - V_T)$ where V_G is the value here. This value is V_G . Now that is inversion charge, to create this charge based on the charges created from generation, so now you know the formula for generation time is N_i/τ_g .

This is a generation rate from (10:16) theory we know. So using the generation time constant here, okay, this is equivalent to the lifetime. Now this is generation rate and generation will happen in the depletion layer as well as within a distance from the depletion layer. So let us say

this is minority diffusion length L_n , now note that this boundary is progressively shifting right so this boundary also will keep shifting.

So always within a diffusion length from the depletion edge at any instant, so you can write here $x_D + L_n$. Within this region the generation will happen and all these carriers will accumulate here and create the inversion layer. So formation of the inversion layer happens in about this much time. Now let us check whether our dimensions are correct. You see that in denominator I have not put Q because numerator contains the charge.

So I have to put a Q here. So n_i is per centimeter cube, this is length, so this is per centimeter square. Numerator also is per centimeter square because C_{ox} is normally in per centimeter square, so this is about the time. So this is how we can describe the formation of inversion layer when you suddenly apply a step voltage at the gate equal to a value more than the threshold voltage.

So first a depletion layer is created in dielectric relaxation time 3 times the τ_D and then within the depletion layer and within a region equal to about diffusion length from the depletion layer, electrons are being generated and those electrons are migrating towards the interface because there is a field that attracted to the interface and deformed the inversion layer. What happens the holes, well the holes move out, so this process can be shown like this, the holes are moving out and the electrons are migrating towards the surface.

Now one more example that we can describe in terms of the characteristic length is capacitance of the MOS. Suppose this is your MOS capacitor or junction, what is the flat band capacitance if you want to know, so we have applied a voltage equal to the flat band voltage here. Under flat band conditions you know that under DC flat band conditions, there is no charge here in the semiconductor.

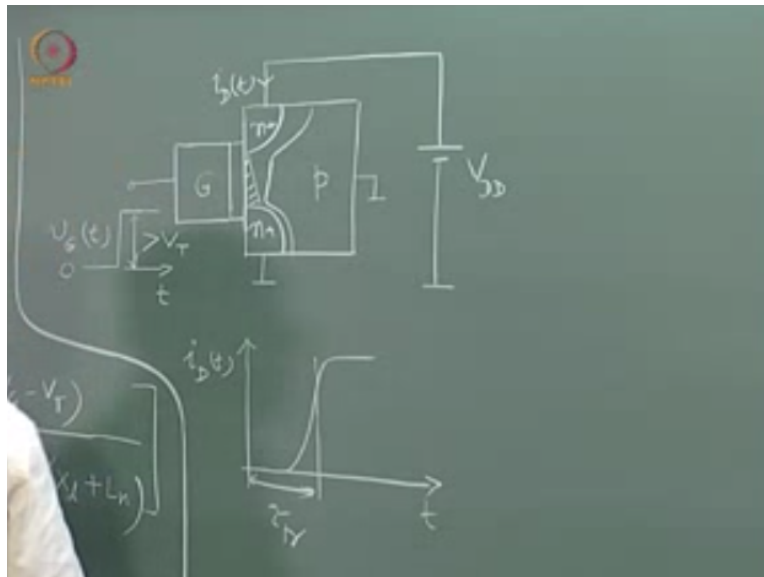
Now when you want to find out the capacitance, it is a response to small signal voltage that is why a small signal voltage has been shown here, so in responses to this voltage a space charge

layer, a modulating space charge layer will be created, now what would be the extent of the space charge region here.

Now that extent would be about a Debye length because we know that in our discussion of Debye length that the space charge regions, they exist within about 3 times of Debye length. So here the region width will be of the order of Debye length and therefore the semiconductor portion will have a capacitance given by $\epsilon S/LD$, so note that this capacitance is not infinity so if you write the flat band capacitance formula, it will be a series combination of oxide capacitance and the semiconductor capacitance.

So it is C_{ox} and C_s in series and that is why at flat band the capacitance is less than C_{ox} , okay because you have this capacitance which is non-infinite because the LD is not 0. Let us take one more example, the example of a MOSFET. Here we want to bring out the utility of transit time.

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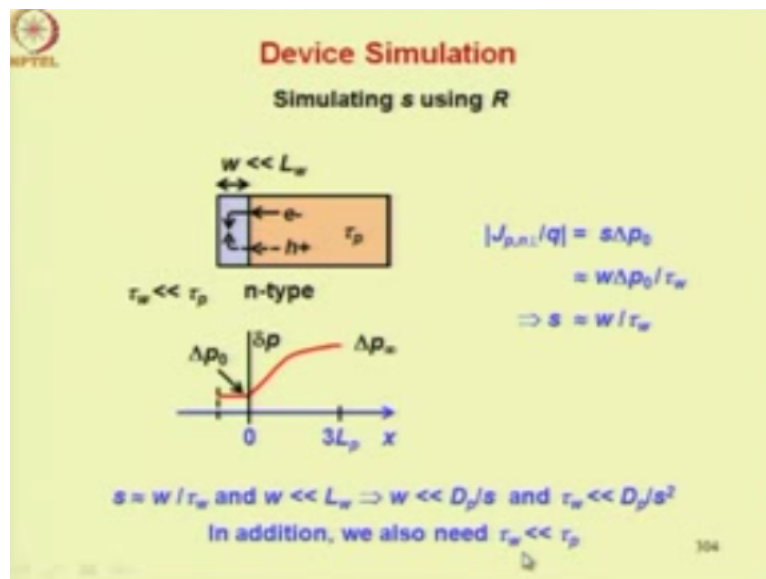
Suppose I have a MOSFET like this, I have applied a drain voltage and I have applied a gate voltage which is suddenly switched from below threshold to above threshold, so from 0 it is switched to a voltage more than threshold voltage. Now when will you feel the drain current. So far, we have not applied a drain source voltage in this case for example even here the voltage was switched from 0.

We talked about formation of inversion layer and so on but there was no drain and source contacts so we are moving to a slightly more complicated situation where we have now put the drain and source contact, so now the issue is not just formation of inversion layer everywhere, when you have a gate to bulk voltage, so you have a drain voltage and you want to feel the current at the drain it when will this happen.

So here you can bring in the concept of transit time, you know that the electrons at the source end will take about a transit time to reach the draining and therefore for this formation of the full inversion layer from source to drain, it will take about a transit time. So moment you apply a gate voltage, the inversion layer will start building up, so electrons in the inversion layer will take about a transit time to reach this draining, and therefore you will find that the IDT in response to VGT will appear after a transit time.

So the IDT versus T would look something like this, where this duration is about a transit time of carriers from source to drain.

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Let us see how our characteristic times and length can be useful in Device Simulation. Consider the problem of simulating a surface recombination using volume recombination, so let us we have a simulator which does not give provision for specifying surface recombination at a particular surface.

But you know that you want to simulate a device in which there is surface recombination, right in practice there is surface recombination, so how will you use the volume recombination provision of the simulator to realize a surface recombination that is what we are discussing here. Now let us say this is an n-type region and you want to simulate a surface recombination velocity S at this left surface.

Let us say the lifetime of this bulk region is τ_p . Now this is a region of the device, we have not shown the other devices here because that does not concern us, but please remember in a simulation situation may be more complex and you will have other regions connected to this region to form a device. Now how do we simulate a surface recombination at S .

Now let us see what is the effect of surface recombination, the effect of surface recombination is electrons and holes will be moving towards the surface and recombining over there, the electron hole pairs. To simulate this situation, we need to actually simulate a carrier concentration variation shown here. We have shown the excess hole concentration as a function of X , you can similarly show excess electron concentration you know that cause a neutrality holes usually and therefore ΔN will be $= \Delta P$, at least approximately.

So we would like to simulate a minority carrier distribution as shown here, the value of this excess hole distribution is ΔP_{∞} for X very large and it dips down to $\Delta P_{surf} = 0$ at $X = 0$, where you have surface recombination to create this tendency for the hole current. Now you know that this variation will occur over a distance equal to about 3 times the diffusion length of holes.

So we really want to simulate electron or a hole current towards the surface perpendicular to the surface given by this formula so the magnitude of the current divided by Q is surface recombination velocity S into the X 's whole concentration at $X=0$ that is this value. Now you can do so using a semiconductor region so what we are doing here is we are extending the semiconductor bulk here by another semiconductor region, right.

This semiconductor region is such that the lifetime of holes in this region is very small compared to the lifetime of holes in the bulk here, now because the lifetime here is smaller there will be a tendency for electrons and holes to move into this region and recombine over there, so please note that similar arrows exist in the bulk of the parent n-type region also. We are not showing those arrows because we are showing only the difference in the recombination between this extension and the parent region, semiconductor region.

Now, how much should this recombination be, this is what we have to see, so this recombination should be so as to simulate the surface recombination velocity. Let us assume the width of that region is W , to start with let us say this W is more than 3 times the diffusion length of holes in the region W that is $L_p \ll W$. Now $L_p \ll W$ will be much shorter than $L_p \ll P$, that is this because lifetime of holes in this region extension is much less than the lifetime of P in the bulk.

Now what will be the effect of this on the excess carrier distribution, so the excess carrier distribution will decay to 0 as shown here in this region, so far away your excess carrier concentration will decay to 0. Now, you might like to think now why is there excess carrier concentration here at all. Now that really does not concern us at this point, there could be many reasons for excess carriers being present there because it is a device.

There may be injection from some other region or there maybe light being shown on the sample and therefore there is volume generation, that does not concern us, right. What we have to concern with is the effect of that, so whatever is the source of excess carriers here please note that source of excess carriers is not here, this is simply a semiconductor extension, so here far away from this surface your excess carrier concentration will go to 0.

Now we want to adjust our W and $\tau_p W$ in such a way that the value of the excess whole concentration at the surface remains $= \Delta P_0$. Now if you do that then we would have simulated the surface recombination velocity S according to this formula. Now let us say we choose a W much less than the diffusion length of holes there. This is to simplify matters. Now

what would happen is this whole distribution now would remain almost approximately constant over this region, okay.

From our knowledge of the characteristic length we know that if the width of any region is much less than the diffusion length there, the excess carrier concentration would remain approximately constant, the decay will be very small, so that is why the whole concentration is now assumed to be approximately constant over W . Now this will help us simplify the formulation of the problem.

So now we can write that the current divided by Q , that is the flux is equal to W that is a width of this region into ΔP_0 divided by τW . Now this is actually the recombination shown by these arrows here, so the flux that is represented by this electron and whole arrows perpendicular to the surface here is the same as the recombination taking place here, taking into account the dimensions properly.

So here whenever you talk about the flux, we talk about per unit area therefore this is the volume recombination per unit area of this surface. So if you want to take into account the actual area then this formula would be W into the area into $\Delta P_0/\tau W$. However, then you will have to change the formula for the flux and you will have to put an area here also. Now equating these 2, what do we get, so we get the surface recombination velocity as $W/\tau W$ because ΔP_0 will cancel.

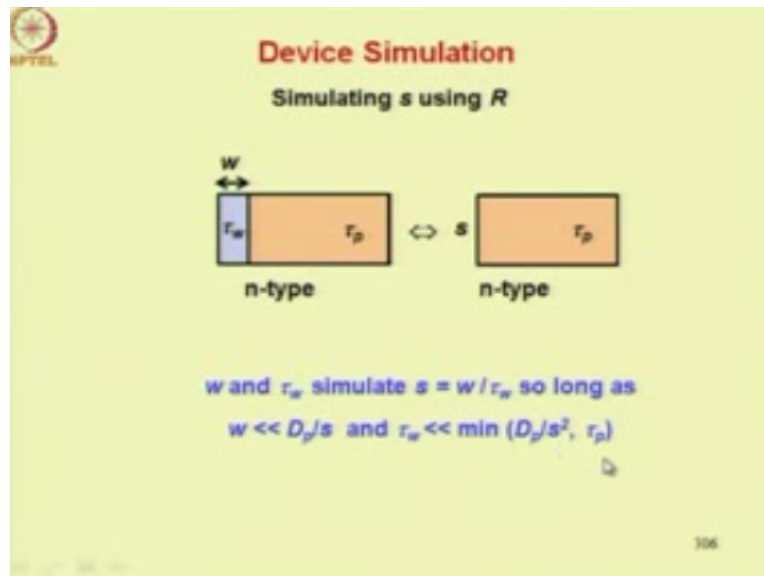
Now this tells you that if I chose an extension W which is much less than LW then by adjusting my lifetime according to this formula I can simulate the surface recombination, so this is what is summarized here, this is approximately $= W/\tau W$ and W is much less than LW , they are the conditions, so this imply the width of your region that simulates surface recombination should be much less than DP/S .

So I can combine these results, okay, because this formula is valid only under this condition, so I leave it as an assignment to you, you substitute in LW square root of the DP into τW because that is what the diffusion length is and then combine these results and you will get this formula

and similarly the condition on τ_w would be it should be much less than DP/s^2 , so this again can be obtained by combining the results here, okay.

Now, you must take care that τ_w remains much less than τ_p also, so that is the additional requirement.

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The summary of our discussion is that a surface recombination S can be simulated using a semiconductor extension with W satisfying the conditions that we have just now outlined and a lifetime τ_w also satisfying the conditions we outline, the important thing about τ_w being, it is much less than τ_p . So W and τ_w simulate $S=W/\tau_w$ so long as W is much less than DP/S and τ_w is much less than minimum of DP/s^2 and τ_p .

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Device Characterization

Determination of doping profile from C-V measurement

- C-V measurements are sensitive to the mobile (majority) carrier concentration rather than doping.
- For a small abrupt change in doping, the mobile carrier concentration varies over a few L_D .

⇒ Abrupt changes in doping occurring in a scale less than L_D cannot be resolved in measurements

Now let us look at device characterization, where can our ideas of characteristic length and time be useful. Let us discuss determination of doping profile from C-V measurement. Now how do you measure the doping profile from C-V measurement, consider P-N junction or a short key junction or a MOS right, capacitor, so what you do is you measure its C-V characteristics, right and you know that the capacitance voltage characteristics will tell you the doping profile.

For example, if you take this MOS capacitor, as you change this voltage, their depletion region changes right and that is how you can find out what is the doping at the edge of the depletion region. There is a formula, you must have studied this formula in the first level course. We are talking about that C-V measurement right to measure the doping profile, so what is the doping variation in the X region, in this bulk.

For example, if you take the P-N junction what is the doping in the likely dope region, what is the profile here. Now C-V measurements are sensitive to mobile majority carrier concentration rather than doping, although you are measuring doping, actually you want to measure doping what the C-V measurement does is monitors the mobile carrier concentration, mobile majority carrier concentration.

And now you know that you cannot always assume the majority carrier concentration is approximately equal to doping concentration why because the doping varies too rapidly there can

be space charge region. So that is the point that we are bringing out here what should the variation of the doping so that you can capture the variation, so for a small abrupt change in doping the mobile carrier concentration varies over a few LD.

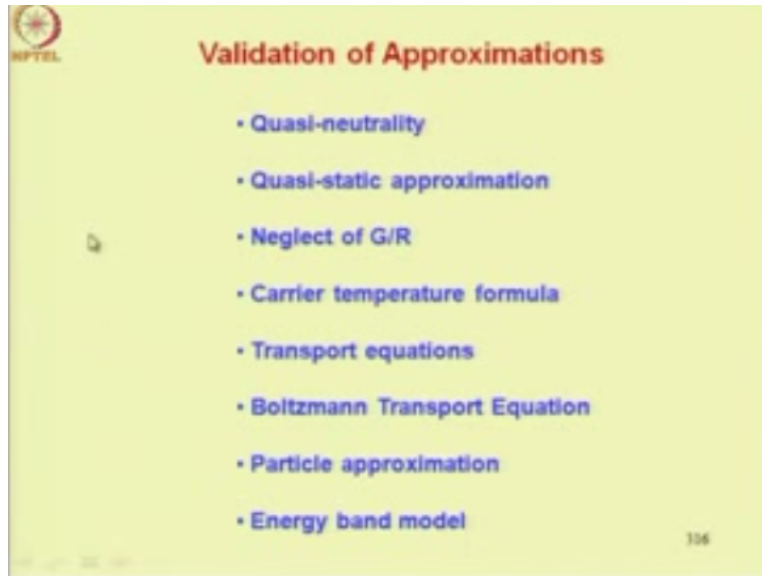
Now, this is the picture, so if you take an N plus N junction, the doping changes abruptly at this surface. Now this is abrupt change in doping. Evidently, the mobile carrier concentration cannot change abruptly because it will result in infinite diffusion currents at the point where abrupt changes occurring and it will change gradually and now from our knowledge of the Debye length, we can easily anticipate that the space charge region over here will extend to about 3 times the Debye length, right.

And similarly on this side also it will be 3 times the Debye length where Debye length corresponds to Debye length of this region over here, which is slightly smaller because doping is higher. So, this is what we mean by saying the mobile carrier concentration varies over a few LD, so now since you are monitoring this mobile carrier concentration and C-V measurement in effect, you cannot capture this rapid variation in doping right.

So this is what is the limitation of the C-V method, so it says that abrupt changes in doping occurring in a scale less than LD cannot be resolved in measurements, okay so if you have a doping profile let us which changes abruptly like this, step like profile, you cannot get the steps out from a C-V measurement data. You will get a more smooth variation, so what the characteristic length Debye length tells you is that the doping variation should not be very rapid over a scale of LD.

So it gives you some idea of what kind of doping variations can be captured.

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Let us come to validation of approximations. Large number of approximations are used in device analysis and modeling, Quasi-neutrality, Quasi-static approximation, neglect of generation recombination, some of these we have discussed already. Then we use a carrier temperature formula, this carrier temperature formula if you recall we have derived using steady state and uniform conditions in the semiconductor.

But still the formula is used for even non-uniform and high-frequency conditions, so that is approximation that we are talking about right, so how is it valid. Then, we are using the various forms of transport equations, division transport equation and balance equations, Boltzmann transport equation and so on. So each of them has their validity range. Then we are making a particle approximation and finally we use an energy band model.

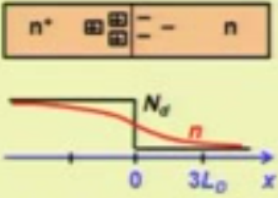
So all these things are often used in device analysis. So let us see how the idea of characteristic times and length enables us to validate these approximations.

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Quasi-neutrality

This approximation holds in a semiconductor region

- which has a high mobile carrier concentration



The diagram shows a rectangular semiconductor region divided into two parts. The left part is labeled n^+ and contains several small squares representing donor ions. The right part is labeled n and contains several small circles representing electrons. Below this is a graph with a horizontal axis labeled x . A vertical line at $x=0$ is labeled N_d . A red curve representing carrier concentration starts at a high value for $x < 0$, drops sharply at $x=0$, and then decays exponentially towards zero for $x > 0$. A point $3L_D$ is marked on the x -axis.

- whose doping does not vary significantly over local L_D
- which is several L_D away from fixed charges


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Let us take Quasi-neutrality, this approximation holds in a semiconductor region which has a high mobile carrier concentration whose doping does not vary significantly over LD, because as we discussed just now if the doping varies rapidly over length scale LD then there will be a space charge region because mobile carrier concentration cannot vary as rapidly, so then causing neutrality, the neutrality will be violated because in this region you have space charge and here also you have space charge.

Now region which is several LD away from fixed charges tends to be Quasi-neutral, you can see here for example a region which is in this place that is beyond 3 times LD from the source of the space charge will be Quasi-neutral and similarly on this side right so that is what is meant here a region which is several LD away from fixed charges. Now in this case there is no specific location of fix charge, but if there is a source of charge and if you are away by several LD from there you will have Quasi-neutral region.

So the Quasi-neutrality will be valid for all these cases. Now look at the high mobile carrier concentration. Why this allows us to use Quasi-neutrality.

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Quasi-neutrality

This approximation holds in a semiconductor region

- which has a high mobile carrier concentration \Rightarrow
 - small $\tau_d \Rightarrow$ operating frequency tends to be $\ll (2\pi\tau_d)^{-1}$
I.e. the carrier concentration does not change much over duration τ_d
 - small $L_D \Rightarrow$ fixed charges tend to be screened within a short distance leaving most of the region quasi-neutral
- whose doping does not vary significantly over local L_D
- which is several L_D away from fixed charges


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This can be understood in terms of the time constants, direct relaxation time and the Debye length L_D , now high mobile carrier concentration means small τ_D and small L_D , you know this from the formula for these characteristic parameters. So small τ_D means operating frequency tends to be much less than $2/\tau_D$ reciprocal of this quantity right.

Because if τ_D is small this quantity would be large and therefore your operating frequency would be much less than this and that means the carrier concentration does not change much over a duration of a τ_D . Now small L_D means fix charges into the screen within a short distance living most of the region Quasi neutral because we have just now seen that if you are several L_D away from fix charge, you are in a Quasi-neutral region and if L_D is small.

This amount by which you need to be away is really very small so most of the region would be Quasi-neutral.

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Quasi-static Approximations

Approximation	Valid for
$\cancel{\partial n} = (1/q)\partial_x J_n + G - (n - n_0)/\tau_{minority}$	$f \ll (2\pi\tau_{minority})^{-1}$
$\cancel{\partial Q_1} = -i_s(t) + (-Q_1/\tau_e) - i_s(t)$	$f \ll (2\pi\tau_e)^{-1}$
$\nabla \times \mathbf{B} = \mu \mathbf{J} = \mu (\mathbf{J}_n + \mathbf{J}_p) + \mu \cancel{\epsilon} \mathbf{E} = \mu \epsilon [(E/\tau_e) + \cancel{\epsilon} \mathbf{E}]$	$f \ll (2\pi\tau_e)^{-1}$
$\nabla \times \mathbf{E} = -\cancel{\partial \mathbf{B}}$	Device size, $L \ll \lambda_{EM\ wave}$ or $f \ll (L\sqrt{\mu\epsilon})^{-1}$

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Let us look at Quasi-static approximations. Let us look at these 2 approximations, now this is a Quasi-static approximation of the carrier balance equation, this is aquatic carrier balance equation, you can neglect $\partial n / \partial t$, so long as your frequency is much less than reciprocal of 2π into minority carrier lifetime that is this time, so in other words we are saying that this term will be negligible compared to this term.

So long as your frequency follows this. Now this is how you get a condition in terms of your characteristic time for a validity of this approximation. Similarly, if you integrate this equation, you get an equation of this type. This is called an equation for stored charges. Now this kind of equation is useful for example in MOSFET, so here let me explain what this equation is. This term is the source current, current into the source terminal.

This term is the bulk current and this term is actually the drain current under steady-state conditions so we have used the transit time concept to write the current. You know that under steady-state conditions the transit time from one into the other end can be written as ratio of charge by current and so the current can be written as the charge that is inversion charge that is causing the current divided by transit time.

This is under steady-state, however, we are writing the formula under transient conditions, so there is a negative sign here because for electrons, the inversion charge is negative right and

therefore there is a negative sign here. Let us spend a few minutes to explain how you get this equation from this equation. In fact, let me first point out the 1-to-1 correspondence, this term is obtained by integrating this term on the left hand side.

These 2 terms emerge from integration of this term, the diversions of the current density when you integrate this, you get these 2 terms for the 2 limits of integration, the drain and the resource. There is really no excess generation in the MOSFET, we are assuming such a condition because of light impact on this shown, all the generations are neglected.

So this term is not there and this bulk current is actually a consequence of the generation recombination current that is coming by integration of this term here. Now let me quickly show you the steps in which the terms emerge.

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The image shows a chalkboard with the following handwritten equations:

$$\int_{y=0}^W -qW \frac{\partial n}{\partial x} dy = \int_{y=0}^W (-qW J_n) dy = \int_{y=0}^W \left(-qW \frac{\tau}{L} \right) \frac{\partial n}{\partial x} dy$$

$$Q_i = -qW \int n dy$$

$$i_n = W \int J_n dy$$

$$Q_D = -qW \int \frac{n - n_0}{L} dy$$

Now let us look at the MOSFET here, the N is the electron concentration here that is causing the inversion layer right so now how do I write a formula for the version charge you know that QI the inversion charge can be written as integral of the electron concentration with respect to DY that is Y so in this case we will assume that this direction is Y and this direction is X, so indicated along this direction.

Then you should also integrate along this direction right because you want the total charge so dQ and you should multiply by $-Q$. Then you must also take into account the fact that there is a width direction here, W okay, so in this direction that is W , so we have to multiply this whole thing by W . So, W into dY into dX , so you integrate in this direction, integrate in this direction, and take into account the fact that there is a W in the direction perpendicular to the board.

So you get a volume W , dY , dX , multiply the volume by a charge concentration, you get the number, multiplied by charge, which is $-Q$ because it is electrons so that is really your QI , now let us look at the current I_D and I_S , so I_S is the current here in the source. Now current anywhere, you want to get the current, how do you get the current from J_N , well your current I_N at any $X = \int J_N dY$ into W okay.

So integrating over and you are taking W that is a width in that direction. So, W into dY is nothing but the area cross-section of the current flow right, the current is flowing like this, it has an area of cross-section, if I show is in 2 dimensions, it would be probably like this, your W is in this way, so something like this, your current flow is in this direction, this is dY and that is W okay, so this is the area, W into dY at each end.

Now what about the bulk current, the bulk current is the current here that AB , now this is because there is a generation of electron hole pairs, electrons move to the positive drain contact in an n-type entire MOSFET and the holes move out here, so that generation is represented by this, so I can write B as so let me write this as small I_N because the reason is that things are changing with time.

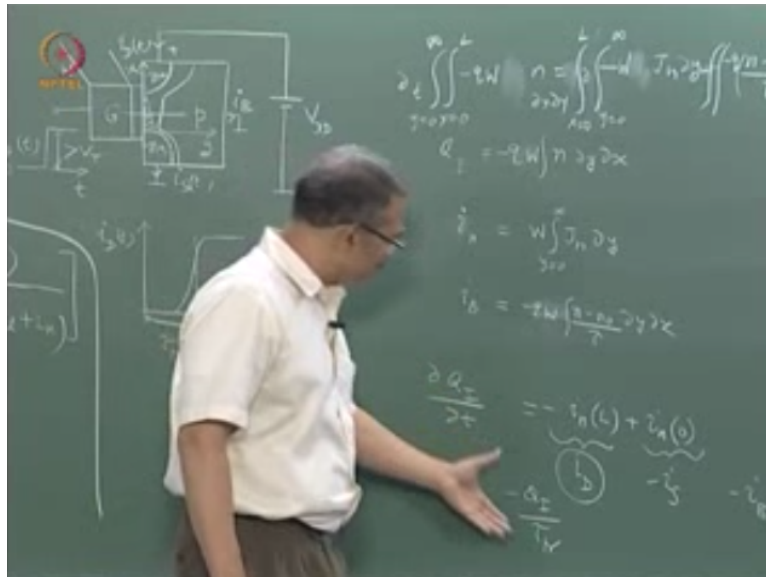
So i_b is $n-n_0/\tau$, again as we have done here we need to integrate over dY , dX and multiply by $-q$ to get the charge over here and w . Then it gives you the current i_b , now note here that this was charge but this is current because there is a time constant in the denominator here to compare these 2, the numerator gives you the charge and this is the time constant that is τ , so now how can I get a relation between these from here so if you integrate now.

So what I do is I multiply this/dou x and dou y right and multiply this also/duo x, dou y. Now, this is dou/dou x, so the dou x in that that dou x has been brought here and I multiply this/dou x and dou y. Then I multiply by -q because I have get -q here. If I multiply by -q, this q goes away, you get a - sign and multiply by -q here. Then I multiply by w, I have to get w, so that gives you a w here and it gives you this w that you require here.

Okay, let me put it on this side, because there is no place here. So now what I do is I integrate, so when I integrate this, when I am integrating with respect to y, it is double integration. I integrate from y=0 to y=infinite. And when I integrate in next direction, I integrate from x=0 to x=l, this is a channel length l, okay. So you have to do the same thing on each of these terms, okay.

Now what will the result be, so this will give you now that by dou/dou t, because its time derivative can be brought out when you are indicating with respect to x and y.

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So I will put it like this, I will remove this out. So this term will reduce to dou qi/dou t. Let us go to right hand side, this term will reduce to you have seen here, so here you are going from y=0 to infinity and there is a dou of jn, so when I move this dou out here. So I can integrate from y=0 to infinity and before when I integrate with respect to x, I take the dou out here, so x=0 to x=l. So this quantity really will turn out to be according to this result.

The value of I_N at $x=L$ minus value of I_N at $x=0$ okay, but there is a negative sign here and therefore it will become minus of I_{NL} plus of I_{N0} . Now what is I_{NL} . So electron current at L . This is nothing but the drain current, the drain current is inward and the electron current is outward, right and therefore this quantity can be written as minus of I_N is I_D , minus of I_N at L is I_D and similarly this quantity can be identified as minus of I_S that is the source current, this again downward whereas your X is upward.

Now what about this here, so this current can be directly related to I_B , there is a minus sign, so you have minus of I_B , so minus of I_B . Now under steady-state conditions, this I_D is written as minus of $q_i/\text{transit time}$. So we use a steady-state result in this transient case to solve this equation in a simple manner, so this kind of an approximation you can do if your dq_i/dt is small that is so-called Quasi-static approximation in the context of MOSFETs, the very important approximation.

So you can see from here that this approximation can be done if this term turns out to be much less than the magnitude of $q_i/\text{transit time}$. Okay, so that is what is shown here, so dq_i/dt , this will turn out to be much less than this quantity and the Quasi-static approximation will hold if your frequency is much less than $1 \times 2\pi \text{ transit time}$.

Let us look at some other Quasi-static approximations, so you take Maxwell's equations, $\text{curl } B = \mu \times j$ that is $\mu \times j_n + j_p$ the j also includes the displacement current. Now often we like to neglect the displacement current because we say the frequency is not very high. Now what is the condition for that, so you can write this sum in this form where we assumed a drift current for j_n and j_p .

So it is expressed in terms of e and from here it is very clear that I can assume $d\epsilon/dt$ of e as negligible if it is small compared to e/τ_d and therefore in terms of the frequency this condition will be valid if f is much less than $1 \times 2\pi \tau_d$. So this is how a knowledge of the dielectric relaxation time helps us to explain the conditions that should be satisfied, the frequency conditions that should remain in the device for the Quasi-static approximation to hold.

Similarly curl E is $-\dot{b}/\dot{t}$. In this case we can neglect the time varying magnetic field when the device size L is much less than the electromagnetic wave length. We will not spend much time in explaining this point. You know that if the device size is small then it does not radiate, right if you want an antenna then the antenna length should be comparable to the wave length of the frequency that you want to radiate out, this you might know from your basic knowledge of electromagnetic fields.

Something like that is being used here, so such a radiation will not happen, we do not have to bother about any radiation or in other words, we do not have to bother about time varying magnetic fields if your device size is less than electromagnetic wavelength and when you convert this into frequency, so writing the electromagnetic wavelength in terms of the velocity of the wave and the frequency you get this in equality.

So frequency should be much less than the device length multiplied by square root of mu epsilon where mu is a permeability and epsilon is the permittivity.

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Approximation	Valid for
$\cancel{\partial_t} W_n = \partial_t F_n + E J_n - (W_n - W_{ext})/\tau_t + S_t$	$f \ll (2\pi\tau_t)^{-1}$
$\cancel{\partial_t} J_n = (2q/m_n)\partial_t W_n + (q^2 E n/m_n) - (J_n/\tau_M)$	$f \ll (2\pi\tau_M)^{-1}$
$\cancel{\partial_t} A = -\gamma \nabla \cdot f - F \cdot \nabla_p f + \partial_t f _{t=0} + s(r, p, t)$	$f \ll (2\pi\tau_t)^{-1}$

Then we have come across the Quasi-static approximations of energy balance equation and momentum balance equation. This is what is shown here, so just repeating those facts, \dot{W}_n can be neglected if your frequency is much less than $1/2 \pi$ energy relaxation time and

$\frac{dJ_n}{dt}$ can be neglected if your frequency is much less than 2π momentum relaxation time reciprocal.

We have used this formula for carrier temperature derivation. Finally, the Boltzmann transport equation, we can neglect the time varying term on the left-hand side if your frequency is much less than $2\pi\tau_c$, $1/2\pi\tau_c$ where this means time Winfrey time between collisions. Now a caution here we are using the F symbol in 2 cases, so here this F is not the frequency as you would have already recognized this F here in the Boltzmann transport equation is the distribution function, whereas this f here is the frequency okay.

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Neglect of G/R

Critical region(s) of modern VLSI devices are \ll the diffusion length of minority carriers in the region(s)

\Rightarrow G / R plays little role in VLSI device operation and can be neglected in the continuity equations

$$\frac{\partial n}{\partial t} = (1/q) \nabla \cdot J_n + \cancel{G} - \cancel{(n - n_0)/\tau_{minority}}$$

$$\frac{\partial p}{\partial t} = -(1/q) \nabla \cdot J_p + \cancel{G} - \cancel{(p - p_0)/\tau_{minority}}$$

\Rightarrow In steady state, the continuity equations in such regions simplify to $\nabla \cdot J_{n,p} = 0$

Before closing this lecture let us look at one more approximation that is commonly very useful, neglect of generation recombination, critical regions of modern VLSI devices are much less than the diffusion length of minority carriers in the regions, so it is the length of the critical region that is what we mean here. Now because this is a fact we can say that generation recombination plays little role in VLSI device operation and can be neglected in the continuity equations.

So this means your continuity equations for VLSI devices could look something like this where both G and $n - n_0 / \tau_{minority}$ have been neglected, so that is a great simplification. In addition, if steady-state conditions prevail than the continued equations in such regions is simply

diversions of JN or $P = 0$, right that is really very, very simple. Now with that we have come to the end of the lecture.

So in this lecture, we have discussed with many examples the utility of the knowledge of characteristic times and lengths. We will consider some more examples in the next lecture and also summarize this module.