

**Semiconductor Device Modeling**  
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**Lecture - 37**  
**Types of Device Models**

In this lecture, we start a new module on the Types of Device Models, now does every problem have a unique solution will the answer is yes and no it depends on the situation or the problem, for example in the school days in physics we have solved the problem of collision between 2 balls, so this 2 body collision problem it has a unique solution and a unique approach of getting the solution.

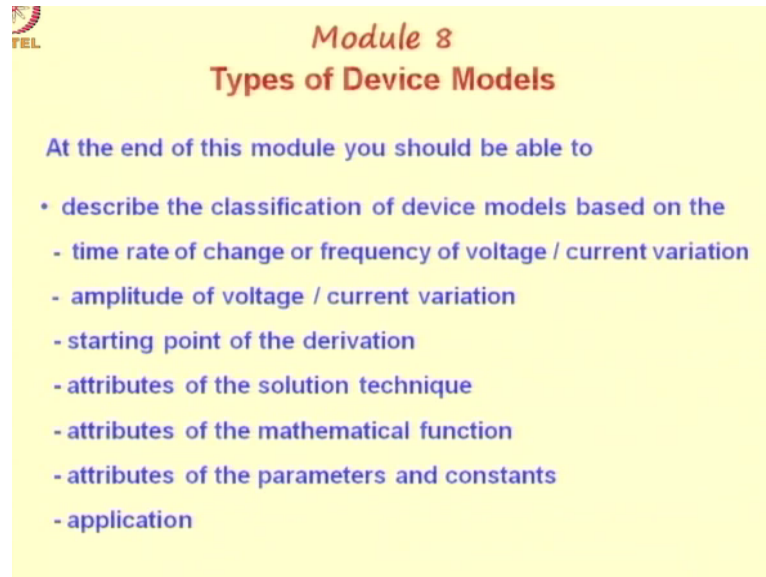
On the other hand, consider the situation in a semiconductor device you have millions of electrons and holes moving about randomly colliding with each other and with other particles like randomly located the dopants, phonons and so on, and then you are applying various forces to the device for example you may be applying an AC voltage, a high frequency voltage or a low frequency voltage, a DC voltage and so on.

So you have variety of situations and the conditions in the device is are complex, in this kind of a problem you do not have a unique solution although the underlying physical laws are unique for example if you want to solve a 2 body problem in the school physics curriculum you use Newton's laws, if you want to solve the problem of deriving the current in response to a voltage in a semiconductor device where millions of particles are colliding with each other moving randomly.

You can start from either Newton's laws or Schrodinger equation, however starting from the fundamental laws and getting an equation for current as a function of voltage this journey is rather difficult and long and therefore many different approaches to getting models for device current as a function of voltage exists.

There are different varieties of approximations to negotiate this difficulty of deriving the model from fundamental laws, and that is why you have various types of device models, now these are the types of models that we are going to look at what are the types of models.

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The slide features a yellow background. In the top left corner, there is a small logo with the letters 'FEL' and a red and yellow circular graphic. The title 'Module 8' is written in a red, serif font, and 'Types of Device Models' is written below it in a bold, red, sans-serif font. Below the title, the text 'At the end of this module you should be able to' is written in a blue, sans-serif font. This is followed by a bulleted list of seven items, each starting with a blue dot and followed by a blue, sans-serif text description.

**Module 8**  
**Types of Device Models**

At the end of this module you should be able to

- describe the classification of device models based on the
  - time rate of change or frequency of voltage / current variation
  - amplitude of voltage / current variation
  - starting point of the derivation
  - attributes of the solution technique
  - attributes of the mathematical function
  - attributes of the parameters and constants
  - application

So at the end of this module you should be able to describe the classification of device models based on the time rate of change or frequency of voltage or current variation, based on amplitude of voltage or current variation, based on the starting point of the derivation or attributes of the solution technique, models based on attributes of the mathematical function in the solution and attributes of the parameters and constants in the solution and finally device models based on the application.

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## Module 8 Types of Device Models

At the end of this module you should be able to

- interpret, from the jargon employed, the type of model being addressed in any literature on device modeling
- use the appropriate jargon to convey a particular type of device model

You should be able to interpret from the jargon employed, the type of model being addressed in any literature on device modelling and finally you must be able to use the appropriate jargon to convey a particular type of device model.

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### Jargon

AC	Implicit	Predictive
Analog	Infinite series	Process aware
Behavioural	Inverse	Quasi-static (QS)
Black-box	Large-signal	Rigorous
Closed-form	Low frequency (LF)	Scalable
Compact	Macro	Small signal
DC	Non-Quasi Static (NQS)	Static
Digital	Numerical	Sub-circuit
Empirical	Parametric	Table look-up
Explicit	Phenomenological	Transcendental
High Frequency (HF)	Physical	Transient

Jargon, now let us look at a number of words that are used in the context of device modelling, the words that we list here most of them refer to some type of device model or the other, a few of them may refer to the type of function that is used in the model solution, we are going to list this words in alphabetical order, so you have AC models, analog model, behavioral model and black-box model.

So against each of these words you must add the word model, closed form, compact, DC and digital are some more varieties of model, then empirical, explicit and high frequency model, so here the word explicit refers to the form of the model equation, implicit this word also refers to the form of a model equation, infinite series form of the model equation and then you have inverse modelling, large signal model, low frequency model.

Macro model, non-quasi-static model and numerical model, parametric, phenomenological, physical, predictive, process aware models, quasi-static model and rigorous model, scalable, small signal model, static model, sub-circuit model and finally table look-up models, transcendental models form of equation of a model and transient models, so you see we have listed a number of words a number of types of models.

This list is not exhaustive but nevertheless if you know this different words and their meanings then you will feel comfortable reading the literature on device modelling, now since there are a variety of models and a large number of words are used in the context of modelling to understand all these different types of models it makes sense to classify and organize them in scientific approach.

Whenever we are dealing with a complex reality and variety of things the first step in understanding them is to organize the things in groups okay now that is what we are doing here.

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## Classification Based on the Time Rate of Change of Voltage / Current

In the context of the DD Model

Static or DC Model: Conditions constant with time

Quasi-static (QS) Model: Conditions varying slowly with time

Flow	Creation	Continuity
$J_n$	$J_n = qD_n \nabla n + qn\mu_n E$	$\partial_t n = (1/q) \nabla \cdot J_n + G - (\delta n / \tau)$
$J_p$	$J_p = -qD_p \nabla p + qp\mu_p E$	$\partial_t p = -(1/q) \nabla \cdot J_p + G - (\delta p / \tau)$
$E$	$E = -\nabla \psi$	$\nabla \cdot E = \rho / \epsilon_s \quad \rho = q(p + N_a^+ - n - N_s^-)$

$$J = J_n + J_p + \epsilon \partial E / \partial t$$

So we begin with classification of models based on the time rate of change of voltage or current. Static or DC models are based on conditions which are constant with time in the device on the other hand quasi-static models are based on conditions which varying slowly with the time, now what are these conditions and how slowly should they be varying in order for the model to be called quasi-static.

Let us answer these questions in the context of the drift diffusion model because it is the drift diffusion model that we have said is the basis of all our device models in this course, the equations of these models are listed here, first the static or DC model is based on the conditions of electrons holes and electric field which are constant with the time, so  $\partial n / \partial t$  that is this term here, the  $\partial p / \partial t$  that is this term here and  $\partial E / \partial t$  in the term here all these terms are 0.

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## Classification Based on the Time Rate of Change of Voltage / Current

In the context of the DD Model

Static or DC Model:  $\partial_t n, \partial_t p, \partial_t E = 0$

Quasi-static (QS) Model: Conditions varying slowly with time

Flow	Creation	Continuity
$J_n$	$J_n = qD_n \nabla n + qn\mu_n E$	$0 = (1/q) \nabla \cdot J_n + G - (\delta n / \tau)$
$J_p$	$J_p = -qD_p \nabla p + qp\mu_p E$	$0 = -(1/q) \nabla \cdot J_p + G - (\delta p / \tau)$
$E$	$E = -\nabla \psi$	$\nabla \cdot E = \rho / \epsilon_s \quad \rho = q(p + N_a^+ - n - N_s^-)$

$$J = J_n + J_p + 0$$

So these are set to 0 here, so this a static or DC model, in other words if you are start with this equation and then derive an equation for current as a function of voltage, then model will be referred to as static or DC model, in contrast let us look at the quasi static model.

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## Classification Based on the Time Rate of Change of Voltage / Current

In the context of the DD Model

Static or DC Model:  $\partial_t n, \partial_t p, \partial_t E = 0$

Quasi-static (QS) Model:  $\partial_t \delta n \ll \delta n / \tau, \partial_t \delta p \ll \delta p / \tau, \partial_t Q_1 \ll Q_1 / \tau_{tr}$

$$\partial_t E \ll E / \tau_d$$

Flow	Creation	Continuity
$J_n$	$J_n = qD_n \nabla n + qn\mu_n E$	<del><math>0 = (1/q) \nabla \cdot J_n + G - (\delta n / \tau)</math></del>
$J_p$	$J_p = -qD_p \nabla p + qp\mu_p E$	<del><math>0 = -(1/q) \nabla \cdot J_p + G - (\delta p / \tau)</math></del>
$E$	$E = -\nabla \psi$	$\nabla \cdot E = \rho / \epsilon_s \quad \rho = q(p + N_a^+ - n - N_s^-)$

$$J = J_n + J_p + \epsilon \partial E / \partial t$$

Now here  $\partial n / \partial t$ ,  $\partial p / \partial t$  and  $\partial E / \partial t$  are not 0 but they are small, now how small are they so  $\partial n / \partial t$  is compared with  $\delta n / \tau$ , so this  $\partial n / \partial t$  compared with this term and now this is very small compared to this term then we say that the conditions are quasi-static, now you might ask why do not you compare with this term or this term well you could compare with these terms.

The advantage of comparing with this term is that you can see it has the form where the numerator is a concentration same as this term and the denominator is a time constant, so this comparison is rather easy, now if this term is very small compared to this that suffices for us to neglect this term right, we need not compare it with other terms to decide whether the term can be neglected okay.

Now here we have used  $\frac{dn}{dt}$  of  $\Delta n$  rather than  $n$  the reason here is that on both sides we want to have the same concentration term, now you know  $\frac{dn}{dt} = \frac{d(\Delta n)}{dt}$  because  $n = n_0 + \Delta n$  the equilibrium value of electron concentration +  $\Delta n$  that is the excess carrier concentration, so if you differentiate  $n$  with respect to  $t$  since the equilibrium concentration  $n_0$  does not vary with time  $\frac{dn_0}{dt} = 0$ .

And therefore  $\frac{dn}{dt} = \frac{d(\Delta n)}{dt}$ , similar commands applied to this inequality in  $\Delta p$ , the  $Q_i$  here refers to the inversion charge in the MOSFET and  $\tau_{tr}$  is the transit time, this  $\tau$  here and here is the same as the  $\tau$  used here which are minority carrier lifetimes and coming back to this  $\frac{dQ_i}{dt} \ll \frac{Q_i}{\text{transit time}}$ , this is the condition that is applicable in the context of MOSFET's.

So quasi-static MOSFET model is one in which the rate of change of inversion charge with time is  $\ll$  the inversion charge by transit time of carriers from source to drain, now this topic was discussed in the module on characteristic lengths and time that is module number 5, so please refer to that module and the discussion, similarly  $\frac{dE}{dt} \ll \frac{E}{\text{dielectric relaxation time}}$  for quasi-static conditions how do you get this relation.


So here you are comparing this term which is being neglected with  $J_n + J_p$ , now  $J_n$  and  $J_p$  are given by diffusion + drift components right, so when you sum this up and put them here you will get a diffusion component and a drift component and the drift component would be  $\sigma E$  where  $\sigma$  is conductivity due to both electrons as well as holes. Now you can compare  $\sigma E$  with this term.

And if this term is very small compared to  $\sigma \tau$  then you can neglect it and this inequality leads you to the inequality  $\frac{dE}{dt} \ll \frac{E}{\text{dielectric relaxation of time}}$  even this fact has been discussed in module number 5 on characteristic lengths and time where we have discussed about the dielectric relaxation time. Thus the quasi-static model assume that the excess carrier concentration  $\delta n$  and  $\delta p$  vary slowly on the scale of minority lifetime  $\tau$  the inversion charge in the MOSFET vary slowly on the scale of transit time.

And the electric field  $E$  vary slowly on the scale of dielectric relaxation time. For clarity let me just mentioned that the inequality  $\frac{dQI}{dt} \ll \frac{QI}{\text{transit time}}$  can be obtained from the equation for electrons that is the continuity equation for electrons or holes depending on the inversion layer is because of electrons or holes, so assuming that inversion layer is because of the electrons you can derive this equation from this electron continuity equation by integrating this equation with respect to distance from source to drain.

So when you integrate left hand side  $\frac{dn}{dt}$  becomes  $\frac{dQI}{dt}$  and when you integrate this term divergence of  $J_n$  over distance you get a term for representing the drain current because you are interesting from source to drain and that drain current term can be written as  $\frac{QI}{\text{transit time}}$ , more details of this please refer to module number 5 on characteristic lengths and time.

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### Classification Based on the Time Rate of Change of Voltage / Current

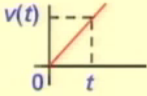
Quasi-static approximations yield a reasonably accurate model for rapidly time varying device phenomena. How ?

- $i(t)$  due to  $v(t)$  is modeled starting from the approximation:
 

$\rho(x), n(x), E(x) \text{ at } t$

 $\approx$ 

steady state conditions corresponding to  $v(t)$



- The model is of the form  $i_{qs}(t) = I[v(t)] + d_t Q[v(t)]$  where  $I(V)$  is the current and  $Q(V)$  is the electron / hole charge stored in the device due to steady state bias  $V$ .



Now majority of the modules are quasi-static, because quasi-static approximations yield a reasonably accurate model for rapidly time varying device phenomena, so even when the conditions vary rapidly with time you can use the solution of these conditions for quasi-static approximation and get reasonably accurate model, now how do you do that.

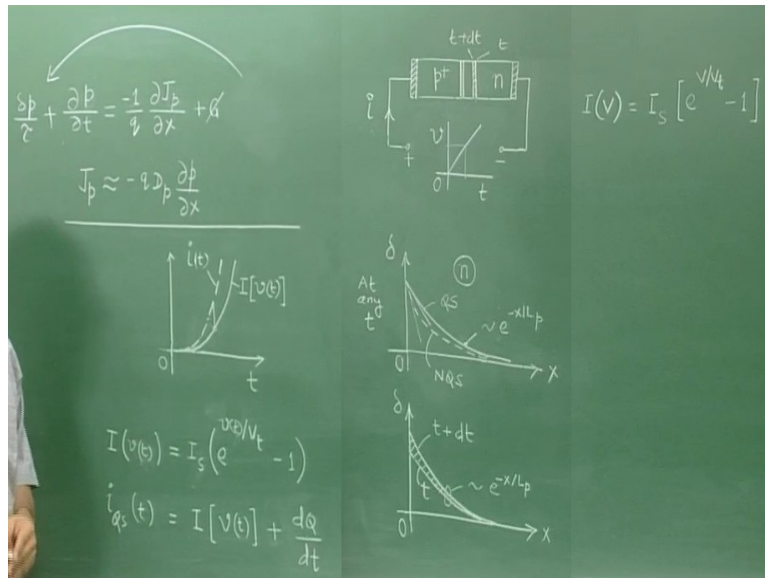
Now this is how you can get the current as a function of time due to a time varying voltage  $v$  is modeled starting from the approximation distribution of holes, electrons and electric field over distance at any instant of time are approximately = the steady-state conditions corresponding to the voltage at that instant of time, let us assume ramp voltage, so voltage is varying linearly with time it is increasing, at any instant of time  $t$  the voltage is  $v(t)$ .

Then what we are saying is we will start with assumption that  $p$ ,  $n$  and  $E$  are the function of  $x$  at this instant of time can be obtained by assuming this voltage or this value of the voltage and assuming DC conditions corresponding to this voltage and then solving for  $p(x)$ ,  $n(x)$  and  $E(x)$  as a function of time.

Now you can start from here and ultimately the module will turn out to be of the form the quasi-static current as a function of time is given by  $I$  as a function of the time varying voltage +  $d/dt$  of a charge which is also a function of the time varying voltage, where  $I$  as a function of  $V$  is the current and  $Q$  as the function of  $V$  is electron or hole charge stored in the device due to steady state bias voltage capital  $V$ .

So how do you start from these conditions and get an equation which is of this form let us illustrate this with the help of diode model.

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So let us say this is our diode and we are applying ramp  $v$  as a function of  $t$  and this is the current as a function of time, I want to get an expression for  $i$  as a function of time, as a function of  $v$  as a function of time, so let us look at the conditions inside the PN junction at any instant of time, we are looking at the distribution of excess carriers on the  $n$  side because  $n$  side is lightly doped,  $p$  side is very heavily doped.

So we neglect the excess carrier concentration on this side it is going to be small, now if I assume quasi-static conditions at any instant of time if I have a certain voltage given by this value for that voltage I am plotting this distributions, so this distribution plotted here is quasi-static meaning it is obtain assuming that in this continuity equation  $\frac{dp}{dt}$  of  $p$  is 0.

And this dotted line excess carrier distribution as a function of  $x$  corresponds to non-quasi-static conditions which means this distribution is obtained by solving this continuity equation taking into account this  $\frac{dp}{dt}$ , the  $J_p$  on right hand side is given by this diffusion current equation, we are using the diffusion approximation okay, from the first level course you know that minority carriers can be assumed to flow due to diffusion.

So when you are solving this equation in the solution the  $J_p$  term will be assumed to be due to diffusion, let us explain these 2 distributions from the first level course you know that this distribution which is quasi-static or static will be of the form exponential  $e^{-x/L_p}$  into a

constant, that constant will depend on the value here, the dotted line however will not be given by this.

Because you will get this exponential distribution from the continuity equation when you remove this term dotted line is obtained when you include this term qualitatively you can appreciate why the dotted line has a higher slope at  $x = 0$  or near this depletion edge, so this  $x = 0$  represents this depletion edge here.

Now why is this slope higher what this slope indicates is that the current small  $i$  instantaneous current small  $i$  including the time variation or time varying effects is more than the steady state current corresponding to the same voltage okay which is given by this slope, so this is this slope gives you the small  $i$ , whereas this slope gives you capital  $I$  that is the current given by this equation which is steady state current voltage equation you are familiar with this.

Let us assume an ideal diode model for simplicity  $I_S$  is the reverse saturation current and  $v_t$  is the thermal voltage, before explaining why is the small  $i$  more than capital  $I$  that is why is the slope here more than this slope, let us plot the effects of this on a graph, so here we show the current  $i(t)$  varying as a function of  $t$  this dotted line and another current capital  $I$  calculated by replacing the DC voltage  $V$ /the time varying voltage  $v(t)$ .

So this is your DC current voltage equation here I replace this  $V$ /this  $v(t)$  so that same equation is written so here I am going to replace this  $V/v(t)$ , so result of this equation is a solid line and this solid line is obtained using a distribution like this whereas this dotted line is obtained using a distribution like this which is non-quasi-static, now the difference between these 2 is what our interest what is the difference why is this more than this.

Now this can be appreciated by from this equation you see when I want to get this distribution I use this term and this term, this time is set to 0 so if I integrate this continuity equation to find out what is the amount of current that is injected from this end which is given by this slope I will find it to be = the integral of this as a function of distance okay which is nothing but the recombination current, that is recombination in this area.

But when the situation is time varying I do not have only this term that can be understood as follows you replace you shift this term to the left hand side then what will happen is it will become  $\Delta p / \tau$ , I remove it from here so now when I integrate this equation with respect to  $x$  to get the current injected at this end I have to integrate both  $\Delta p / \tau$  as well as  $d p / d t$ , so I have to take into account an additional term okay that is  $d p / d t$ .

Now because of this the initial slope here is more in other words my current is going to be more under non-persistent conditions as compared to under quasi-static conditions okay, so now my goal is to find out how much is this slope, because of dash line or dotted line more than the slope because of quasi-static conditions, now that I can easily appreciate as follows let me plot under quasi-static conditions the distribution of excess carriers for 2 different instance of time.

So this distribution is for instant  $t$  and for  $t + \Delta t$  you have a higher excess carrier concentration because the value of voltage goes on increasing as your time increases, so this is for  $v(t)$  and this is for  $v(t) + \Delta t$ , now you see that between  $t$  and  $t + \Delta t$  the device has to store this much charge shown by the shaded region in the end region so called neutral region.

Therefore, the current flowing into the device should not only supply for recombination in this region but also it should supply the extra charge which is being stored between the instance  $t$  and  $t + dt$  that is why we can write an expression for the time varying current  $i(t)$  as the current that you calculate assuming DC conditions +  $dQ/dt$  where  $dQ$  is the extra charge you have to store during the interval  $t$  and  $t + dt$  inside the device.

Now please note the extra charge you have to store happens to be not only in the neutral region that is this region here but also at the edge of the space charge for example when my voltage increases from  $v(t)$  to  $v(t) + dt$  your depletion region shrinks okay, so this is the depletion region for  $t$  this is the depletion region for  $t + dt$ , so you have to store an extra charge because of shrinking of depletion region, depletion region shrinks on this side also right it is not shown here.

Because when you draw the diagram to scale the depletion region here is very small that is why I am not able to show, now the extra holes because of the shrinking of the depletion region which happens to be the holes on this edge of the depletion region are the same as the extra electrons which are coming in here and therefore this shaded area + this shaded area represents the excess holes being stored in the device from the instant  $t$  to  $t+3T$  and this is same as the shaded area.

So this  $dQ$  that we are writing here is sum total of this shaded area + this shaded area or this shaded area, now that is how you get your time varying current as the current obtained from the DC current voltage characteristics + a term  $dQ/dt$ , so that is the form of an equation wherein you find that time varying current has been calculated using static or quasi-static conditions.

So this current voltage equation is under static conditions and this  $Q$  is obtained from distributions under static conditions, so this is the so called quasi-static approach in which the time varying conditions are modeled based on static or very slowly varying very slowly time varying conditions.

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**Classification Based on the Time Rate of Change of Voltage / Current**

*Assignment-8.1*

*We wish to replace the "distributed" circuit on the left by its "lumped" equivalent on the right.*

Now here is an assignment for you we wish to replace the distributed circuit on the left that is this by its lumped equivalent on the right that is this one, so here you have an equivalent resistance  $R$  suffix  $e$ , an equivalent capacitance  $C$  suffix  $e$  and these are in parallel, the

impedance  $Y$  looked into these 2 terminals should be the same as the impedance  $Y$  looked into the terminals of this distributed circuit, now that is the goal.

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**Classification Based on the Time Rate of Change of Voltage / Current**

*Assignment-8.1*

Express  $R_e, C_e$  in terms of  $R, C$  in two ways:

QS method -  $R_e = [\Delta I / \Delta V]^{-1}$      $C_e = \Delta Q / \Delta V$ ;

NQS method -  $R_e = [\text{Re } Y(j\omega)]^{-1}$      $C_e = [\text{Im } Y(j\omega)] / \omega$ .

Compare the NQS expressions at  $\omega \rightarrow 0$  with the QS expressions.

Express  $R$  suffix  $e$  and  $C$  suffix  $e$  that is these 2 components in terms of  $R$  and  $C$  so  $R$  and  $C$  are the components of the distributed network in 2 ways quasi-static method,  $R_e$  is given by  $\Delta DC / \Delta DC$  voltage  $V$  reciprocal right to the power - 1 and  $C$  suffix  $e$  is given by  $\Delta Q / \Delta V$  where  $V$  is the DC voltage, so what you are doing is you are applying a DC voltage  $V$  finding out the current and charge through this circuit.

So you are applying a DC voltage  $V$  find out the current in this circuit and the charge stored on this capacitor for that voltage, change the DC voltage to  $V + \Delta V$  find out the new current and the new charges, find out the difference in the 2 DC currents and the difference in the 2 DC charges those are what are  $\Delta I$  and  $\Delta I$  and  $\Delta Q$  here.

And take the ratio of the difference in the DC currents and difference in the voltage and difference in the charges and difference in the voltage that is the quasi-static method, the non quasi static method it would be finding out  $R$  suffix  $e$  in the following way as real part of  $Y$  of  $j\omega$  reciprocal and  $C$  suffix  $e$  is imaginary part of  $Y$  of  $j\omega$  divided by  $\omega$ , what is  $Y$  of  $j\omega$  if you apply a sinusoid to this network.



Low frequency is the region over here and high frequency is the region beyond a certain frequency and in between you have the intermediate frequency, now how do you quantify this ranges let us denote the transition between intermediate frequency and high frequency as  $f_0$ , then the transition between low frequency and intermediate frequency can be assume to be  $1/10$  of  $f_0$ , now what is this  $f_0$ , the  $f_0 = \text{reciprocal of } 2\pi \text{ into time constant}$ .

And that time constant depends on context it could be for example in devices = the minority carrier lifetime or it could be transit time or it could be dielectric relaxation time, so for different conditions the range of high frequencies and low frequencies are different it depends on the situation and what is the appropriate time scale for that and the time scales could be either minority carrier lifetime or dielectric relaxation time or transit time and so on.

Now this time constant has been discussed in the module on characteristic time and length, let us place the quasi-static and non-quasi-static models on this frequencies scale and connect these descriptions to the frequency dependence descriptions, so S here stands for static, QS for quasi-static and NQS for non-quasi-static, so you see the boundary between the quasi-static and non-quasi-static occurs somewhere in the intermediate frequency range.

In other words, the quasi-static models are valid for a frequency range beyond the low frequencies okay, on the other hand beyond the certain frequency in the intermediate frequency range you have to use the non-quasi-static model even before you encounter high frequencies, the low frequency and quasi-static models are frequency independent but not necessarily identical. Now this part will be evident when we take the example of a diode.

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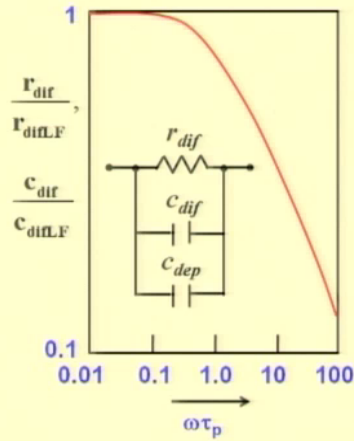
## Classification Based on the Frequency of Voltage / Current

Example: p\*n diode

$\omega \tau_p$	$< 0.1$ (LF)	$> 10$
$r_{dif}$	$\frac{V_t}{I}$	$\frac{V_t}{I} \sqrt{\frac{2}{\omega \tau_p}}$
$C_{dif}$	$\frac{I \tau_p}{2V_t}$	$\frac{I \tau_p}{2V_t} \sqrt{\frac{2}{\omega \tau_p}}$

$$C_{difQS} = 2C_{difLF} = I \tau_p / V_t$$

$$r_{difQS} = r_{difLF}$$



Here is an example of high frequency and low frequency models of a diode, the derivation of these equations it is assumed that you have done in first level course if not please refer to the derivations in any book to understand the equations given here, now diffusion resistance of a diode given by  $V_t/I$  in the low frequency model, where low frequency here is the frequency  $\omega \tau_p < 0.1$ ,  $\tau_p$  is the minority carrier lifetime in the n region.

In the n region minority carriers are holes that is why you have this suffix p here, however for high frequencies in the range  $\omega \tau_p > 10$  in that region the equation happens to be something like this, here we are not concerned with the derivation of the equations but just to show you what difference does it make to the model equation or form of the model equation when the frequencies are low and when the frequencies are high.

So you find the high frequency region the resistance is frequency dependent whereas in low frequency it is frequency independent same thing applies to the diffusion capacitance, now note the difference between low frequency and quasi static models, the quasi static diffusion capacitance = 2 times the low frequency diffusion capacitance that is illustrated here and so it is  $I \tau_p / V_t$ .

When you derive the PN junction capacitance using a formula  $dQ/dV$  where  $dQ$  is the difference in the charge stored in the device for DC voltage  $V$  and DC voltage  $V+dV$  that approach is

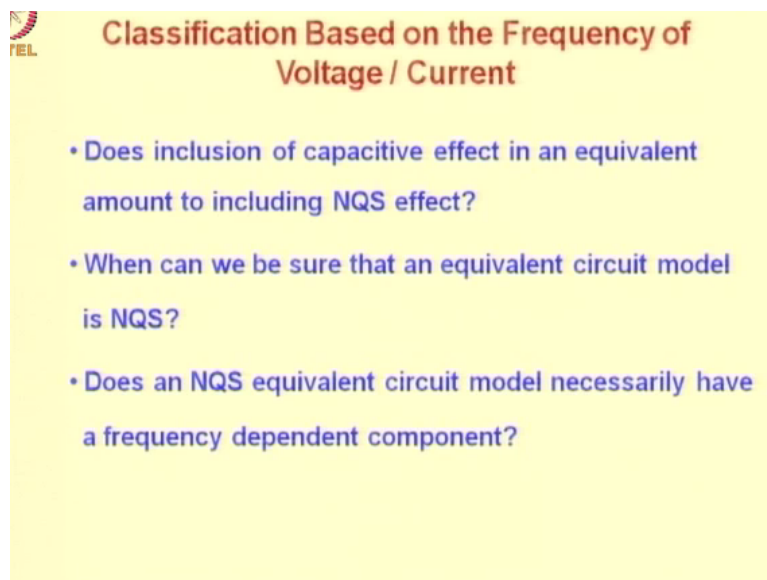
referred to as quasi-static approach, on the other hand if I use a continuity equation of this type wherein I assume the hole is to be varying sinusoidally with voltage and then solve this equation, then that is what is a non-quasi-static approach.

And in that case you will get a frequency dependent capacitance, now that is what is happening here and in that frequency dependent expression when you turn  $\omega$  to 0 then you will get this expression, so this is an example to illustrate the fact that quasi-static and low frequency values of a parameter need not be identical, however both of them will be independent of frequency, now R diffusion quasi-static on the other hand is identical to R diffusion low frequency.

Now here is a plot of the ratio of the diffusion resistance to low frequency diffusion resistance and capacitance that is the diffusion capacitance to low frequency diffusion capacitance this is how it looks like it has been plotted as a function of  $\omega \tau_p$ , consider the range  $\omega \tau_p < 0.1$  you can see that the line here is almost flat  $\omega \tau_p 0.1$  is this here, in other words these parameters are not varying with frequency.

This  $\omega \tau_p > 10$  refers to the region shown here, so it is in this range that these expressions will apply, in between range however the expressions will be somewhat more complex if you want to know them please refer to the books.

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**Classification Based on the Frequency of Voltage / Current**

- Does inclusion of capacitive effect in an equivalent amount to including NQS effect?
- When can we be sure that an equivalent circuit model is NQS?
- Does an NQS equivalent circuit model necessarily have a frequency dependent component?

Let us answer some questions after this discussion. Does inclusion of capacitive effect in an equivalent circuit amount to including non-quasi-static effect? Evidently the answer is no, we have just now seen in the previous slide that the capacitance can also be calculated from quasi-static conditions, so inclusion of capacitance does not mean that model is quasi static, however if the capacitance is frequency dependent then you can say the model is quasi-static.

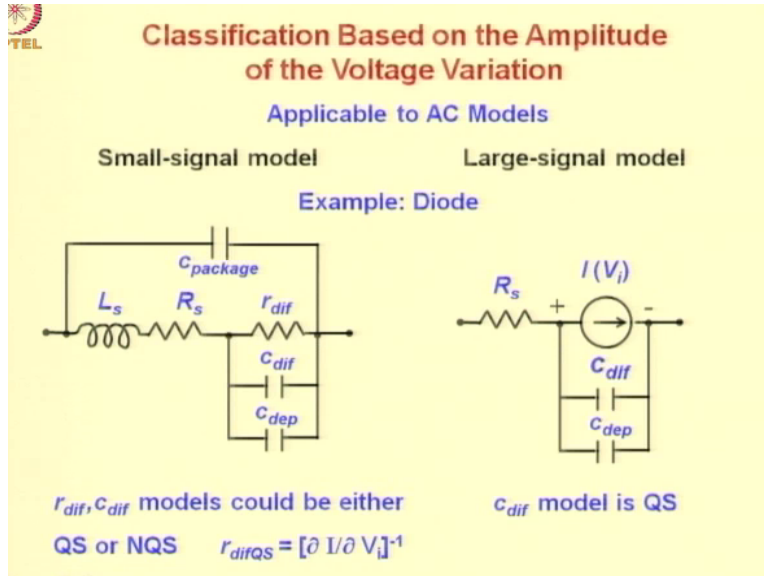
But mere presence of capacitance in an equivalent circuit does not mean that the model is quasi-static or it is including quasi-static effect. When can we be sure that an equivalent circuit model is non quasi-static? From the discussion so far it is clear that if you have a component in an equivalent circuit which is frequency dependent then you can say that the model is non quasi-static please understand one point.

When we say capacitance is frequency dependent this statement is not the same as saying impedance due to capacitance is frequency dependent, impedance due to the capacitance is always frequency dependent, for a capacitance  $C$  the impedance is  $1/j\omega C$ , so evidently  $\omega$  is coming in the expressions for the impedance and the impedance of the capacitance will always be frequency-dependent.

Independent of whether the capacitance has been derived under quasi-static conditions or non quasi static conditions, what we are saying is the capacitance itself will be frequency dependent if it is a non quasi static model. Does a non quasi static equivalent circuit model necessarily have a frequency dependent component? Well the answer to this question is no.

Because consider a resistor a simple resistor no matter what is your frequency the resistance of this element will always be frequency independent and the value will be the same as that under low frequencies.

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Next, let us move on to classification based on the amplitude of the voltage variation, this classification is applicable to AC model, so you have small-signal models and large-signal model let us consider the example of a diode to illustrate the difference between them, here is a small-signal model of a diode it consists of elements such as inductances, capacitances and resistances, now the resistance and capacitance models could be either quasi-static or non quasi-static.

For instance, the quasi-static diffusion resistance model is  $dou DC \text{ current} / dou DC \text{ voltage}$  where here  $I$  stands for internal voltage that is the voltage drop across the depletion layer and reciprocal of this. The large-signal model of the same diode would look something like this where this  $C_{diff}$  and  $C_{depletion}$  are coming here in this large-signal model also, however these in this equivalent circuit the diffusion capacitance is quasi-static.

Whereas in small-signal circuit it could be quasi-static or non quasi-static, further the diffusion resistance is not there here, package capacitance or any series inductance in the large-signal model, often this model is used for lower frequencies, this diffusion, resistance has been replaced by current source dependent on the voltage the  $V_i$  here refers to the voltage across the depletion layer, so in the context of the diode here.

For example, the  $V_i$  means the voltage drop across this whereas  $V$  is the voltage drop between these 2 terminals some voltage can drop across the neutral regions and that is why you have the

series resistance which accounts for this voltage drop in the neutral regions  $V$  suffix  $i$  would be the voltage across this, so generally in the large signal model you will find not resistances but the current source dependent on the voltage.

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**Classification Based on the Amplitude of the Voltage Variation**  
Applicable to AC Models

Large-signal model  
Example: Diode

$$I \approx K_{III} I_s \left[ \exp\left(\frac{V_i}{N V_t}\right) - 1 \right] + I_{GR} - I_B$$

$$V_i = V - I R_s$$

$$I_{GR} = I_{SR} \left( 1 - \frac{V_i}{V_j} \right)^M \left[ \exp\left(\frac{V_i}{N_R V_t}\right) - 1 \right]$$

$$I_B = I_{BV} \exp\left(-\frac{V_i + BV}{N V_t}\right)$$

$$K_{III} = \sqrt{I_{KF} / (I_{KF} + I_D)}$$

$C_{dif}$  model is QS

The equation for current as a voltage function of large-signal model is shown here this is a SPICE model, we have introduced this equation in the second module that is introduction module earlier.

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**Classification Based on the Starting Point of the Derivation**

Rigorous Model	Phenomenological Model
Derivation begins from the fundamentals or first principles, e.g. from Newton's laws, Maxwell's equations, Quantum transport equation, Boltzmann transport equation etc.	Derivation begins from the appearance of the phenomena rather than from the fundamentals, but the model is capable of predicting a range of observations, and so, is reasonably consistent with fundamentals

Let us move on to classification based on starting point of the model derivation, so far our classification was based on the signals apply to the device voltage or current signal, so we

consider frequency, we consider time varying nature of the signal and then we consider the amplitude of the signal. Now let us move on to the model derivation, there are 2 approaches that can be called as the Rigorous approach which give the rigorous model.

And Phenomenological approach that gives a phenomenological model, so what is the difference in starting point of these 2 approaches, in the rigorous model the derivation begins from the fundamentals or first principles example from Newton's laws, Maxwell's equations, Quantum transport equation, Boltzmann transport equation etc.

In a phenomenological model however derivation begins from the appearance of the phenomena rather than from the fundamentals, but the model is capable of predicting range of observations and so is reasonably consistent with fundamentals, so you are interested in the equations for current as a function of voltage as we have remarked it is the very challenging exercise to start from fundamental laws like Boltzmann transport equation or Quantum transport equation or Schrodinger equation.

And then get an equation for current as a function of voltage in a device where there are millions of carriers moving randomly and colliding with many particles, that is why you are looking at other approaches, so people say why not you start from appearance of phenomenon and think of an equation that seems to match the experimental data over a wide range and it has some content of Physics, let us illustrate this idea using some examples.

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## Examples of Phenomenological Approach

### • DD current density equations

Flow	Creation	Continuity
$J_n$	$J_n = qD_n \nabla n + qn\mu_n E$	$\partial_t n = (1/q) \nabla \cdot J_n + G - (\delta n / \tau)$
$J_p$	$J_p = -qD_p \nabla p + qp\mu_p E$	$\partial_t p = -(1/q) \nabla \cdot J_p + G - (\delta p / \tau)$
$E$	$E = -\nabla \psi$	$\nabla \cdot E = \rho / \epsilon_s \quad \rho = q(p + N_a^+ - n - N_s^-)$

### • Mathesian mobility model

So one example of phenomenological approach is the drift diffusion current density equation, equations for  $J_n$  and  $J_p$  listed here now these equations can be derived rigorously starting from the Boltzmann transport equation for example, we have seen this in the modules on equations of carrier transport.

There we showed how Boltzmann transport equation can be reduced to a set up balance equation and how from the momentum balance equation by doing a set of approximation you can get the current density expression, so this is the rigorous approach, on the other hand I can use a different approach something that we have adopted in our first level course on solid state devices.

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$$\begin{aligned}
 J_{ndrift} &= -q n v_d \\
 &= q n \mu_n E \\
 J_{ndiff} &= q D_n \nabla n \\
 J_n &= q D_n \nabla n + q n \mu_n E
 \end{aligned}$$

There we said that I can assume the drift current of electrons as proportional to the carrier concentration  $n$ , further we can assume that all the carriers have the same velocity drift velocity, now this may not really be true because you know that the picture is random and all carriers may not have the velocity I am just making an approximation to derive the simple equation, then I can multiplied by  $-q$  which is the charge to get the expression for the drift current.

Now further we can say that like the ohm's law kind of behaviour which we often observe between current and voltage we can assume that the drift velocity is proportional to electric field, so if I assume this is proportional to electric field and put in a proportionality constant called the mobility of the electrons I put a negative sign because the drift velocity is opposite to the direction of electric field for electrons.

And now I put all these terms together to get the formula  $q n \mu_n E$  for the drift current of electrons, I adopt a similar approach for the diffusion current so I just like that say that the diffusion current of electrons will be proportional to the driving force namely the concentration gradient because we often observe the effects to be proportional to the excitations for small values of excitations, so we carry forward that experience of appearance of phenomena here.

Then I put an equality sign and put in a constant of proportionality and multiplied by the charge to get the diffusion current, next I use a simple logic that when electric field and concentration



gradient of carriers both present at the same time in that case the diffusion current can be superposed over the drift current to get the total current.

Now this superposition principle may not actually hold in practice right, you may get some terms proportional to let us say the product of the concentration gradient and electric field we do not know if I do a rigorous derivation some other terms can also arise, but then we use a simple logic of superposition of effects and then we write  $J_n = \text{the diffusion current } q D_n \text{ grad } n + \text{the drift current } q n \mu_n \text{ into } E$ .

Now this method of writing down the current density equation of electrons will be referred to as phenomenological approach, because we have made a number of assumptions and approximations based on our experience of the wide variety of phenomena to put together several terms in the form of a simple equation.

So the same equation can be written in 2 ways one starting from the fundamental equations and then detailing a series of approximation to get the equation or another in a phenomenological manner where we put down the various terms of the equations based on our observation of a wide variety of phenomena without bothering too much about how the equation that we write down is connected to the basic equation.

Another example of a phenomenological approach is writing the mobility according to the Matheson rule, so when you say that reciprocal of mobility due to a number of scattering mechanism is given by  $1/\text{mobility due to 1 scattering mechanism} + 1/\text{mobility due to another scattering mechanism} + 1/\text{mobility due to another scattering mechanism}$  and so on, so when you write this kind of an equation you are writing it phenomenologically.

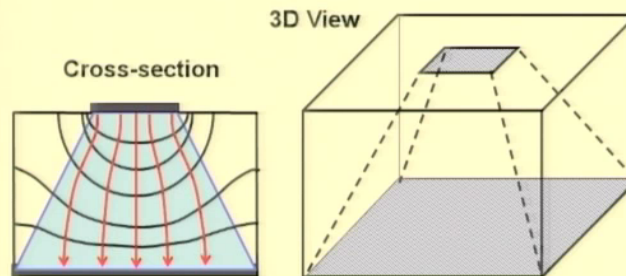
Because you are assuming that when several scattering processes are present together the mobility because of this processes all present together would be the same as superposition of the results, because of process considered individually which may not actually hold right.

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## Examples of Phenomenological Approach

- Current flow in spreading resistance



- Charge sharing model for threshold voltage of a short channel MOSFET

Another example of phenomenological approach is the assumption of a constant spreading current flow from the smaller contact of a spreading resistance to the larger contact okay, so this kind of a spreading assumption is phenomenological because you cannot connect it to any fundamental equation, finally an example of phenomenological modelling is the charge sharing model of threshold voltage of a short channel MOSFET.

So this model we will discuss in the module on MOSFETs you might have come across short channel threshold voltage model based on charge sharing approach where you assume that the charge in the channel of a MOSFET can be assumed to be partitioned into a part controlled by the source, another controlled by the drain and another controlled by the gate, this kind of partitioning is phenomenological because you cannot really show some fundamental equation how you can get this 3 parts.

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## Classification Based on the Solution Technique

	Analytical Model	Numerical Model
S	• Includes device specific simplifications of the "EB" by neglecting some effects	• No simplifications of the "EB" ⇒ model works for a wide range of devices/bias; model physically more accurate.
Q		
EB	⇒ model is device specific; model physically less accurate	• Transforms the "E"s using finite element, finite difference or monte carlo methods (⇒ numerical inaccuracy present but controllable) so that purely arithmetic operations yield values of current / charge for specific values of voltage
(A)		
(S)	• Uses techniques such as calculus, algebra, geometry, trigonometry to yield the current / charge as an analytic function of the voltage	• Computationally intensive
T		
I		
P		
	• Computationally economical	

Let us talk about classifications based on the solution technique, the 2 types of models it is the major classification are Analytical models and Numerical models, this classification has to do with the approximation and solution steps of the device modelling procedure which has 9 steps SQEBASTIP.

In analytical model the approximation step you include the device specific simplifications of the equations and boundary conditions EB stands for equations and boundary conditions by neglecting some effects, so you neglect some effects and simplify the equations, so your approximations step will consist of the approximation done in the qualitative stage as well as additional approximations of the equations and boundary conditions.

But in numerical model you do not make any simplification of the equations and boundary conditions, the only simplifications or approximation are those made during the qualitative step and as far as the solution step is concerned in analytical model we use techniques such as calculus, algebra, geometry, trigonometry etc. to yield the current or charge as an analytical function of the voltage we will shortly see what is the meaning of analytic.

Now in the numerical model what will you do is you transform the equations E here stands equation using finite element, finite difference or Monte Carlo methods, so that purely arithmetic operations yield values of current or charge for specific values of voltage, in analytical you have

an equation and expression or a function for current as a function of voltage, whereas here you have a table of values in the numerical model of table of values of currents for different voltages.

Now let us look at the consequences of these differences, so because the analytic model includes device specific simplifications of equations and boundary conditions the model is device specific and it is likely to be physically less accurate as compared to numerical model which works for a wide range of devices and bias, because you are not making any simplifications of the equations and boundary conditions and also the model is physically more accurate because all effects can be considered.

Now because the numerical model transform the equations using finite element, finite difference or Monte Carlo methods the numerical inaccuracy is present in this model, however the important thing is it can be controlled by increasing the amount of computation, finally an analytical model is computationally economical whereas a numerical model is computationally intensive that is the price you pay for including all the different effects and for having a model that works for a wide range of devices or bias.

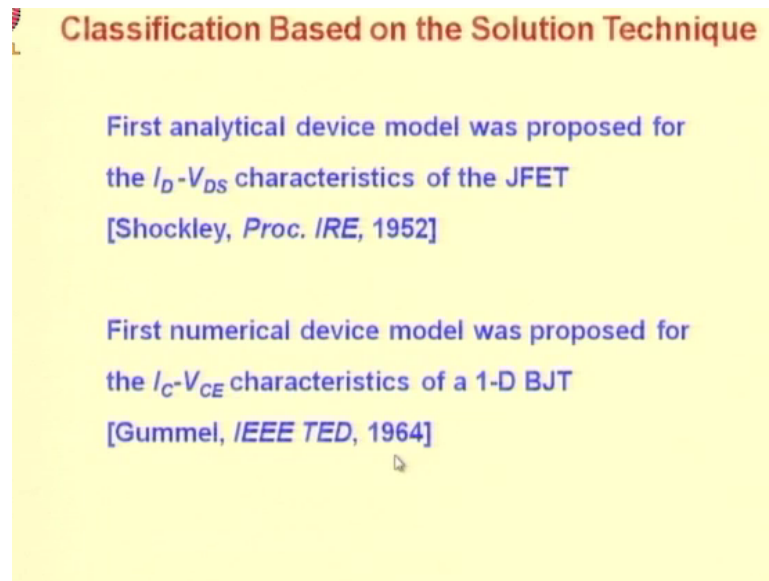
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Classification Based on the Solution Technique	
Analytical Model	Numerical Model
<p><b>S</b> • Includes device specific simplifications of the "EB" by neglecting some effects</p> <p><b>Q</b> ⇒ model is device specific; model physically less accurate</p> <p><b>EB</b> ⇒ model is device specific; model physically less accurate</p> <p><b>(A)</b></p> <p><b>(S)</b> • Uses techniques such as calculus, algebra, geometry, trigonometry to yield the current / charge as an analytic function of the voltage</p> <p><b>T</b></p> <p><b>I</b></p> <p><b>P</b> ⇒ model provides insight</p>	<p>• No simplifications of the "EB" ⇒ model works for a wide range of devices/bias; model physically more accurate.</p> <p>• Transforms the "E"s using finite element, finite difference or monte carlo methods (⇒ numerical inaccuracy present but controllable) so that purely arithmetic operations yield values of current / charge for specific values of voltage</p> <p>⇒ model provides no insight</p>

Important the analytical model provides insight because you have a current voltage expression on the other hand because you are having a table of data points for current as a function of

voltage and numerical model, the numerical model provides no insight, so that is the downside of a numerical model.

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**Classification Based on the Solution Technique**

- First analytical device model was proposed for the  $I_D$ - $V_{DS}$  characteristics of the JFET [Shockley, *Proc. IRE*, 1952]
- First numerical device model was proposed for the  $I_C$ - $V_{CE}$  characteristics of a 1-D BJT [Gummel, *IEEE TED*, 1964]

Some interesting historical tips the first analytical device model was proposed for the drain current versus drain source voltage characteristics of the Junction FET by Shockley in 1952, on the other hand the first numerical device model was proposed for the collector current versus collector to emitter voltage characteristic of a 1-dimensional BJT by Gummel in 1964, with that we have come to end of this lecture and let us summaries the important points.

In this lecture will began a discussion of the various types of device models because of the situation in a device is complex it consists of millions of charge carriers moving about randomly and colliding with several particles and with each other and because the conditions apply to the device can also vary widely, you can have DC, low frequency, high frequency conditions, the modelling of devices is a complex exercise.

Therefore, many approaches exist some of these approaches we are considering in this module on types of device models in the particular lecture we first gave a variety of words which are used in the context of device modelling and then we began as discussion of classification of device models, so we consider classification based on the time varying nature of the voltage or current, based on the frequency of voltage or current.

And then based on the amplitude of current or voltage, after that we considered classifications of device models based on the starting point of model derivation to be considered as rigorous and phenomenological models, thereafter we considered classification of models based on the methodology adopted for solution. So you have analytical and numerical models, we shall consider further types of device models in the next lecture.