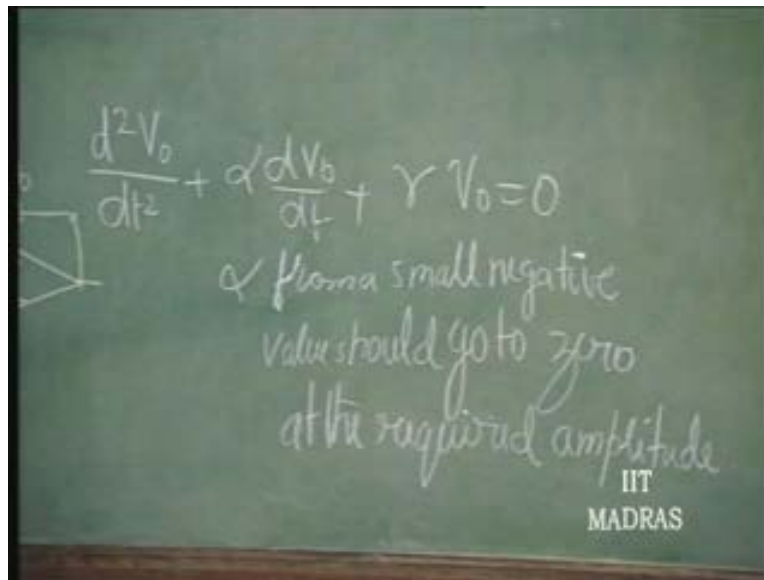


Electronics for Analog Signal Processing - II
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Department of Electrical Engineering
Indian Institute of Technology – Madras

Lecture - 14
Oscillators

Let us consider sinusoidal oscillators. We had already seen that sinusoidal oscillator...or that function can give us a solution, straightaway sinusoidal value, is $d^2V_{naught} / dt^2 + \gamma V_{naught} = 0$, which is the harmonic equation; and solution to that is $V_{naught} = V_t \sin \Omega t$. Ω is equal to $\sqrt{\gamma}$. We have seen this and we said such a thing can be simulated; but if we want really a practical oscillator, we must have dV_{naught} / dt term. But the co-efficient α should start from a small negative value and should go to zero at the required amplitude of oscillation. This is the basic requirement for buildup of oscillation.

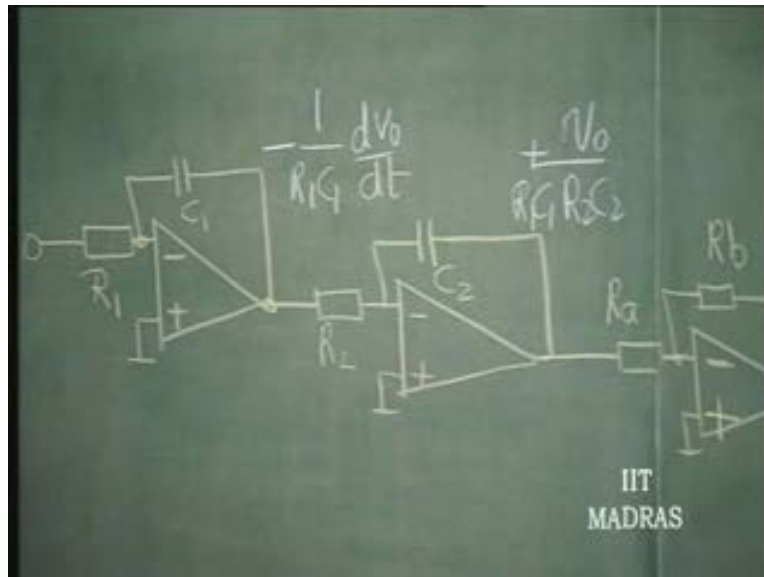
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So, how do we really come out with simulation of such a thing? This, we had seen. Earlier, I had explained to you how to simulate second order differential equation.

I start with the assumption that $d^2 V_{in} / dt^2$ is available. Then, this will be $-1 / R_1 C_1 dV_{in} / dt$, one integration; and this will be again $+1 / R_1 C_1 R_2 C_2$, again integration, V_{in} .

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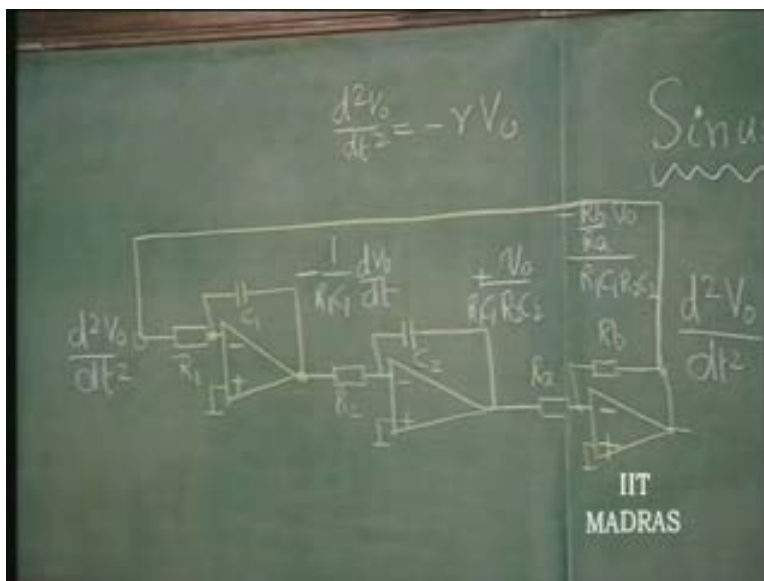
And at this point, it will be just inversion with $R_b / R_a V_{in}$ divided by...this is going to be inverted; $R_1 C_1 R_2 C_2$.

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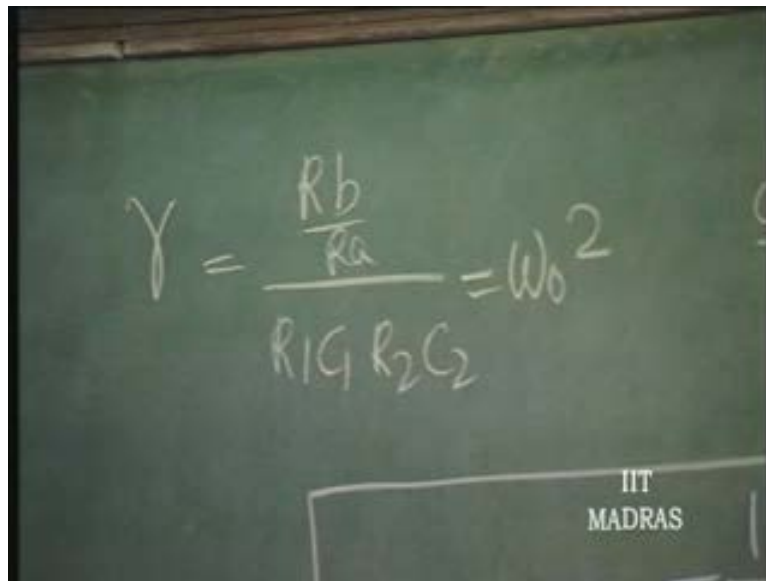
So, if you start with $d^2 V_{\text{naught}} / dt^2$, one differentiation integration will give you $-1 / R_1 C_1 d V_{\text{naught}} / dt$; another integration will give you $V_{\text{naught}} / R_2 C_2$ and then $-R_b / R_a$ is the gain this stage; and ultimately, we have an equation here $-d^2 V_{\text{naught}} / dt^2 = \dots$ if you ignore this, $\text{minus } \gamma V_{\text{naught}}$. So, that $\text{minus } \gamma V_{\text{naught}}$ we have got here...

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So, we can connect this; and this connection will make $d^2 V_{naught}$ by dt^2 equal to minus ΓV_{naught} , where Γ according to us is nothing but R_b by R_a divided by $R_1 C_1 R_2 C_2$. This is also called Ω_{naught}^2 , the frequency of oscillation being Ω_{naught} ; $\sqrt{\Gamma}$ is the frequency of oscillation.

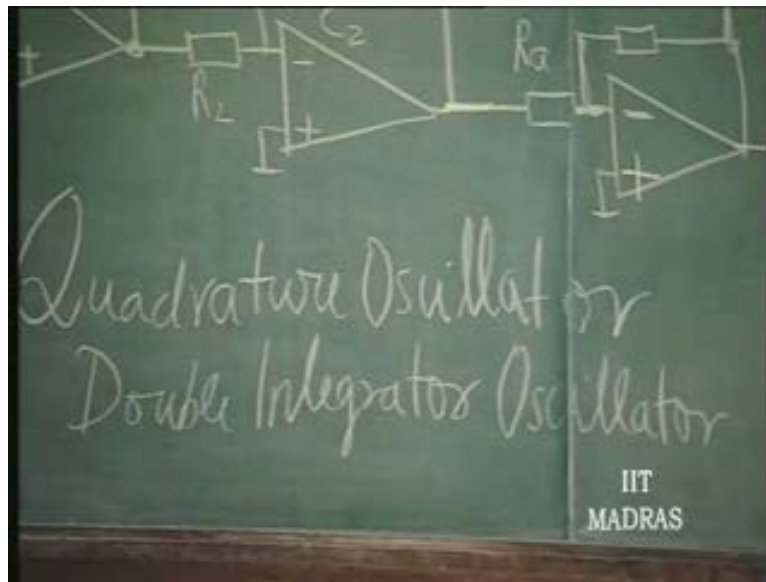
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$$\gamma = \frac{\frac{R_b}{R_a}}{R_1 C_1 R_2 C_2} = \omega_0^2$$

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So, we get, using this structure and oscillator, which is called quadrature oscillator or this is also called double integrator oscillator.

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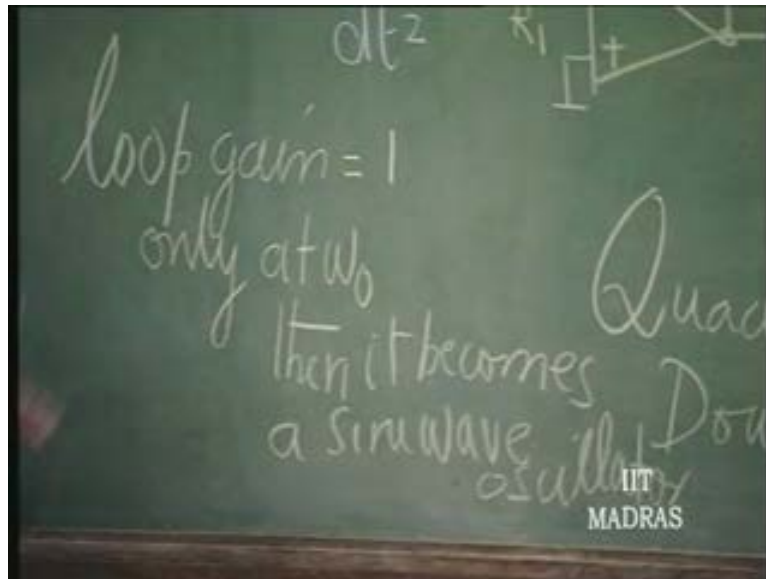
Quadrature because if this is $V \sin \Omega t$, this will be $V \cos \Omega t$. $V \cos \Omega t$ dash $\cos \Omega t$; and therefore, frequency of oscillation, Ω in this case, is root of R_b by $R_a R_1 C_1 R_2 C_2$.

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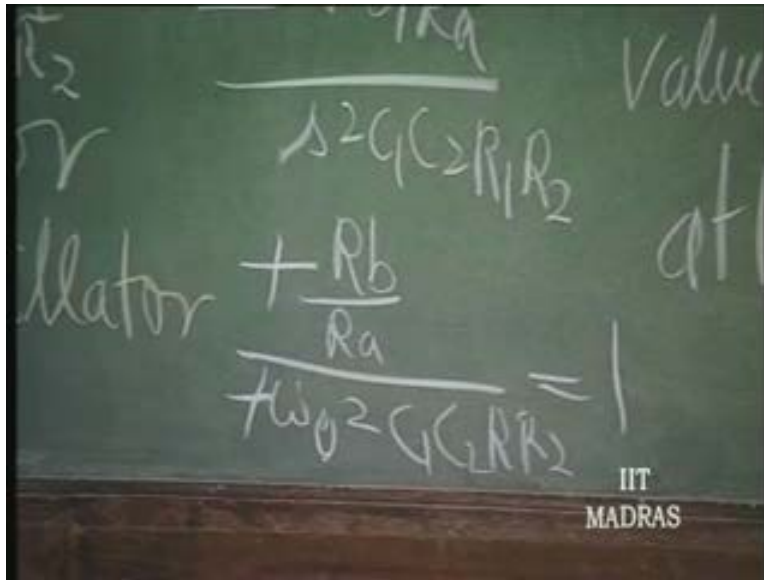
Now, if therefore output can become equal to input in magnitude...by connecting this, if I break the loop and then at a certain frequency if output can become equal to input in magnitude and have zero phase, then I can close the loop and make an oscillator out of it; or, if the loop gain, the loop gain becomes equal to 1 at Ω_{naught} , only at Ω_{naught} , then it becomes a sine wave oscillator at Ω_{naught} .

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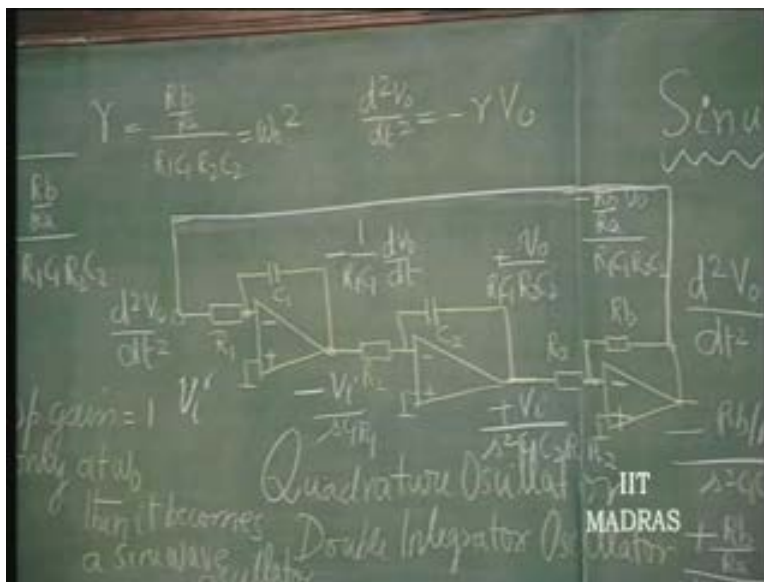
We can see here. This is...to start with, if you call this as V_i , I break the loop, call this V_i ; this is minus V_i by $S C_1 R_1$. This is plus V_i by $S^2 C_1 C_2 R_1 R_2$. This is minus R_b by R_a divided by $S^2 C_1 C_2 R_1 R_2$. S^2 - you replace it by Ω . Therefore, that factor becomes minus R_b by R_a minus Ω_{naught}^2 . This becomes plus, $C_1 C_2 R_1 R_2$. That is the loop gain. It becomes equal to 1 at Ω_{naught}^2 equal to R_b by $R_a R_1 C_1 R_2 C_2$.

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So the loop gain...that is, now I can close the loop and it will act as an oscillator. So, this is another way of looking at the oscillator.

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If I can make the loop gain in any loop equal to 1, that means magnitude should equal 1, phase shift should equal zero. That means output should become equal to input, in a loop,

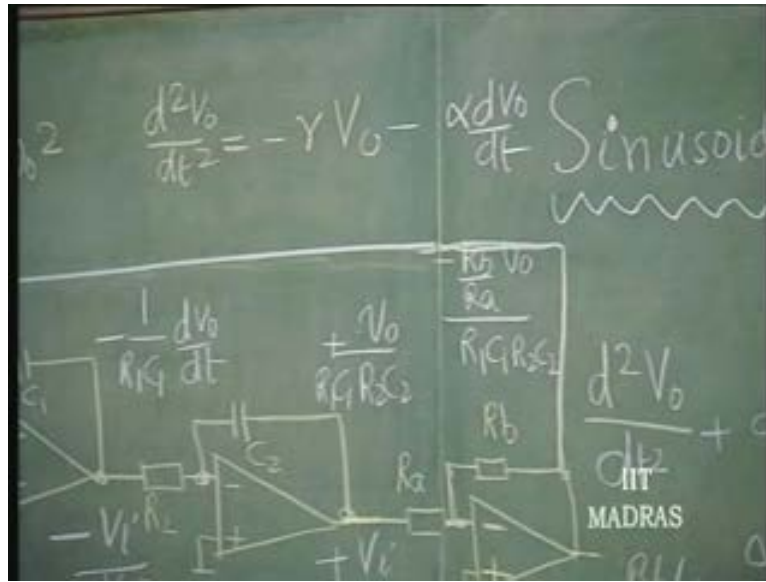
when it is broken, only at a certain ω , then it becomes a sine wave oscillator. This is a classic case of such a circuit.

There is another way of looking at oscillators. One way of looking at oscillators is this way. That it is simulating a second order differential equation with α going to zero. Another way of looking at it is in terms of loop gain by saying that if I break the loop, if I apply voltage here, output becomes equal to input, both in phase as well as in magnitude. Then, I can close the output and input and it becomes an oscillator.

So, what it means is, initially I apply voltage. Output becomes then equal to input at a certain frequency. Then this voltage source need not be connected at all. First, we can connect the output input and then take away this input voltage. It does not know that input voltage has been taken away; and therefore, it sustains the oscillation ω . So, this kind of analysis also can be adopted in coming up with new oscillator circuits. And, only one thing... In this circuit, suppose we have to introduce this α . What should I do?

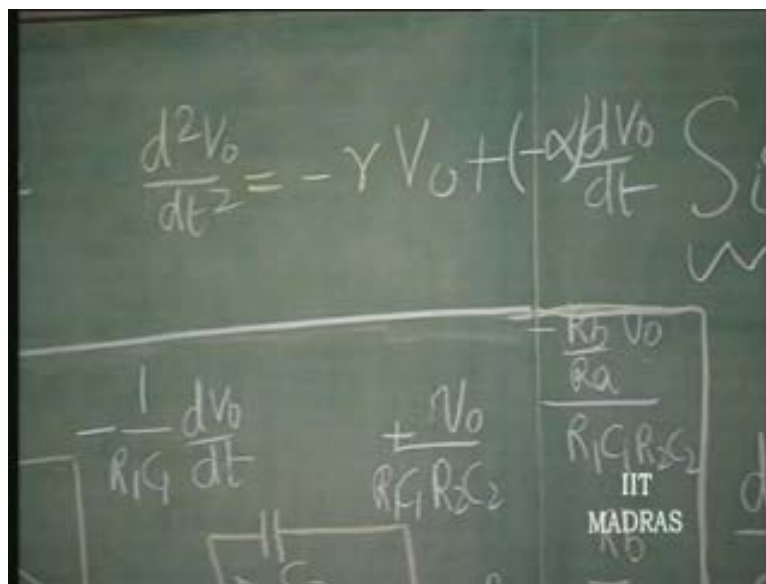
That means $d^2 V / dt^2$ is minus γV and minus $\alpha dV / dt$. In order to make it go into oscillation, in practice, we need a small negative value here which will ultimately go to zero, at the required amplitude.

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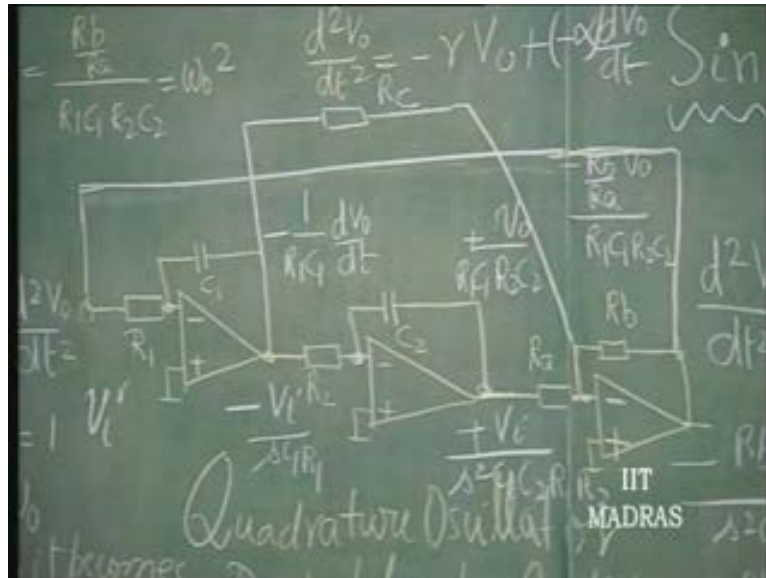
So, how do I make that? I have minus 1 over $R_1 C_1$ $\frac{dV_o}{dt}$ here. I already have minus γV_o here. Therefore, I had to take portion of this and bring it over here. So, this is what is necessary. This is $\frac{d^2V_o}{dt^2}$. This is minus 1 over $R_1 C_1$ $\frac{dV_o}{dt}$. If α is to be negative, then this should be negative; and this will be plus. α is to be negative.

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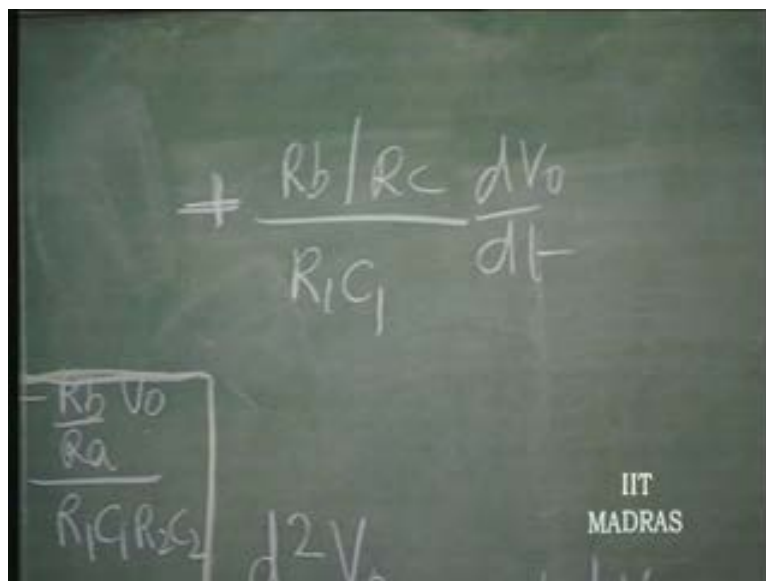
So, I can simply add from here, resistance. Let us call this R C.

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So, what happens to the output here? It is now...this is going to have, apart from this, plus Gamma...this thing...plus...from here, one inversion. Let us call this R C. So, R b by R c divided by R 1 C 1 into d v naught by d t.

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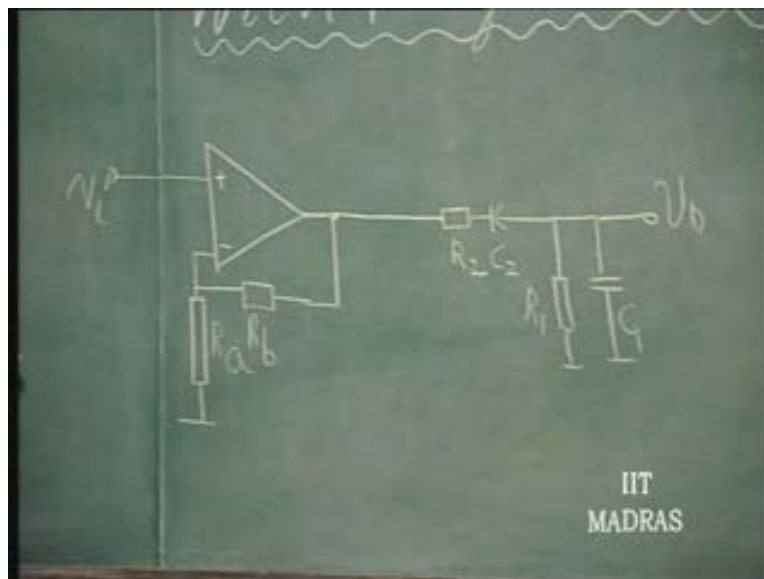


So, you can get negative value for the entire thing now, through this kind of feedback. So, this is going to introduce the negative Alpha on to the...this side; and already we had seen how a positive sort of Alpha can be obtained by feeding back at this point. So, that will make a filter get design. This will make an oscillator get design for you. So, if it does not oscillate, simply put a large value resistor between this and this and adjust the value of resistor until it starts oscillating.

So, this resistance R C should go to infinity as amplitude builds up, ultimately locating the poles of this system on the imaginary axis. By putting R C, I have made sure that the system pole to start with is on the right half of the S plane. So, this is one type of oscillator.

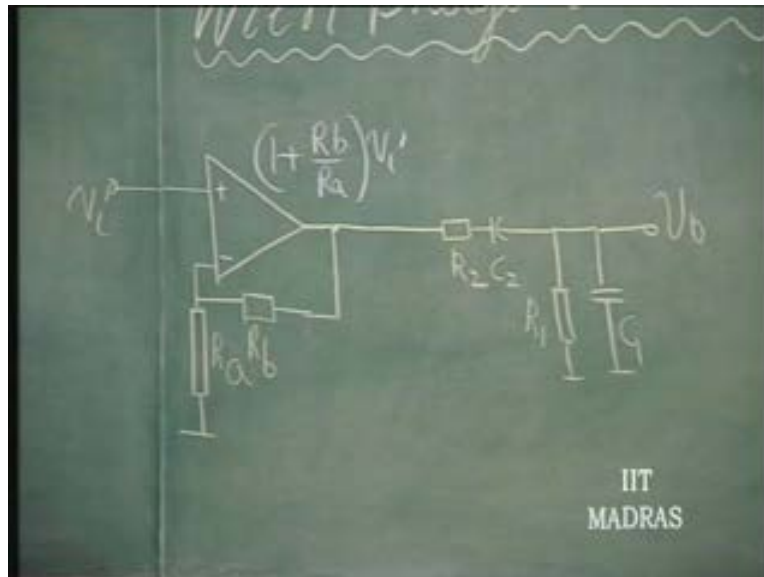
Let us now use the concept that we have just now learned about making the loop gain equal to 1 at a certain frequency; only at one frequency. That is, output is in phase with input and output is equal to input in magnitude.

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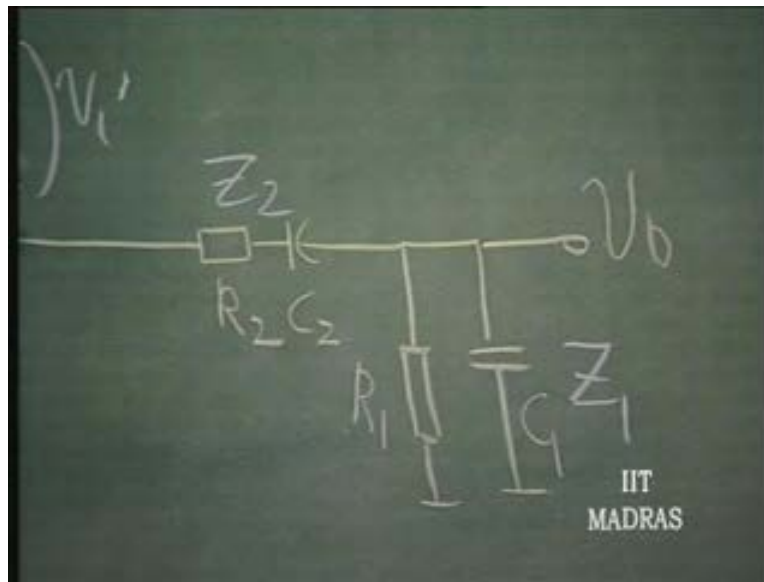
If that is the case, then I can connect the output to input and have an oscillator. So, if this is V_i , we know that I use a non-inverting amplifier here. I get a gain of $1 + \frac{R_b}{R_a}$ times V_i here. This is a non-inverting amplifier of gain $1 + \frac{R_b}{R_a}$.

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As far as this network is concerned, if you call this as Z_1 and this as Z_2 , the attenuation here is going to be Z_1 divided by $Z_1 + Z_2$.

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So, output V_o is going to be $1 + \frac{R_b}{R_a}$ times V_i , which is the input to be Z network, into Z_1 by $Z_1 + Z_2$; or, this is also equal to $1 + \frac{R_b}{R_a}$ into V_i ; dividing by $Z_1 + Z_2$ into Y_1 . So, this is equal to $1 + \frac{R_b}{R_a}$ into $V_i \dots 1 + \frac{R_b}{R_a}$ Z_2 is $R_2 + \frac{1}{sC_2}$; $R_2 + \frac{1}{sC_2}$. Y_1 is $\frac{1}{R_1 + sC_1}$; $\frac{1}{R_1 + sC_1}$.

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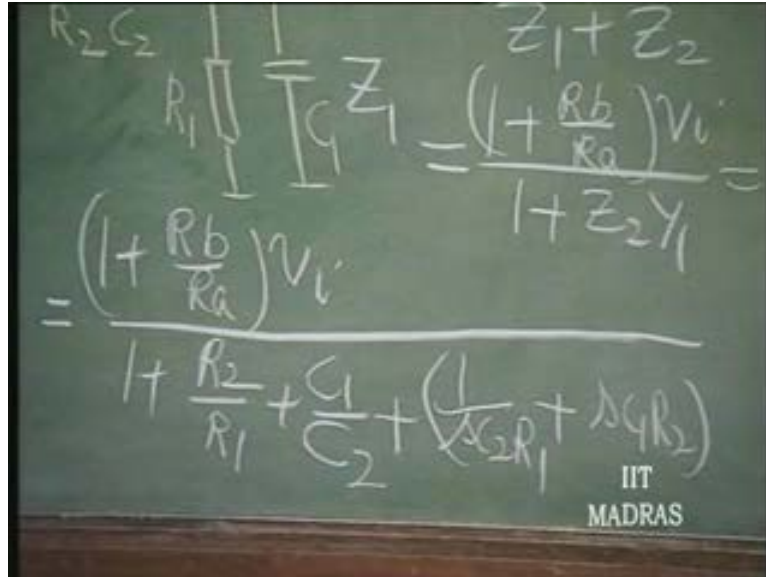
$$V_o = \frac{\left(1 + \frac{R_b}{R_a}\right) V_i Z_1}{Z_1 + Z_2}$$

$$= \frac{\left(1 + \frac{R_b}{R_a}\right) V_i}{1 + Z_2 Y_1} = \frac{\left(1 + \frac{R_b}{R_a}\right) V_i}{1 + \left(R_2 + \frac{1}{sC_2}\right) \left(\frac{1}{R_1 + sC_1}\right)}$$

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So, you get this...here V_{naught} , as being equal to $1 + R_b$ by R_a into V_i divided by $1 + R_2$ by $R_1 C_1$ by C_2 plus 1 over $S C_2 R_1$ plus $S C_1 R_2$.

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And now you can substitute S equal to $j\Omega$. $j\Omega$. This is also $j\Omega$; or you can actually bring this j out here. So, this becomes minus. So, you can make this quantity within j zero at one frequency.

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$$Z_1 = \frac{1}{\frac{1}{R_1} + j\omega C_1}$$

$$Z_2 = \frac{1}{\frac{1}{R_2} + j\omega C_2}$$

$$V_0 = \frac{Z_2}{Z_1 + Z_2} V_i = \frac{\frac{1}{\frac{1}{R_2} + j\omega C_2}}{\frac{1}{\frac{1}{R_1} + j\omega C_1} + \frac{1}{\frac{1}{R_2} + j\omega C_2}} V_i$$

$$= \frac{\left(1 + \frac{R_2}{R_1}\right) V_i}{1 + \frac{R_2}{R_1} + \frac{C_1}{C_2} + j\omega \left(\frac{R_2 C_1}{R_1} + \omega C_2 R_2\right)}$$

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That means it will be in phase; output will be in phase with the input at that frequency. So, this quantity is the one which contributes to phase shift. So, that happens equal to zero when ω is equal to $1/\sqrt{C_1 C_2 R_1 R_2}$. That is the frequency of oscillation.

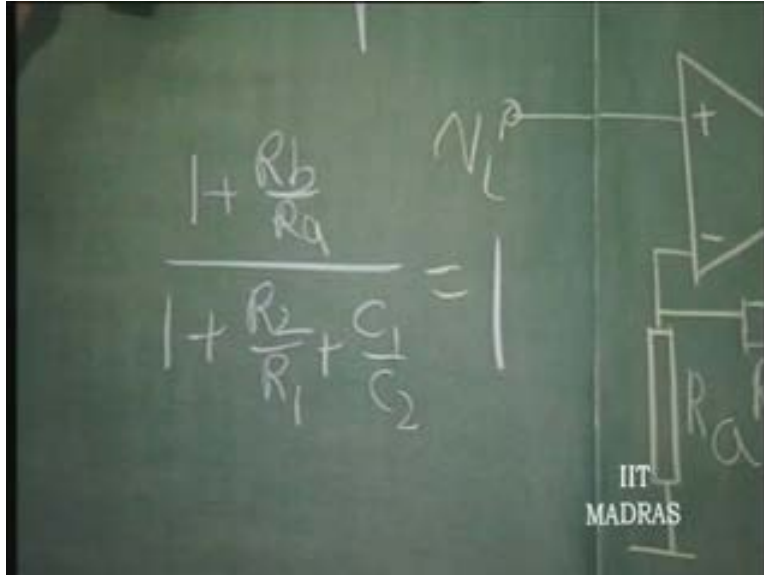
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$$\omega_0 = \frac{1}{\sqrt{C_1 C_2 R_1 R_2}}$$

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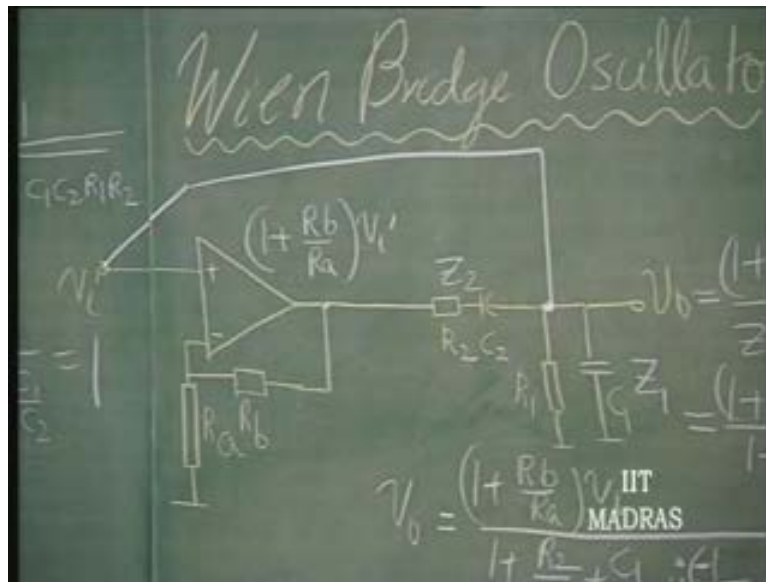
At that...when this is becoming equal to zero, the attenuation is $1 + \frac{R_b}{R_a}$ divided by $1 + \frac{R_2}{R_1} + \frac{C_1}{C_2}$. If this becomes equal to 1, then output is equal to input and it is in phase.

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So, I can now connect the output to the input. It will oscillate at $\omega = \frac{1}{\sqrt{C_1 C_2 R_1 R_2}}$.

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So, the condition for oscillation is that R_b by R_a has to be made equal to R_2 by R_1 equal to C_1 over C_2 . This is easily done by setting R_2 equal to R_1 , C_1 equal to C_2 . R_b by R_a has to be then equal to 2. So, if R_2 equals R_1 equals R , C_2 equals C_1 equals C , then the frequency of oscillation Ω is 1 over $R C$; and the condition for oscillation is that this is equal to 2.

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$$1 + \frac{R_2}{R_1} + \frac{C_1}{C_2} = 2$$

$$\frac{R_b}{R_a} = \frac{R_2}{R_1} + \frac{C_1}{C_2} = 2$$

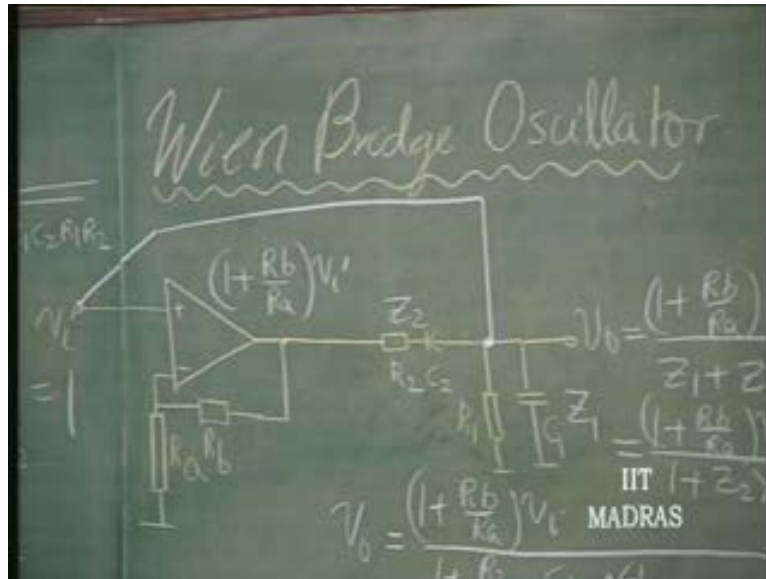
$$R_2 = R_1 = R$$

$$C_2 = C_1 = C$$

So, if it is greater than 2, R_b by R_a is greater than 2, it will start building up oscillation. Gain is greater than 2. If it is less than 2, it will start decaying. If it is exactly at 2, it is having poles on the imaginary axis. This system with this kind of feedback will have poles on the imaginary axis. So, this is an oscillator.

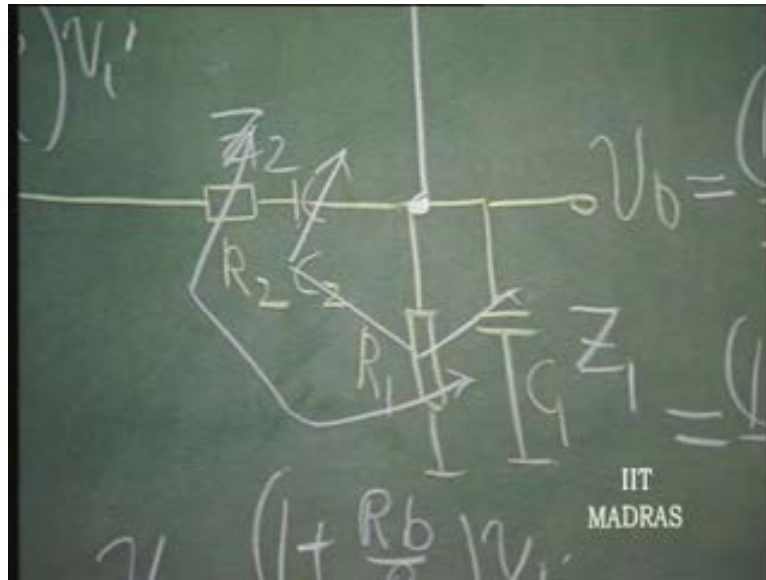
This is called Wien bridge because this arm and this arm, and this arm and this arm, form a bridge between which we have introduced an operational amplifier. So, this arm and this arm, this arm and this arm, form a Wien bridge; very famous oscillator.

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This is the oscillator circuit which is commonly used in all test oscillators, sine wave oscillators available as test oscillators, wherein you can gang these capacitors and vary it in steps for differ...getting different ranges and gang these resistors and vary this continuously in order to vary the frequency of oscillation continuously.

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This ganging becomes necessary because R_2 by R_1 plus C_1 by C_2 should be equal to 1; R_2 by $R_1 C_1$ by C_2 , both should be equal to 1 and total should be equal to 2; so the ganging becomes necessary.

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$$1 + \frac{R_2}{R_1} + \frac{C_1}{C_2}$$

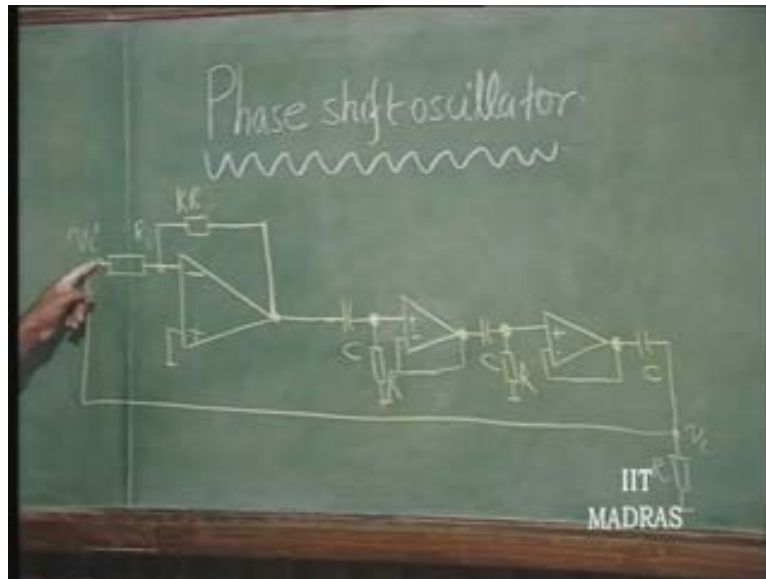
$$\frac{R_b}{R_1} = \frac{R_2}{R_1} + \frac{C_1}{C_2} = 2$$

$$R_2 = R_1 = R$$

$$C_2 = C_1 = C$$

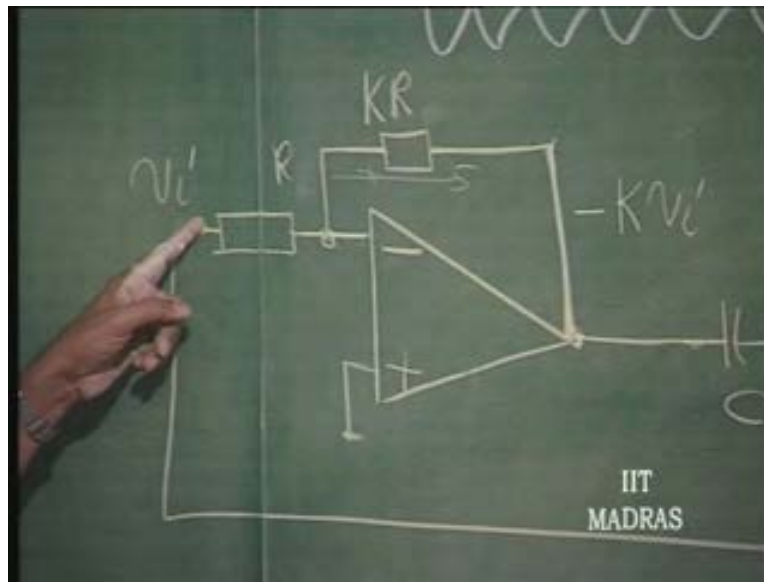
Now let us consider another oscillator. Once again, I am going to break the loop here. Let us assume that this loop is broken here. This resistance of R is going to really appear as resistance to ground here; this much of ground. So, in effect, this loading effect of the input resistance of the amplifier is coming here as R . So, actually speaking, so V_i is going to be developed there.

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We have to see whether V naught developed here is same as V_i . So, if V ...this is V_i ; this is minus K times V_i . This we have seen. As an inverting amplifier, if this is V_i , this is ground; this is virtual ground. Current in this is V_i by R ; the same current will flow through this and develop the potential here, which is negative; and minus K times V_i is the output here.

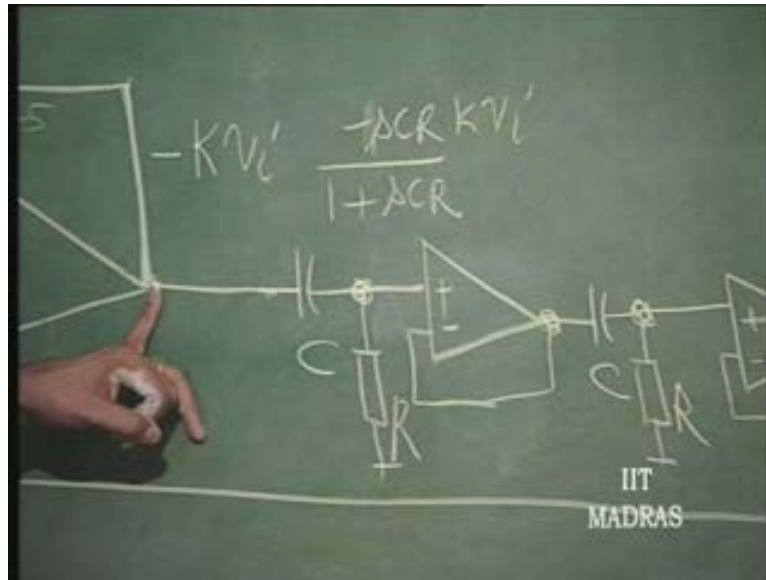
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So, we have got a 180 degree phase shift which is independent of frequency here. In order to make it now come in phase, we have to provide additional phase shift of 180 degrees. That is why this is called phase shift oscillator. Additional phase shift can be offered by three such R C networks, in order to prevent this from loading. Otherwise, I might have to use a higher gain amplifier here. I am using buffer stages. This is only illustrating the principle of phase shift oscillator.

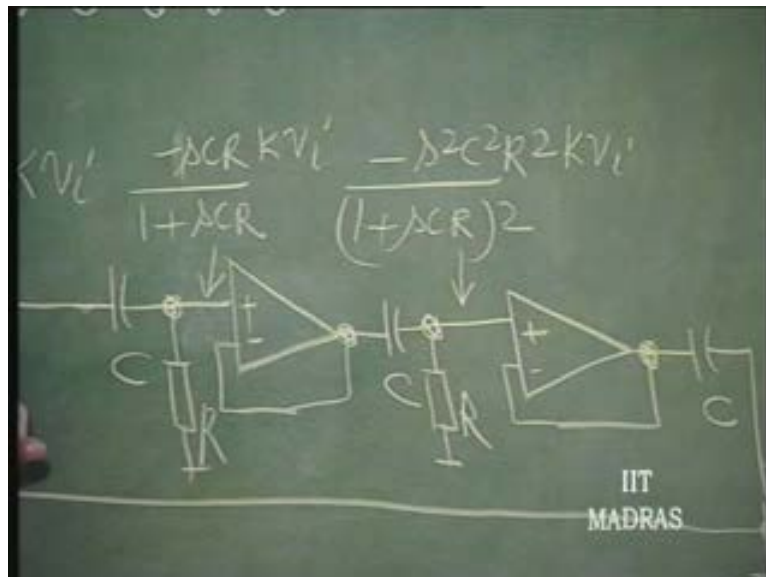
So, one by one, that is, this phase shift is $S C R$ divided by $S C R$ is the transfer function from here to here. So, that...this is minus $K V_i$...into minus K times V_i . So, this single network can give a phase shift of...from starting from, let us say, 90 degrees at low frequency to let us say, at very high frequency, it can go up to 180 degrees; but it cannot... that is, going to be only at infinite frequency.

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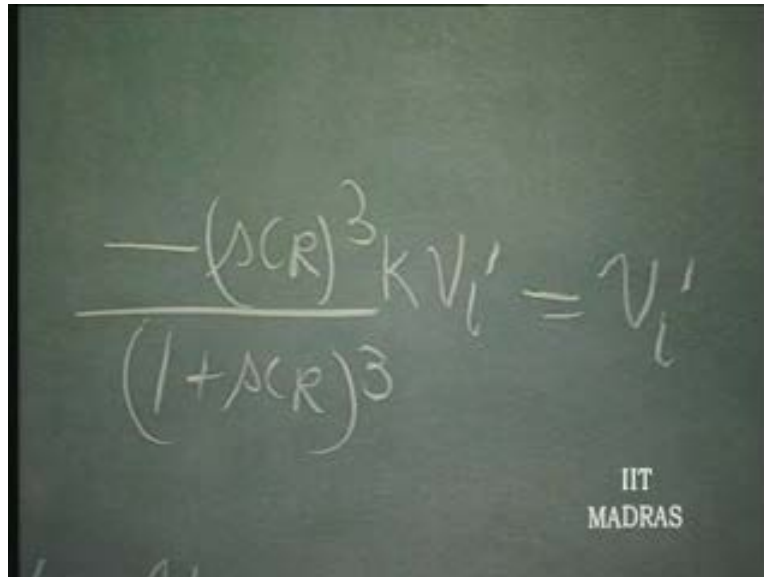
So, I put two such networks. So, we get here minus S squared C squared R squared divided by 1 plus $S C R$ whole square, the gain; into K times V_i here; and at...that is at this point also, it is going to be the same because of the buffer.

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And then, at this point it will be minus S C R whole cube by 1 plus S C R cube K times V i. That should become equal to V i. That is...

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$$\frac{-(SCR)^3 K V_i'}{(1 + SCR)^3} = V_i'$$

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So, let us now rewrite the whole thing. Minus S C R whole cube into K divided by 1 plus S C R whole cube should become equal to 1; V i V i getting cancelled. So, 1 plus S C R whole cube plus S C R cube into K should be equal to zero.

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$$\frac{-(\Delta CR)^3 K}{(1 + \Delta CR)^3} = 1$$

$$(1 + \Delta CR)^3 + (\Delta CR)^3 K = 0$$

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This cubic equation $1 + 3 S C R$ squared plus $3 S C R$ plus $S C R$ whole cube will come out of this, plus K . So, $K + 1$ into this should be equal to zero. Now put S is equal to $j \Omega$. So, we get $1 - 3 \Omega^2 C^2 R^2 + 3 j \Omega C R$ plus... That is actually minus, $j S^3 C^3 R^3$. One j square will give you minus and another j will remain $K + 1$.

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$$(1 + \Delta CR)^3 + (\Delta CR)^3 K = 0$$

$$1 + 3\Delta CR^2 + 3\Delta CR + (\Delta CR)^3 (k+1) = 0$$

$$1 - 3\Omega^2 C^2 R^2 + 3j\Omega CR - j\Delta^3 C^3 R^3 (k+1) = 0$$

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So, this is going to be equal to zero. If the real part becomes equal to zero, simultaneously, the imaginary part also should become equal to zero. So, we get Omega naught equal to, from this. 1 divided by 3 , root 3 , C into R ; making this equal to zero, we will get Omega naught, frequency of oscillation, is equal to 1 by root 3 into $C R$.

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The image shows a chalkboard with the following handwritten equations and a circuit diagram:

$$\omega_0 = \frac{1}{\sqrt{3}CR}$$

$$\frac{-(sCR)^3 K}{(1+sCR)^3} = 1$$

$$(1+sCR)^3 + (sCR)^3 K = 0$$

$$1 + 3s^2 C^2 R^2 + 3s^3 C^3 R^3 + (sCR)^3 (K+1) = 0$$

$$1 - 3\omega_0^2 C^2 R^2 + 3j\omega_0 C R - j^3 \omega_0^3 C^3 R^3 (K+1) = 0$$

The IIT Madras logo is visible in the bottom right corner of the chalkboard image.

And then here $\omega_0 C R$. This is ω_0 cube C cube R cube. So, $\omega_0 C R$ goes, j goes. So, 3 minus ω_0 squared C squared R squared into 1 plus K also should become equal to zero; or this should become equal to this. Already we know that ω_0 squared C squared R squared has become equal to 1 over 3 . ω_0 squared C squared R squared has already become equal to 1 over 3 , from this equation. So, K plus 1 becomes equal to 9 ; or K has to be equal to 8 . Amplifier gain has to be equal to 8 . Then, the frequency of oscillation is going to be equal to be 1 over root 3 into C into R .

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$$3 = \frac{1}{3} (1+k)$$

$$k+1 = 9 \text{ or } \underline{k=8}$$

$$\omega_0 = \frac{1}{\sqrt{3RC}}$$

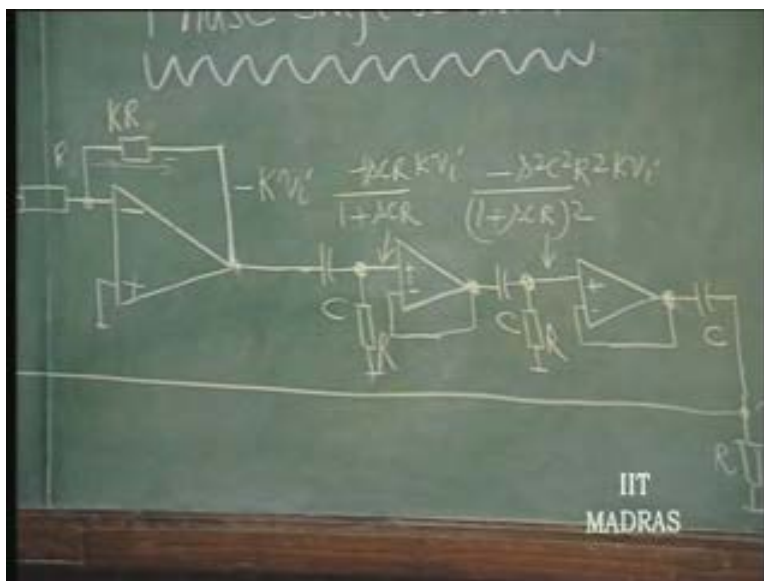
$$\frac{-(\Delta CR)^3 K}{(1+\Delta CR)^3} = 1$$

$$(1+\Delta CR)^3 + (\Delta CR)^3 K = 0$$

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So, this is the phase shift oscillator which, of course, gets modified. I would like you to therefore work out this problem where this buffer stage is removed. So, this is going to load this and this also is removed so that, in fact, use only one op amp and three R C networks like this, in order to come up with... So, this circuit has this R already being provided here. Once you close the loop, this is there and this satisfies this relationship.

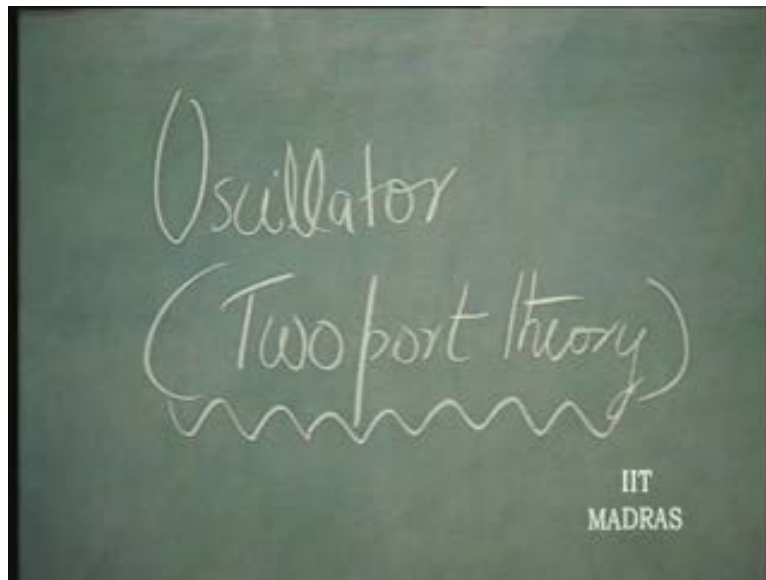
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This is a phase shift oscillator. Again, you can have the phase shift as being brought about by R here, C here, R here, C here, all that. That will bring about, instead of this, $\sqrt{3}$ divided by C R; and the K is going to be same as 8.

Now, let us try to understand this oscillator in terms of the two port theory that we have studied in the case of amplifiers.

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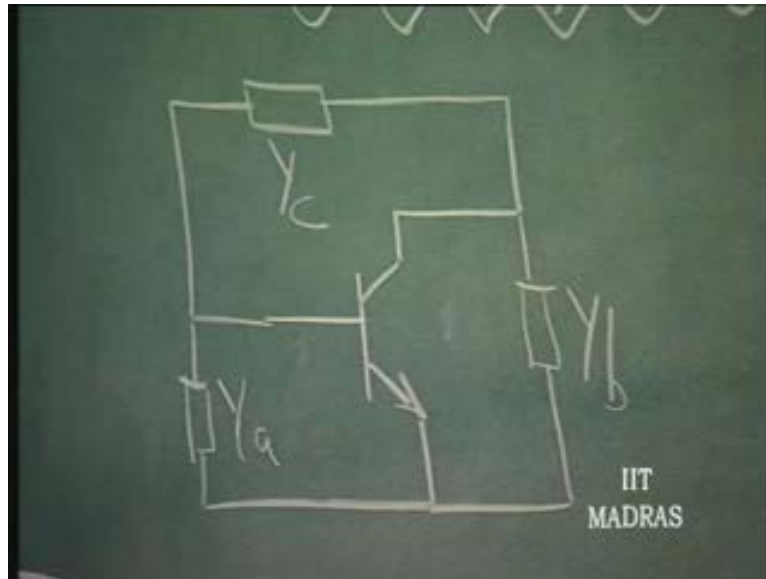


So, let us consider that my active device which is a transistor, for example, with all this biasing and all other things put here, which will comprise of the admittance now. It may be now, not just resistance because it obviously...we have to make the loop gain equal to 1 only at a certain frequency. That is possible only by putting resistors and inductors. So, this is an admittance.

Let us say, this we will call as Y_a . This will contain the load and admittance at the output, etcetera. We will call it altogether Y_b ; and this may have a feedback also. So, Y_c . So, this is the active device which is supposed to make it going to oscillation. This does not have any feedback. It is biasing circuit and input reactances and output secured load

and output reactances and the feedback reactance, in general, have been incorporated here.

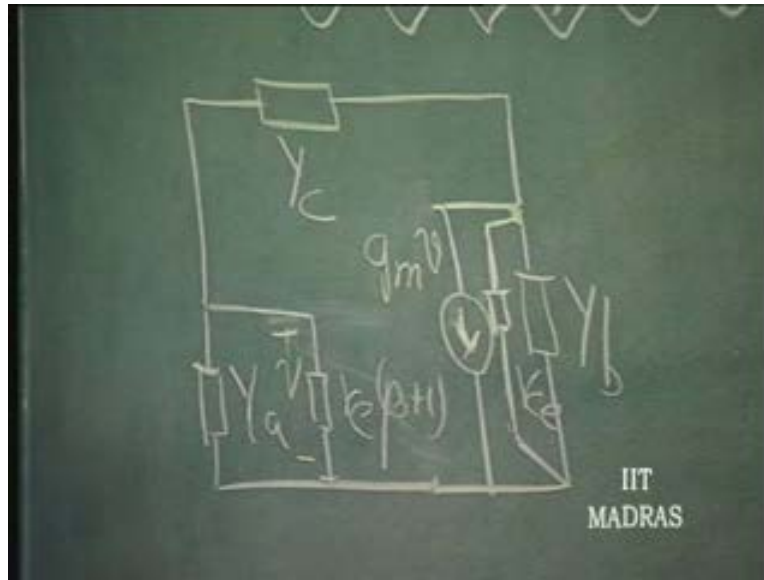
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I would like to know whether such a circuit can be made into an oscillator. How do I analyze such a thing? Obviously, let us say, as far as this equivalent circuit is concerned, it could be, FET or a bipolar structure here. If it is, let us say, a bipolar structure, we will put it as an equivalent circuit here. We will put r_e into Beta plus 1 and then here we will put this. If this V , we will put g_m into V here as the current, source current. If you want to include the output impedance of this structure, you can also put r_{ce} .

In the case of a FET, this will be infinity and again g_m into V and R_{ds} will be put here. So, that is the only difference. So, for such a circuit now, I would like to analyze and find out the condition for making an oscillator out of this. This is a general two port.

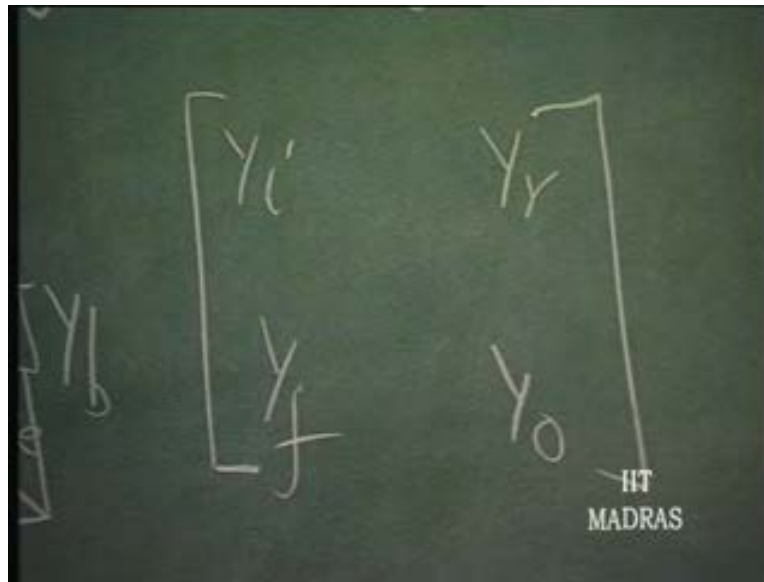
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So, this is, let us say, input port; this is output port; this is the feedback. So, in a two port theory, how do I make sure that the loop gain is equal to 1 at a certain frequency? Loop gain, when it comes to loop gain, we saw that if I write down the matrix of the entire parameter... now I consider this a Y parameter and I have Y_i here Y_{naught} here.

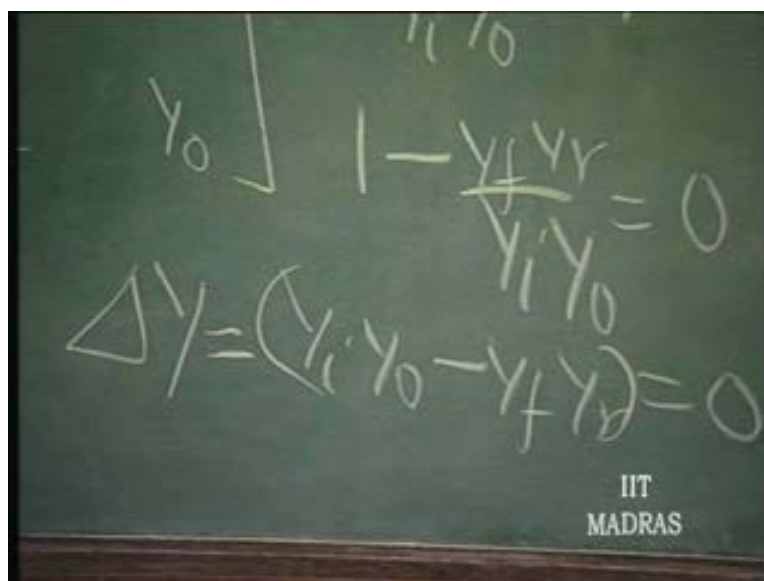
This Y_i can include the source impedance and Y_{naught} can include the load impedance. In this case, since it is an oscillator, it is not going to be driven by a source. So, this will be the self input admittance short circuit. This will be self output admittance short circuit and this is the feedback and this is the feed forward.

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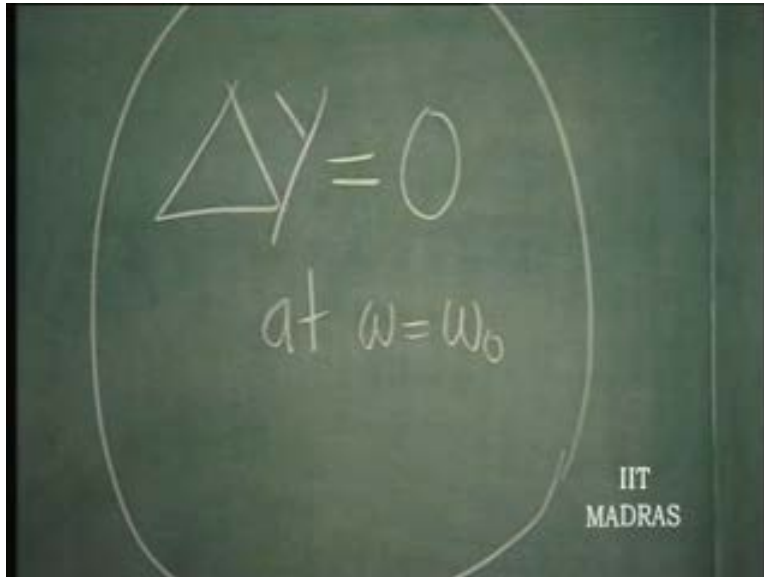
In such a situation, what is the loop gain? We have seen that $Y_{if} Y_{ir}$ divided by $Y_{ii} Y_{io}$ is the loop gain; $Y_{if} Y_{ir}$ divided by $Y_{ii} Y_{io}$ is the loop gain; and that loop gain has to be made equal to 1; or $1 - Y_{if} Y_{ir}$ divided by $Y_{ii} Y_{io}$ should be equal to zero; or $Y_{ii} Y_{io} - Y_{if} Y_{ir}$ should become equal to zero, Assuming that Y_{ii} and Y_{io} do not go to zero at any frequency.

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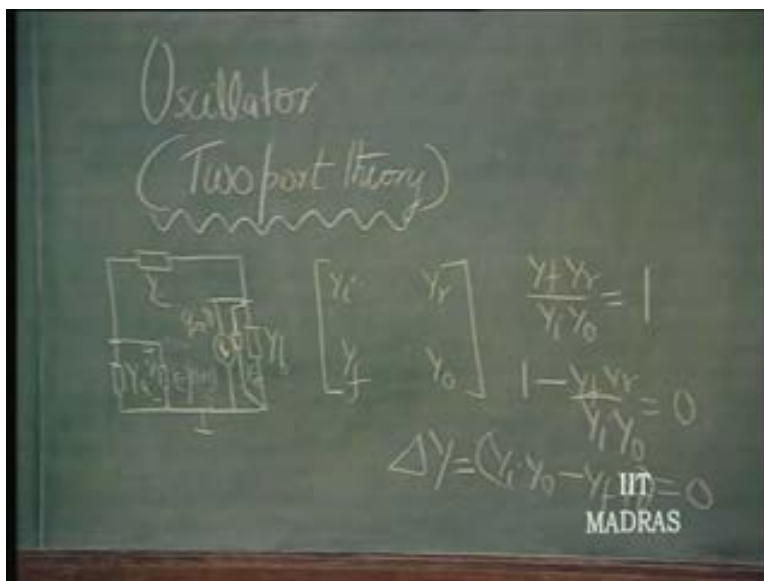
That means what is this? This is nothing but capital Delta Y. So, the condition for any oscillator according to two port theory is that the Delta Y should go to zero at Omega equal to Omega naught; a single frequency.

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This is an important condition. So, let us consider that for this network.

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So, as far as this network is concerned now, the composite Y matrix can be written here. Input Y i is Y a plus 1 over r e into Beta plus 1 plus Y c; and this is shorted. That is all. And feedback factor is only minus Y c. This is the only admittance connected; so, minus Y c. So, this is from V naught. Even if I apply V naught, what is the current in this? Short circuit; V naught by Y c in the opposite direction to the positive direction, so, minus Y c.

Now, feed forward part. When I apply V and short circuit, what will be the current? I have a V applied here. So, this is the same V and g m into V is the short circuit current coming in. So, it is positive, g m. Apart from that, minus Y c is also going to be there. That is all. Then, admittance at the output when this is shorted will be again 1 over Y b. That is Y b plus 1 over r c e plus Y c. That is all.

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$$\begin{bmatrix} Y_a + \frac{1}{Y_e(\beta + 1)} + Y_c & -Y_c \\ g_m - Y_c & Y_b + \frac{1}{r_c e} + Y_c \end{bmatrix}$$

So, what is Delta Y now? Delta Y is this into this minus this into this. So, Y a plus 1 over r e into Beta plus 1 plus Y c into Y b plus Y c plus 1 over r c e minus... minus, minus, becomes plus. So, Y c g m minus Y c. So, this should be equal to zero at a certain frequency. This will give you both frequency of oscillation and condition for oscillation, depending upon how you select Y a, Y b and Y c.

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$$\left(Y_a + \frac{1}{Y_c(s+1)} + Y_c \right) \left(Y_b + Y_c + \frac{1}{Y_{ce}} \right) + Y_c (g_m - Y_c) = 0$$

$$\left[Y_a + \frac{1}{Y_c(s+1)} + Y_c - Y_c \right]$$

$$g_m - Y_c$$

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So, this is what is going to be equal to be zero. Simplifying this, this will give me Y a into Y b; Y a into Y b; Y a into Y c. That is exhausting. This plus Y c into Y b plus, Y c squared gets cancelled with Y c squared here. So, that is all.

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$$\left(Y_a + \frac{1}{Y_c(s+1)} + Y_c \right) \left(Y_b + Y_c + \frac{1}{Y_{ce}} \right) + Y_c (g_m - Y_c) = 0$$

$$Y_a Y_b + Y_a Y_c + Y_c Y_b$$

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This can be put totally as $\sum Y_a \text{ into } Y_b$. $Y_a Y_b$ plus $Y_b Y_c$ plus $Y_c Y_a$. What... That is what it means. $\sum Y_a \text{ into } Y_b$ means $Y_a Y_b$ plus $Y_b Y_c$ plus $Y_c Y_a$. Then, the rest of the factors will be dependent upon this; $1 \text{ over } r e \text{ into } \text{Beta plus } 1$, into Y_b plus Y_c plus $1 \text{ over } r c e \text{ into } Y_a$ plus Y_c .

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The image shows a chalkboard with the following handwritten mathematical expressions:

$$r - Y_c) \sum Y_a Y_b$$

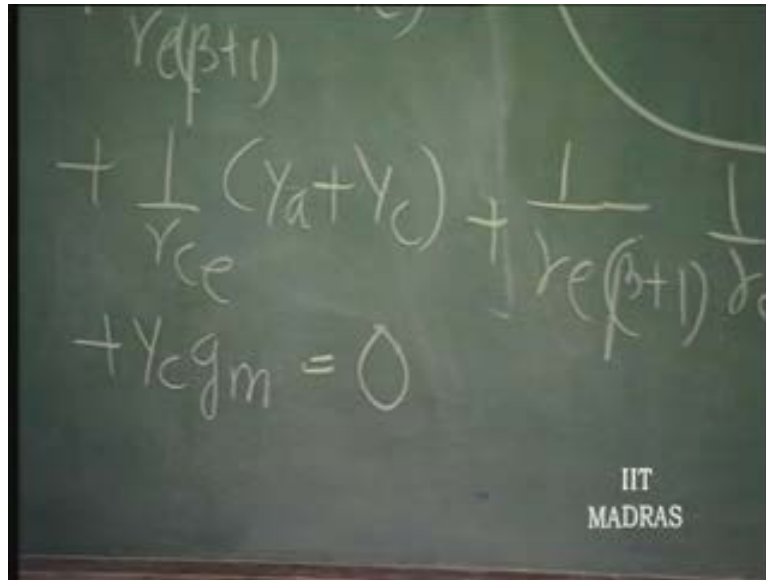
$$+ \frac{1}{Y_c(\beta + 1)} (Y_b + Y_c)$$

$$+ \frac{1}{Y_c \epsilon} (Y_a + Y_c)$$

In the bottom right corner of the chalkboard, the text "IIT MADRAS" is visible.

And then a factor which is independent of this Y , which is $1 \text{ over } r e \text{ into } \text{Beta plus } 1$, $r c e$. So, those are the factors, I think... Yes. All these things are taken into account. Next, Y_c into $g m$ that should be equal to zero.

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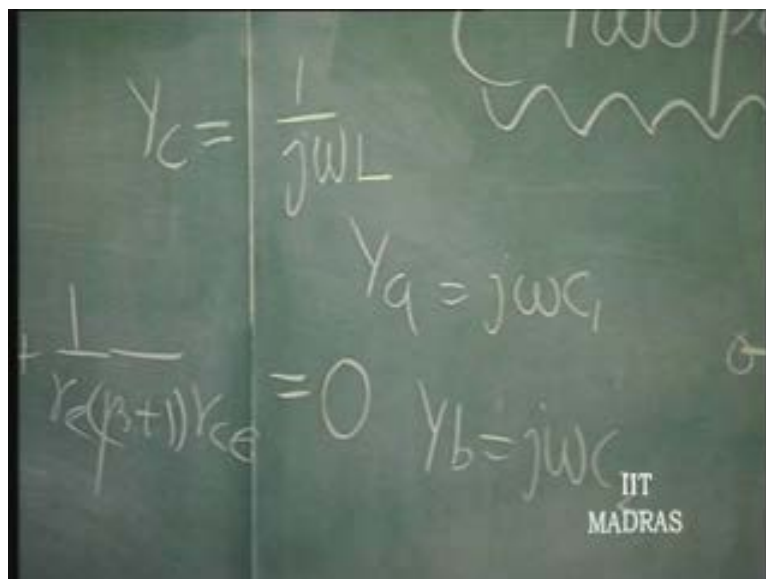
The chalkboard shows the following equations:

$$Y_c(\beta+1) + \frac{1}{Y_{ce}}(Y_a + Y_c) + \frac{1}{Y_c(\beta+1)} + \frac{1}{Y_c} + Y_{cgm} = 0$$

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Now, when such a condition is to be satisfied, we have to see that it is possible. What does it mean? This will obviously have real part and imaginary part and this should be possible to make the real part go to zero and imaginary part also go to zero. Now, that can be done by selecting one possible combination – Y_c as being equal to 1 over j Ω L and Y_a equal to, let us say, j Ω C_1 and Y_b as j Ω C_2 .

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The chalkboard shows the following equations:

$$Y_c = \frac{1}{j\omega L}$$
$$Y_a = j\omega C_1$$
$$Y_b = j\omega C_2$$
$$\frac{1}{Y_c(\beta+1)Y_{ce}} = 0$$

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That means two of them are capacitive and one is inductive; or, this is in a capacitive and the other two are inductive. The feedback element should be of one type and the other two should be of the opposite type. So, that way, we will see that we have two types of oscillators. One – Y_c is inductive, Y_a and Y_b are capacitive; another – Y_c is capacitive, Y_a and Y_b are inductive.

We will now consider the first one. So, because of this, you get here, both real part positive as well as negative, which can actually independently go to zero; and the imaginary part which is going to be remaining here because this, these terms contribute to only imaginary part; that also has to be independently going to zero. Apart from that, this will also contribute to some amount of real part.

So, let us see that $Y_a Y_b \dots Y_a Y_b$ is going to give me $\Omega^2 C_1 C_2$ which is negative; and Y_b plus Y_a into Y_c , Y_b plus Y_a which is C_1 plus C_2 divided by L .

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$$Y_a Y_b + Y_b Y_c + Y_c Y_a$$

$$+ \frac{Y_b + Y_c}{Y_c (1 + \beta)} + \frac{Y_c + Y_a}{Y_c e} + g_m$$

$$- \omega^2 C_1 C_2 + \frac{C_1 + C_2}{L}$$

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So, that is the real part which is positive. This is negative. Apart from this, we will also have this $1/r_e$ into $\beta + 1/r_c$. This should be equal to zero.

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The image shows a chalkboard with handwritten mathematical equations. At the top, it says $Y_a Y_b + Y_b Y_c + Y_c Y_a$. Below that, the equation is written as $\frac{(Y_b + Y_c)}{r_e(1+\beta)} + \frac{(Y_c + Y_a)}{r_{ce}} + g_m Y_c + \frac{1}{r_e(\beta + 1)}$. The final equation shown is $1 + \frac{(G_1 + G_2)}{L} + \frac{1}{r_e(\beta + 1)r_{ce}} = 0$. The IIT MADRAS logo is visible in the bottom right corner of the chalkboard image.

Or, ω^2 which is the frequency of oscillation; we will take that, that side, is going to be equal to $C_1 + C_2$ by $L C_1 C_2$. This is the first part; plus, what about this non-ideality is? Because, $1/r_c$ can be equal to zero. So then, this will vanish. It will be purely dependent upon passive. Otherwise, it will depend upon the active parameter there. So, $1/r_e$ into $\beta + 1/r_c$ divided by $C_1 C_2$. That is the non-ideality. So, ω^2 therefore is root of this.

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$$\omega_0 = \sqrt{\frac{C_1 + C_2}{L C_1 C_2} + \frac{\left(\frac{1}{\gamma_a(\beta+1)\gamma_e\gamma} \right)}{C_1 C_2}}$$

$\Delta\gamma = 0$

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It is going to be approximately equal to root of 1 by L into C 1 C 2 C 1 plus C 2. So, this is the frequency of oscillation.

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$$\frac{C_1 + C_2}{C_1 C_2} + \frac{\left(\frac{1}{\gamma_a(\beta+1)\gamma_e\gamma} \right)}{C_1 C_2} \approx \sqrt{\frac{1}{L C_1 C_2}}$$

$\Delta\gamma = 0$

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That takes care of completely the real part. This is gone. Now, the imaginary part is going to be due to these three.

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So g_m into Y_c will result in g_m by ΩL minus j . Then, other things... plus Y_b plus Y_c , Y_b plus Y_c . So, $j \Omega C_2$ plus 1 over minus $j \Omega L$ divided by r_e into β plus 1 , plus Y_c plus Y_a again. Or, Y_a is $j \Omega C_1$ minus j by ΩL divided by r_e . This is equal to zero.

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$$\Delta I = 0$$

$$Y_a Y_b + Y_b Y_c + Y_c Y_a$$

$$+ \frac{Y_b + Y_c}{r_e(1+\beta)} + \frac{Y_c + Y_a}{r_{ce}} + g_m Y_c + \frac{Y_c}{r_{e(1+\beta)} r_{ce}} = 0$$

$$Y_c = \frac{1}{j\omega}$$

$$- \frac{g_m}{\omega L} + \frac{(j\omega C - \frac{1}{\omega L})}{r_e(1+\beta)} + \frac{(j\omega C - \frac{1}{\omega L})}{r_{ce}} = 0$$

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So basically, we can see here - this is only the imaginary part. So, all these things become 1 and this can be made equal to zero.

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$$\Delta I = 0$$

$$Y_a Y_b + Y_b Y_c + Y_c Y_a$$

$$+ \frac{Y_b + Y_c}{r_e(1+\beta)} + \frac{Y_c + Y_a}{r_{ce}} + g_m Y_c + \frac{Y_c}{r_{e(1+\beta)} r_{ce}} = 0$$

$$Y_c = \frac{1}{j\omega}$$

$$- \frac{g_m}{\omega L} + \frac{(j\omega C - \frac{1}{\omega L})}{r_e(1+\beta)} + \frac{(j\omega C - \frac{1}{\omega L})}{r_{ce}} = 0$$

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So essentially, we can see that we can collect the co-efficient of 1 over Omega on the other side. So, 1 over Omega into g m by L. This will go to the other side. All of them are

negative. So, when they go to the other side, they become positive. 1 over ΩL can be taken out. g_m plus 1 over r_e into $\beta + 1$. Still, this can be ignored because g_m is 1 over r_e ; that divided by $\beta + 1$...you can just compare; plus 1 over r_{ce} , because of this.

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$$\frac{g}{\Omega L} \left[\frac{g_m}{\beta + 1} + \frac{1}{r_e(\beta + 1)} + \frac{1}{r_{ce}} \right]$$

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So, this is taken care of; this is taken care of; and so essentially in this, the quantity of interest is only g_m . These become negligibly small.

So, that is equal to this and that, ω into C_2 divided by r_e into $\beta + 1$ plus C_1 by r_{ce} . So, this is the condition for oscillation.

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$$\omega = W \left[\frac{C_2}{r_e(\beta+1)} + \frac{C_1}{r_{ce}} \right]$$

So, this ω can go here. This will become ω squared. So, 1 over ω squared is equal to... ω naught squared actually. This, we will say, this ω naught and that ω naught should be the same.

1 over ω naught square should be equal to $L C_1 C_2$ by C_1 plus C_2 ; $L C_1 C_2$ by C_1 plus C_2 ; and therefore, you see that L gets cancelled and essentially this is negligible. g_m into $C_1 C_2$ by C_1 plus C_2 should be equal to C_2 by $r_e \beta + 1$ C_1 by r_{ce} .

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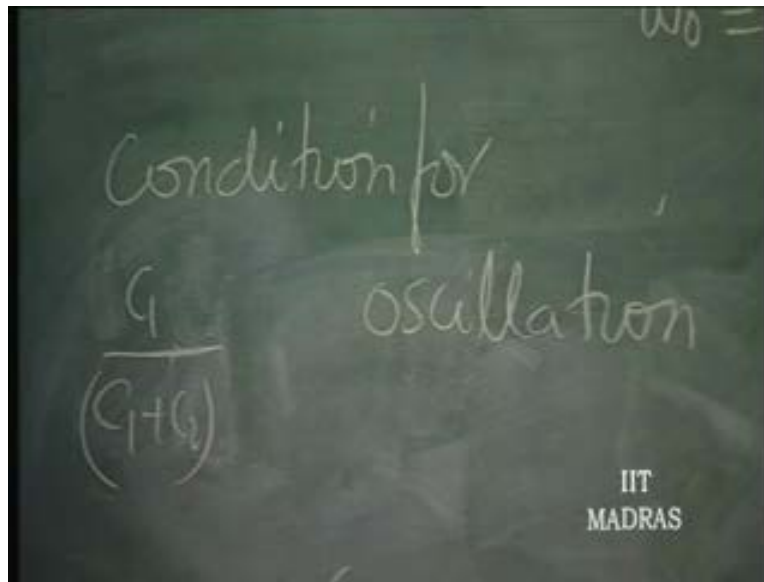
$$\frac{g_m}{(C_1 + C_2)} [g_m] = \left[\frac{C_2}{r_e(\beta + 1)} + \frac{C_1}{r_{ce}} \right]$$

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Essentially, this can be ignored compared to this, normally. Or, you can take that also into consideration. So, g_m into C_1 by C_1 plus C_2 - this is the condition to be satisfied. Now, g_m can be taken... g_m into r_e is very nearly 1. g_m is equal to 1 over r_e . So, this divided by g_m into r_{ce} . So, 1 over $\beta + 1$ should be equal to C_1 by C_1 plus C_2 .

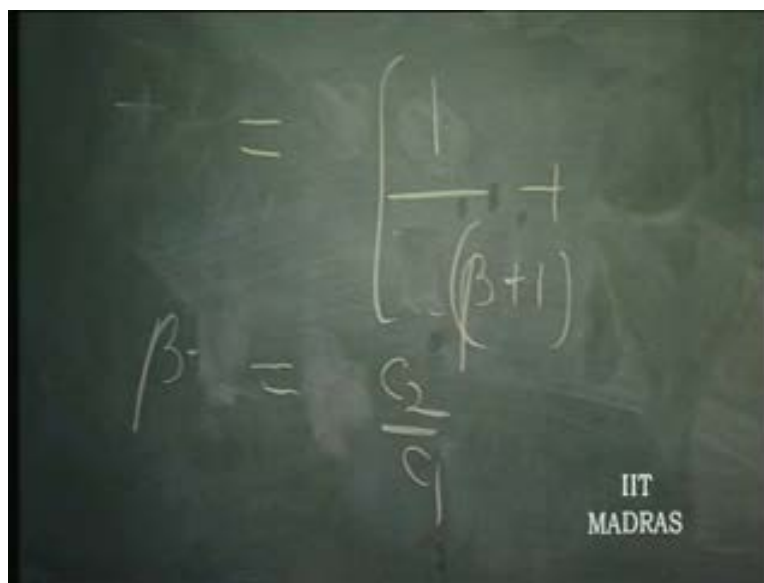
C_1 C_1 plus C_2 is always a quantity less than 1 and that can be easily satisfied by 1 making it equal to 1 over $\beta + 1$ plus C_1 by C_2 g_m into r_{ce} . So, this is the equation; condition for oscillation.

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If r_c is infinity, then this also goes. The frequency of oscillation becomes exactly this. The condition for oscillation becomes very simple. Beta plus 1 becomes equal to 1 plus C_2 over C_1 . So, 1, 1 get cancelled. So, Beta becomes equal to C_2 over C_1 . Or, C_2 over C_1 has to be chosen to be equal to Beta. That is the condition for oscillation.

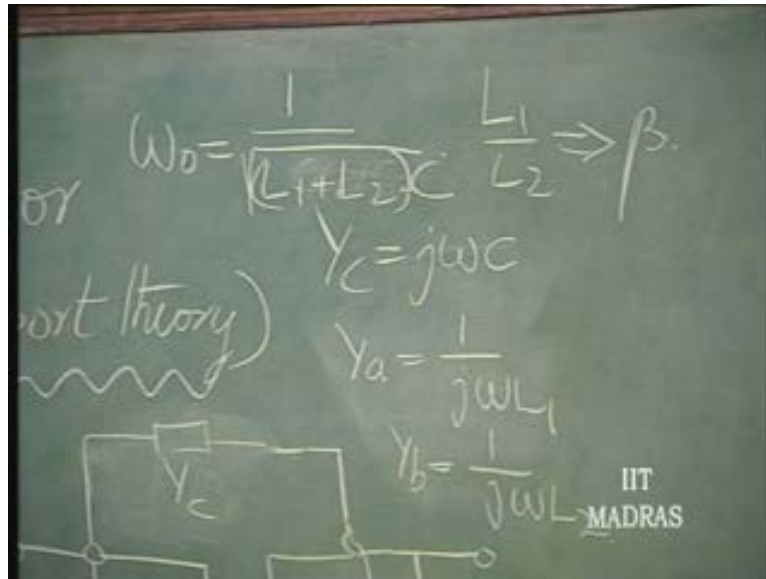
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I would like you to work out as a problem when Y_c is $j\omega C$ and Y_a is 1 over $j\omega L_1$ and Y_b is 1 over $j\omega L_2$. The frequency of oscillation can be shown to be ω_0 . Show that ω_0 is equal to 1 over root of L_1 into C plus...sorry. L_1 plus L_2 into C .

And, condition for oscillation is now governed by...here it was C_2 over C_1 . There it will be L_1 over L_2 becoming equal to β . That is the only difference. So, condition L_1 over L_2 - how it is related to β .

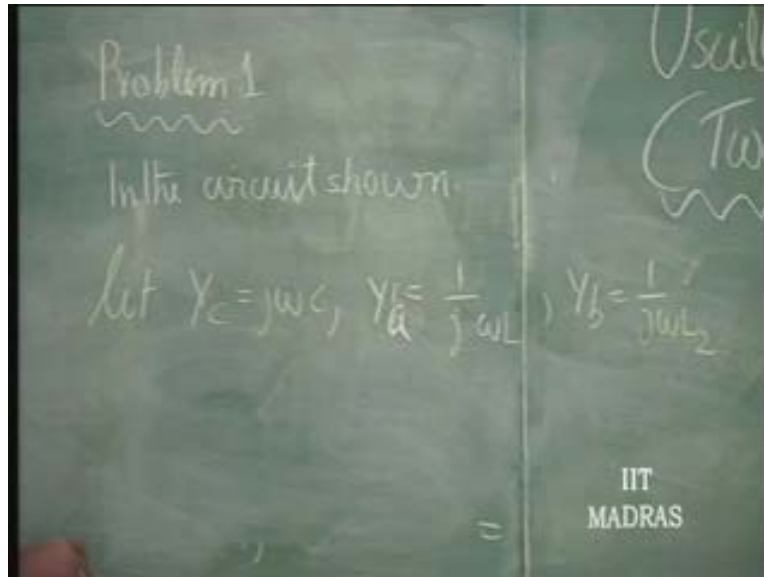
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This can be determined by replacing Y_c by $j\omega C$, Y_a by 1 over $j\omega L_1$, $2 Y_b$ by 1 over $j\omega L_2$. So, please work this out as a problem. So, let me now give you the complete problem.

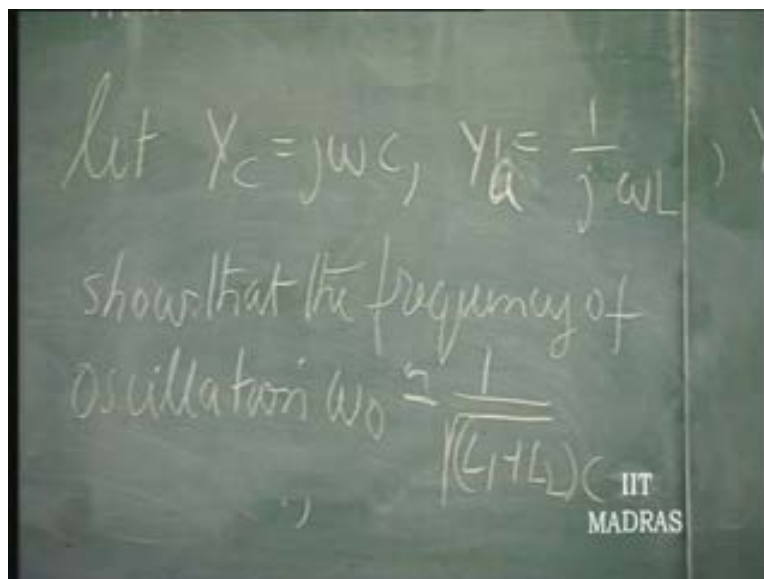
So, in the circuit shown, let Y_c equal to $j\omega C$, Y_b equal to... Y_a equal to 1 over $j\omega L_1$, Y_b equal to 1 over $j\omega L_2$.

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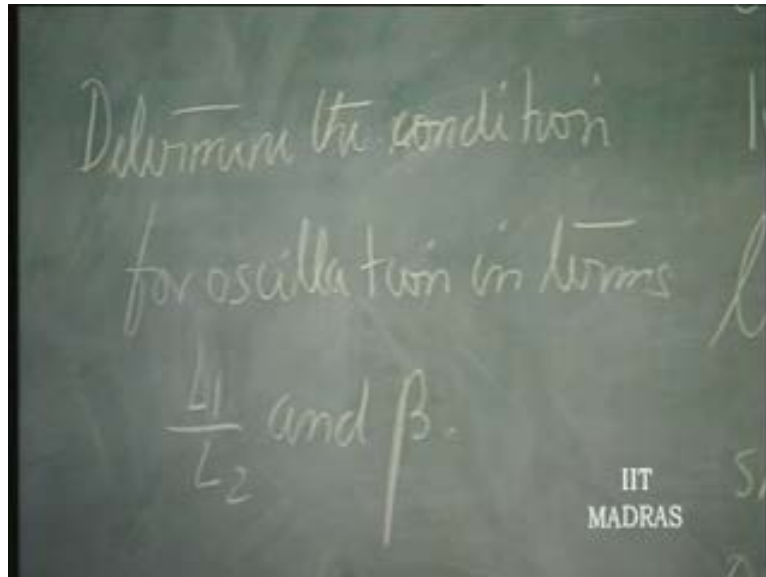
Show that the frequency of oscillation ω_0 is very nearly equal to 1 over root L_1 plus L_2 into C .

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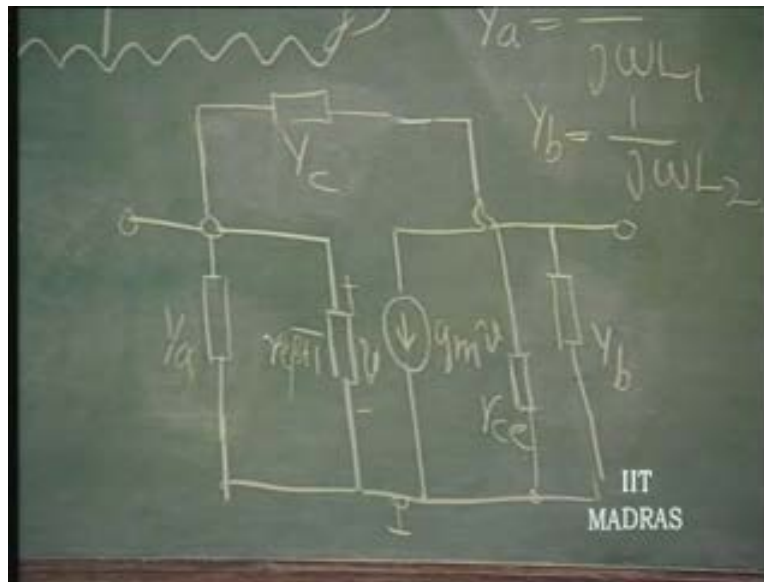
Determine the relationship...determine the condition, for oscillation in terms of L_1 by L_2 and β .

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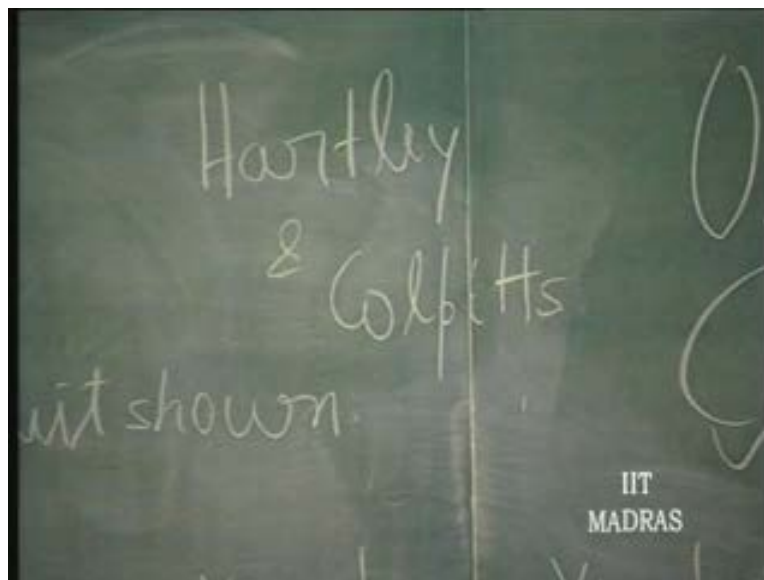
So, these are two of the most popular LC oscillator, transistor, or FET oscillator; we can replace. The same analysis is valid for transistor as well as FET, except that r_e into $\beta + 1$ is replaced by open circuit and rest of the thing remain same. This is g_m . This is going to be replaced by r_{ds} . Except for that, rest of the circuitry remains the same; and therefore, the same analysis can be adopted for the field effect transistors, oscillator also.

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These are respectively called Hartley and Colpitts oscillators. These are the most popular LC oscillators existing today.

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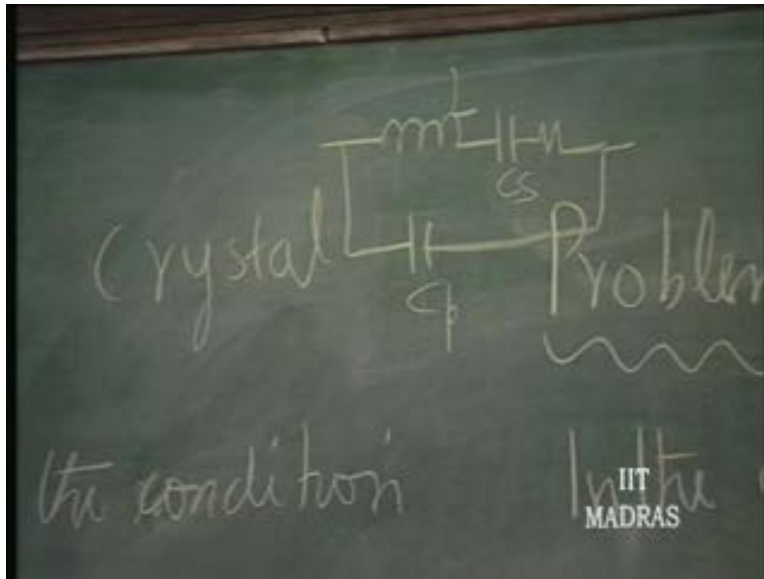


Apart from that, you have tuned, let us say, collector and the transformer coupled base type of oscillators also available; but again, the same type of analysis is valid there also.

The coupling between output and input is a transformer coupling. So, it is a tuned collector transformer coupled base which is used as an oscillator.

Then we have crystal itself being used as an L C. Lock crystal can be used both for series resonance as well as parallel resonance. It is nothing...Equivalent of a crystal is...that means it has both series capacitor as well as parallel capacitor. So, it can resonate with this capacitor in terms of series resonance and the parallel resonance frequencies, very close to series resonance.

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And therefore, it is going to result in either a short circuit or an open circuit. That means impedance variation can be drastic and phase variation also can be drastic, around the resonance frequency. And therefore, such a circuit can be used as a feedback structure where it can take on inductive or capacitive reactance at the frequency of resonance; and therefore, it will always, invariably, oscillate at the crystal frequency.