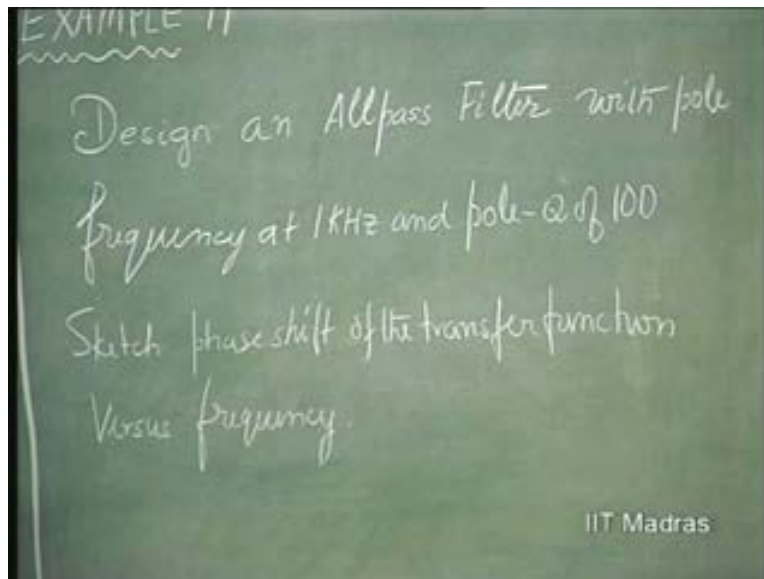


**Electronics for Analog Signal Processing - II**  
**Prof. K. Radhakrishna Rao**  
**Department of Electrical Engineering**  
**Indian Institute of Technology – Madras**

**Lecture - 13**  
**Simulation of Harmonic Oscillators**

Today, we will consider Example 11 in which we will design another important filter. Design an all pass filter. All pass - that means it will pass all the frequencies. So, what is it going to be used for? Let us see later...with pole frequency at 1 Kilo hertz. Now we have understood pole frequency and pole Q of 100. Sketch phase shift of the transfer function versus frequency. That is important. Here it is primarily used for obtaining a phase shift variation with respect to frequency.

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So, we will see how such a network which passes all the frequencies without any attenuation or with same amount of attenuation or amplification, but will subject each frequency component to a certain phase shift. So, such a transfer function, second order transfer function, will look like this. The denominator is always represented in a standardized form in the case of a filter; second order.  $s^2$  by  $\Omega$  naught

squared, Omega naught being the pole frequency, Q being the pole Q. Alright. So, this we have understood. How this... it is s here, not... s squared, s plus 1.

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Handwritten equation on a chalkboard:

$$\left[ \frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q} + 1 \right]$$

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So, numerator... now, for all pass - that means, all the frequencies should be transmitted; but there should be a only phase shift. So, only sign wise it will change; coefficients will be exactly same, or of the same ratio. So, this is an all pass transfer function.

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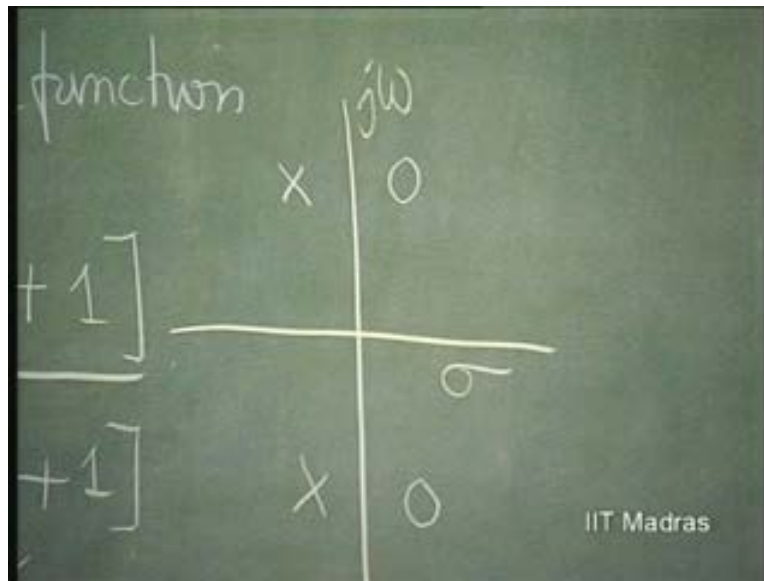
Handwritten equation on a chalkboard:

$$\frac{\left[ \frac{s^2}{\omega_0^2} - \frac{s}{\omega_0 Q} + 1 \right]}{\left[ \frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q} + 1 \right]}$$

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The coefficients remain the same or they are of the same ratio. So, it could be that this is minus, this is plus, this is minus; or, this is plus, this is minus, this is... That means all the even functions will have the same sign and all the odd functions of  $s$  will have the same sign. Here, of course, all will have positive sign. These are therefore the zeros. If you plot these zeros, actually, in  $s$  domain for an all pass, you will see that there would be complex conjugate pair of poles to the  $Q(x)$  and there will be a pair of zeros; mirror image of poles. So, this is going to be in  $s$  domain, the location of poles for an all pass transfer function.

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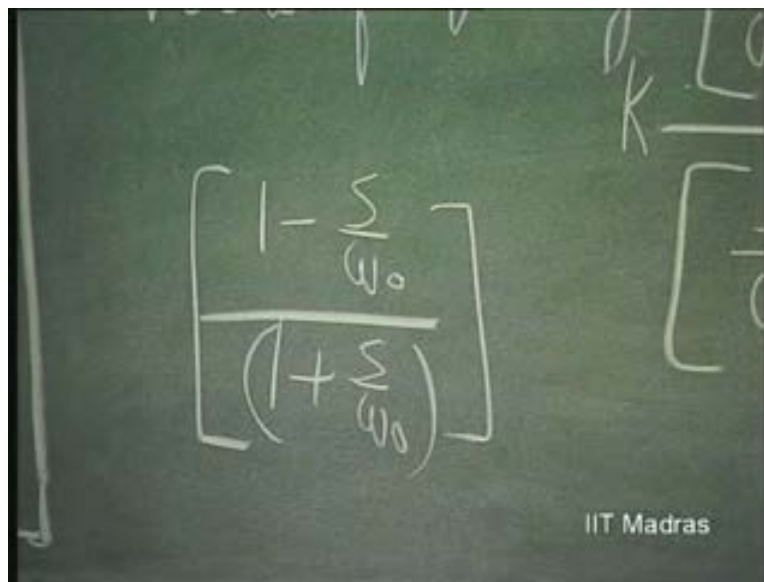
What is it going to be used for, is first something that we will discuss. Here since...if you take the magnitude, magnitude is unity throughout; or, if there is a  $K$  factor here, it is going to be  $K$  throughout for all frequencies because, if you take the magnitude, this will give you a contribution of unity. Only change of sign occurs; whereas, the phase shift is going to be offered by both poles as well as zeros.

So, contribution to the phase shift is going to be there and therefore, this is going to be very useful for obtaining phase shift. If you consider the phase shift at  $s$  is equal to  $j\omega$ ,  $\omega$  equal to zero, the phase shift is zero; and ultimately again, at  $s$  is equal to

infinity, put  $s$  is equal to  $j\Omega$  and  $\Omega$  equal to infinity, the phase shift is becoming 360 degrees, or, zero again.

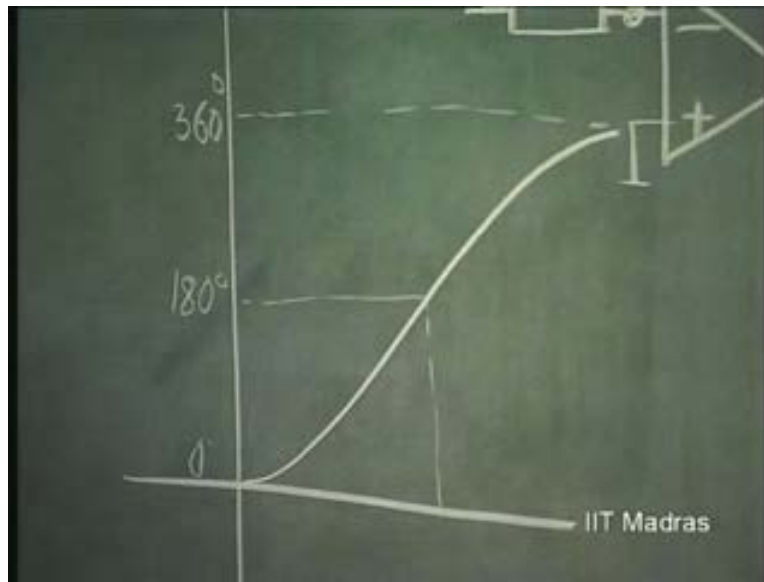
So, this will give you one full phase coverage of zero to 360 degrees. If it is a first order all pass, just for comparison, first order all pass will be... So, this will give you a full coverage from zero to 180 degrees. Zero to minus 1 - 180 degrees. A second order all pass will give you a phase coverage of zero to 360 degrees.

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$$H(s) = \frac{1 - \frac{s}{\omega_0}}{1 + \frac{s}{\omega_0}}$$

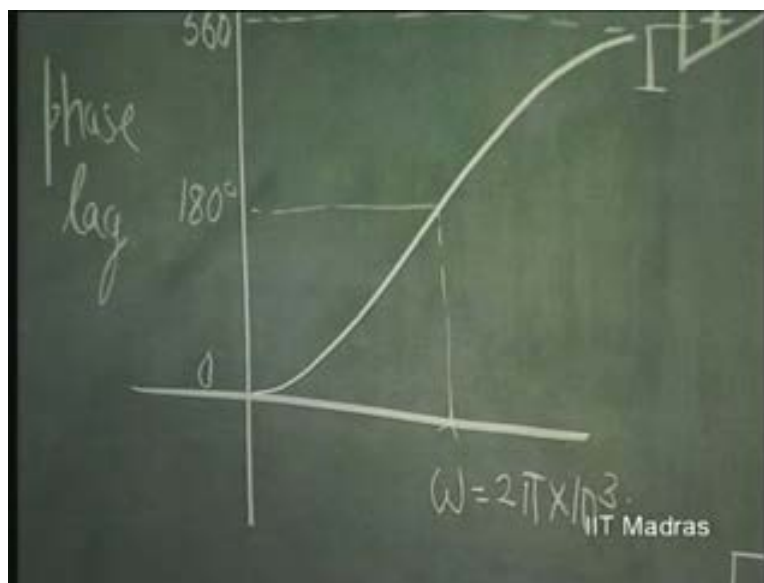
Now, that can be concluded. The pole alone can give you a phase shift from zero to  $s$  squared; means minus  $\Omega$  squared, 180 degree; and addition will have zero also contributing another 180 degree. So, the nature of variation of this phase with respect to frequency is same; both for poles and zeros, because the coefficients are the same. And therefore, you will see that the phase shift varies all the way from zero to 360 degrees; and we will note that if this is absent only due to pole at  $s$  is equal to  $j\Omega$ ,  $\Omega$  equal to  $\Omega$  naught, this gets cancelled with this. It is  $1$  over  $s$ . That is phase lag of 90 degrees.

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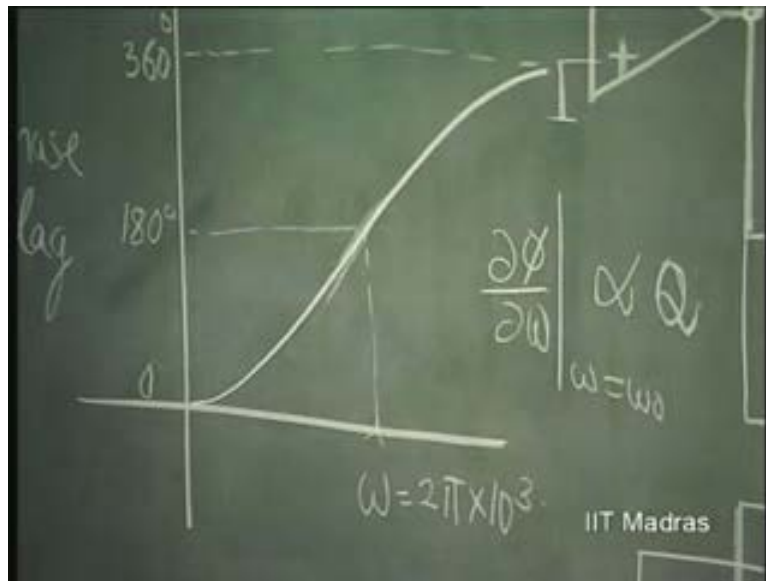
Now, this is... this is phase lag. Actually, the entire thing can be. So, the zero also contributes to 90 degrees at the same point; phase lag of 90 degrees. So, 180 degree. So, this is going to be occurring at a frequency which is  $2\pi \times 10^3$ ,  $\Omega$  naught f naught.

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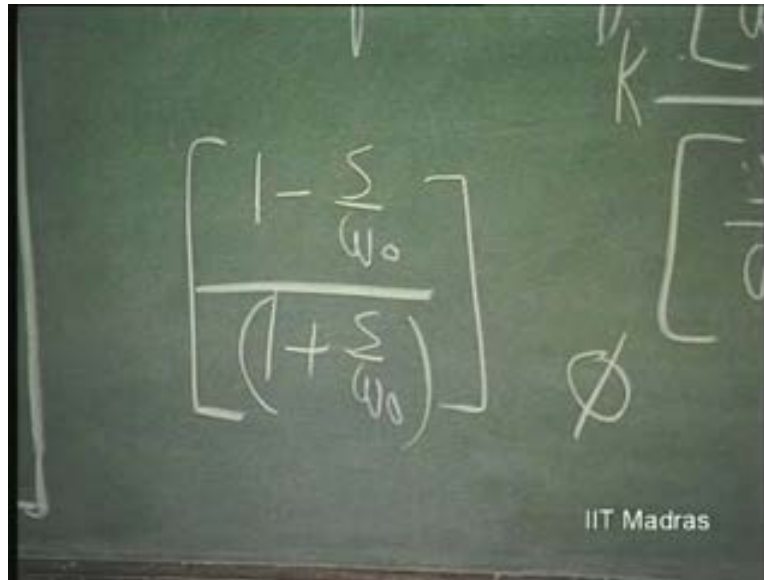
So, this is going to be occurring at the pole frequency. This phase shift of 90 degree contributed by the pole, 90 degree contributed by the zero; 180 degree is going to occur at this; and it can be shown that the phase variation, that is Delta phi by Delta Omega is maximum at Omega equal to Omega naught and it is directly proportional to Q.

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This, I would like you to write...write down the phase phi for this. Write down phi for this and show that Delta phi by Delta Omega is maximum at Omega equal to Omega naught and is directly proportional to Q.

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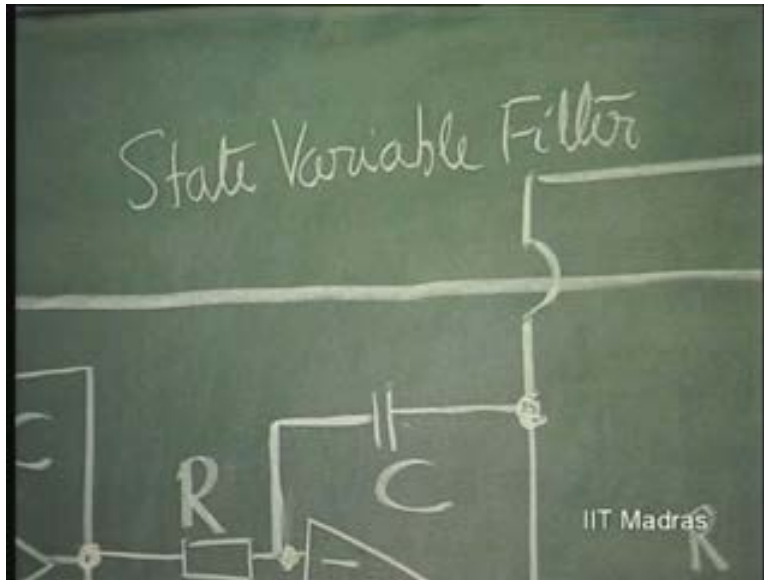
The image shows a chalkboard with a transfer function written in white chalk. The transfer function is a fraction with a numerator of  $1 - \frac{s}{\omega_0}$  and a denominator of  $(1 + \frac{s}{\omega_0})$ . To the right of the transfer function, there is a circled Greek letter phi ( $\phi$ ). In the bottom right corner of the chalkboard, the text "IIT Madras" is visible.

That means this slope at this point is going to be steep, if  $Q$  is very high. That is the function of  $Q$ . If you make  $Q$  very high, the phase will vary rapidly with frequency around  $\Omega$  equal to  $\omega_0$  so that, in this particular case, we wanted the phase variation to be maximum around 1 Kilo hertz and we wanted the phase variation to be very rapid. That is why we have chosen the pole  $Q$  to be 100.

So, the purpose of designing this so called all pass circuit so as to have the required phase variation at the required frequency is served in this manner. So, this gives you uniformly, the phase varying from  $\Omega$  equal to zero to  $\Omega$  equal to 360 degrees. So, the design of this circuit is for this purpose; to get this kind of phase variation.

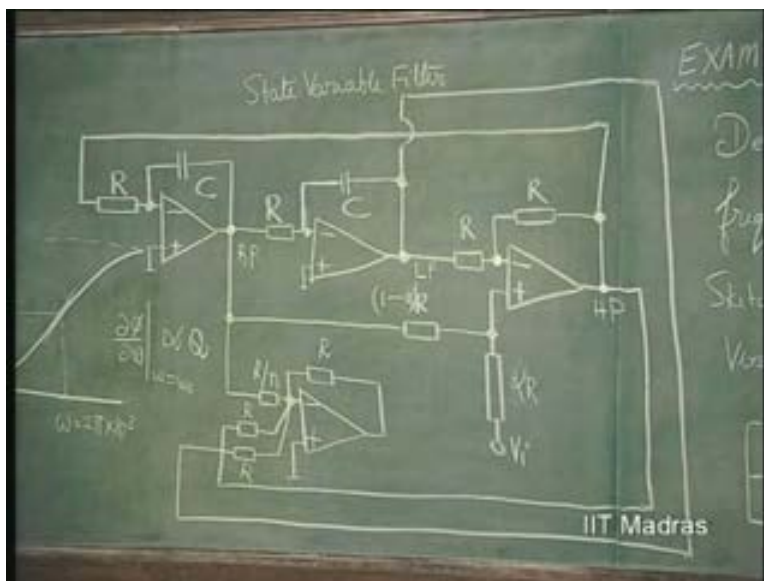
Now, let us see how this can be synthesized using the K H N network. This circuit is also called, let us say, universal active filter or also state variable filter. Different people call it differently.

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So, how do we therefore get this all pass function? We know that band pass, low pass, high pass outputs can be obtained at the output of these three operational amplifiers in the case of the state variable filter. So, we have to simply add these because high pass comes without any sign change; low pass comes without any sign change; band pass already has a sign change.

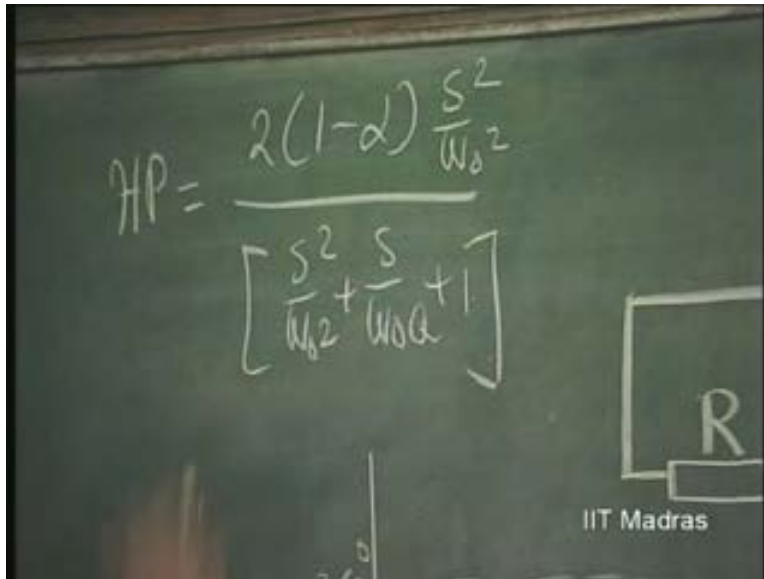
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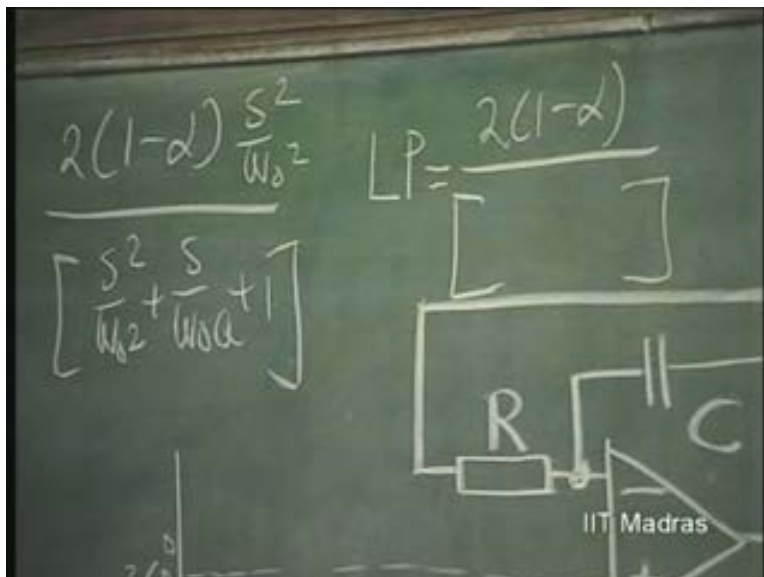
In our expression, if you see, high pass comes without any sign change. So, high pass is actually equal to  $2(1-\alpha) \frac{s^2}{\omega_0^2}$  divided by the same denominator;  $s^2$  by  $\omega_0^2$  squared plus  $s$  by  $\omega_0$  cube plus 1. This is high pass.

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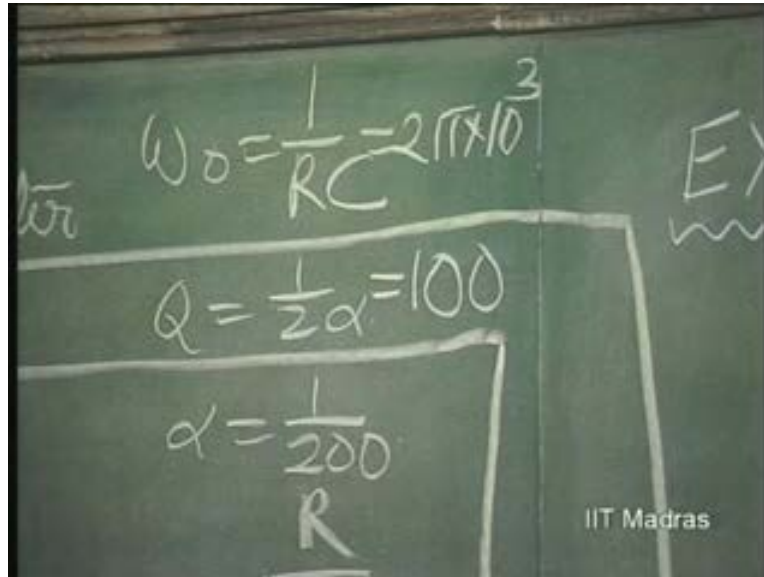
Low pass is going to be  $2(1-\alpha)$  divided by the same denominator.

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Where  $\omega_0$  is equal to  $1/RC$ , this we already know; and  $Q$  is equal to  $1/2\alpha$ . This is required to be made equal to 100. So,  $\alpha$  is  $1/200$ . This is to be made equal to  $2\pi \times 10^3$ .

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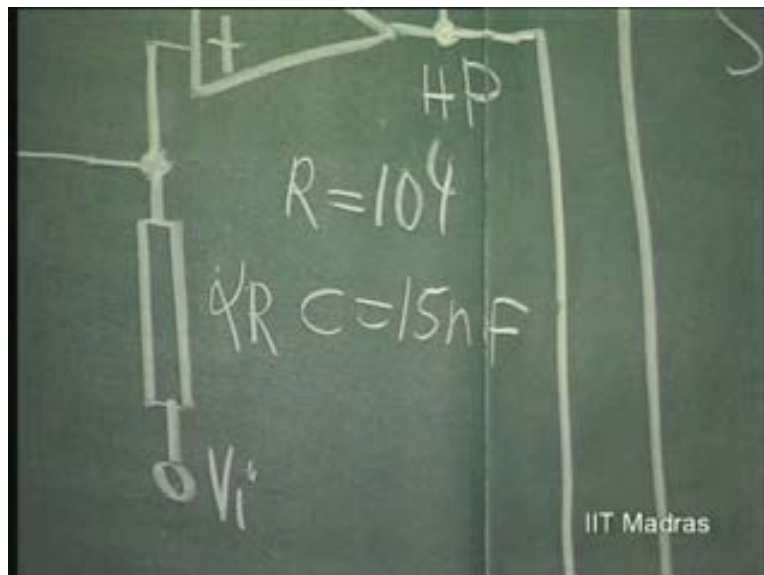
Handwritten equations on a chalkboard:

$$\omega_0 = \frac{1}{RC} = 2\pi \times 10^3$$
$$Q = \frac{1}{2\alpha} = 100$$
$$\alpha = \frac{1}{200R}$$

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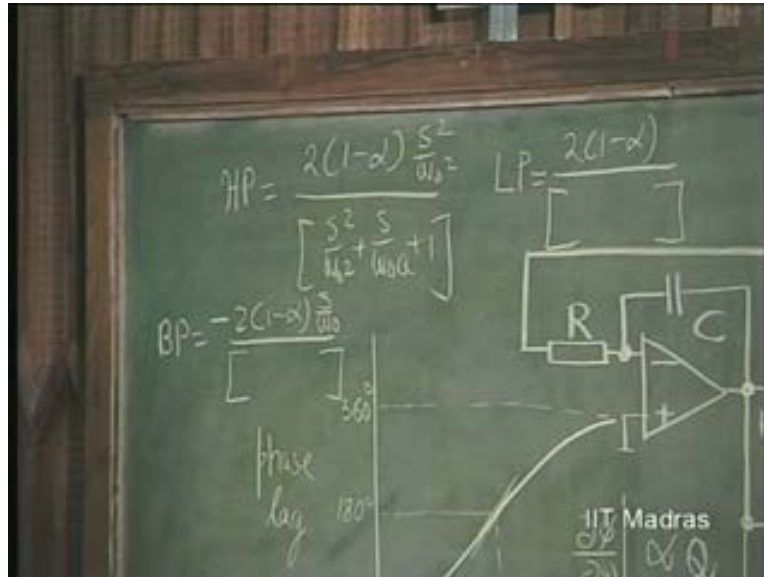
So  $R$ , if I select as  $10\text{ K}$  like last time,  $C$  becomes  $15\text{ nanofarad}$ . This, we had already calculated earlier.

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So, with that kind of thing, we have high pass here like this, low pass here like this, and band pass is minus 2 into 1 minus Alpha s by Omega naught divided by the same denominator.

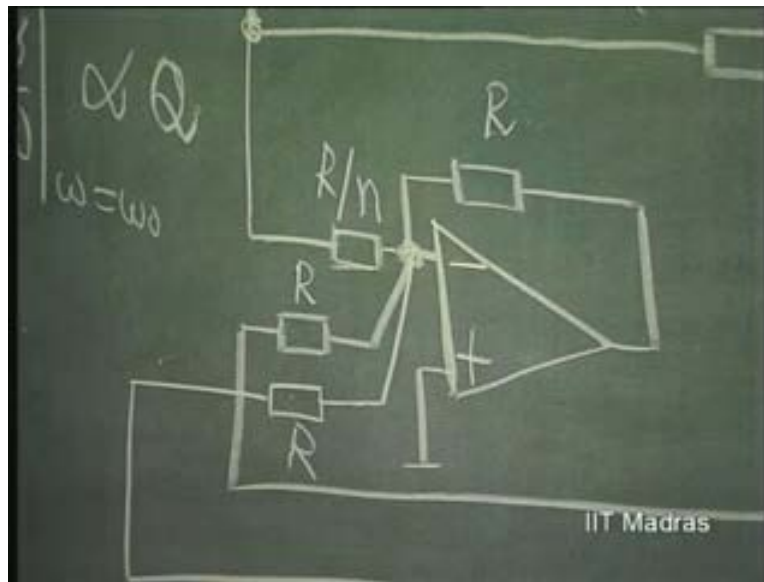
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So, what do I want? I want to simply add high pass with low pass and band pass. Only band pass has to be added with suitable modification of the coefficient because band pass coefficient should not be the same as the coefficient of low pass and high pass.

Here, the coefficient of this thing is 1; here it is s squared by Omega naught squared, which is 1 again. Here it is 1 over Q, s by Omega naught. So, you can see here, we have the coefficient here, same; 2 into 1 minus Alpha; 2 into 1 minus Alpha; minus 2 into 1 minus Alpha. So, we can simply add. So, that is the arrangement. This is a summing amplifier. I am adding low pass with high pass and band pass with a modified coefficient. Here, it is going to give inversion for both low pass and high pass; just minus 1.

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So, we get a coefficient here which is going to be minus 2 into 1 minus Alpha. Because of the inversion for both of this, minus 2 into 1 minus Alpha; into  $s^2$  by  $\omega_0^2$  plus 1 divided by the same denominator;  $s$  by  $\omega_0 Q$  plus 1.

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$$-2(1-\alpha) \cdot \frac{\left[ \frac{s^2}{\omega_0^2} + 1 \right]}{\left[ \left( \frac{s}{\omega_0} \right)^2 + \frac{s}{\omega_0 Q} + 1 \right]}$$

in Allpass filter with pole  
at 1KHz and pole-Q of 100

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This is what we are getting at the output of this. If you simply add this one with this, with minus sign, then this one we already have taken out, 2 into 1 minus Alpha, with the minus sign. So, that will be again, minus.

So, because this originally itself minus; so, it becomes plus here. So, s by Omega naught comes there; 2 into 1 minus Alpha has been taken out and here the coefficient is n. R divided by R by n. That is n. So, n can be made equal to 1 over Q. So, this is the output, let us say, V naught 4. So, this is equal to V naught 4 by V i. So, V naught 4 divided by V i is that and the design is over.

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$$\frac{V_o}{V_i} = -2(1-\alpha) \cdot \frac{\left[ \frac{s^2}{\omega_0^2} - n \frac{s}{\omega_0} + 1 \right]}{\left[ \left( \frac{s}{\omega_0} \right)^2 + \frac{s}{\omega_0} + 1 \right]}$$

an Allpass filter with pole  
at 1 kHz and pole-Q of 100

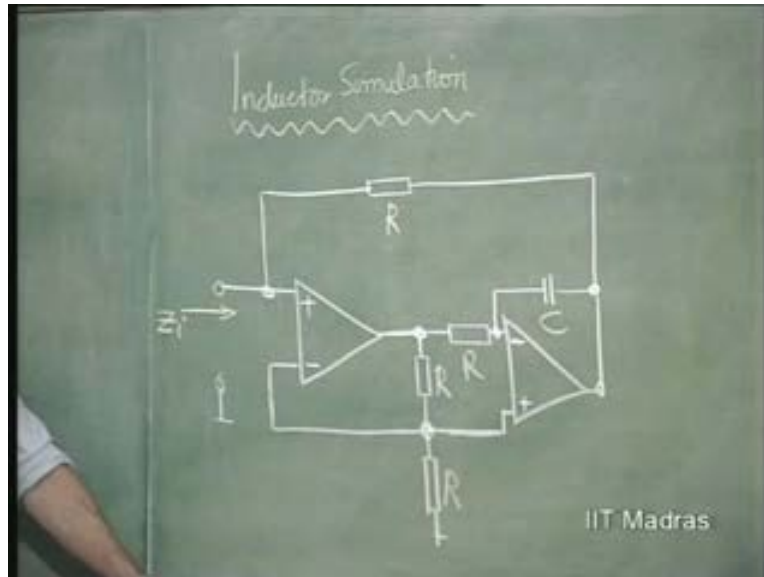
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We have already made Q equal to 100; Omega naught equal to 2 p i into 10 to power 3; and if I make n equal to 1 over Q, I would have achieved the all pass filter design. So, this is the usefulness of this summing amplifier, here. It can therefore make you locate the zero anywhere on the S plane. Now, we have located the zero here. So, you can locate the zero anywhere you like and the pole gets fixed by Omega naught n cube.

So, let us now consider another important part of active filters. That is, simulation of inductor. Now, this comes into picture... Inductor, for example, in the micro

miniaturization, becomes a very difficult component to deal with - the coil. In monolithic integrator circuit, it is not possible to fabricate it at all.

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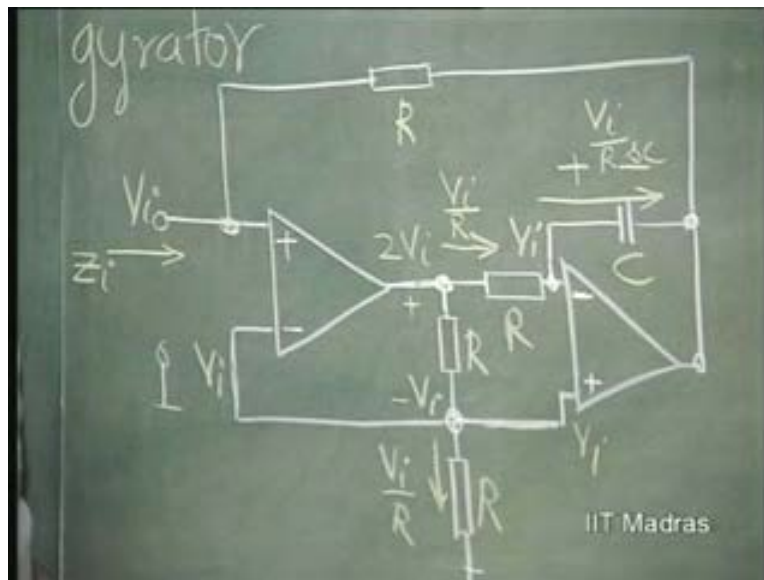
So, almost all circuits which are to be made out of monolithic IC require that inductor be not existing. Suppose therefore, we want to simulate such an inductor using resistors and capacitors and active elements. How is it possible? Now, this circuit which is called a gyrator; gyration means rotation. What it means is a capacitor has a certain relationship with voltage and current and that relationship is made use of in order to change this voltage to current relationship for the simulated component. That is the inductor.

So,  $Z_i$  here should become inductive. So, let us suppose that this is  $V_i$ . I am going to analyze the circuit and we will see how to analyze therefore, op amp circuit, in a very simple manner. If this is  $V_i$ , if this is a circuit which is supposed to work with negative feedback, then this has to be  $V_i$ . This we said, nullor. If this is  $V_i$ , this has to be  $V_i$  and if this is... So, this is  $V_i$  and the current in this is  $V_i$  by  $R$ . If that is the current in this,  $V_i$  by  $R$ , the same current should be flowing through this.

So, the potential across this R is also going to be  $V_i$ . So, this is going to be  $V_i$ . That means this is twice  $V_i$ . That can be easily concluded also by the fact that this is nothing but a non-inverting amplifier of gain equal to  $1 + \frac{R_2}{R_1}$ ;  $R_2$  is equal to R and  $R_1$  is equal to R.

So, the gain from here to here should be equal to 2. So, this is  $2V_i$ . If this is  $2V_i$  and we already know that this is  $V_i$ , this has to be  $V_i$ ; it is nullor. So, the current in this has to be  $V_i$  by R. This current cannot go anywhere else other than into the capacitor. So, the drop across this has to be  $V_i$  by R into SC, with this kind of thing.

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So, the voltage here is going to be this  $V_i$  minus the drop here. This is the voltage here. I am applying  $V_i$  here; this is R. So, the potential across this is, this  $V_i$  minus this  $V_i$ ; those  $V_i$ ,  $V_i$ , get cancelled; plus  $V_i$  by SC R. So, potential across this is  $V_i$  by SC R and the current through this is in this direction. This divided by R. So, the impedance  $Z_i$  is nothing but  $V_i$  divided by... let us call this  $I_i$ . This is equal to  $Z_i$ . So,  $V_i$  by  $I_i$  is  $V_i$  by SC R square. So, this is nothing but SC R square.

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A chalkboard showing the derivation of input impedance. The equations are written in white chalk on a dark green background. The first equation is  $Z_i = \frac{V_i}{I_i}$ . The second equation is  $= \frac{V_i}{V_i} SCR^2$ . The third equation is  $= \underline{\underline{SCR^2}}$ . To the right, there is a diagram showing a voltage source  $V_i$  and a current  $I_i$  flowing through an impedance  $Z_i$ . The text "IIT Madras" is visible in the bottom right corner.

So, impedance is S C R square which is not anything other than that of the inductor. So, L simulated is equal to C into R square.

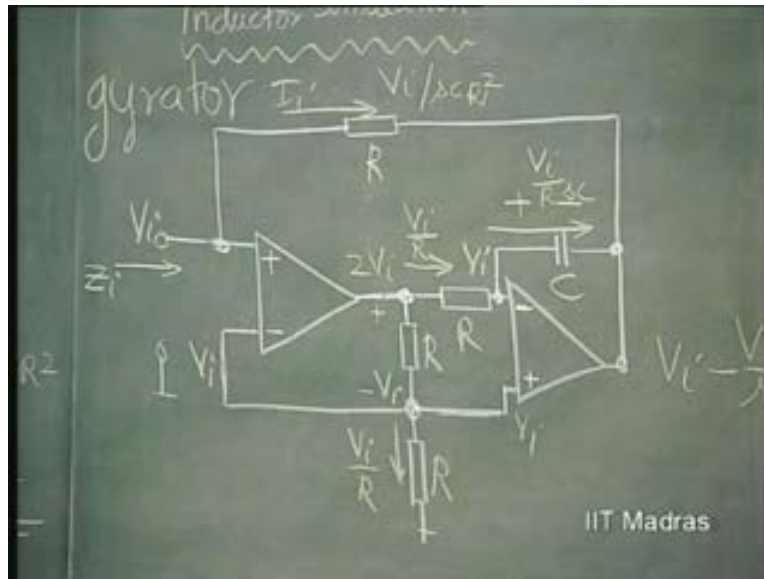
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A chalkboard showing the derivation of the simulated inductor value. The first equation is  $L = CR^2$ . Below it, there are some scribbles. The second equation is  $Z_i = \frac{V_i}{I_i}$ . To the right, there is a diagram showing a voltage source  $V_i$  and a current  $I_i$  flowing through an impedance  $Z_i$ . The text "IIT Madras" is visible in the bottom right corner.



So, any such circuit which is terminated by a capacitor and sees an inductor is called gyrator. This is one such circuit which is able to use a capacitor for simulating an inductor of magnitude equal to  $C R^2$ .

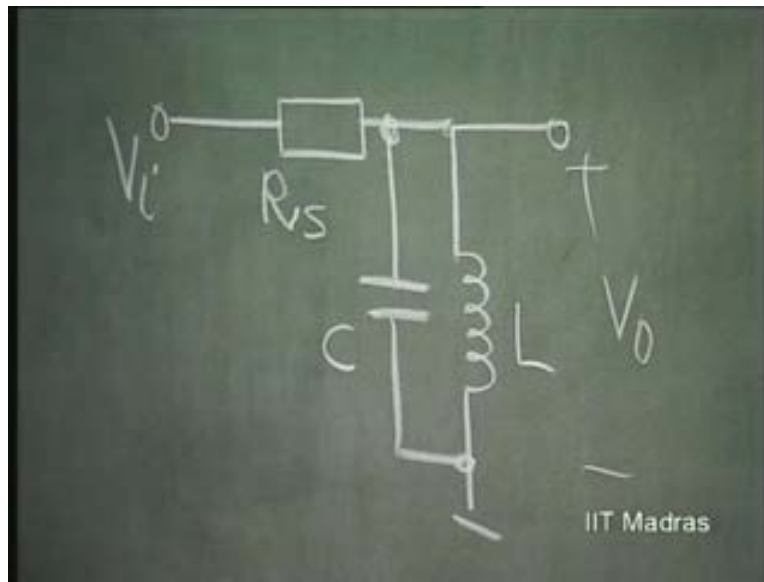
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One thing you should note, however, is that this inductor is between this point and ground; therefore, this is a grounded inductor. Now, what is the use of this?

Let us therefore go to the good old networks that we have understood earlier in the networks course. This is what is called as a tank circuit which can store energy. That is being excited by, let us say, a  $V_i$  through a resistance  $R$ . So, if I now take the output here,  $V_o$ ,  $V_o$  divided by  $V_i$  is the transfer function of this; is always equal to...  $V_o$  and  $V_i$ , the conductance linking that. This is one **one** way to write down easily - this conductance linking the output with the input divided by the total admittances.

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So,  $1$  over  $R s$  plus  $s c$  plus  $1$  over  $s L$ . This is the transfer function; easy way of writing.

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The handwritten transfer function equation on a chalkboard is:

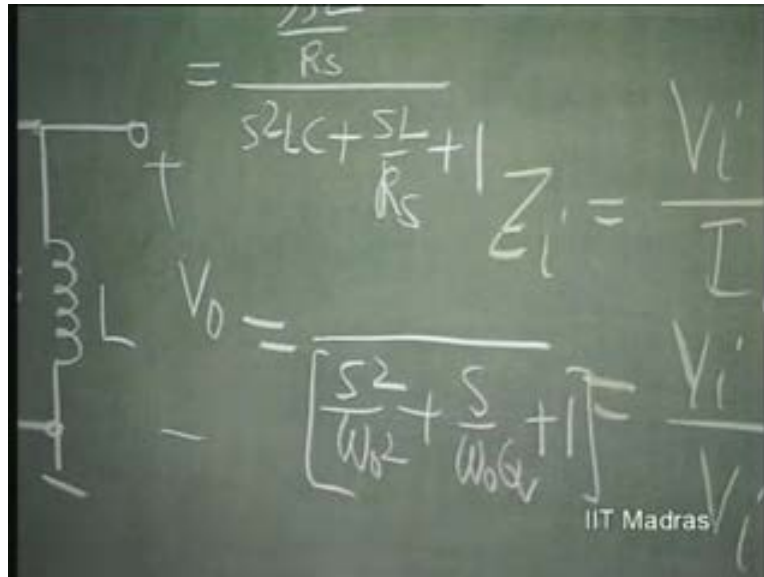
$$\frac{V_o}{V_i} = \frac{\frac{1}{R_s}}{\frac{1}{R_s} + sC + \frac{1}{sL}}$$

The IIT Madras logo is visible in the bottom right corner.

The conductance  $R$  admittance, linking the source with the output, divided by the summation of all admittances. So, we get here this as  $s L$  by  $R s$  divided by, multiplying by  $s L$  throughout;  $S$  squared  $LC$  plus  $S L$  by  $R s$  plus  $1$ . Now, this is rewritten according to us as... Normalization. Just as I did in the case of our  $K H N$  filter or universal active

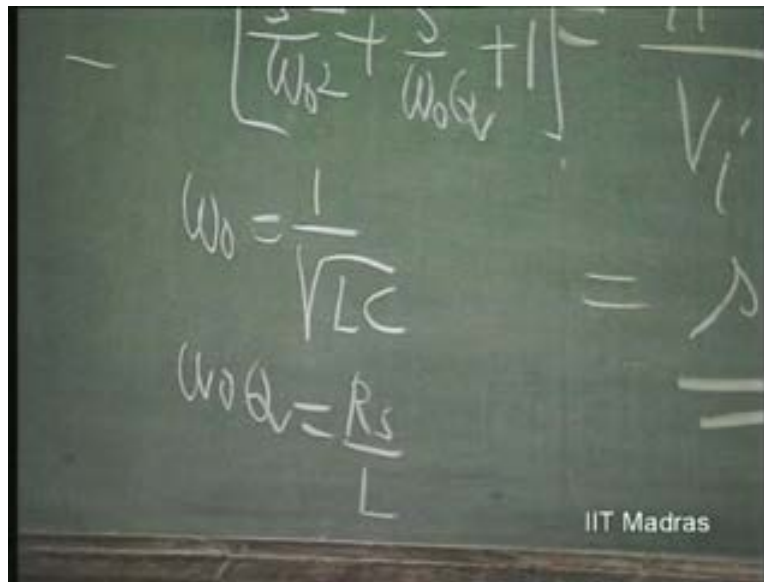
filter or state variable filter, this is put as natural frequency,  $s$  squared by  $\Omega_0$  squared plus  $s$  divided by  $\Omega_0$  into  $Q$  plus 1.

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So,  $\Omega_0$  is equal to  $1/\sqrt{LC}$ ; and that is by comparison, you can see  $\Omega_0^2$  is equal to  $1/LC$ .  $\Omega_0$  is equal to  $1/\sqrt{LC}$ ; and we have  $Q = R_s/L$ .  $Q$  is nothing but  $R_s$  divided by  $L$ .

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So, you can write down Q, quality factor, as  $R_s$  divided by  $L$  into  $\omega_0$ . So, please remember here; this is the conventional definition for Q of a parallel resonant circuit. Q is always equal to the resistance shunting the parallel resonance circuit divided by the inductive impedance; or, this is also equal to  $R_s$  into  $\omega_0 C$  because  $\omega_0$   $1 \dots C$  is equal to  $1$  over  $\omega_0 L$ .

So, this is equal to  $R_s$  by  $L$  divided by  $\omega_0$ . **Omega naught**.  $1$  by  $\omega_0$  is equal to  $\sqrt{L}$  into  $C R$ . This is  $R_s$  into  $\sqrt{C}$  divided by  $L$ . So, this is known to you for a passive network.

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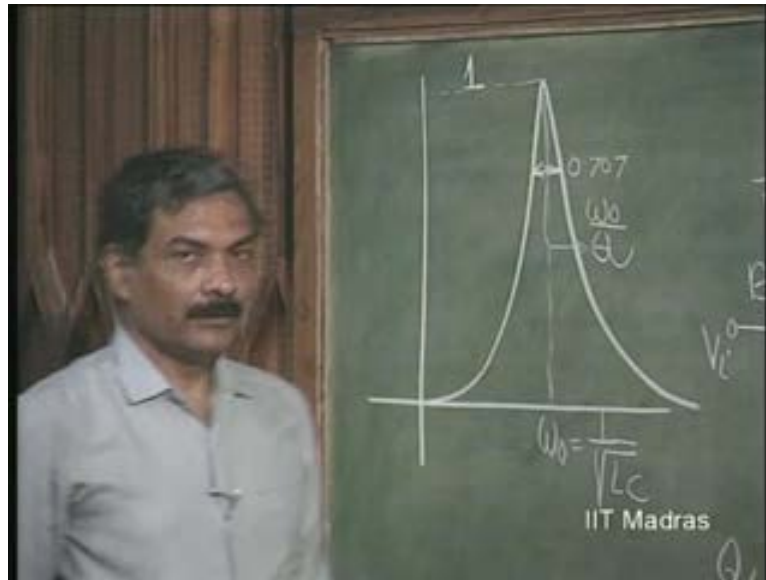
$$Q = \frac{R_s}{L \omega_0} = R_s \omega_0 C$$
$$= \frac{R_s}{L} \sqrt{LC} = R_s \sqrt{\frac{C}{L}}$$

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So, you can therefore fix the center frequency of this band pass. This is called a band pass filter. So, how will the response look like? The center frequency  $\omega_0$  is going to be  $1/\sqrt{LC}$  and at resonance this is going to be 1 because  $\omega_0$  is equal to  $1/\sqrt{LC}$ . In fact, the numerator is also going to be  $s$  by  $\omega_0$   $Q$  because it is same as  $sL$  by  $R_s$ .

So, at resonance, this particular thing becomes infinite, resistance; and output will be equal to input. So, this is unity and the bandwidth obviously, it is at point  $\omega_0$ , put a value. This bandwidth corresponds to  $\omega_0$  divided by  $Q$ , by definition. So, all these things are known to you from your networks course.

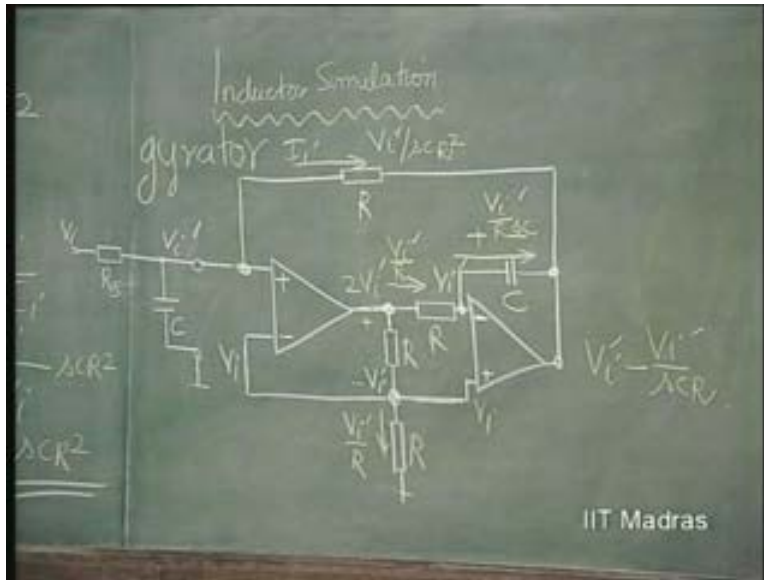
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Now, if I want such a circuit without using an inductor, I simply use this, remove this inductor and use this gyrator circuit. So, what happens here? Just remove this. This will simulate an inductor I know, of magnitude equal to  $C$  into  $R$  squared. Then I put another capacitor because I have to put another capacitor. Let us say, that also is  $C$ ; and I put a resistance which is  $R$  s.

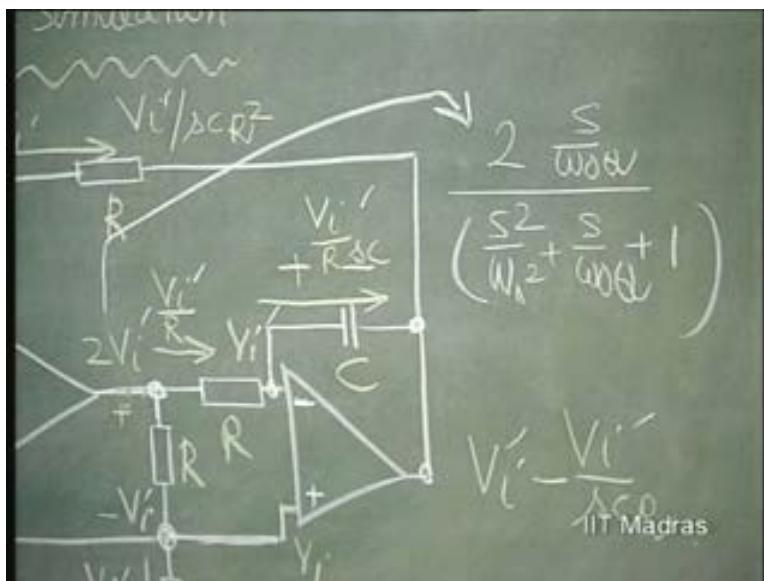
So now, at this point, if I put  $V$ ...let us say, you call this  $V_i$ ; then this  $V_i$  prime is going to be my  $V$  naught here which is going to have a transfer function like this. If I take the output here, this will be  $V_i$  prime. Just distinguish it from this  $V_i$ . This will be twice  $V_i$  prime.

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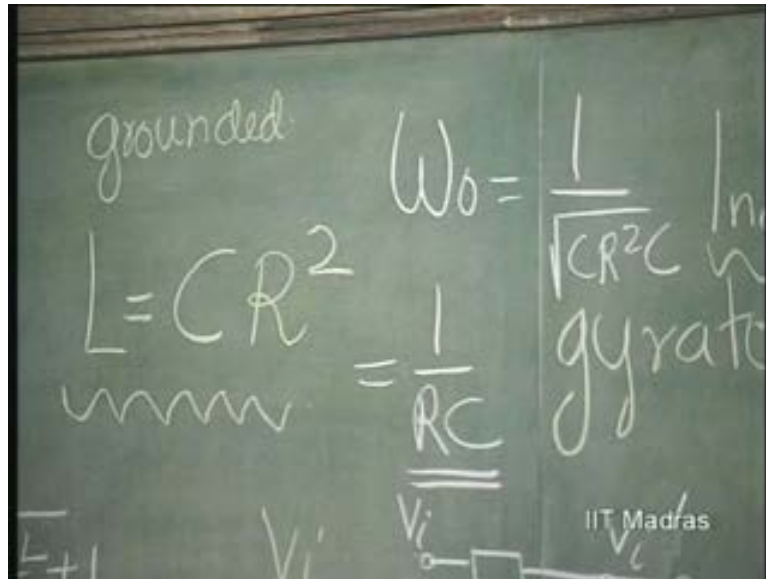
So, it will be again band pass. The output here also is going to be band pass. So, you can as well take the output here because here it is buffer. That means you can load this without affecting the Q of the circuit. So, it is better to take the output here which is going to give you twice  $V_i$ . That means twice  $S$  by  $\Omega$  naught  $Q$ ,  $S$  squared by  $\Omega$  naught squared,  $S$  by  $\Omega$  naught  $Q$  plus 1 is got here.

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And what is Q? Let us find out. First of all, let us find out what Omega naught is. Omega naught for this is 1 over root L C; root of... L is equal to C R squared; C R squared into C; or, this is equal to 1 over R into C. Very simple. Just like the case of state variable filter.

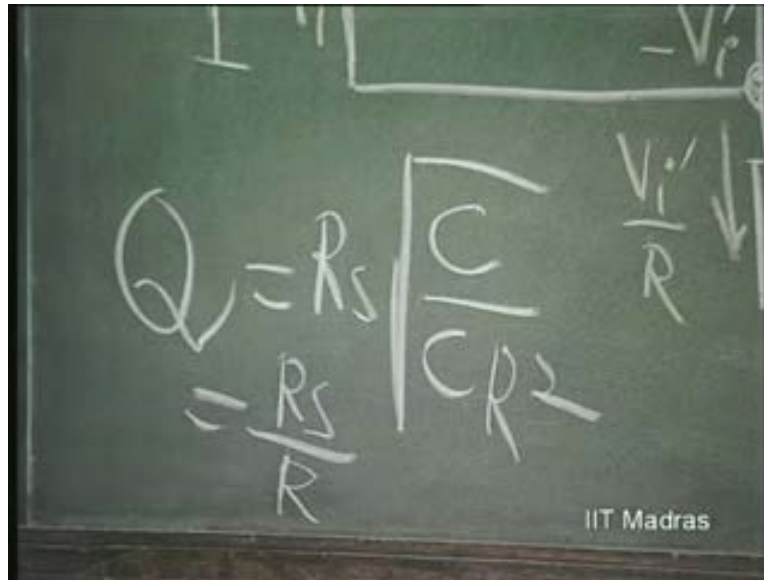
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So, if I fix R and C's accordingly, I can fix Omega naught easily. Next, we would like to know what Q is. Q is given by R S into root of C by L, which is C into R squared, which is nothing but R s by R. So, this is a very simple design.



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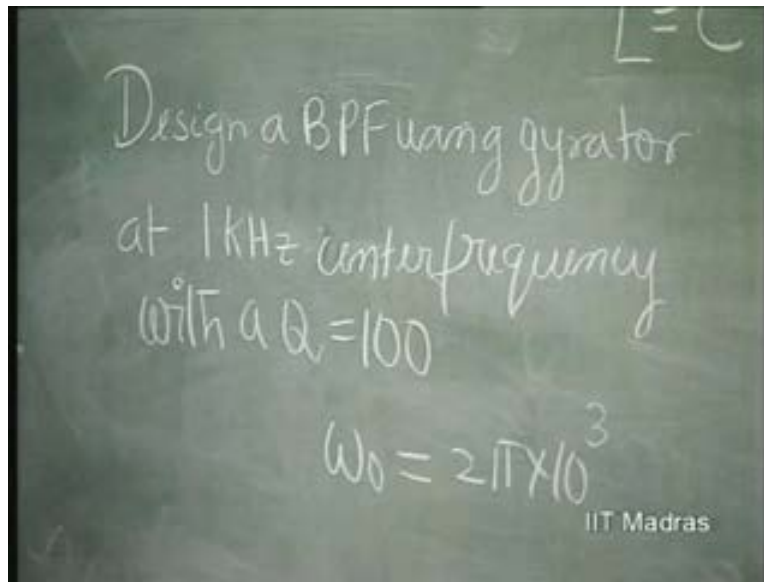


$R_s$  has to be equal to  $Q$  into  $R$ .  $R_s$  value should be equal to  $Q$  into  $R$  and  $\Omega_{naught}$  has to be equal to  $1$  over  $R C$ .

So, let us now consider an example so that this problem is clear, of simulation of inductor.

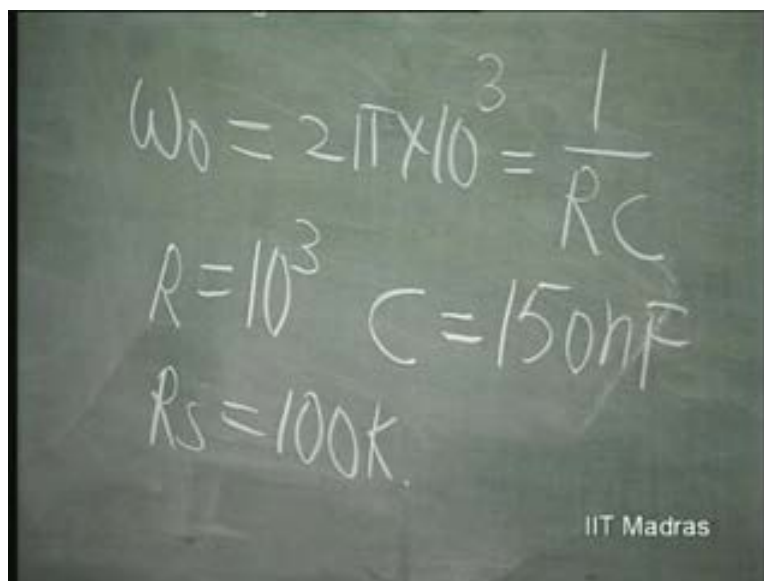
Design a band pass filter using gyrator at 1 Kilo hertz center frequency with a  $Q$  equal to 100. So, how do we proceed with this? Circuit is the same.  $\Omega_{naught}$  is given as 1 Kilo hertz. That means  $2\pi$  into 10 to power 3.

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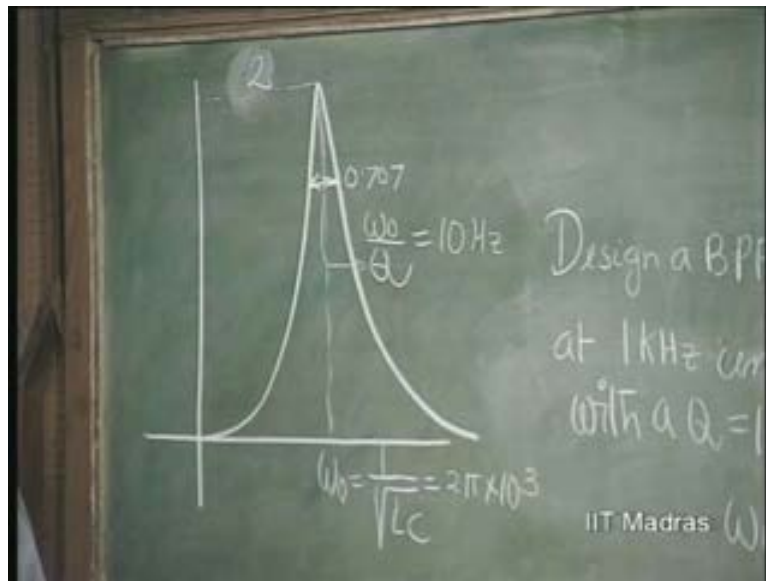
This is equal to  $1/R \times C$ . For a change, we will take  $R$  as 1 Kilo ohm. Last time we took it as 10 Kilo ohm. Then  $C$  will be larger; 150 nanofarads. Earlier, it was 50 nanofarads; now 150 nanofarads. Then,  $R_s$  has to be 100 times 1 Kilo. 100 K. So, that finishes the design of the band pass filter.

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Gain at the center frequency is going to be 2 because we are taking the output at this point. Gain at the center frequency is going to be 2 instead of 1. So actually for our circuit now, this will be... if you take the output there – 2, and this is going to be  $2\pi$  into 10 to power 3 and this is going to be 10 hertz.

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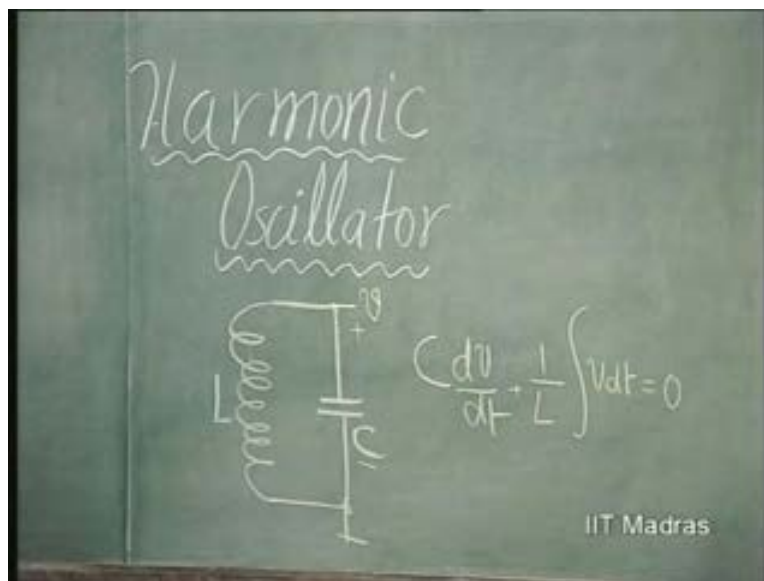
So, this is a very simple design. Now, we have understood what is meant by...this particular thing of design has to be given as filter with gyrator. This has to be given Example... What is it? Example 12. So, please make that correction.

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Now, let us come to an important topic - Harmonic Oscillator, which we have already touched upon. When we discussed state variable filter, we said, solution of  $d^2 V$  by  $dt^2$  plus  $\gamma V$  naught equal to zero, is nothing but  $V$  naught equal to some  $a \sin \Omega t$  plus  $\phi$ . So, it is a harmonic oscillator. This is a very important thing. This might have been taught to you in mechanics already. This simple pendulum.

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So here, in using L and C, it has been taught to you that this is nothing but a tank circuit. If ideal L and ideal capacitor is connected like this and you pump in certain amount of energy, that energy is retained in the form of oscillation; and that oscillation frequency is  $\frac{1}{2\pi\sqrt{LC}}$ . All of you know. How do we obtain this solution? It is very simple. If L and C are connected like this and V is the voltage here, load voltage, then  $C \frac{dv}{dt}$  is the current in the circuit, which is also same as  $\frac{1}{L} \int V dt$ .

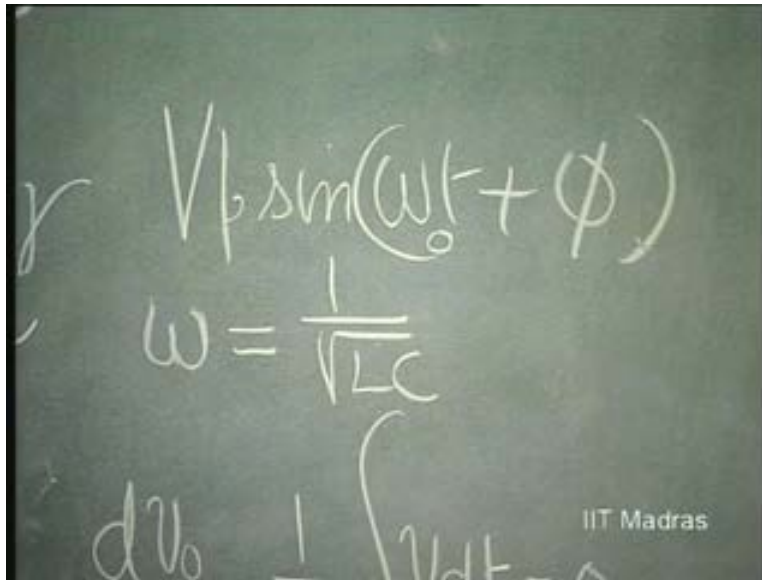
So, current entering and current leaving...so from that,  $C \frac{dv}{dt} + \frac{1}{L} \int V dt$  is equal to zero; or we can write this as... like this. Or,  $\frac{d^2V}{dt^2} + \frac{V}{LC}$  is equal to zero. This is the same equation that we get. This, if this is V naught, let us call this V naught, so that we are consistent with our symbol used in state variable filter. So, you get the same equation  $\frac{d^2V}{dt^2} + \gamma V$  naught equal to zero.

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The image shows a chalkboard with two equations written in white chalk. The top equation is  $\frac{dV_0}{dt} + \frac{1}{LC} \int V_0 dt = 0$ . The bottom equation is  $\frac{d^2V_0}{dt^2} + \frac{V_0}{LC} = 0$ . In the bottom right corner of the chalkboard, the text "IIT Madras" is visible.

Solution of this is nothing but, well, some  $V_0 \sin \Omega t + \text{some } \phi$ .  $\Omega$  naught is the resonant frequency, is equal to  $\frac{1}{\sqrt{LC}}$  root of  $\gamma$ . That is known as the frequency.

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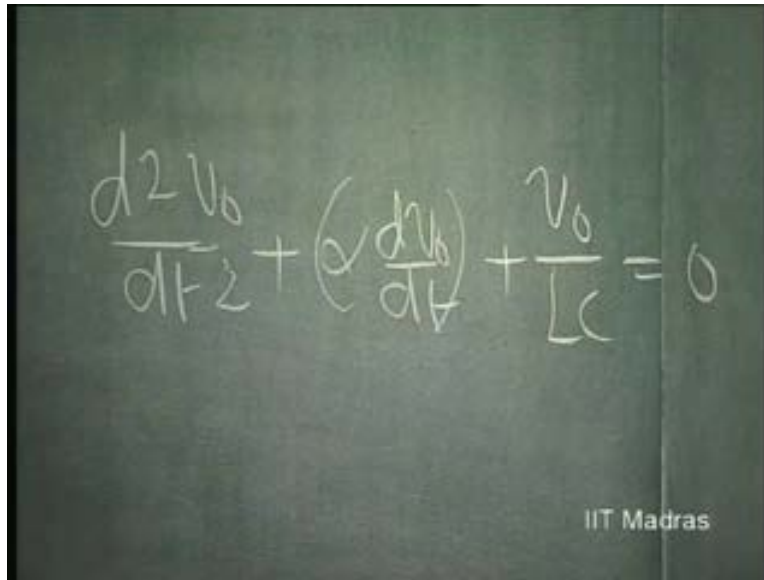

$$V_p \sin(\omega_0 t + \phi)$$
$$\omega = \frac{1}{\sqrt{LC}}$$

$d^2 v_0 / dt^2 = 0$  IIT Madras

In mechanics, we might be using some  $K$  or something like that, there. So,  $\Omega$  is equal to  $\sqrt{LC}$ . This as a resonant circuit, it can sustain sinusoidal oscillation with certain amplitude and it does not tell anything about amplitude. It depends upon the energy that is pumped into it. So, that will be sustained throughout. At that magnitude, it will not decay or grow; so, this is the case. If...

Let us understand this.  $d^2 v_0 / dt^2 + \dots = 0$  gives you a sinusoidal oscillator. If this coefficient of here,  $d v_0 / dt$ , which we had earlier put as some  $\alpha$  or something like that... So, if this is zero, then it is a harmonic oscillator. If this is negative, the poles will be lying on the right half of the  $S$  plane and the amplitude of our solution will keep on growing. The growth is characterized by  $e$  to the power some  $K$  times  $t$ ,  $K$  being positive. This is what...this I am saying that...

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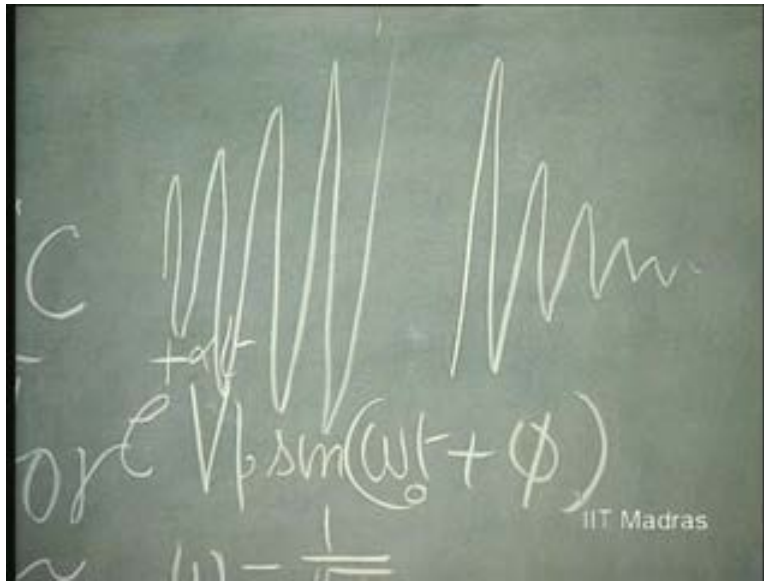

$$\frac{d^2 V_0}{dt^2} + \alpha \left( \frac{dV_0}{dt} \right) + \frac{V_0}{LC} = 0$$

IIT Madras

So, if Alpha is positive, then it will decay and go to zero. So basically speaking, this Alpha is going to determine whether it is going to be growing with respect to time or decaying with respect to time. If Alpha is zero, it is constant. If Alpha is negative, it will decay. If Alpha is positive, it will grow.

So, this is the way you have studied in networks. So, it will grow like this, if Alpha is positive. If it is negative, it will decay; ultimately become equal to zero. Otherwise, it will remain constant. So, this property is important to understand in the case of oscillators.

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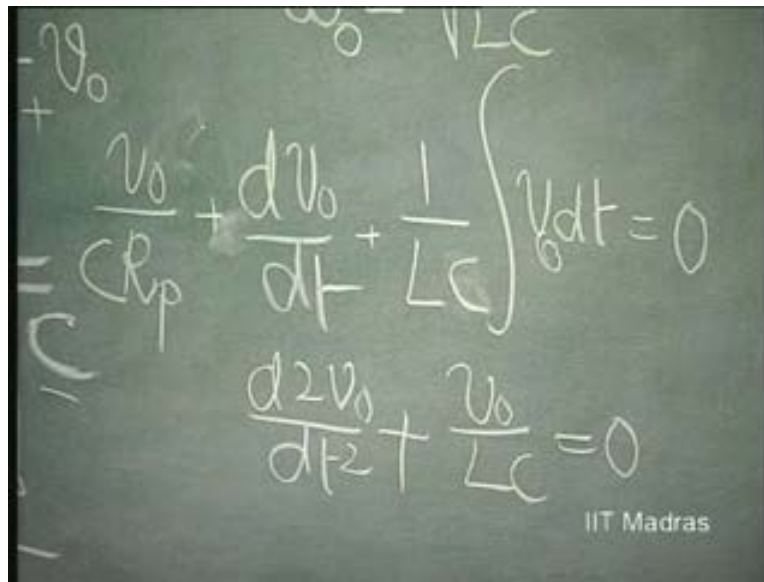


So, how can I therefore say that in practice, this circuit definitely is going to have inductor with some loss component in it? So, **so** will the capacitor have some loss component?

If I can represent this loss component in terms of a resistance dissipative, that is the thing, then in this equation, the current that you have to add will be, sum of all currents,  $V$  naught by  $R_p$  and in fact, it was  $C \frac{dv}{dt}$ ;  $V$  naught by  $R_p$  plus  $\frac{1}{L} \int V$  naught  $dt$ . Now dividing throughout by  $C$ , this will be getting  $C$ , this will be getting  $C$ .



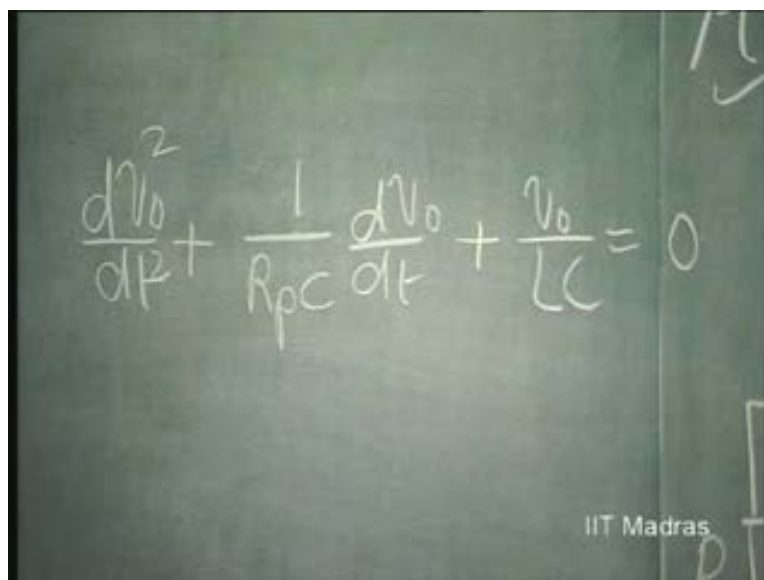
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A chalkboard showing a differential equation. The equation is written as  $\frac{v_0}{RC} + \frac{dv_0}{dt} + \frac{1}{LC} \int v_0 dt = 0$ . Below it, the equation is differentiated to  $\frac{d^2 v_0}{dt^2} + \frac{v_0}{LC} = 0$ . The text "IIT Madras" is visible in the bottom right corner.

This therefore gets modified as...  $\frac{dv}{dt} + \frac{v}{RC} + \dots = 0$  plus  $\frac{d^2 v}{dt^2} + \frac{v}{LC} = 0$ . So, how to make it an oscillator now? Because, this is going to cause decay of whatever oscillation amplitude that exists. So ultimately, it will become zero.

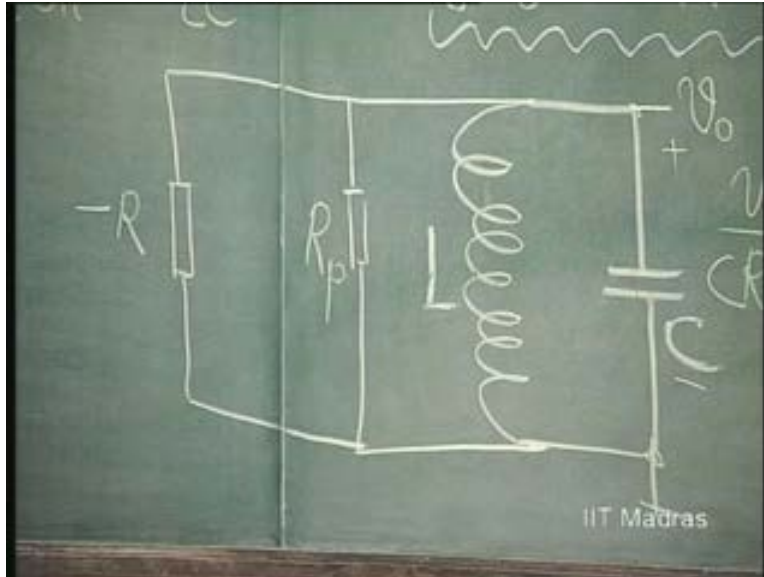
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A chalkboard showing a differential equation. The equation is written as  $\frac{d^2 v_0}{dt^2} + \frac{1}{RC} \frac{dv_0}{dt} + \frac{v_0}{LC} = 0$ . The text "IIT Madras" is visible in the bottom right corner.

So, that means, I will have to connect another resistance across this whose value is negative so that it cancels with this.

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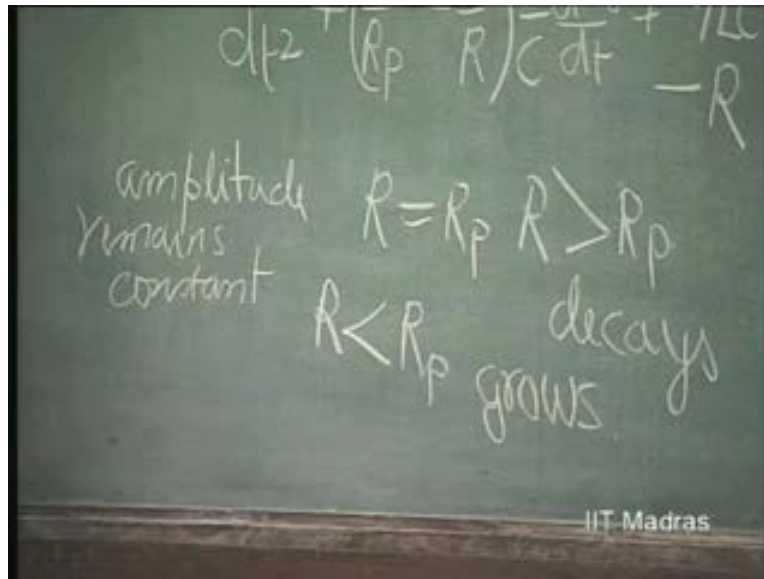
So that means, this will then be modified and  $d^2 v_{\text{naught}} / dt^2 + 1 / R_p - 1 / R$  into  $1 / C d v_{\text{naught}} / dt + v_{\text{naught}} / LC = 0$ .

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$$\frac{d^2 v_0}{dt^2} + \left( \frac{1}{R_p} - \frac{1}{R} \right) \frac{1}{C} \frac{dv_0}{dt} + \frac{v_0}{LC} = 0$$

So, I require a negative resistance to make the resistance equal to infinity. When will it happen? When  $R$  is equal to  $R_p$ . So, and then I make  $R$  equal to  $R_p$ . You notice that if  $R$  is greater than  $R_p$ , the oscillation decays. If  $R$  is less than  $R_p$ , here, oscillation grows. When exactly  $R$  is equal to  $R_p$ , oscillation amplitude is remaining constant, amplitude remains constant.

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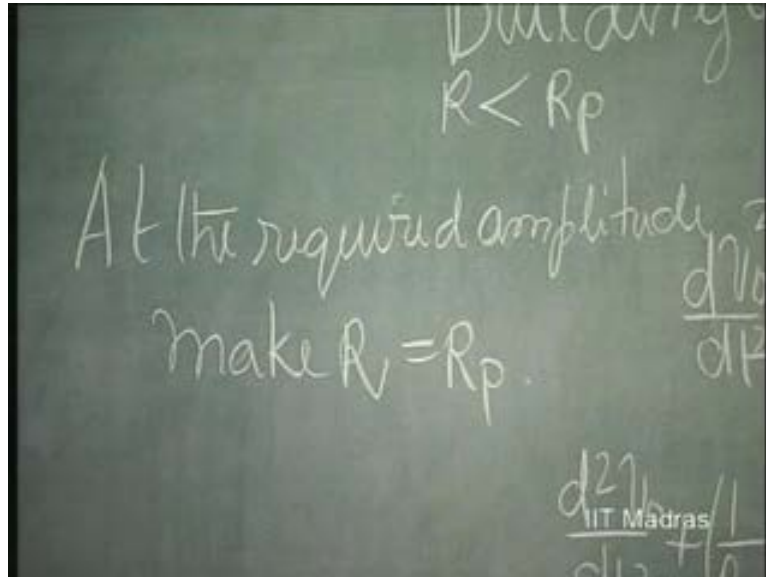


This basic principle has to be understood in terms of time domain. In terms of frequency domain, what does it mean? This will have roots lying on the left half of the  $S$  plane because of this being positive, if this  $R_p$  is positive. If effective resistance is negative, it will have the roots lying on the right half of this plane. If  $R$  is exactly equal to  $R_p$ , the roots will be lying on the imaginary axis. Then it is a harmonic oscillator.

How do we have to design oscillators? Because initially, there **there** will not be any amplitude. When you switch on, there will not be any amplitude. A practical oscillator should have amplitude building up. So, building up of oscillation... if oscillation amplitude has to be built up, what should be the case? The poles of the resistant should lie initially on the right half of the  $S$  plane; or in this case,  $R$  should be initially less than  $R_p$ .

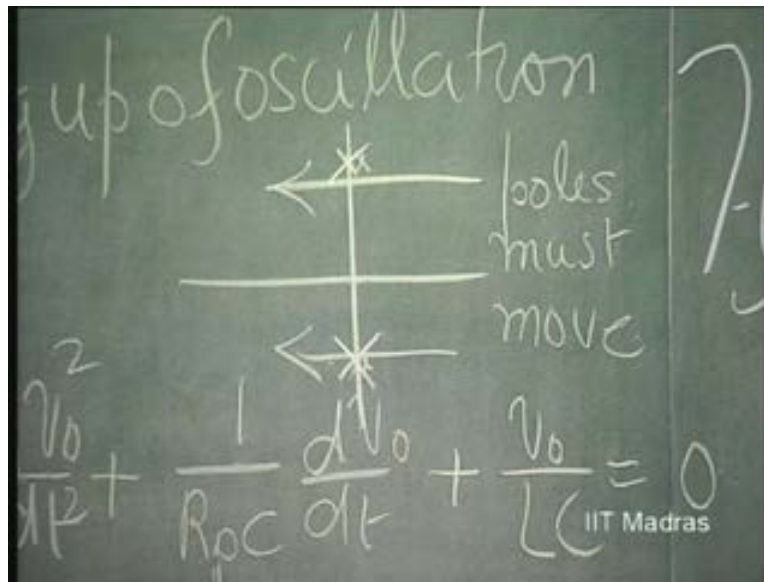
Then, this keeps on growing. Then I should change the value of R such that R is exactly equal to  $R_p$  at the required amplitude I desire. That is what is called amplitude stabilization. At the required amplitude, at the required amplitude, make R equal to  $R_p$ .

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So, this is the strategy which is adopted in every harmonic oscillator so that the oscillation starts. Invariably, it will be pretty close to the imaginary axis, but it will be on the right half of the S plane. Then gradually, if this is the S plane, somewhere here, let us say, and it will come and lie exactly on the imaginary axis. This is how poles have to move, of the system which is... As amplitude builds up, the poles must move this side and lie exactly on the imaginary axis at the required amplitude of oscillation.

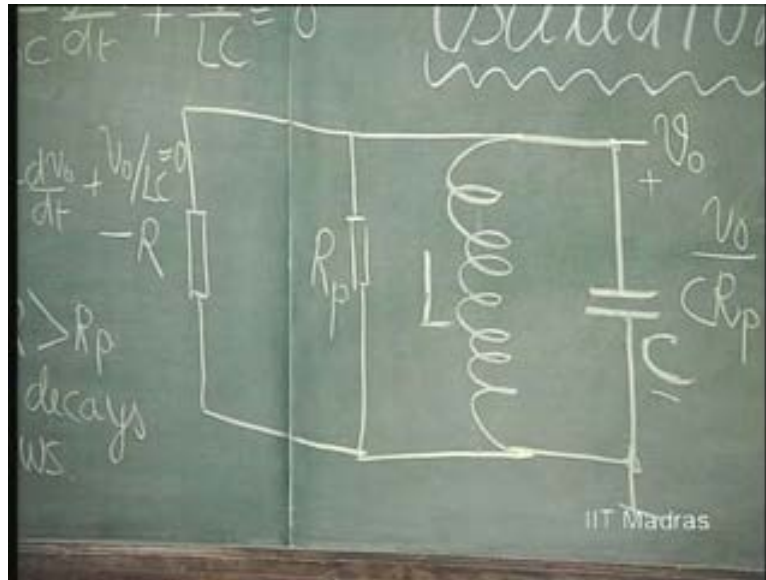
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So, this covers the basic theory of amplitude stabilization for any oscillator. Now we have discussed what is called as two terminal oscillator; negative resistance oscillator it is called, wherein I have put an inductor and a capacitor practical value whose loss component is represented as  $R \vee R_p$  and I am putting a negative resistance.

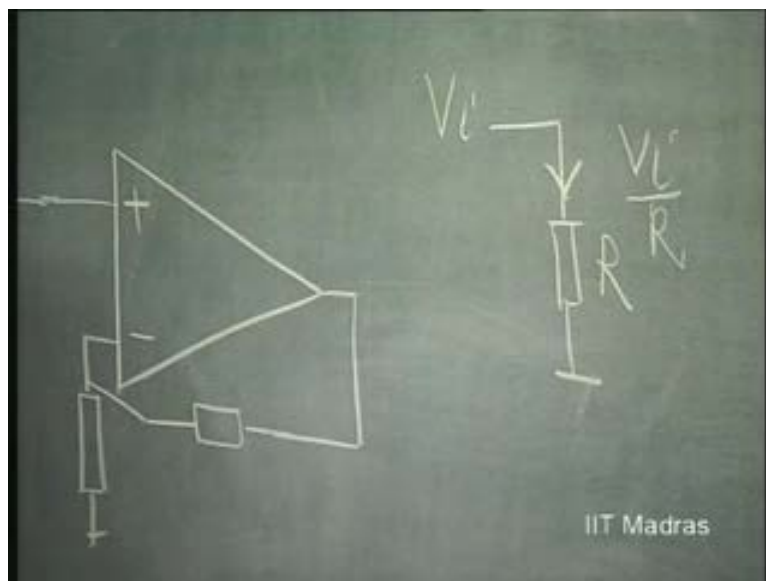
So now, I would like to know how to simulate negative resistances. We already know how to simulate inductor. So, we have capacitors anyway. These are going to be lossy. So, let us now try to understand how to simulate negative resistance.

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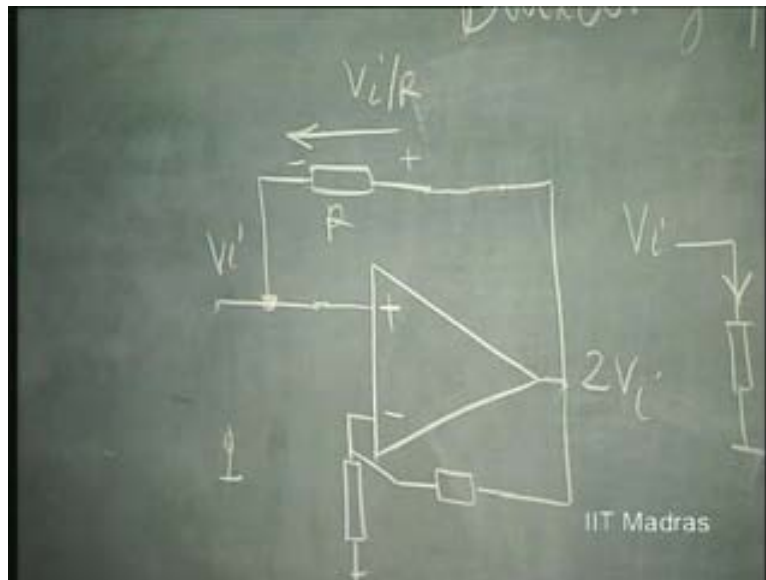
This simulation process is very simple. Let us once again assume that we can use the op amp. If this is  $V_i$ , if this is  $V_i$  and I connect a resistance like this to ground and that resistance is  $R$ , it is a positive resistance. How do I make out? Because a current of  $V_i$  by  $R$  will flow in this direction. When will it be a negative resistance? When a current of  $V_i$  by  $R$  flows in the opposite direction.

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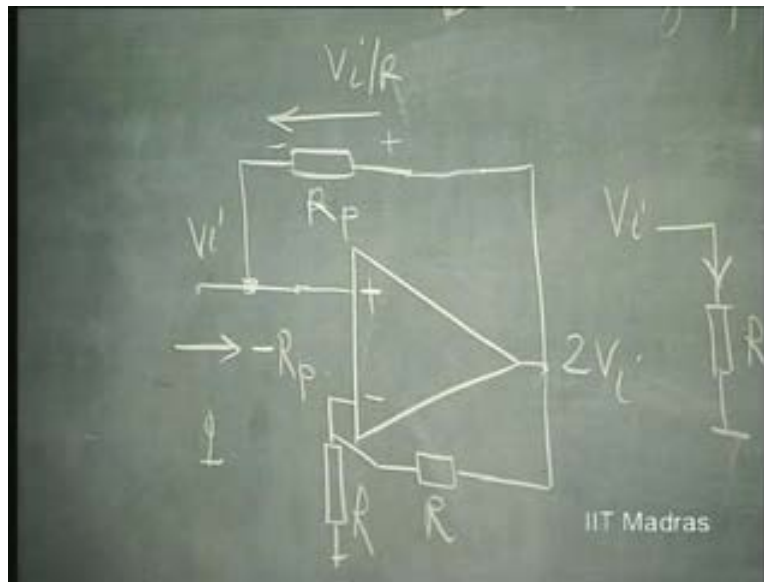
So, that is a very simple... If this is  $V_i$ ... So, if this is to see a negative resistance and I put  $R$  and it has to see a current which is in this direction, which is half magnitude  $V_i$  by  $R$ , obviously, the potential here should be... This is  $V_i$  and the potential across  $R$  should be  $V_i$ , but in the opposite direction. That means the potential here has to be twice  $V_i$ . Is this... This is twice  $V_i$ . This is  $V_i$ . So, the potential difference here is going to be plus here, minus here, of  $V_i$ ; and the current will be  $V_i$  by  $R$  in this direction.

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So, how do I get a gain of 2? I can put  $R$  and  $R$ . So, this simulates a negative resistance of magnitude  $R$ . So, I want to simulate a negative resistance of magnitude how much?  $R_p$ . So, I simply put here  $R_p$ . So, this simulates a negative resistance of magnitude  $R_p$ .

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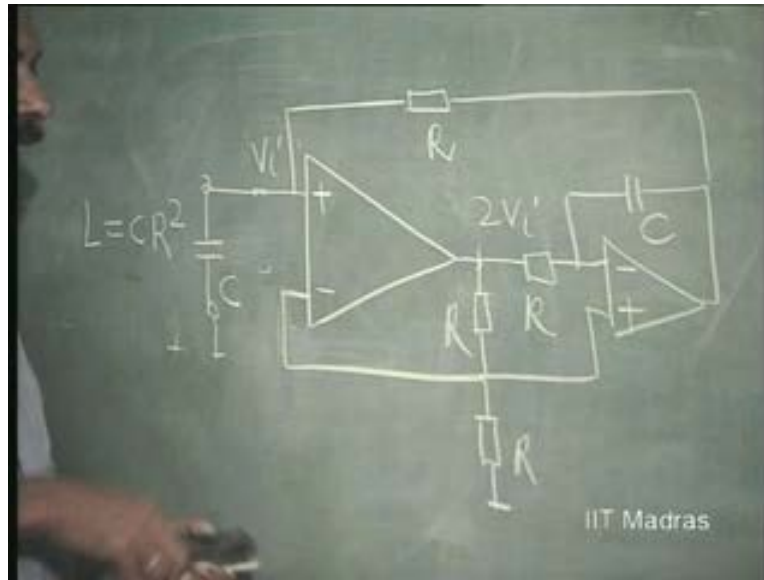
So, if I connect this circuit between this point and ground and I use a practical L and C with loss component  $R_p$ , then it will invariably go into oscillation. If I do not want to use an inductor, I can simulate that inductor using the gyrator.

So, let us look at that circuit of ours – that is, the gyrator circuit. How will it look like? In fact, we had used this kind of resistance there, if you remember. The first circuit was an amplifier with gain of 2. So, this was  $V_i$ . This was twice  $V_i$ . Then we put an integrator R, R, R, here like this; and we had collected another resistance from here to here; and it was simulating an inductor between this point and ground whose magnitude L was equal to C into R square.

Now I want the capacitor to come here. The capacitor is connected like this. Now, it will oscillate. Unfortunately, we have assumed that these are ideal operational amplifiers with infinite gain. That is how we are able to derive this as C R square.



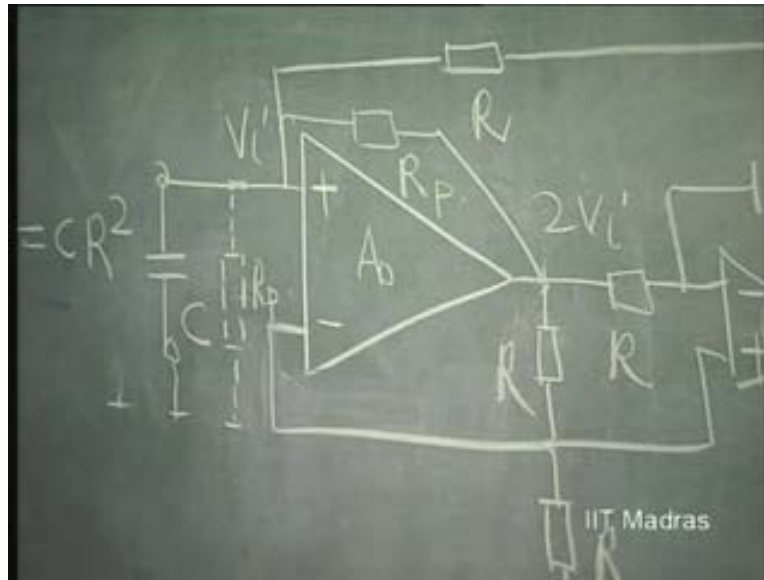
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But actually, if you take finite gain which is non-infinity, then you will see that this inductor will be shunted by a resistance of magnitude which is dependent upon  $A$  naught into  $R$ . So, there will be a loss component. Even otherwise, the capacitor also will have a loss component.

So necessarily, I must put a negative resistance of suitable magnitude to make it go into oscillation. Otherwise, the oscillation will invariably die. So, how do I now create a negative resistance? It is very simple. Now, from here, it is  $2V_i$ . So, I put a resistance here which is  $R_p$ . That will compensate for the so called loss component which is already existing, whose magnitude is equal to  $R_p$ .

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So, this circuit will invariably burst it. You can therefore start with very large value of  $R_p$ , keep on reducing it. At a particular point, you will see that the whole circuit will burst into oscillation. Here, there is nothing limiting the amplitude. So, it will go up to the supply voltage.

So, it will burst into beautiful sinusoidal oscillation at frequency  $\Omega$  naught equal to  $1/RC$ . The poles therefore lie exactly on the imaginary axis. So, this is what is called as a gyrator oscillator; uses only resistors and capacitors; and you can vary the capacitor and make the frequency vary. So, this is the basic principle of such oscillators.

We will discuss in the next class other types of oscillators which normally cannot be, cannot only be represented in this manner, but also can be represented in another manner. That is, straightaway as synthesis of a second order differential equation with  $d^2v/dt^2$  being absent.