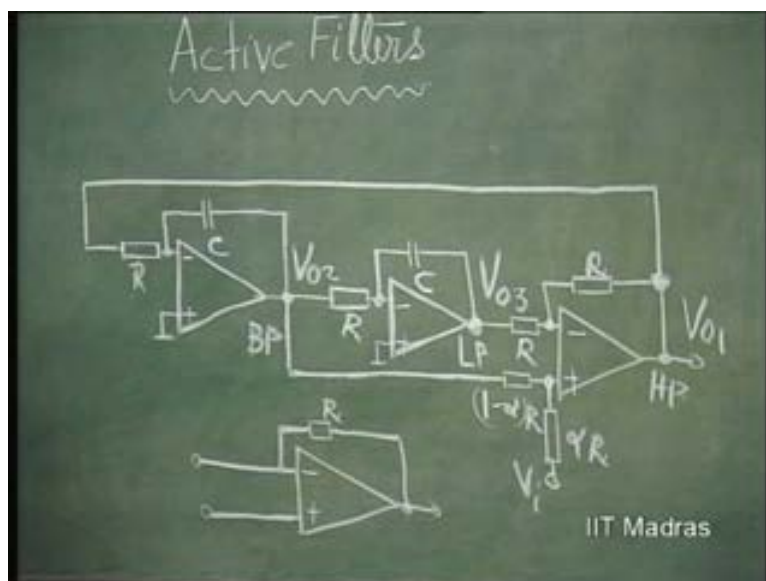


**Electronics for Analog Signal Processing - II**  
**Prof. K. Radhakrishna Rao**  
**Department of Electrical Engineering**  
**Indian Institute of Technology – Madras**

**Lecture - 12**  
**Active Filters**

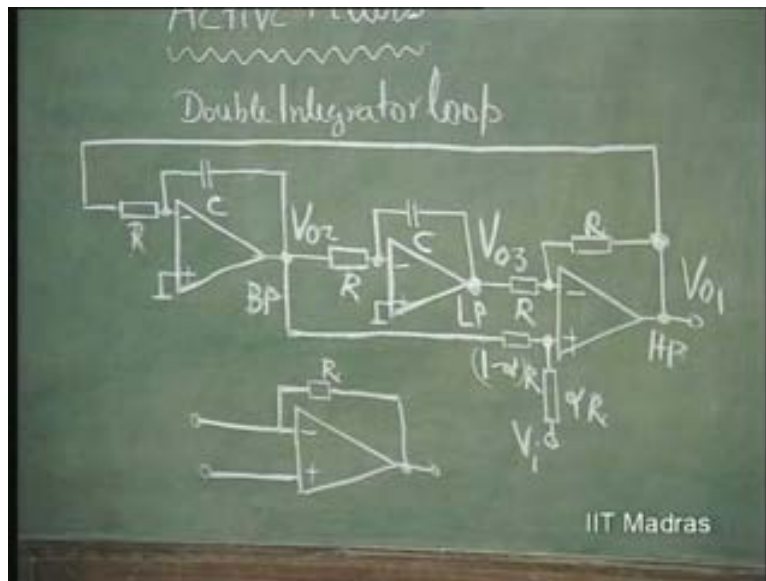
So, we had discussed this circuit. We had even synthesized this circuit. This is the circuit that simulates a second order differential equation.

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And it is also called Kerwin-Huelsman-Newcomb network or universal active filter and this is available as an IC with four amplifiers inside, op amps inside and the integrator double, this is called double integrator; two integrators are used. So, this is also called double integrator loop.

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This integrator, these two integrators are in a loop and the whole thing is available as an IC so that it can synthesize any second order transfer function which is stable.

That means the poles of the system can lie anywhere on the left of a plane of the  $s$ , the main, and zeros can lie anywhere. So, this kind of structure can be synthesized using this; and here the transfer functions are all given here, respective;  $Q$  being equal  $1$  over  $2\alpha$ ,  $\omega_0$  equal to  $1$  over  $RC$ .

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The equations are written on a chalkboard:

$$Q = \frac{1}{2\alpha}$$
$$\omega_0 = \frac{1}{RC}$$

The IIT Madras logo is visible in the bottom right corner.

So, this we had derived in the last class. Let us see what these responses will look like. This is also called pole frequency. This particular system has poles given out by the denominator here and if you find out  $S_{1,2}$  divided by  $\Omega_0$ , this is called normalized frequency; so,  $S_{1,2}$  by  $\Omega_0$ , the  $\Omega_0$  being this.

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pole frequency

$$P \left( \frac{S_{1,2}}{\Omega_0} \right) = \frac{-\frac{1}{Q} \pm \sqrt{\frac{1}{Q^2} - 4}}{2}$$

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You can solve this quadratic equation. Equate it to zero and get minus b plus or minus root of b square minus 4 divided by 2. This is,  $S$  by  $\Omega_0$  is put as  $x$ . So,  $x^2 + x/Q + 1 = 0$  is here quadratic equation. So, you get  $x^2 + x/Q + 1 = 0$ . Solving that quadratic equation, the roots will be  $S_1$  and  $S_2$ ; that is,  $S_1$  and  $S_2$ , pairs; and this will be real if as you see here,  $1/Q^2 > 4$  or  $1/Q > 2$  or  $Q < 1/2$ , the poles will be real.

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pole frequency

$$s = \frac{-\frac{1}{Q} \pm \sqrt{\frac{1}{Q^2} - 4}}{2}$$

$\frac{1}{Q} > \sqrt{4}$   
 $Q < \frac{1}{\sqrt{4}}$   
 $\frac{1}{2}$   
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We are normally not interested in that kind of circuit. If they are greater, then this will become complex.  $4$  minus  $1$  over  $Q$  square by  $2$ ; it becomes a complex conjugate pair if  $Q$  is greater than half.

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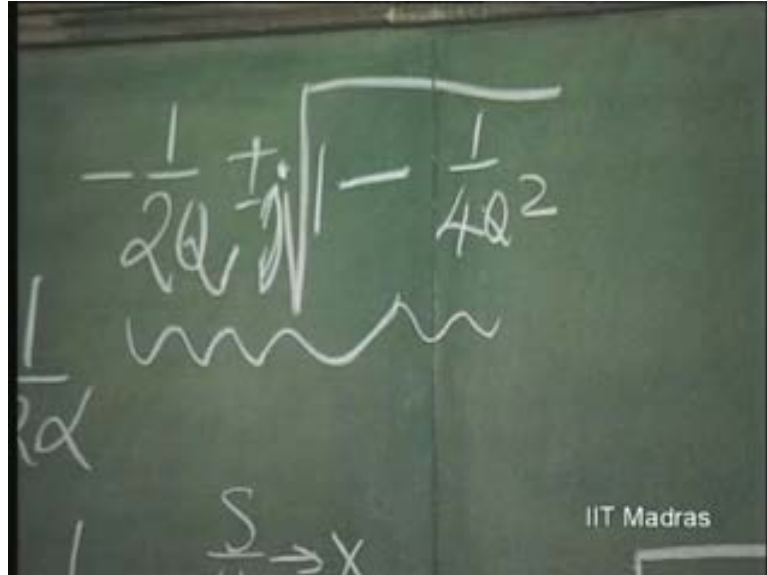
pole frequency

$$s = \frac{-\frac{1}{Q} \pm \sqrt{\frac{1}{Q^2} - 4}}{2}$$
$$= \frac{-\frac{1}{Q} \pm j\sqrt{4 - \frac{1}{Q^2}}}{2}$$

$\frac{1}{Q} > \sqrt{4}$   
 $Q < \frac{1}{\sqrt{4}}$   
 $\frac{1}{2}$   
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So, the poles will be  $-\frac{1}{2Q} \pm j\sqrt{1 - \frac{1}{4Q^2}}$  with  $j$ . It is a complex conjugate pair of poles you get.

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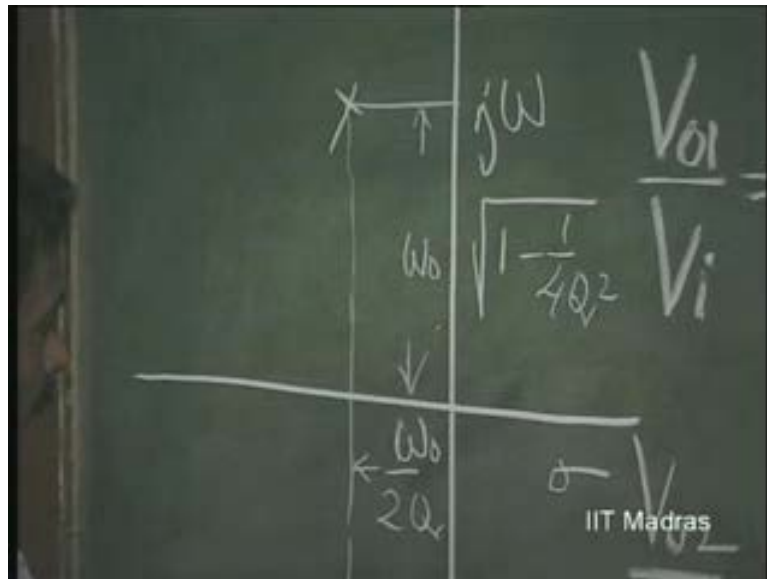
The real part is given by this. The imaginary part is given by this. So, this is going to be  $S_1$  by  $\Omega$  naught and  $S_2$  by  $\Omega$  naught. If you want the actual location of the pole, you have to merely multiply by  $\Omega$  naught here. So, this will be  $\Omega$  naught, actually  $S_1$  and  $S_2$ . This will be  $\Omega$  naught, this will be  $\Omega$  naught. So,  $\Omega$  naught is called the pole frequency and  $Q$ , this  $Q$  is called the pole  $Q$ .

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Now, if I plot this in the S domain...this is the S domain. It has a negative real part always as long as Q is positive. Negative real part which is minus 1 over 2 Q; or actually speaking, if we plot in terms of frequency, it is Omega naught over 2 Q, and it can be located. And imaginary part is going to be this one which is Omega naught into root of 1 minus 1 over 4 Q square; a complex conjugate pair of poles.

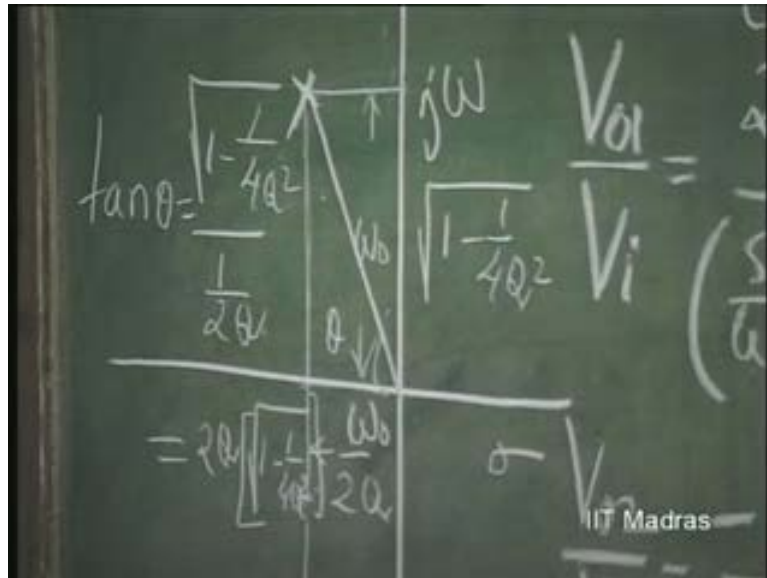
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These are the poles and you can see this is Omega naught by 2 Q. This is Omega naught into 1 minus 1 over 4 Q square. Actually speaking, this is going to be unity; or Omega naught in this case. If it is normalized, it is unity and it is called unit circle; if it is not normalized, this radius is going to be Omega naught because this is going to be square of this plus square of this; square of this plus square of this.

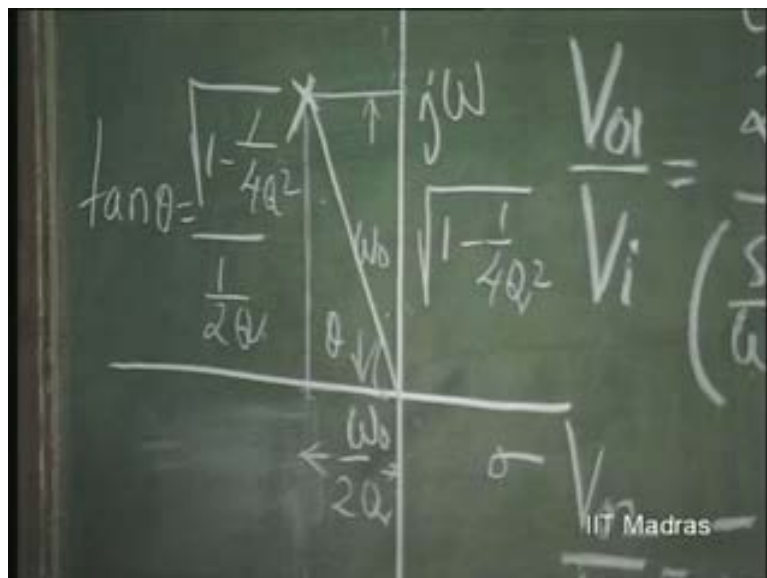
This Theta angle is going to be nothing but this divided by this, tan Theta. This divided by this. So, this value divided by this 1 over 2 Q. So, it tells us how close this is to the imaginary axis. If Theta is 90 degrees, it is very close to the imaginary axis, very nearly. So, that information is given by this Q because Theta, tan Theta...if this is Theta, tan Theta is root of 1 minus 1 by 4 Q square by 1 over 2 Q. So, this is nothing but 2 Q into root of 1 minus 1 over 4 Q square. So, you see that it is directly proportional to Q, if Q is high. So, if it is very nearly 90 degrees, it means, simply means that the poles are near the imaginary axis. That is indicated by the extent of Q.

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You can see when the Q is very high. When the Q is very high, this distance is going to very low and Omega, this j Omega is going to be very close to Omega naught because this is going to zero and this becomes Omega naught. That is called the resonant frequency.

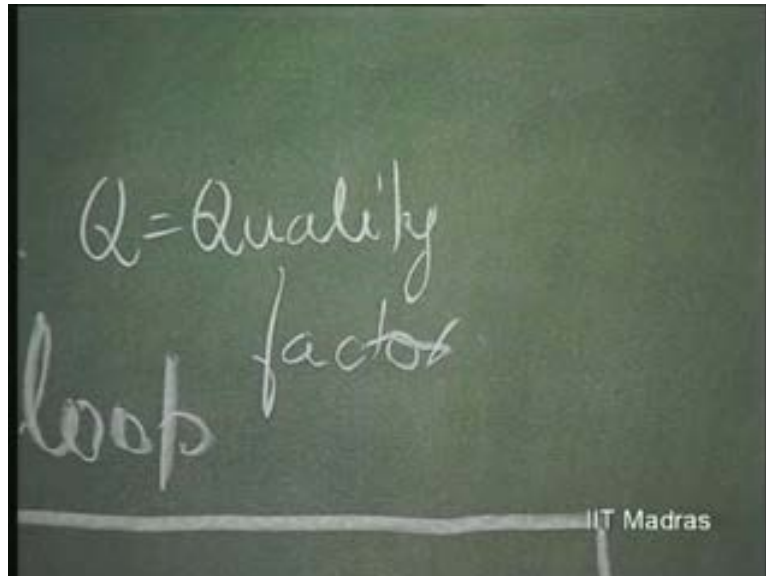
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When the poles lie on the imaginary axis, that is called the resonant frequency. If the pole Q is high, the actual frequency is close to the resonant frequency. So, this gives us information about the system. This is a second order system, the pole being second

order. So, this is a second order system and the pole Q high means the system has high quality factor. Q is quality factor.

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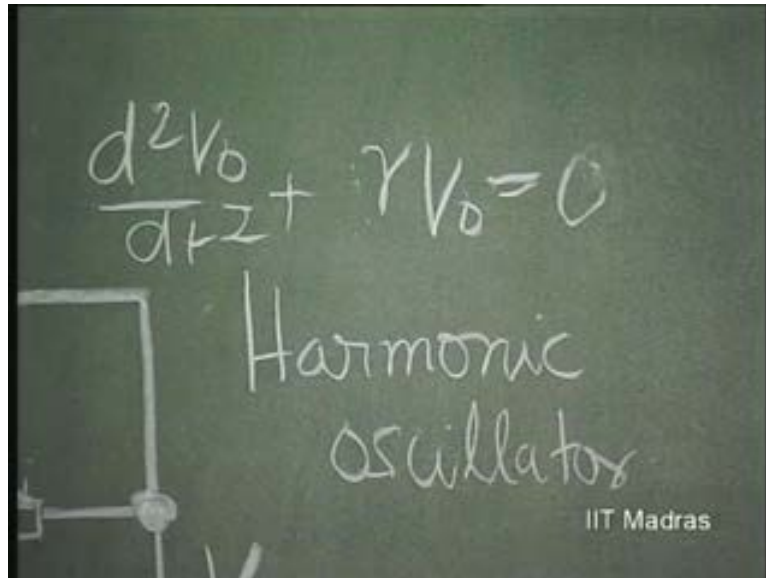
If this Q is infinity...when will Q be infinity? You can see here, when Alpha is zero, Q is infinity, means, there is no real part; the poles are on the imaginary axis. That means for the system, the poles are on the imaginary axis, when Q is infinity, corresponding to Alpha zero. That means there is no feedback here. This is removed. This... that is, this is zero. This is disconnected because this is contributing to Alpha. Alpha is zero means this is zero.  $V_i$  is connected here; but this is zero.

That means this whole thing is not getting fed back here. That means it is just having 2 integrators and inversions. That means, actually speaking, if you look at the differential equation, it has only  $d^2 V_{naught}$  by  $d T^2$  plus, we said, Alpha here. This is zero. That means  $d V_{naught}$  by  $d t$  is not present; only  $V_{naught}$ , some  $\gamma V_{naught}$  equals to  $V_i$  or zero.

So, this is a second order system which is that of the harmonic oscillator. When  $d V_{naught}$  by  $d t$  term is absent, this is called harmonic oscillator. It can by itself, without any input, give an output at a certain frequency called resonant frequency. The output will be corresponding to resonant frequency.

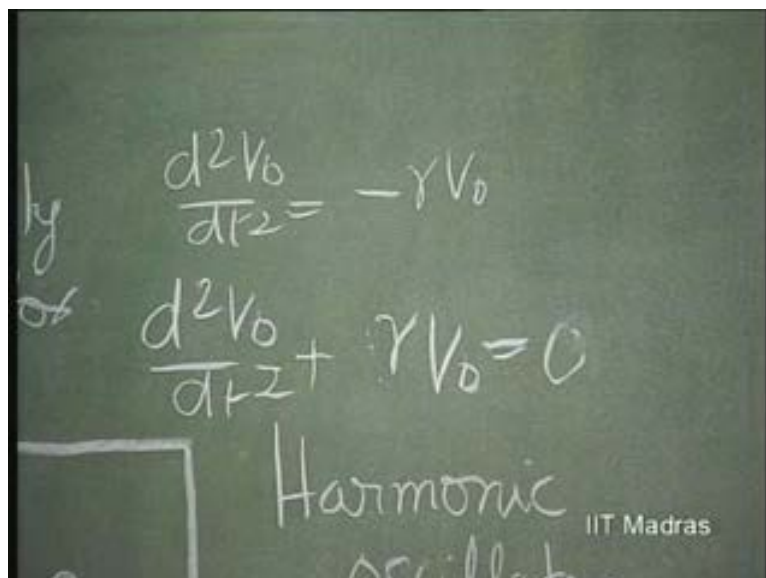


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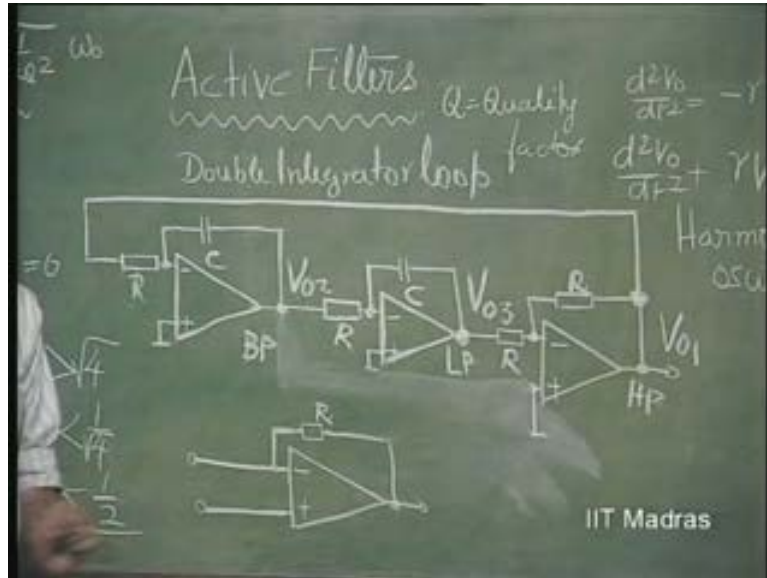
So, this should be absent. That means this has to be disconnected and whether  $V_i$  is there or not, it will by itself go into oscillation. So, you can as well connect  $V_i$  to ground. So, this circuit modified in this manner when  $\alpha$  is zero; this is corresponding to  $\alpha$  equal to zero. This resistance will be there; but this is connected to ground. That is zero. So, which means we can as well say that there is circuit like this. What does it simulate? This simulates simply this harmonic oscillatory equation. You can just see.  $d^2V_0$  by  $dt^2$  equal to minus  $\gamma V_0$ .

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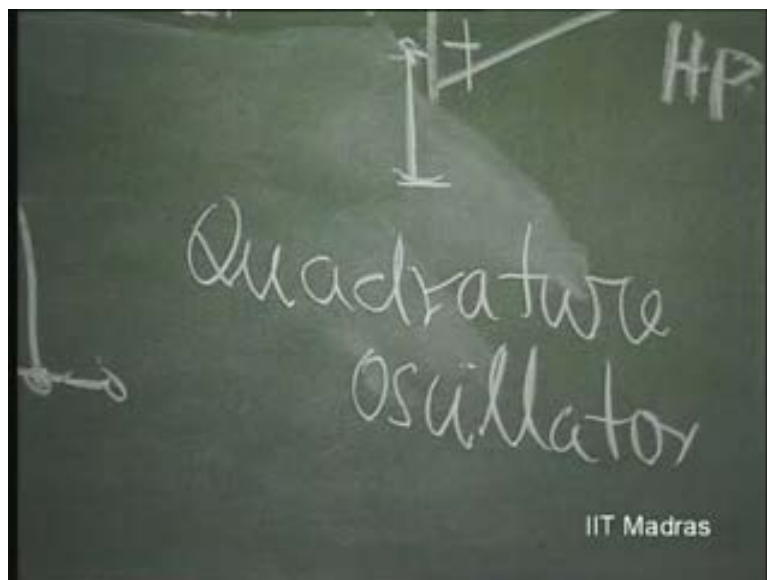


So, let us see whether this is what it is.  $d^2V$  naught by  $d t^2$  square is this. This will be  $d V \dots$  minus  $d V$  naught by  $d t$ . This will be  $V$  naught. This will be minus  $V$  naught into some constant. So, that is made equal to  $d^2V$  naught by  $d t^2$  square.

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So, this is in double integrator loop; is very famous as a harmonic oscillator; or it is called, also, quadrature oscillator.

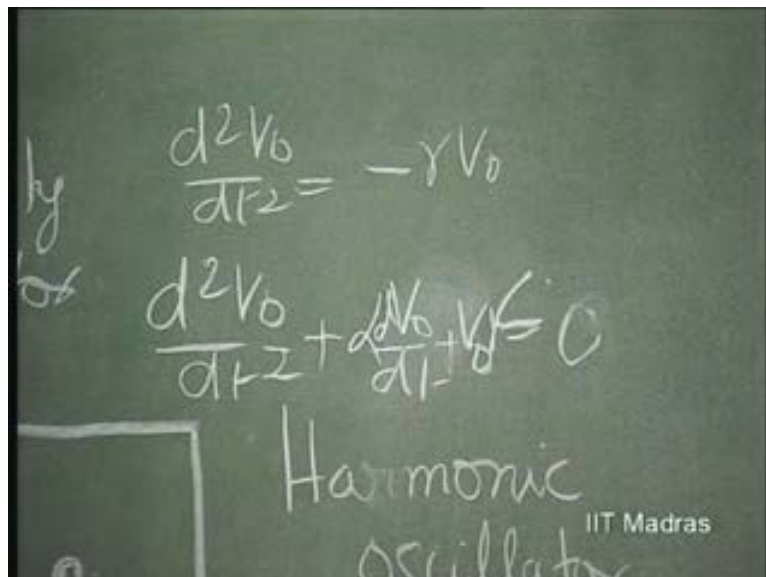


Analog computer people used to use this commonly to generate sine wave and cosine wave. Because it is using integrators, if this is sine, this will be cosine, because of a phase shift of 90 degrees given by the integrator.

So, this is called quadrature oscillator. It is commonly used as an oscillator to generate wave forms. How do you generate wave forms? The poles of the system should lie on the imaginary axis. That means there should not be Alpha feedback. That is Alpha should be zero; which means these 2 poles will be... Now, I mean Q equal to infinity; which means that these are going to be simply located at Omega naught and Omega naught here. That is that of the quadrature oscillator.

But we do not want it to oscillate. We want it to work as a filter. So, now you can see that. I have bring in, I have brought in Alpha times d V naught by d t plus Gamma times V naught. This was the equation that we had simulated earlier by synthesizing this circuit; and this gives you poles; always located on the left half of the plane because you will see that Q is always positive and this will result in always a negative real part.

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If Q is also made greater than half, this can result in complex conjugate pair of poles. Depending upon the Q and Omega naught, you can locate it anywhere on the left half of the plane. So, the design parameters are going to be Omega naught, pole frequency

and Q, pole Q. By designing this independently using Alpha and R C, you can locate the pole anywhere on the left half of the plane. That is, as far as pole location is concerned.

Now, as far as zero location is concerned, in this case, there is no zero; or zeros are at infinity. And here, there are 2 zeros at S is equal to j Omega equal to zero. That means there will be 2 zeros here and in this case there is one zero at S is equal to zero. So, there will be a zero here. So, this is low pass. What does it mean?

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The image shows a chalkboard with two transfer functions written in white chalk. The top equation is labeled 'LP' and the bottom equation is labeled 'HP'. Both equations have a denominator of  $(\frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q} + 1)$ . The LP equation has a numerator of  $\frac{2(1-\alpha)}{V_i}$ . The HP equation has a numerator of  $\frac{2(1-\alpha)s^2}{V_i}$ . The IIT Madras logo is visible in the bottom right corner of the chalkboard image.

$$\frac{V_{o3}}{V_i} = \frac{2(1-\alpha)}{(\frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q} + 1)} \quad LP$$

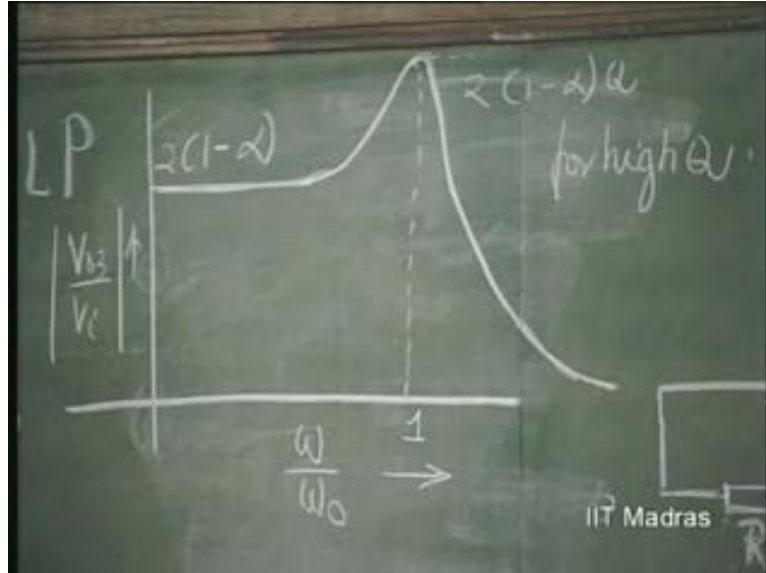
$$\frac{V_{o1}}{V_i} = \frac{2(1-\alpha)s^2}{(\frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q} + 1)} \quad HP$$

If I plot this characteristic as magnitude function  $V_{o3} / V_i$ , I request you to plot this as a function of  $\Omega / \Omega_0$ , let us say, normalized frequency. You will get this... at very low frequency, it is  $2(1-\alpha)$ . You can put  $S = j\Omega$  and  $\Omega = 0$ . So, at very low frequency, it is  $2(1-\alpha)$ . At very high frequencies, this becomes dominant and it goes to zero.

At  $S = j\Omega$ ,  $\Omega = \Omega_0$ , this gets cancelled with this. This gives you  $Q \times 2(1-\alpha)$ . If  $Q$  is very high, this can go to very high values. This is going to be very nearly at 1. This is going to be very nearly equal to  $2(1-\alpha) \times 2$ , for high  $Q$ . this is going to be  $1 - \Omega / \Omega_0^2$ . At  $\Omega = \Omega_0$ , this will get cancelled with

this and this will become 1 and Q goes above. So, 2 into 1 minus Alpha times Q. So, this is called low pass.

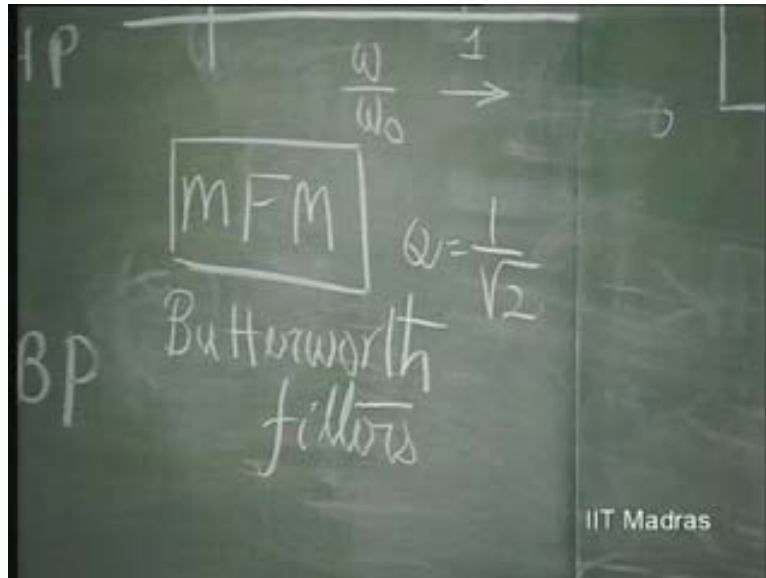
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This peaking can be reduced based on the value of Q. I can make it make it maximally flat also. That flat over the maximum distance can come about for a Q equal to 1 over root 2. So this, you can... Q equal to 1 over root 2 - maximum flatness. That is, it will be flat even here. This is not illustrated here because the maximum of this is not really equal to 2 into 1 minus Alpha into Q. That is, the maximum becomes equal to this only at very high Q. This is the value of the transfer function at Omega equal to Omega naught and Omega equal to Omega naught may be, may not be, the frequency at which maximum occurs for low Qs.

So, you can show that this becomes very flat when Q is equal to 1 over root 2 and therefore you can design filters which become maximally flat amplitude, Maximally Flat Magnitude. These are called M F M; by making Q equal to 1 over root 2. These are also called Butterworth filters.

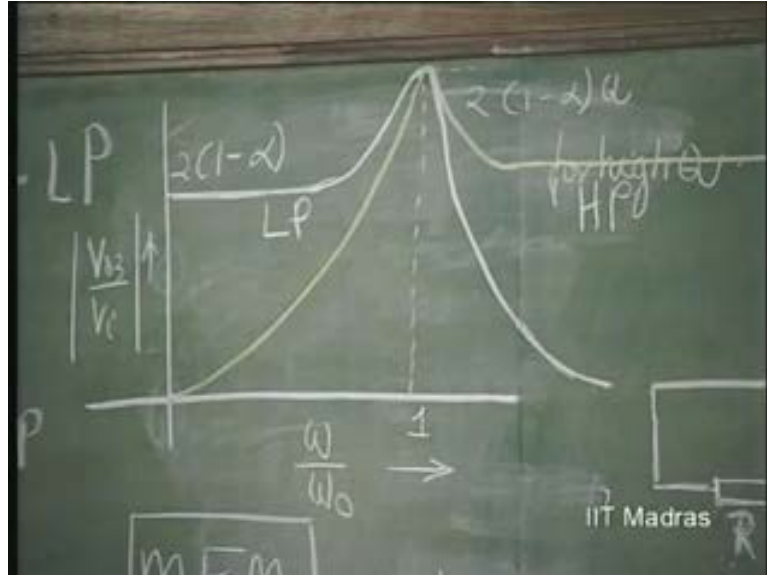
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The people have already mathematically decided for maximum flatness what kind of  $Q$  you should have for second order filter; what kind of poles you should have for third order filter. All these things have already been done mathematically and such filters with specific values of  $Q$ s are called Butterworth filters. For second order, the Butterworth filter will have a  $Q$  of  $1/\sqrt{2}$ . So, that is as far as low pass is concerned. As far as high pass is concerned, you can see, at  $S$  is equal to  $j\Omega$ ,  $\Omega$  going to infinity, this becomes  $2$  into  $1$  minus  $\alpha$ . These  $2$  get cancelled. So, it will be following, this is the asymptote at high frequencies. And at low frequency  $S$  equal to  $\Omega$ ,  $\Omega$  equal to zero, it should be zero because this comes in the numerator. It has double zeros at  $S$  is equal to zero.

So, this is the nature of high pass. This corresponds to low pass, this corresponds to high pass.

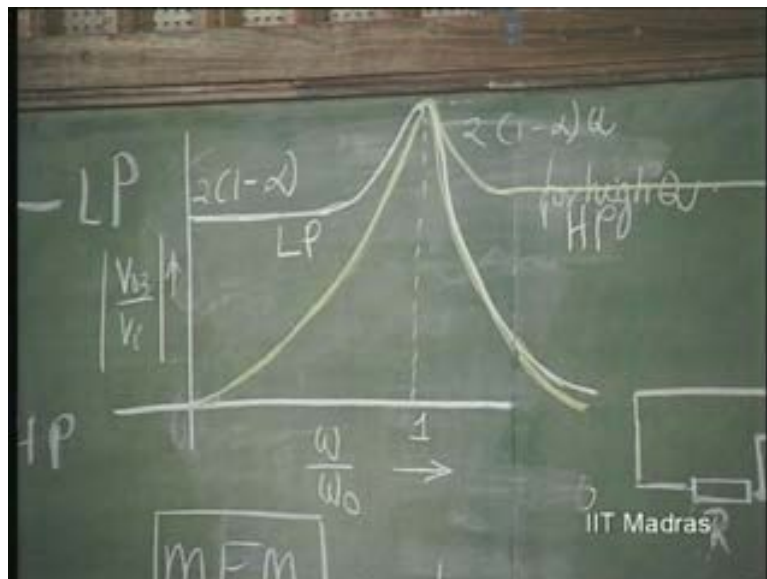
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And as far as band pass is concerned, at  $S$  is equal  $j\Omega$ ,  $\Omega$  equal to zero, it is zero at  $S$  is equal to infinity; because of this  $S$  square, it becomes zero.

So, we get zero at this point, zero at this point and it is going to follow this. This is going to be the band pass.

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So, low pass, high pass, band pass; so you can make filters which are selective. This band pass filter selects a band of frequencies. So, if you want to select a band of frequencies, you use band pass. Higher the Q, narrower will be the band width. This is called band width.

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What is band width? Band width of a band pass is defined as centre frequency  $\Omega_0$  divided by the difference between upper cut-off frequency and the lower cut-off frequency.

That can be defined as  $1/\sqrt{2}$ . So, find out the peak and find out the points at which  $1/\sqrt{2}$  times this occurs; one is called the upper cut-off frequency; another is called the lower cut-off frequency; and that you can prove in this case equals, equal to Q.



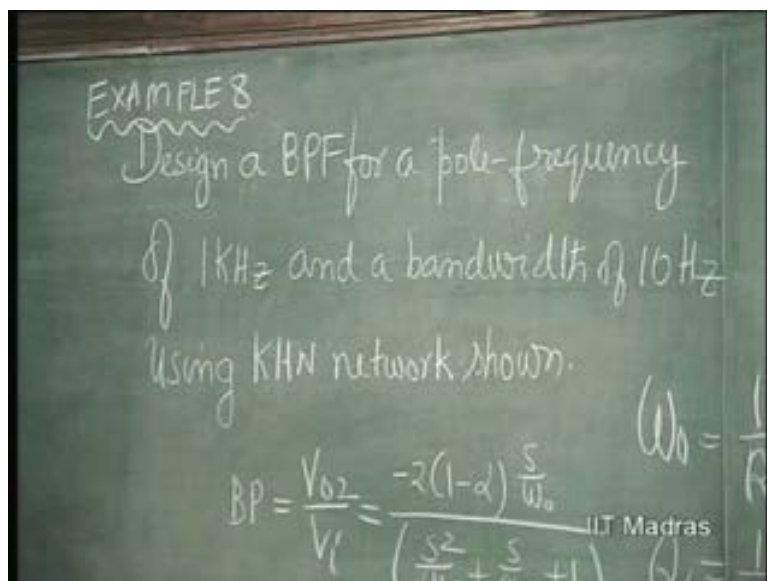
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So, that means the centre frequency divided by the band width is always equal to quality factor in a band pass filter of this type. So, it gives you the quality of the band pass. That is why it is Q factor, it is called. It indicates how narrow this band of frequency is going to be. This is the one that is selective.

So, let us now consider a design wherein I would like to use this structure to design a band pass filter. So, let us now discuss an example of filters - Example 8.

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Design a band pass filter for a pole frequency of 1 Kilohertz; pole frequency, let us call it  $\omega_0$  of 1 Kilohertz; and a bandwidth of 10 hertz, bandwidth of 10 hertz.

$\omega_0$  is equal to  $2\pi$ . This sign you have to remember because  $f_0$  is what is given. So, in fact, this is not  $\omega_0$ . What is given is  $f_0$  because it is given as 1 Kilohertz.  $\omega_0$  is radians per second. That you can get by multiplying  $f_0$  by  $2\pi$ . So,  $\omega_0$  is  $2\pi$  into  $10^3$ . Band width is 10 hertz; into  $2\pi$ . This is radians per second; this also is radians per second.  $Q$  is  $\omega_0$  by band width or... this is going to be equal to  $10^3$  by  $10$  which is 100.

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$$\omega_0 = 2\pi \times 10^3$$

$$BW = 10 \times 2\pi$$

$$Q = \frac{\omega_0}{BW} = \frac{10^3}{10} = 100$$

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This is...in our circuit which is nothing but this; this complete circuit. We know that  $\omega_0$  is equal to  $1/RC$ ;  $Q$  is equal to  $1/2\alpha$  and the transfer function is which we had already noted -  $\frac{2}{\omega_0^2 s^2 + \alpha \omega_0 s + 1}$ . That means if the output is taken at this point which is  $V_0$ ,  $V_0/V_i$  is going to be this, with  $Q$  equal to  $1/2\alpha$ ;  $\omega_0$  equal to  $1/RC$ .

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network shown.

$$= \frac{-2(1-\alpha) \frac{s}{\omega_0}}{\left(\frac{s^2}{\omega_0^2} + \frac{s}{\omega_0} + 1\right)}$$

$$\omega_0 = \frac{1}{RC}$$

$$\alpha = \frac{1}{2Q}$$

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So, this is equal to 1 over 2 Alpha; or Alpha is equal to one by 200. So, that is a design. Alpha is made equal to 1 over 200. That will facilitate design of resistance Alpha as R over 200 and another resistance as 199 by 200 into R. These are the resistance values. So, Omega naught is equal to 2 pi into 10 to power 3, which is given. That should be made equal to 1 over R C.

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BW = 10

$$= 100 = \frac{1}{2\alpha}$$

$$BP = \frac{V_{02}}{V_1} = \frac{-2(\dots)}{\left(\frac{s^2}{\omega_0^2} + \frac{s}{\omega_0} + 1\right)}$$

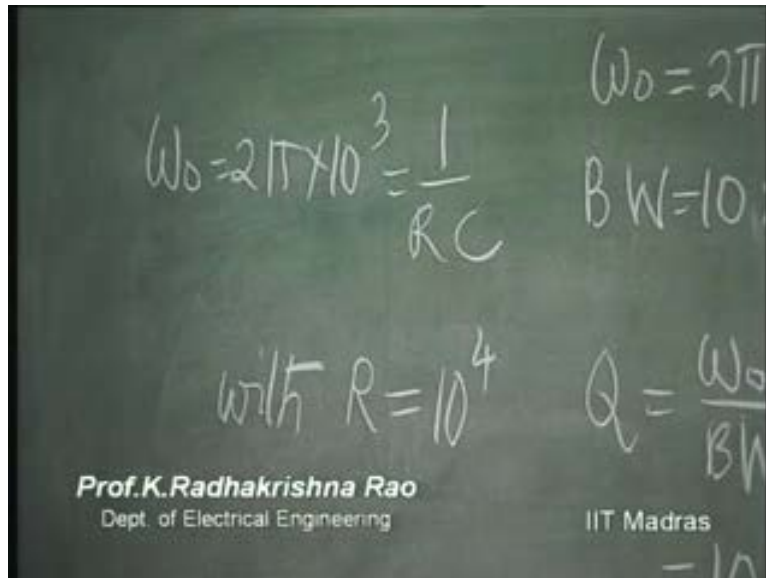
$$\alpha = \frac{1}{200}$$

$$\omega_0 = 2\pi \times 10^3 = \frac{1}{RC}$$

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Now you have a choice. So, Rs are not specified. Suppose Rs are specified. Let us say, Omega naught is equal to 2 pi into 10 to power 3 is 1 over R C, with let us say R equal to typically 10 K; I would like to take, 10 to power 4 ohms.

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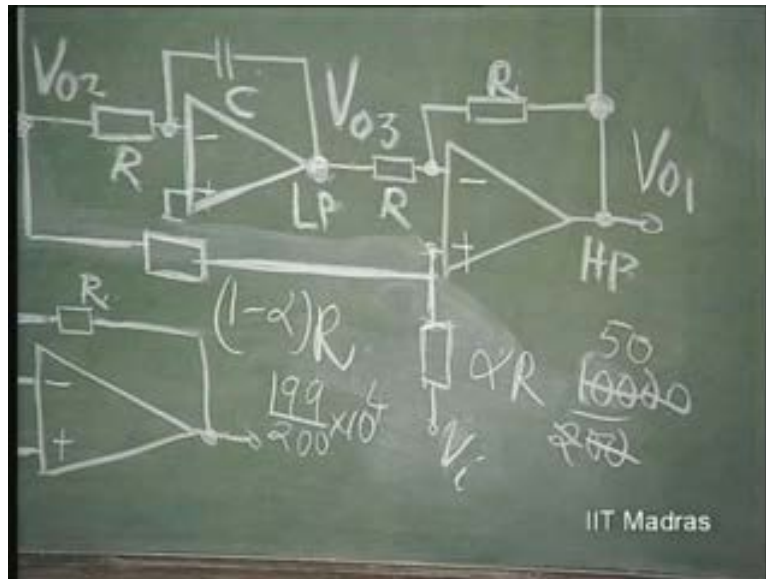
So, what will be C? 1 over 2 pi into 10 to power 3; his will come down; into 10 to power 4; or this is 1 over 2 pi into 10 microfarads. 15? Now, this is 15 nanofarads.

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So, you can see that by using capacitors of the order 15 nanofarads and resistance of the order of 10 K in this network... this is R obviously. All these resistors are 10 Ks. All these capacitors are 15 nanofarads and this resistance is going to be R by 200. That is 10 K divided by 200, which means, actually it is 50 ohms. This is 50 ohms and this is very nearly 10 K because this is 199 by 200 of 10 K.

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So, these are the values. This is 50 ohms. This is very nearly 10 K. You can just build this circuit and see that it will exactly function as designed. That means if you take the response of this, you take the centre frequency, it will be 1 Kilohertz and band width will be exactly equal to 10 hertz. This is the design of the band pass filter.

What will be the gain at centre frequency? You can obtain that. It will be  $Q$  into  $2$  into  $1$  minus  $\alpha$ . So, the gain at center frequency...  $1$  by  $200$  into  $Q$ , that is, what is the value of  $Q$ ? That is  $100$ . So, gain at centre frequency is going to be nothing but you can see  $199, 199$ . So, the characteristic will look like this.

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$$\text{gain at } \omega_0 = 2\left(1 - \frac{1}{200}\right)100$$

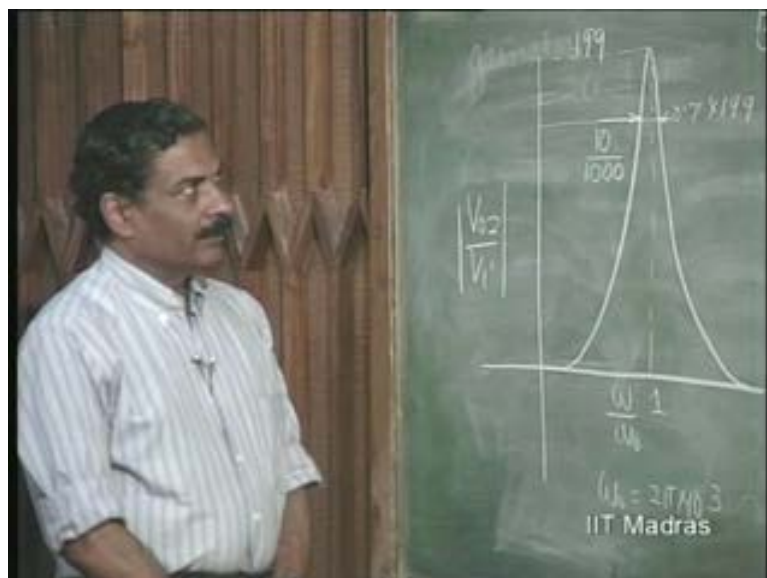
$$= 199$$

$$\omega_0 = 2\pi \times 10^3 = \frac{1}{RC}$$

$$BW = 10 \times 2\pi$$
 with  $R = 10^4$

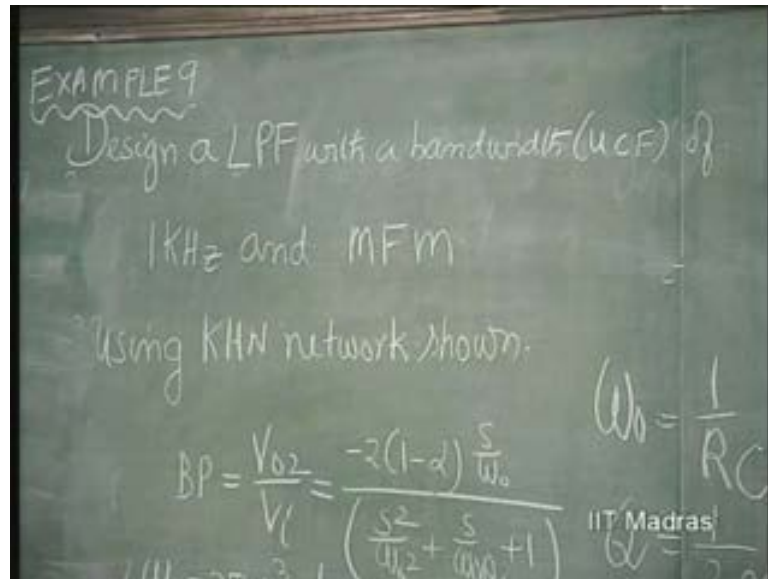
After you have designed, you can plot this response of the filter. That is, what you do is,  $V_{\text{out}}/V_{\text{in}}$  magnitude of this versus  $\Omega/\Omega_0$ .  $\Omega_0$  being  $2\pi \times 10^3$ . Then you get...this is going to be at 1;  $\Omega$  equal to  $\Omega_0$ ; and this is going to be 199. You take the bandwidth which is point 7 times 199 at these 2 points. This will be 10 hertz. Actually, this width if you take, it will be also divided by 1000. So, this is going to be 10 by 1000 according to... because all these things will be normalized with respect to  $\Omega_0$ . So, 10, 10 hertz by 1000 or  $10 \times 2\pi$  divided by 1000 into  $2\pi$ . So, this will be  $1/100$ . This width. This is the band pass filter which we have designed.

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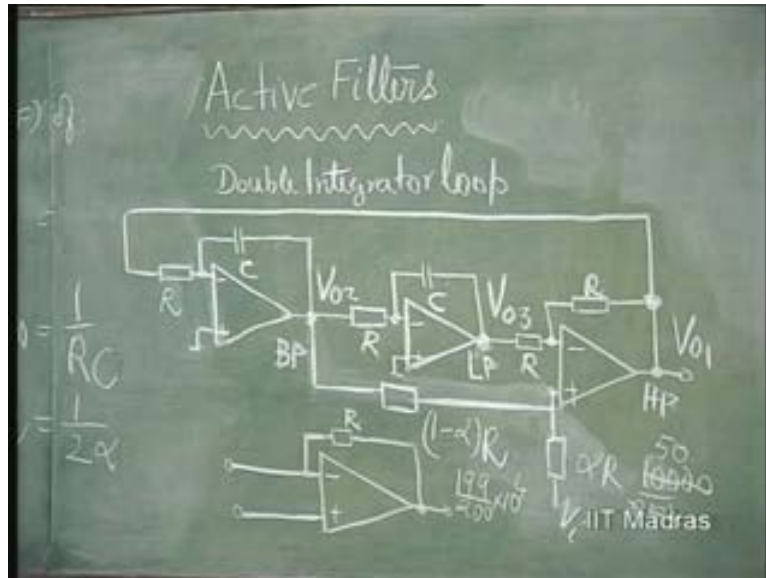
Next, let us consider another example. I would like to design another filter which is also quite popular. Example 9, we will say. Design a low pass filter at a frequency of 1 Kilohertz. That is, now we want, say, at a frequency, low pass filter with a bandwidth or upper cut-off frequency, because this has only upper cut-off frequency, of 1 Kilohertz and maximally flat magnitude using K H N network shown.

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So, this is Example 9. Design a low pass filter with a band width or upper cut-off frequency of 1 Kilo hertz and maximally flat magnitude using K H N network shown. So, we have to now take some other output which is low pass output which corresponds to...

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There in the figure, this point, low pass,  $V_{03}$ . So,  $V_{03}$  which is going to be plus 2 into 1 minus Alpha. This is the low pass output.

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Using KHN network shown.

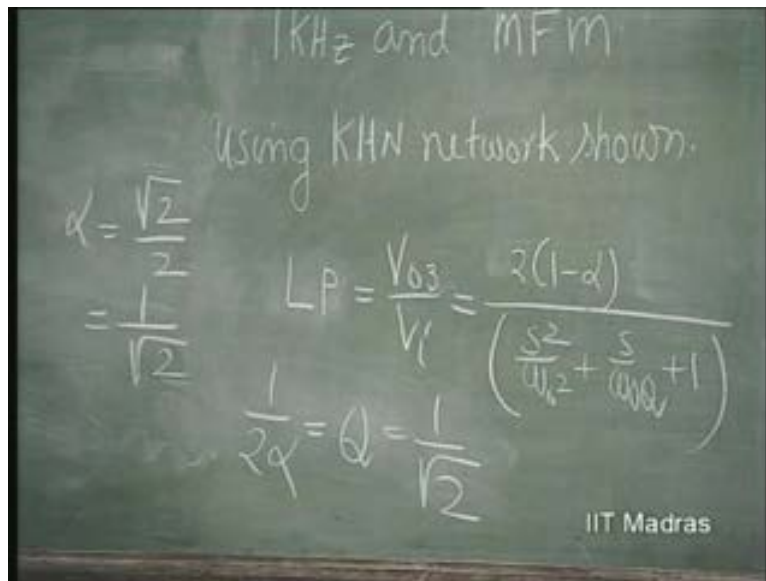
$$LP = \frac{V_{03}}{V_i} = \frac{2(1-\alpha)}{\left(\frac{s^2}{\omega_c^2} + \frac{s}{\omega_c Q} + 1\right)}$$

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And, as I told you, since maximally flat magnitude requirement or Butterworth filter requirement says that  $Q$  should be  $1/\sqrt{2}$ , which means,  $1/2\alpha$  is equal to  $1/\sqrt{2}$ ; or  $\alpha$  is equal to  $\sqrt{2}/2$ ; or  $\alpha$  is equal to  $1/\sqrt{2}$ . It is...  $1/2\alpha$  is the  $Q$  factor and that is equal to  $1/\sqrt{2}$ . So,  $2\alpha$  is equal to  $\sqrt{2}$ ;  $\alpha$  is equal to  $1/\sqrt{2}$ .

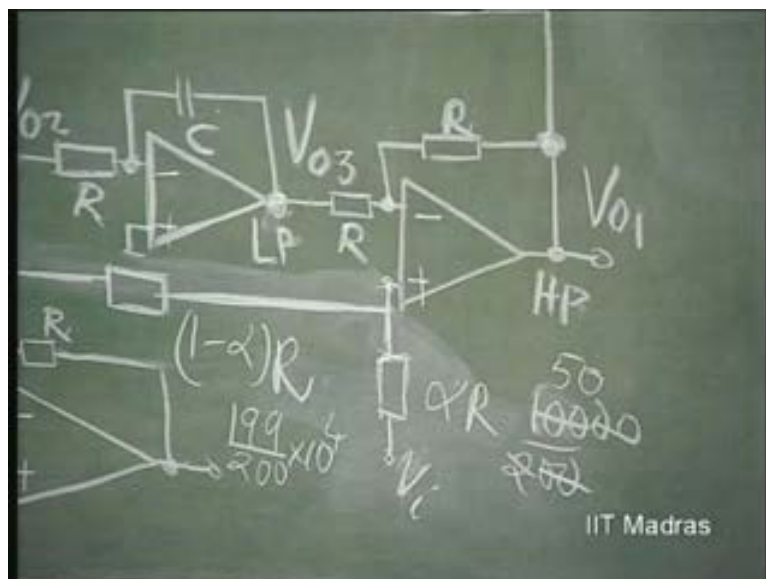


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This can be designed. R resistance, R, becomes Alpha by... that is the... R by Alpha is equal to root 2. So, R by root 2. This becomes 1 minus 1 by root 2 into R.

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Omega naught is the same. In this particular case, the band width is going to be determined by this Omega naught here. Omega naught is equal to 2 pi into 10 to power 3, which is 1 over R C. Again, we can take the same value. R equal to 10 K would give me C equal to 15 nanofarads. I can make use of the same information that I had earlier taken because the frequency remains the same - 15 nanofarads.

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The image shows a chalkboard with the following handwritten equations:

$$\frac{1}{RC} = \omega_0 = 2\pi \times 10^3 \text{ rad/s}$$
$$R = 10k \quad \alpha = \frac{\sqrt{2}}{2}$$
$$C = 15nF = \frac{1}{\sqrt{2}}$$

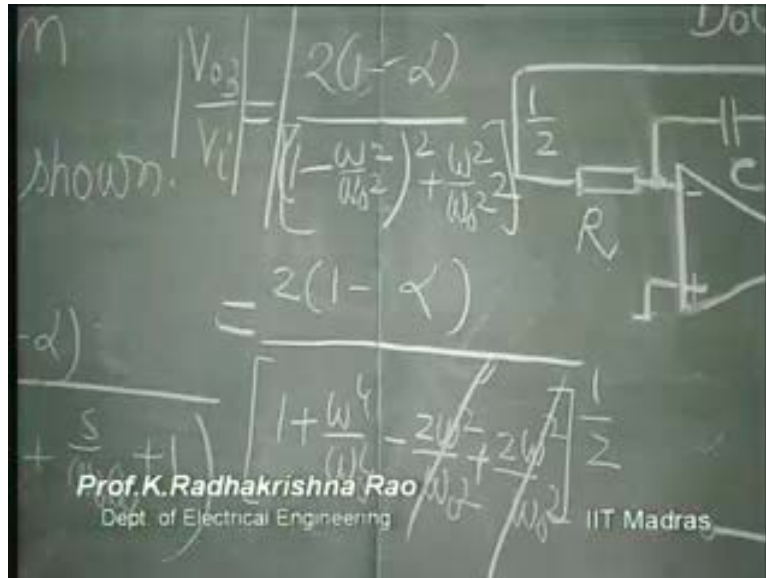
The text "IIT Madras" is visible in the bottom right corner of the chalkboard image.

So, that is the solution to a low pass filter. How will it look like? Let us actually plot. This is for your information.

Let us get  $V_{out}$  by  $V_{in}$  because we have not proven anything here, that it is maximally flat. I will show you that it is maximally flat. This is  $2 \times 10^{-1} \alpha$  divided by  $1 - \dots$  magnitude of this is same as magnitude of this. You put  $S$  is equal to  $j\omega$  here;  $1 - \omega^2$  by  $\omega^2$ .

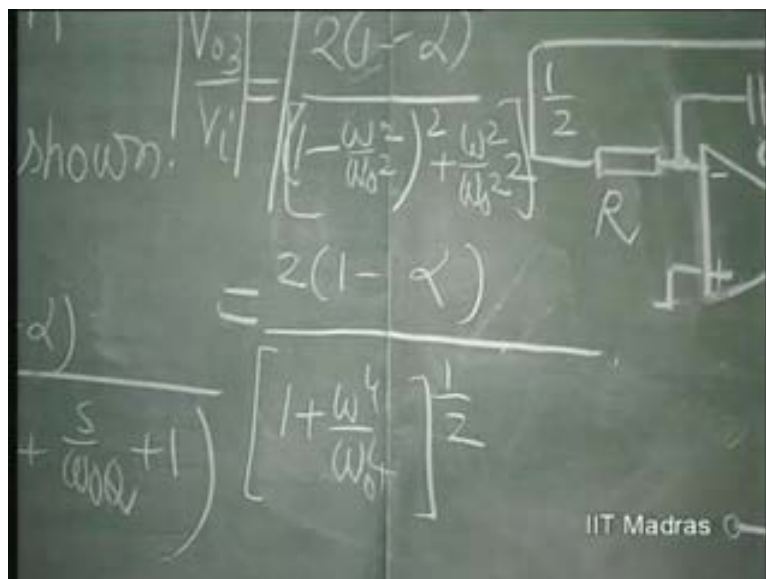
This whole square plus... this becomes  $\omega^2 Q$ . So,  $\omega^2$  by  $\omega^2$  is  $1$  over root... See,  $Q$  is equal to  $1$  over root  $2$ . So, root  $2$  comes in the denominator,  $S$  coefficient. So,  $2$ . So, this becomes  $2$  into this whole thing to the power half. That is the magnitude of this. So,  $2 \times 10^{-1} \alpha$  divided by... to the power half.  $1 + \omega^4$  by  $\omega^4$ . This square minus... look at this  $2 \omega^2$  by  $\omega^2$ . This is square of that; plus  $2 \omega^2$  by  $\omega^2$ . This cancels exactly this; in a polynomial of  $\omega^2$  by  $\omega^2$ , which is what is going to be contained in the denominator at all times, only the highest polynomial function remains. In a maximally flat magnitude function, all the other polynomial functions of  $\omega$ ,  $\omega^2$  will go to zero. That is the basis.

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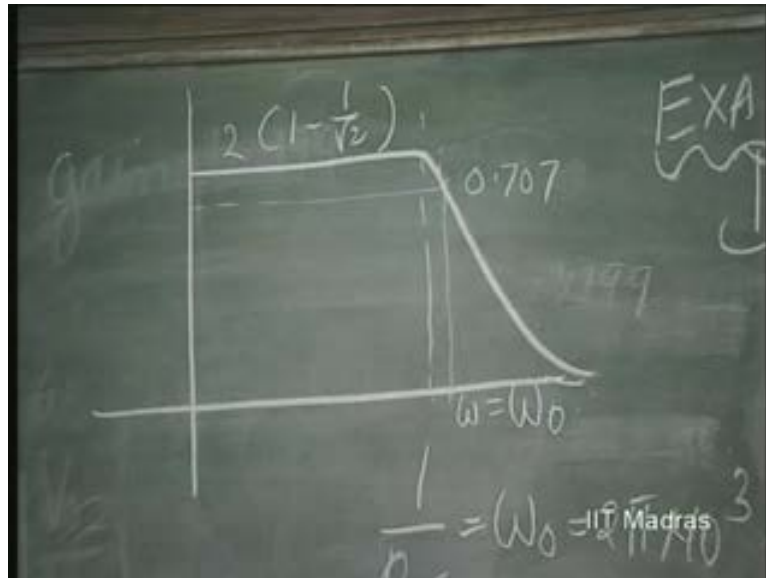
That means it will become as close to unity as possible, except that the highest order is going to remain; this to the power half.

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So, you can see that the band width corresponds to Omega equal to Omega naught; and it is going to be 1 over root 2. That is the fact. This is... this is going to be maximally flat. At this point, Omega equal to Omega naught, where it is 1 over root 2. Point 707 times whatever it is, 2 into 1 minus Alpha which is what it is; 2 into 1 minus 1 by root 2. That is the K.

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So, this is the maximally flat magnitude transfer function which is normally used for obtaining a low pass filter of good quality.

Next, let us consider Example 10. Example 10 is going to be a special filter. Design a notch filter. This is also called band elimination filter, at 1 Kilohertz; notch at 1 Kilohertz and a pole Q of 100, using K H N network shown.

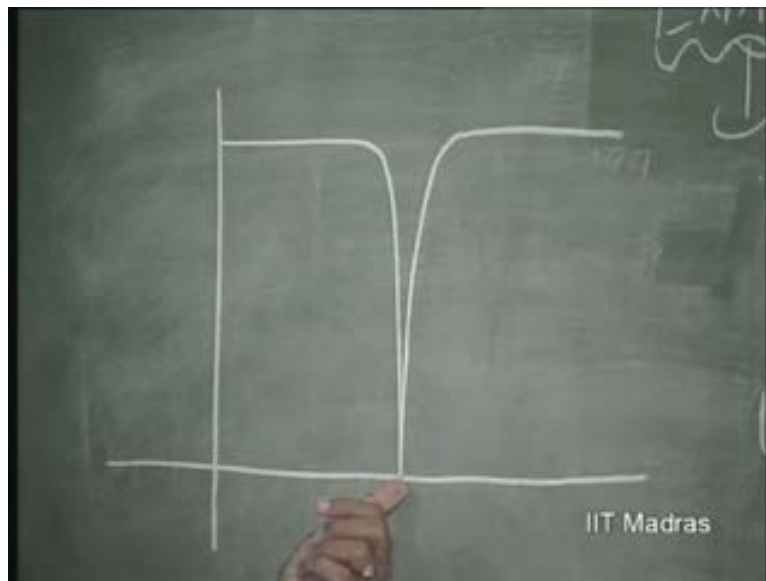
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EXAMPLE 10  
Design a notch filter (band elimination) at 1 kHz  
and a pole Q of 100  
using KHN network shown

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So, what is a notch function? Let us understand what this filter function is. What is required is that output should go to zero at a certain frequency. It should be constant at all other frequencies. Output should go to zero only at a specific frequency. It should be as constant as possible at all other frequencies. This is what is called a notch filter. It should go to zero. That can be done only by zeros because zeros by definition indicate that transfer function goes to zero.

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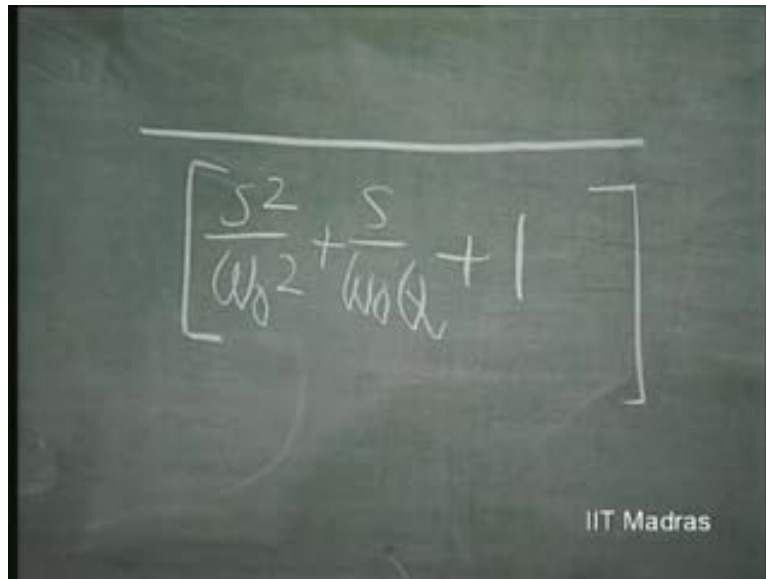
So, I have to locate a finite zero in my transfer function so that it becomes zero at a certain frequency. This kind of filter is very useful in particularly bio-medical applications where you are taking the signal from the human body; and the human body picks up lot of power line frequency which is 50 hertz; and therefore this bio-medical signal is always mixed with this kind of 50 hertz pick up.

How to separate this? Such a separation can be done by a notch filter. So, this notch, obviously, at... should occur at 50 hertz in the case of bio-medical application. In our example, it is supposed to occur at 1 Kilohertz. So, this is what is given. Let us find out how we can use this network which is giving you band pass, low pass and high pass.

In order to obtain a notch filter or band elimination filter, we said as far as this filter is concerned, the denominator is never changed because the loop is fixed. Loop - you

cannot change. You can now take the outputs and add or subtract using another operational amplifier so that the zeros can be anywhere. So, the denominator always remains the same, once the loop is fixed. This is, as far as you are using this network, denom...poles remain the same. Please... Poles are characteristic of the feedback. They cannot be changed.

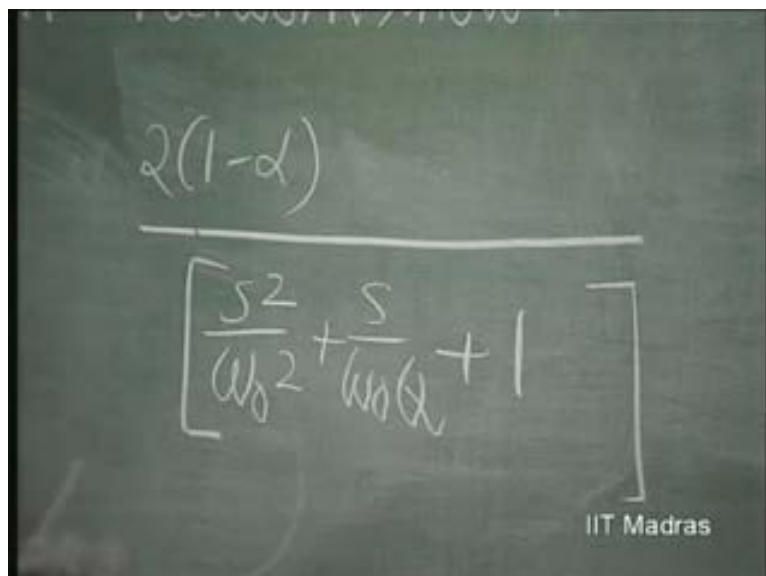
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A chalkboard showing a handwritten transfer function enclosed in large square brackets. The function is  $\frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q} + 1$ . The text "IIT Madras" is visible in the bottom right corner of the chalkboard image.

Now, zeros can change, the numerator can change. For example, if I take the low pass, it is just this.

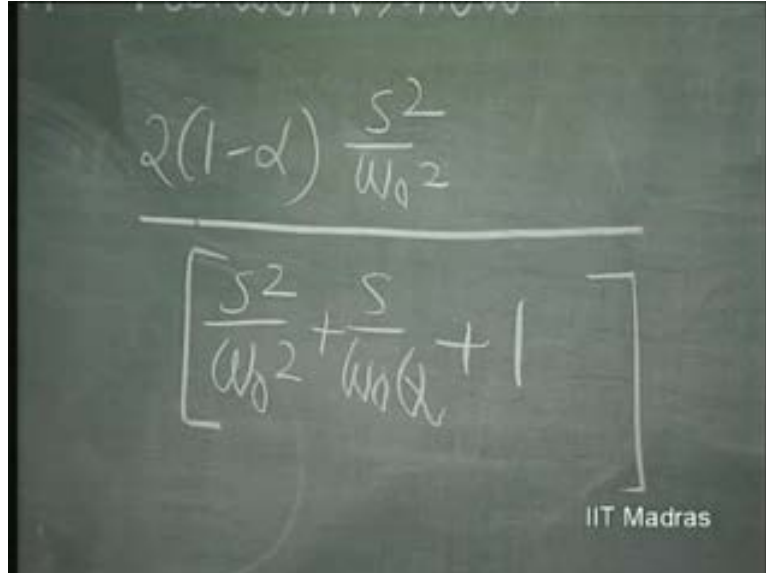
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A chalkboard showing a handwritten transfer function. The numerator is  $2(1-d)$ , followed by a horizontal line, and then the denominator  $\frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q} + 1$  enclosed in large square brackets. The text "IIT Madras" is visible in the bottom right corner of the chalkboard image.

And the high pass is this into S square divided by Omega naught square.

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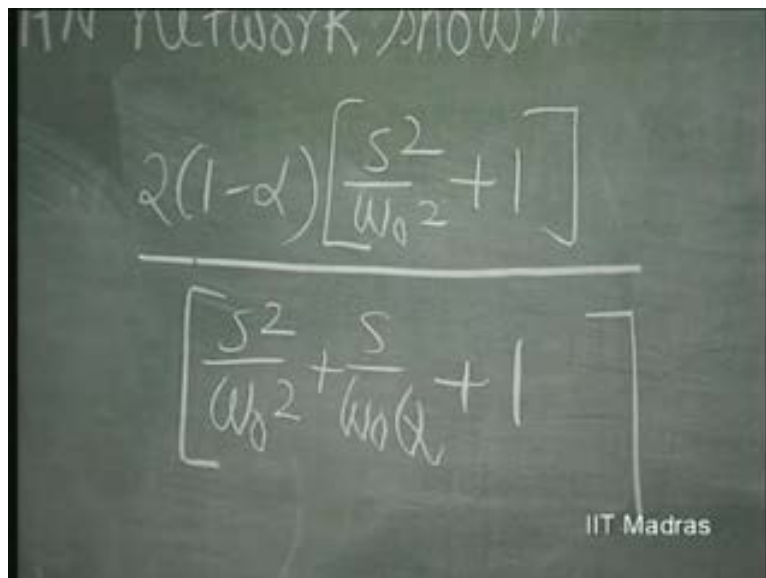


A chalkboard showing the transfer function for a high-pass filter. The numerator is  $2(1-\alpha) \frac{s^2}{\omega_0^2}$ . The denominator is  $\left[ \frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q} + 1 \right]$ . The IIT Madras logo is visible in the bottom right corner.

$$\frac{2(1-\alpha) \frac{s^2}{\omega_0^2}}{\left[ \frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q} + 1 \right]}$$

So, if I add low pass to high pass, I get a notch at Omega naught.

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A chalkboard showing the transfer function for a notch filter. The numerator is  $2(1-\alpha) \left[ \frac{s^2}{\omega_0^2} + 1 \right]$ . The denominator is  $\left[ \frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q} + 1 \right]$ . The IIT Madras logo is visible in the bottom right corner.

$$\frac{2(1-\alpha) \left[ \frac{s^2}{\omega_0^2} + 1 \right]}{\left[ \frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q} + 1 \right]}$$

So, one way is, add low pass to high pass, output. Low pass is available here, high pass is available here. Add this. Then you will get 2 into 1 minus Alpha. That is, the low pass divided by the same denominator; 2 into 1 minus Alpha into s squared by

Omega naught square is the high pass. So, if I add high pass and low pass, I get a zero at what frequency? Put S is equal to j Omega. This becomes 1 minus Omega square by Omega naught square. At Omega equal to Omega naught, it goes to zero.

So basically, this is going to be looking like this. 2 into 1 minus Alpha, 1 minus Omega square by Omega naught square. That divided by the denominator function which is always the same.

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$$|G(j\omega)| = \frac{2(1-\alpha) \left[ 1 - \frac{\omega^2}{\omega_0^2} \right]}{\left[ \left( 1 - \frac{\omega^2}{\omega_0^2} \right)^2 + \frac{\omega^2}{\omega_0^2} Q^2 \right]^{1/2}}$$

That is the magnitude of this. So, it is going to zero at Omega equal to Omega naught. What is the low frequency transmission? You can see. Low frequency transmission is the same as high frequency transmission and it is equal to 2 into 1 minus Alpha, in this case. And that also can be changed by adding suitably with a gain, if you want.

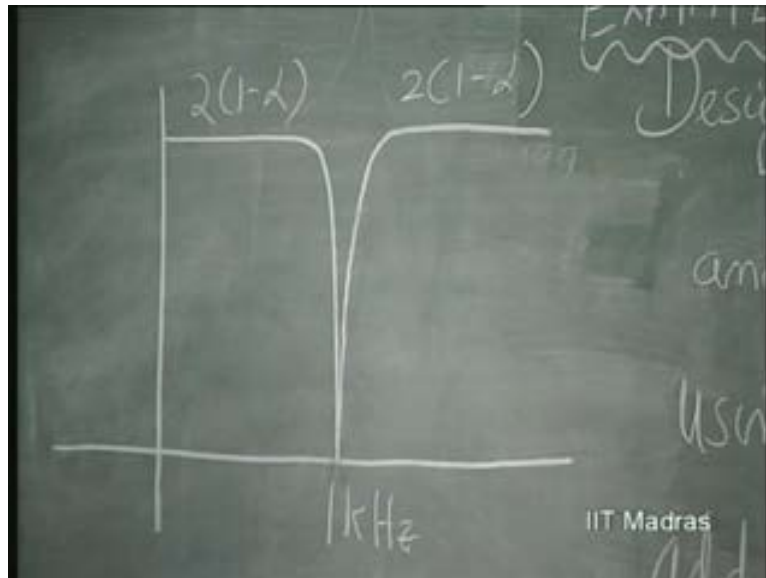
But, for the time being, let us say this is 2 into 1 minus Alpha. This is also 2 into 1 minus Alpha. That remains constant. At Omega equal to Omega naught, it goes to zero. The higher the Q of the pole, the narrower is this. What? You can see that even the... this introduces simply as zero at Omega equal to Omega naught; but the shaping of the zero depends upon the peaking here.

You are trying to make the pole give a peak here exactly at the point where there is a zero. So, even though it is trying to peak to an extent of 2 into 1 minus Alpha into Q,



since it is going to be multiplied by zero, it is just brought down; and if the narrow peak occurs, then this will be also narrow. So, the Q really governs how narrow this is going to be. So, higher the Q, narrower is this.

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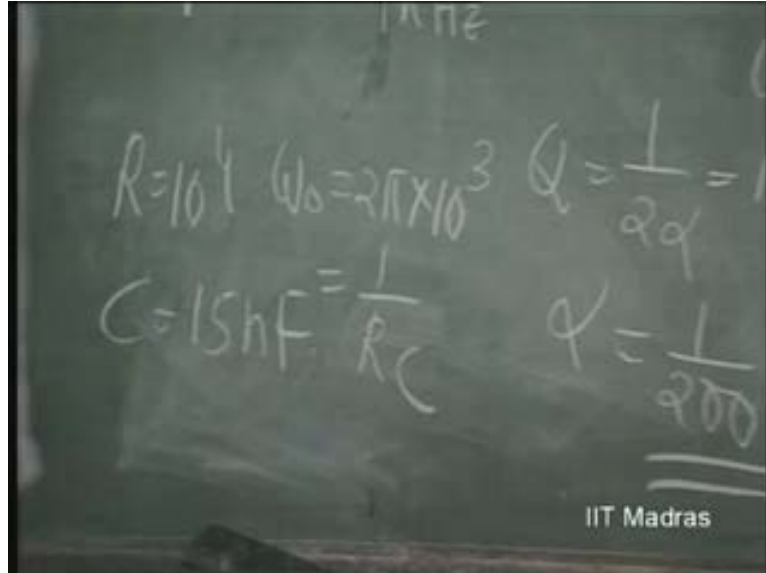
So, in a real notch filter, I would like to remove only the unwanted frequency; and other frequency components should be retained. So, the Q of... the whole Q of the system should be made very high. Now, if you make it pretty high, it will remove only 1 Kilohertz and that frequency should be accurately located. That means Omega naught equal to 1 over R C should be accurately adjusted. Otherwise, instead of 1 Kilohertz, it might remove 1.001Kilohertz, very accurately.

So, this is important. The stability of the pole frequency is very important. What I mean is, if you are getting rid of 50 hertz, you should adjust it exactly to 50 hertz; and you should have only 50 hertz component getting eliminated. But, if the frequency changes to 49 hertz, it will still remove only 50 hertz and not remove 49 hertz. That means it is not enough if you put a high Q filter. It should track the incoming unwanted frequency of 50 hertz. Such things are called adaptive filters, where the R C will track itself so that it will keep adjusting to the frequency that is to be eliminated.

So, we have 2 into 1 minus Alpha here. So, Q is equal to 1 over 2 Alpha. This, let us say, we will adjust it to be of the order of 100, like earlier. Alpha is equal to 1 over

200. Omega naught is equal to 2 pi into 10 to power 3, which is equal to 1 over R C. Again, R is equal to 10 to power 4, C is equal to 15 nanofarads.

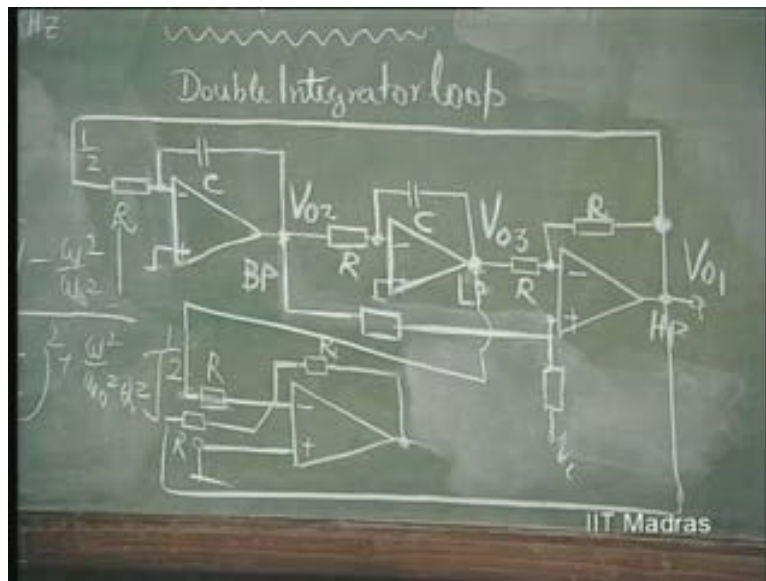
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This design is over except that I have to practically implement it using this circuit. So, let us see... How do... how do we implement this as such. All these resistances and capacitances are fixed now.

So, I have to merely now take this output high pass and take this output low pass and simply add. This addition is facilitated by grounding this. So, just... it gives you an inversion, let us say, here; and other addition is going to be just this.

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Except for changing sign now, this will be simply added. So, this is an adding circuit... something circuit that we had earlier used. So, this is grounded. You put equal resistors. Connect one to low pass, another to high pass; and we get an addition of this; except that, there is a change in sign here. That is all, because of the inversion. So, this is the physical implementation here. We get notch output.

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