Electronics for Analog Signal Processing - II
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Lecture - 15
Oscillators (Continued)

We had discussed a variety of oscillators so far. R C oscillators - under this category, we had discussed Wien bridge oscillator, phase shift oscillator. L C oscillators - under this category, we had discussed Hartley Colpitts oscillator and also variegated simulated oscillators.

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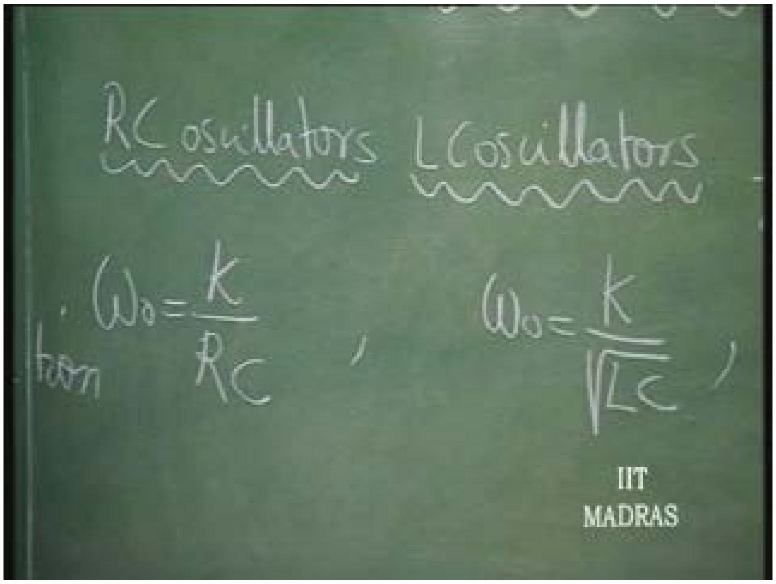

And crystal oscillators - we just mentioned at the end of the last class that the crystal can replace an L C circuit because basically, a crystal is a combination of series resonance and parallel resonance circuit, actually represented...

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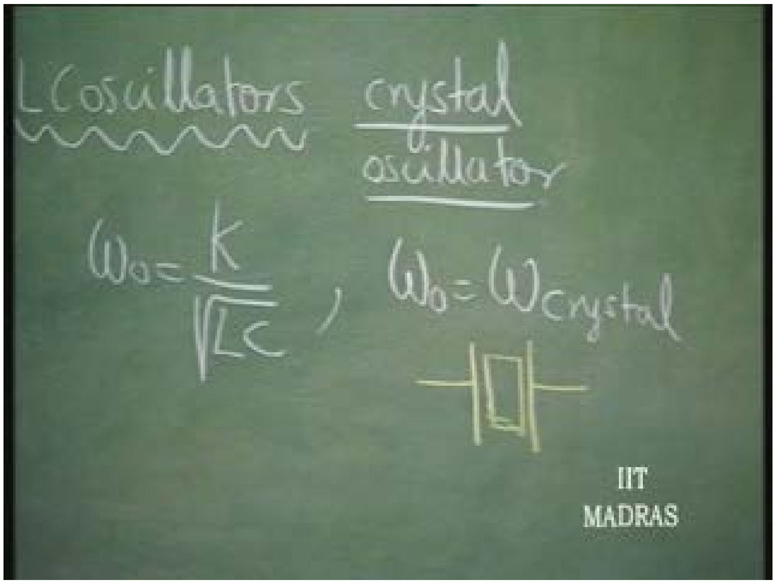

So, this crystal has an inductor in series with a capacitor and a small resistance and then a shunt with... Of course, this is not --- This is going to be the electrode capacitance here. So...and also this ---- capacitance. It has a series resistance and a series capacitance and an inductance. And therefore, at the series, resonance frequency is Omega naught is equal to 1 over root L s C s. That is the series resonance frequency.

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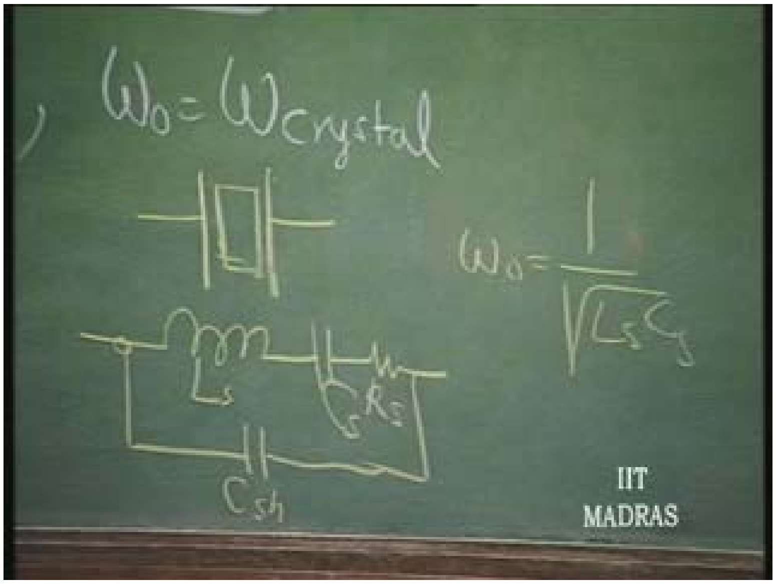

And the parallel resonance frequency is, Omega naught is equal to 1 over root L... Across L, you have C s coming in series with C shunt. So, C s C shunt...we will put down this elsewhere.

This is series resonance. This is parallel resonance. 1 over root of L C s C shunt divided by C s plus C shunt. They are coming in series. So, effective capacitance is there. That is the parallel resonance circuit. Parallel resonance. It can be seen that this is very nearly equal to 1 over root L s C s primarily because you can take this out; root of 1 by 1 plus C s by C shunt.

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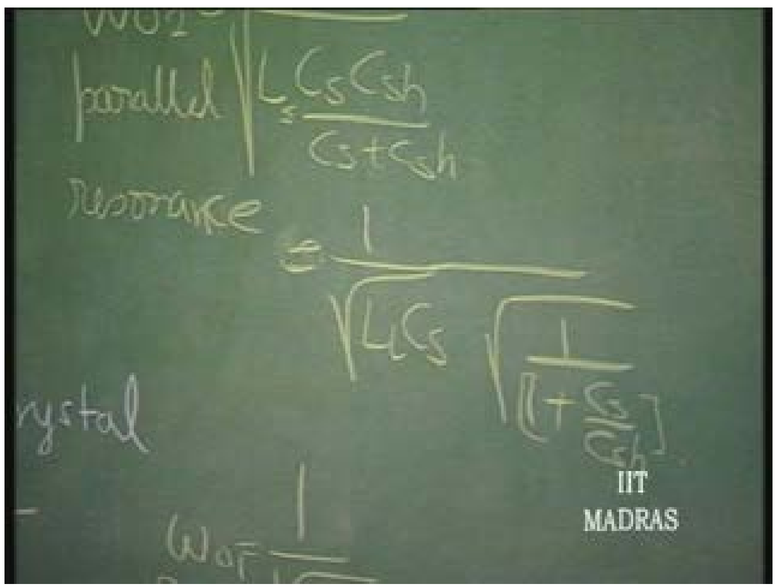

This L s C s you take out, then divide by C shunt. Because this factor is going to very small value, that means actually, if you put additional C shunt, it is closer to the actual. That means, basically, this ratio is very small compared to 1, by structure. So, additional C shunt will again make it closer to this series resonance.

So, this fact has made this very popular because you can get a wide range. For example, when it is in series resonance, it is a short circuit basically. That means, actually, it is a small resistance in series. That is R s. And a parallel resonance, it is going to be an open circuit. That means a huge resistance value which is Q square times R s. Basically, the quality of this is very high.

So, it is an open circuit. So, the impedance level can go from short circuit to open circuit around a small change in frequency. That small change in frequency depends upon this deviation here.

So, you can see that this is a very versatile circuit which will act as capacitive or inductive or short circuit or open circuit around the same frequency. This is an important property in what we call now as frequency stability in oscillators. Let us understand this basic concept. Why? What is frequency stability? The frequency of oscillation in all these oscillators, for example in R c oscillators, is determined by the resistance and the capacitance this way. Omega naught is invariably equal to K by R c.

You need a minimum of 2 R c networks, 2 time constants in order to make any oscillator. Not single...with single time constant, you cannot make a harmonic oscillator because second order is the basic requirement of second order differential equations.

So, 2. That means basically, this is going to be some K divided by root of some R 1 R 2 C 1 C 2, for any oscillator; K differing depending on the type of oscillator. That means sensitivity of Omega naught to R 1 R 2 C 1 C 2, they are all same. Sensitivity in this case is equal to half. What is sensitivity? Sensitivity is defined as, Omega naught to R 1, is defined as Delta Omega naught by Omega naught for Delta R 1 by R 1. This is the definition of sensitivity.

What is the change, percentage change in frequency, for a change in the component value? That is defined as sensitivity. Percentage change in the frequency for percentage change in the component value; that is defined as the sensitivity of Omega naught to R 1. Similarly, sensitivity of Omega naught to R 2; this R 2 will come here, Delta R 2 by R 2.

Now, this one if you determine for this, will be equal to minus half for all these components. That is because there is a root coming. So, you will see that sensitivity is going to be minus because it is coming in the denominator. So, whenever anything comes in the denominator we have negative coming. That...when that component increases, this other parameter decreases. That is indicating negative and it is half.

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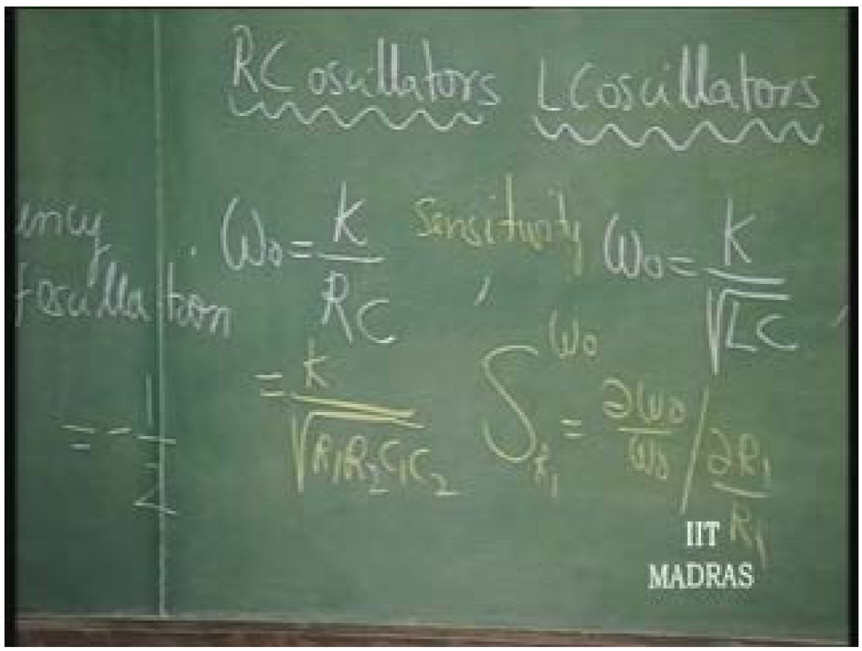

You can see this. Therefore, the sensitivity of passive component here for L C oscillators is also the same – half, minus half; L and C. This is the same; R 1 R 2 C 1 C 2; L C. So, these are all... So, sensitivity to passive components remains the same in all these cases. L s and C s also is half except that for C shunt, it is not sensitive at all. This you can see. For sensitivity of Omega naught for C shunt is very nearly zero.

So, I am discussing this passive parameter sensitivity in order to illustrate that apart from this passive parameter sensitivity, it is bound to be sensitive. There is no way avoiding it. What it means is if resistance varies with respect to time, temperature, etcetera, the frequency is going to drift; but therefore, we will make sure that these resistances are chosen in such a manner that there is no drift in the frequency of oscillation, because they are pretty sensitive to these variations.

So, passive parameters are made stable in their value. They are selected in such a manner that these values do not change with temperature or time so that the frequency does not change with temperature or time. That is one aspect of frequency stability. Whether it is R C oscillator or L C oscillator, the performance is the same in terms of passive parameter sensitivity.

It is the same. What it means is if I am designing a L C oscillator, the capacitor I put should be more domining than the parasitic capacitor so that the frequency stability of my circuit is good; or the inductor I put in my design should be greater than any lead inductance in order that the stability of oscillator is good.

So, this is illustrated in the crystal situation also. The parasitic normally comes as a shunt capacitor and the sensitivity to shunt capacitor should be very low. That is one aspect. Next aspect is...this is different. Active parameter sensitivity.

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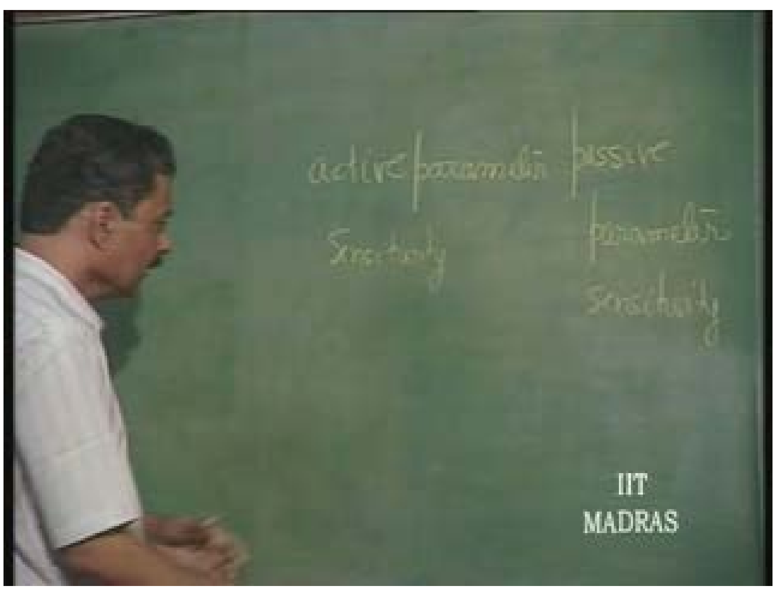

In realizing these oscillators, we had used op amps, transistors, etcetera. What is active parameter sensitivity? It is the same thing. If Omega naught is sensitive to, let us say, the parameter, active parameter of the device that you are using...let us say, Beta of the transistor, for example, one active parameter is Beta of the transistor; or, g m of the transistor. These are all active parameters. In the case of --- self for transistors; FET and this thing. For...and also input resistance, output resistance. These are all the parameters which will influence the...in fact, I should not put input resistance. I should put as input impedance, output impedance; these are parameters which will affect the performance of the oscillators.

Similarly, in the case of op amp, A naught, open loop gain, and actually, A naught also depends upon frequency. Let us say this is called the band width; let us say dominant pole. We will discuss this later. It has a frequency dependence; the gain has a frequency dependence; so, its frequency dependence. Again, input impedance, output impedance. These are the parameters which are responsible for changing this.

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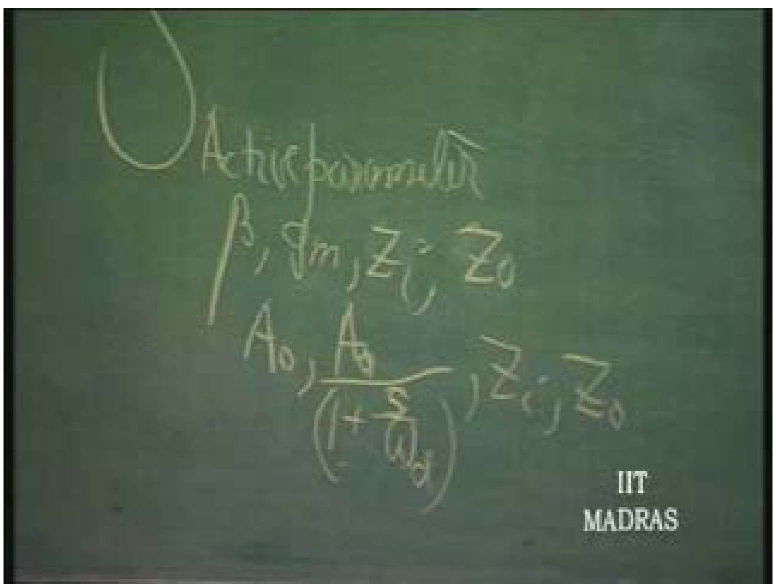

If the frequency is dependent upon these parameters, then these parameters in turn will depend upon supply voltage, temperature, and the device itself; when I change the device, these may change because these properties are different for the other device. So, the stability of such oscillator which is designed using these components is now poor, if the dependence is heavy. If the dependence is not so heavy, the stability is good. Now, how does stability come into picture in oscillators? This part we will discuss now.

In any oscillator we have seen that...we have discussed it in terms of two things: one is the loop gain. This is the loop gain. The loop gain becomes equal to 1. This is both in phase as well as magnitude. That is, there is no phase, zero phase; and the magnitude becomes equal to 1 at a certain frequency Omega equal to Omega naught. Then it oscillates. This is what we have shown. At that frequency, it oscillates. This is the basic principle.

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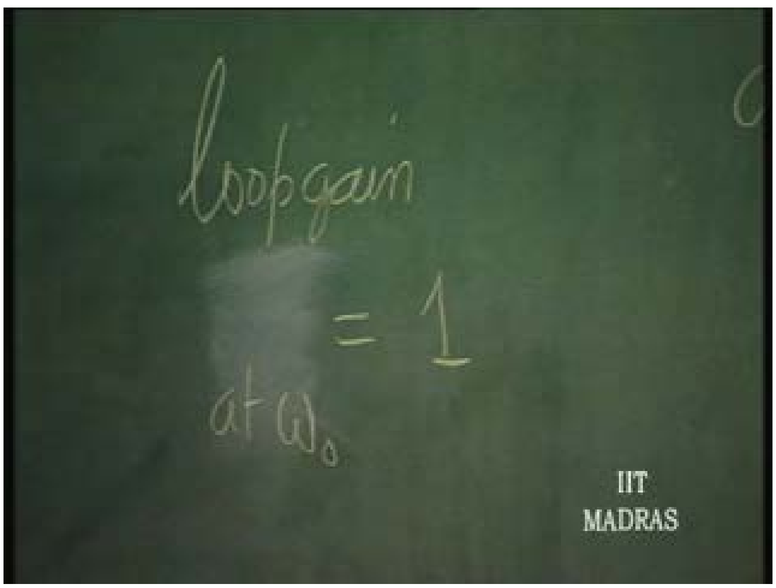

Now, let us consider this. I am, for illustration, taking Wien bridge oscillator so that you can understand this clearly.

We have put here resistances, let us say, R a and R b. The gain was 1 plus R b by R a and we had put here resistors. In fact, it could be just R 1 C 1 R 2 C 2. You remember this.

Then we had closed this loop. So, this was our Wien bridge oscillator and we had got some condition for oscillation, etcetera. We again derive that.

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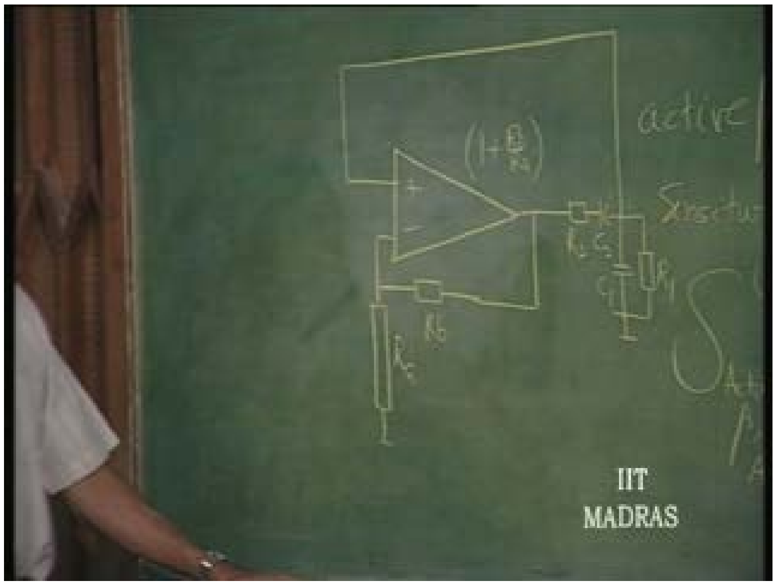

1 plus... if this is V i, we just said this is 1 plus... I will break this loop here. This is V i; this is 1 plus R b by R a times V i. That multiplied by... this is how we had derived. Let us say, Z 1 by Z 1 plus Z 2 is the voltage here and I divided by Z 1, 1 plus Z 2 by 1 it became. And then, we considered this as Z 2 is R 2 plus 1 by S C 2 and Z 1 is 1 over R 1 plus S C 1; and we just wrote this as 1 plus R b by R a V i by 1 plus R 2 by R 1 C 1 by C 2 plus 1 over S C 2 R 1 plus S C 1 R 2.

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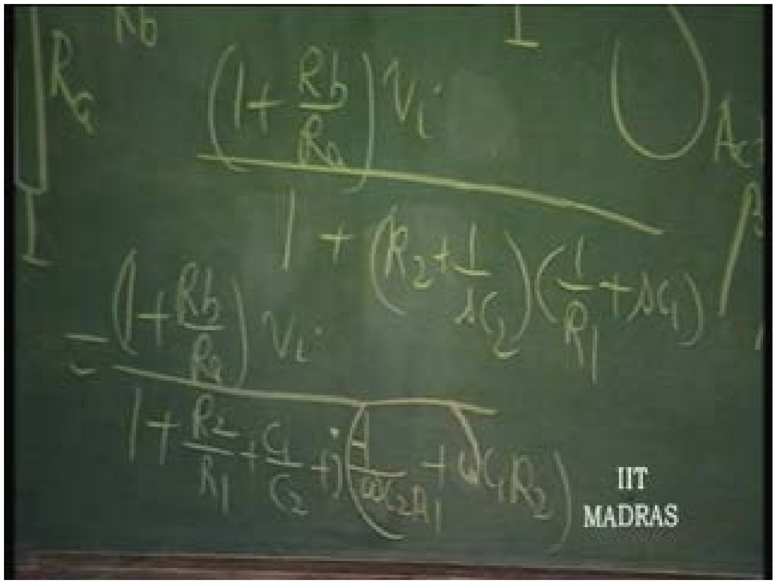

This was frequency dependent and this was... I brought out the j here, let us say. This became minus Omega and this became plus Omega.

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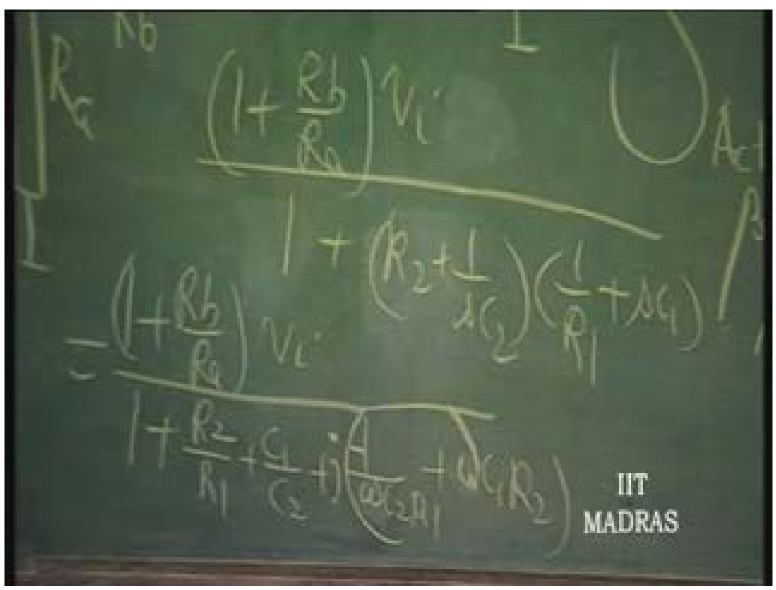

So essentially, we had this going to zero at Omega naught equal to 1 over C 1 C 2 R 1 R 2. This all vanishing; and we made R b by R a...that was the condition for oscillation, equal to R 2 by R 1 plus C 1 by C 2.

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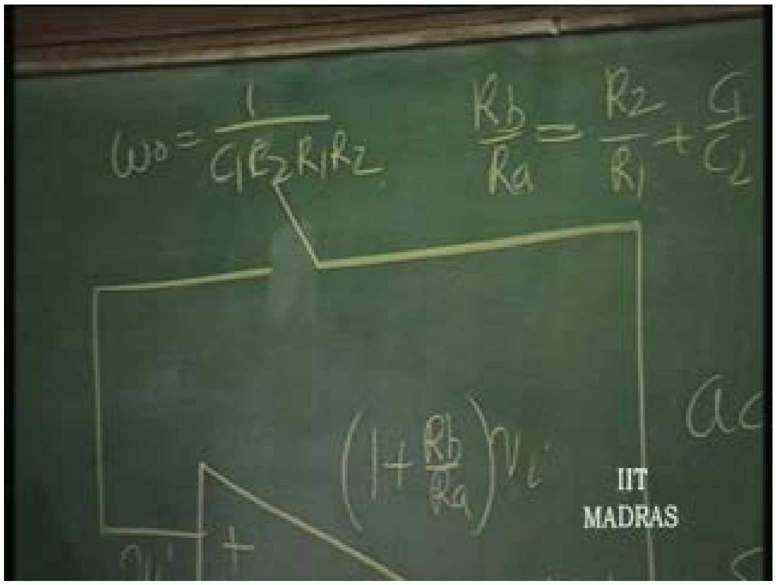

You remember this. And we made it all equal. R b by R a equal to R 2 by R 1 equal to 1; C 1 by C 2 equal to 1; R b by R a equal to 2. When R 1 equals to R 2, C 1 equal to C 2, this was R, this was C.

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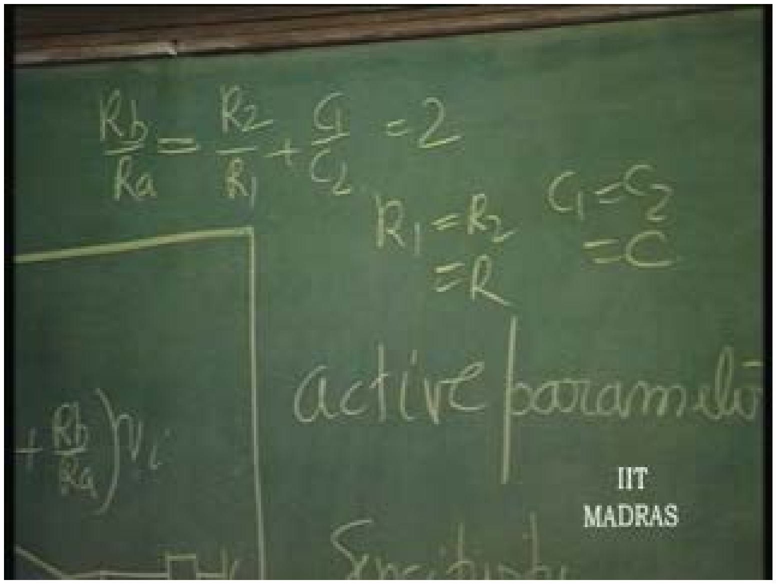

And therefore, this was Omega naught square, this was C square R square.

So, Omega naught was 1 over C R. This, I am just repeating for completion sake.

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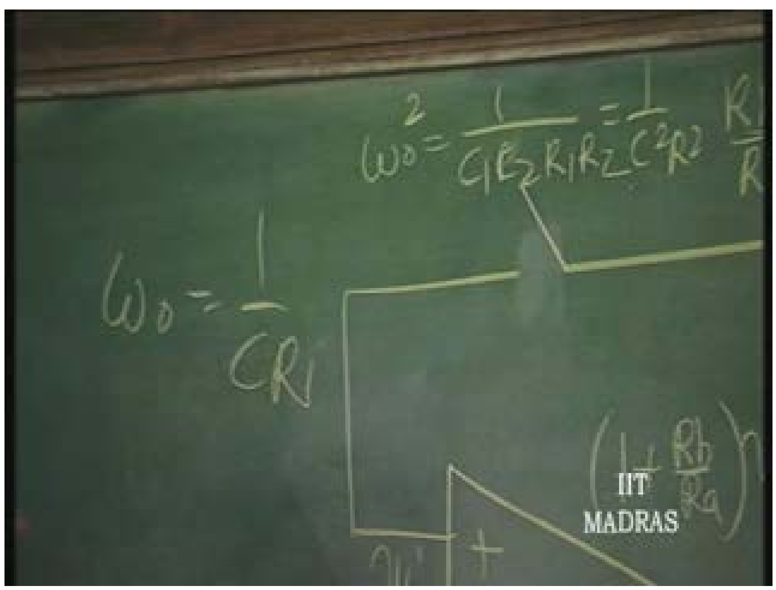

Now, what is active parameter sensitivity? Passive parameter sensitivity is clear. What is active parameter sensitivity?

The gain here is finite; let us say, A. If that is the case, it is not infinite. I am considering only one aspect here. The gain is not infinite. We are not considering input impedance, output impedance. That will further add to our troubles because output impedance will add in series with R 2 and input impedance will add in shunt with R a.

So definitely, if input impedance and output impedance become comparable to R a and R 2, it is going to cause stability problem. In this case, we will not consider that. We will consider only the gain aspect and show... So, if this is A, we had earlier derived that the gain is not this; the gain is A divided by 1 plus A into R 1 R a by R a plus R b. Do you remember this? For the non-inverting amplifier, we had derived this gain, when it is not infinite. A by 1 plus A into Beta, Beta being R a by R a plus R b.

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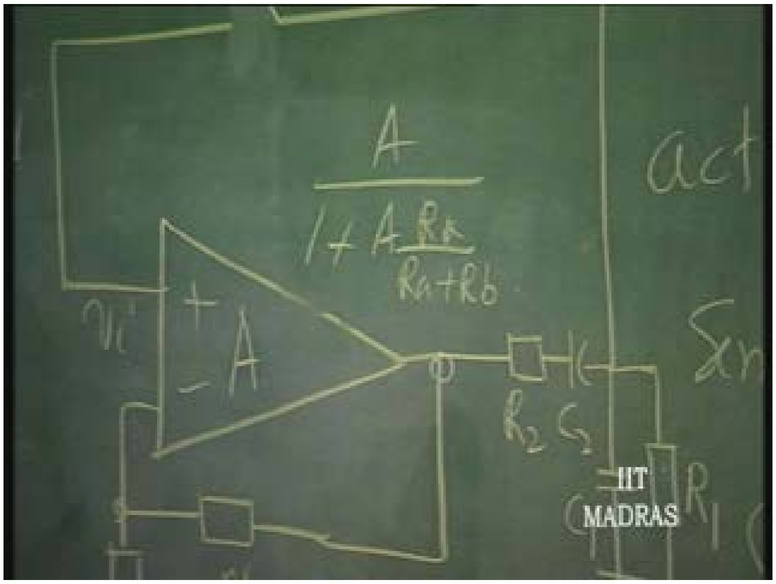

So, actually speaking, we will divide it by A Beta throughout. So, the gain is really 1 plus R b by R a which is correct, divided by 1 plus 1 over A Beta. A Beta is considered as the loop gain for this. So, the loop gain A into Beta, it can be rewritten this way. That is the error.

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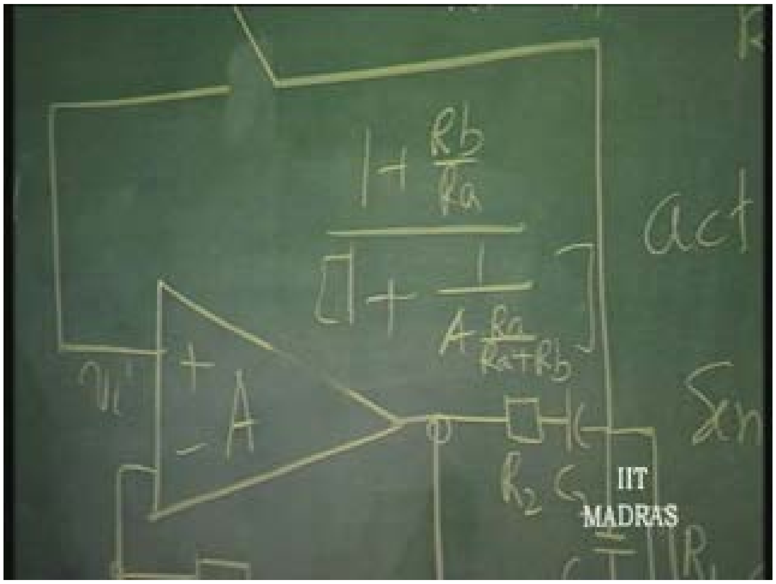

It will...if A is real and equal to A naught, there is no problem. It is less than 1 plus R b by a R a by certain amount. I have to make R b by R a slightly higher than 1; still frequency stability is not going to be disturbed. Is this clear?

If A is real, this will be simply A naught and this whole factor has to be made equal to, let us say, 2. That is, this whole factor has to be made equal to 1. R b by R a should be close to 2 --- higher than 2. That is all that has to be done. So, this whole factor should be made equal to 1 plus, let us say, R 2 by R 1 plus C 1 by C 2. This is the condition.

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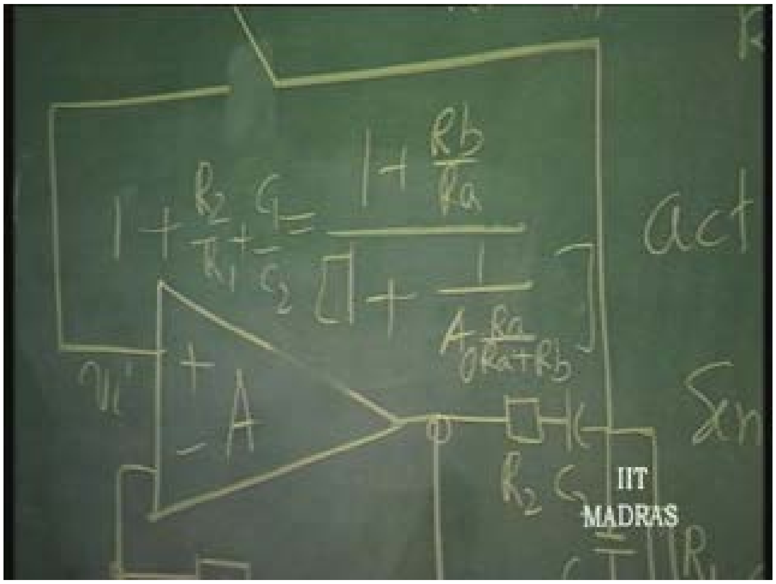

And therefore, if A naught is not infinite, there is no problem of satisfying this condition. That is not coming into picture in the frequency of oscillation. Nothing comes into picture as far as A naught is concerned; but if A naught is also frequency dependent, then there is a problem.

Let us say A naught... A is frequency dependent and it is A naught by 1 plus s by Omega d, as we have put.

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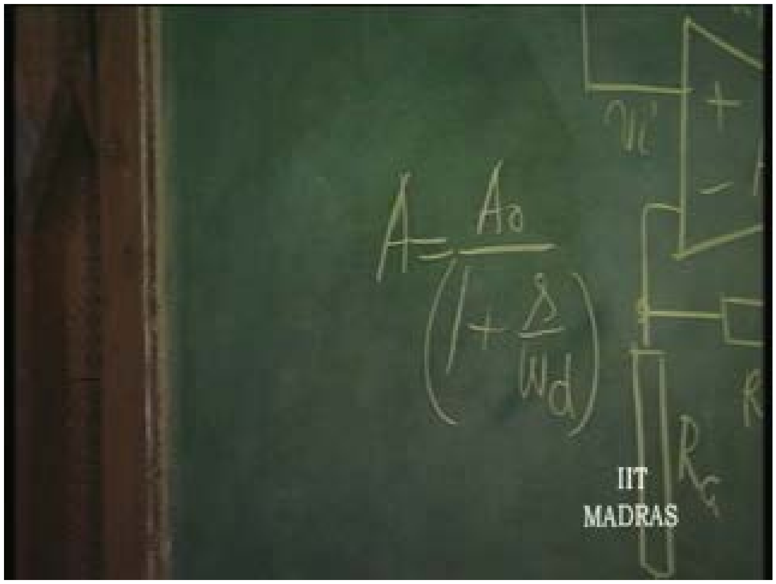

If this amplifier gain is frequency dependent, then we will see that this A naught divided by n... now, this is not the case. This is not the real part at all because this is also contributing to phase shift. Do you see that? That means this was purely real from here to here. Earlier it was purely real; either it was 1 plus R b by R a or 1 plus R b by R a divided by 1 plus 1 over loop gain, as long the loop gain was real.

But now, the loop gain is contributing the phase shift. The whole thing therefore says that the output is not exactly in phase with input. This is not an exact non-inverting amplifier. There is an error. To that extent... Earlier, from here to here, the phase was zero; and therefore, at the frequency Omega naught, we could just make the phase equal to zero for this network also. So, it was oscillating with output being equal to input or loop gain being equal to 1. So, I could close this exactly.

But now, what happens? Let us say there is a phase error, phase lag of 1 radian. I mean, I am just giving it an as an example. At the frequency of interest to me, it is giving as much phase shift as 1 radian, let us say. That depends upon the Omega d; 1 radian. Then the frequency at which I should select this network is not Omega naught any longer. It should be deviating from Omega naught such that it will give a phase lead of 1 radian.

So the phase shift of this should be such that it will give you a phase lead. That means, it will be different from Omega naught. How much it is going to be different is going to depend upon this phase shift here. So, we have to study the phase variation of this network, passive network, Z 1 by Z 1 plus Z 2, in order to say how much it should be different from Omega naught, in order to give a phase contribution of 1 radian phase lead, so that the phase lag there by the active device is compensated for by the phase lead here.

Suppose that I compensate at certain frequency and for 1 degree it gives you the frequency of oscillation slightly different from Omega naught, so as to give a phase lead of 1 degree. Next, it will change to 1 point 5 degrees because of temperature variation. Then this will change correspondingly to a phase lead of 1 point 5. That means the frequency has to change again.

So, now you see how frequency of oscillation of this Wien bridge oscillator is directly dependent upon the frequency; that is, variation and the phase variation with respect to frequency here. So, let us plot the phase variation of this network with frequency. If you plot that...

You can see that this part of the network gives you this kind of thing. 1 plus 1... R 2 over R 1 C 1 over C 2 plus j into Omega C 1 R 2 minus Omega C 2 R 1. This is the expression.

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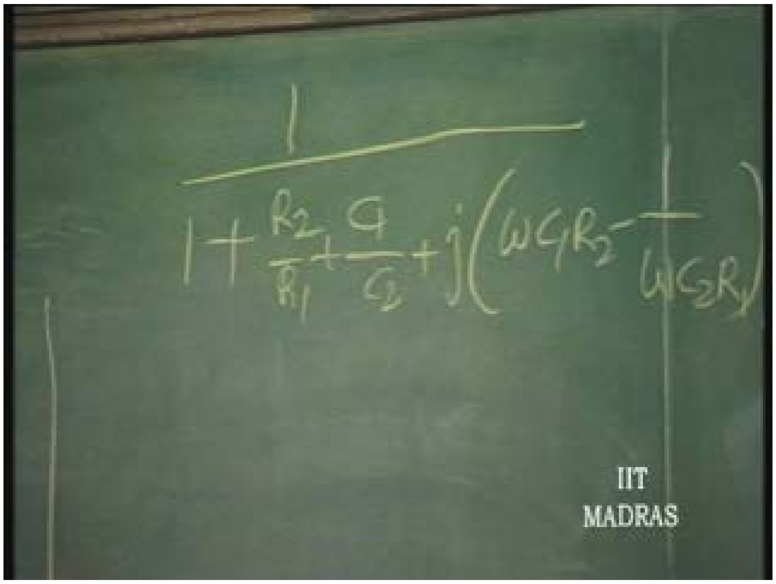

R 2 is equal to R 1, nominally. So, consider the nominal values. R 2 is always made equal to R 1. So, this is equal to 3. So, this is C, this is R. This is the condition.

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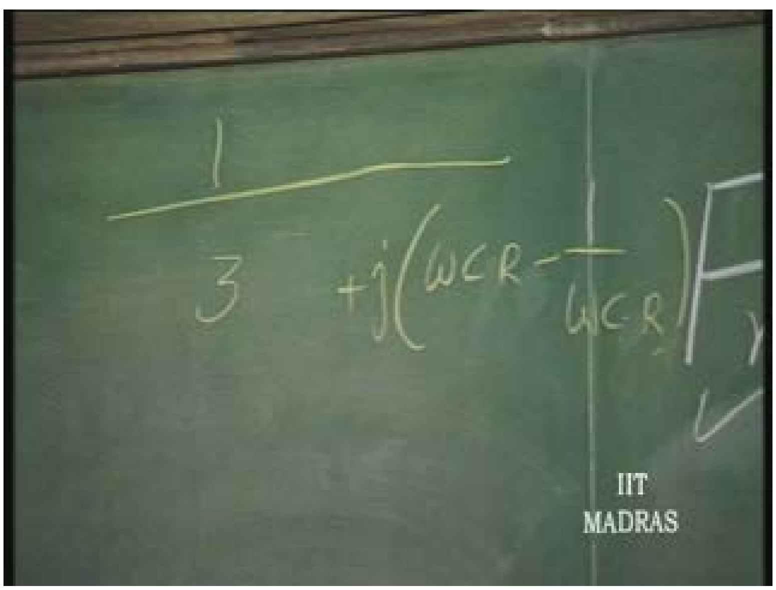

You will see that at Omega equal to 1 over C R, at Omega equal to 1 over C R, this phase is zero. It is just 1 over 3. So, at Omega equal to 1 over C R, which we are calling as Omega naught, this is not the actual frequency of oscillation. This is the theoretical frequency of oscillation. At Omega naught, the phase is zero. Let us say, phi is zero here. At very low frequencies, just consider; at very low frequencies, this is going to zero. This is going to become very huge.

So, it is only the contribution due to this. This is minus... 1 over minus j; or, plus j. That means it will give you a phase shift of 90 degrees. At very low frequencies, it gives you a phase shift of 90 degrees. At very high frequencies, this goes. This becomes dominant. This becomes negligible. So, at very high frequencies, this gives you a phase shift of minus 90 degrees. So, it is going to change in this manner.

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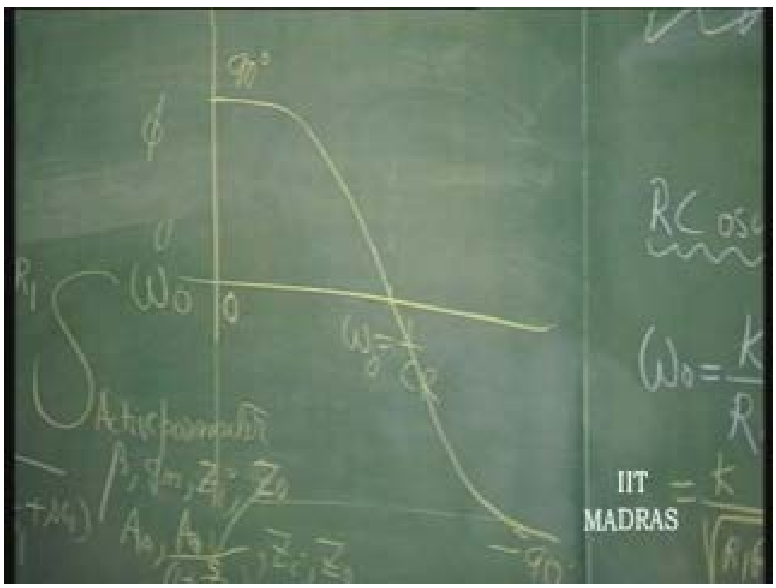

How it changes at this point is important. Now, you can see. If this 90 degree is lag, this 90 degree is lead. That means if it has to contribute to lead angle, then the frequency fixed by this network should be less than Omega naught, which is 1 over C R.

So obviously, if amplifier network is giving you lag, this frequency at which this is going to give lead is going to be less than Omega naught. That is invariably the case. By how much it is going to be less depends upon the phase that you want; lead that you want to give; to compensate for the phase lag that it has suffered through the active device.

How much it is going to deviate here depends upon this slope here. That means Delta phi by Delta Omega is an important factor because this is one curve; another curve may look like this. You know. Another curve may look like this. Let us now see this curve.

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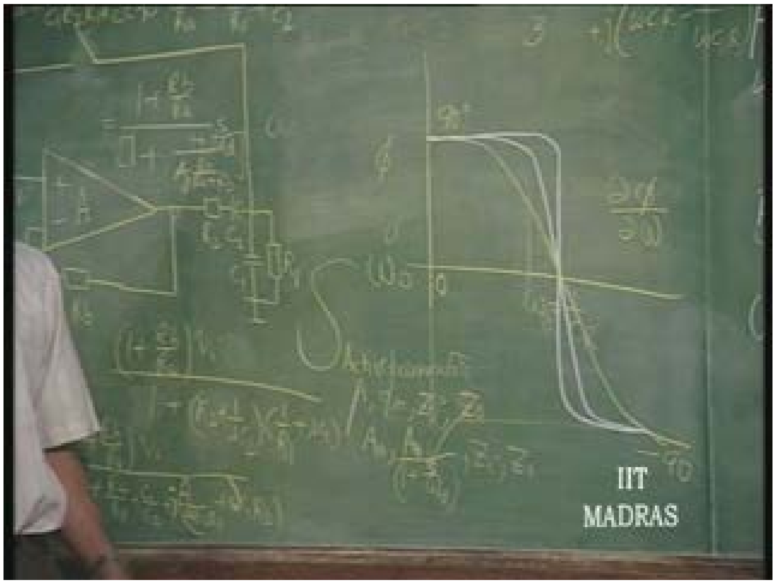

If Delta phi by Delta Omega is very high, Omega oscillation, actual oscillation will be very close to Omega naught irrespective of the phase lag contributed by my active device. This is an important aspect. That means if this is a steep thing like this, then the frequency stability is good. If the passive network can give a steep thing, whereas, in the case of passive R C network, this you can find out. For this phi is minus tan inverse Omega C R minus 1 by Omega C R; this divided by 3. So, this is the phi.

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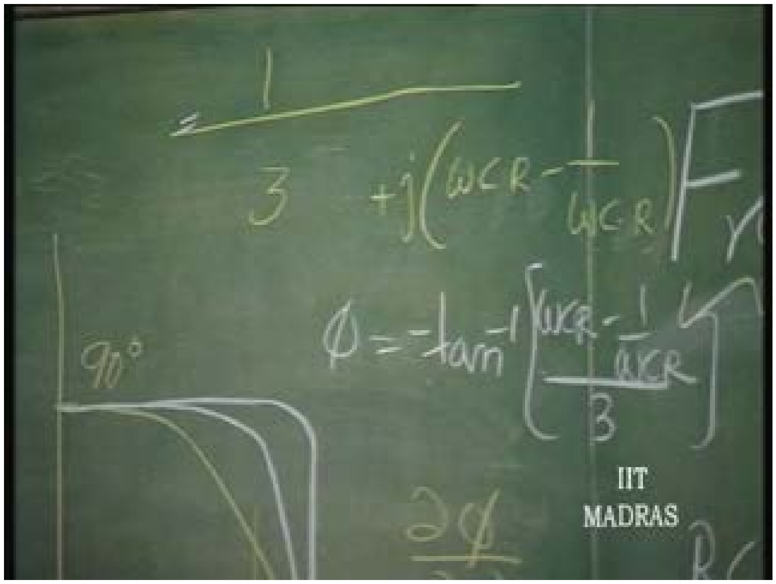

So, you can differentiate and find the maximum at Omega equal to 1 over C R; and you will find that this quantity is not much for any passive network, for that matter. Whether it is phase shift or Wien bridge, you can always find out Delta phi by Delta Omega. It is not going to be minus; whereas in the case of a crystal or an L C oscillator, this is going to be pretty high. It is directly proportional to Q; and Q for the passive R C network is never greater than half.

That means all the poles of these passive networks always lie on the negative real axis. You cannot get complex conjugate pair of poles and you can prove that Q cannot be greater than half; whereas, in the case of L C network and crystal, Q can be pretty high. Complex conjugate pair of poles will occur and these pair of poles can be very close to the imaginary axis. If they are very close to the imaginary axis, the resonance frequency is close to the actual frequency of oscillation; and therefore, the phase shift variation is directly proportional to Q. The higher the Q, the steeper is the phase variation. And that is why frequency stability of any oscillator directly depends upon the Q, quality factor of the passive network that composes the frequency determining network.

So, irrespective of the device that you use...in the case of crystal, apart from the phase varying so rapidly, even the magnitude of the impedance goes from zero to infinity. So, this facilitates the crystal being used in the loop; and invariably, the crystal oscillator oscillating at the crystal frequency.

At any temperature or any situation, extreme situation, the crystal still makes the oscillator oscillate at the crystal frequency, as long as what? - the loop gain is greater than one in magnitude, at that. That has to be provided by the active device. At all temperatures, if this is satisfied, that is enough; but the phase part of it is automatically getting satisfied. Even the magnitude part is going to be somewhat getting satisfied because the impedance value itself changes in the case of a crystal. So, this is an important aspect of frequency stability.

In order to understand what we have discussed so far in terms of frequency stability, let us try to solve this problem. This problem may also illustrate a typical situation of an amplifier used with L C oscillator or a crystal.

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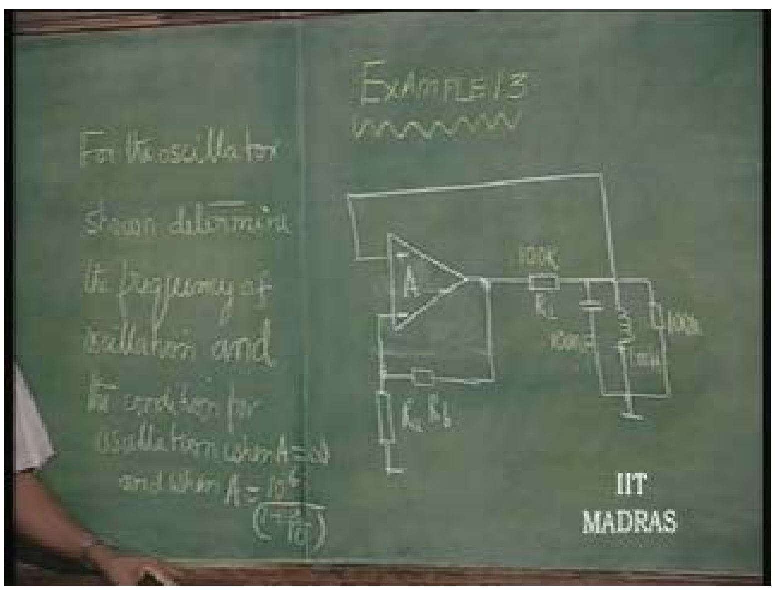

The crystal can be connected here or in place of this 100 K; and the 100 K can be put there. That kind of illustration we can do.

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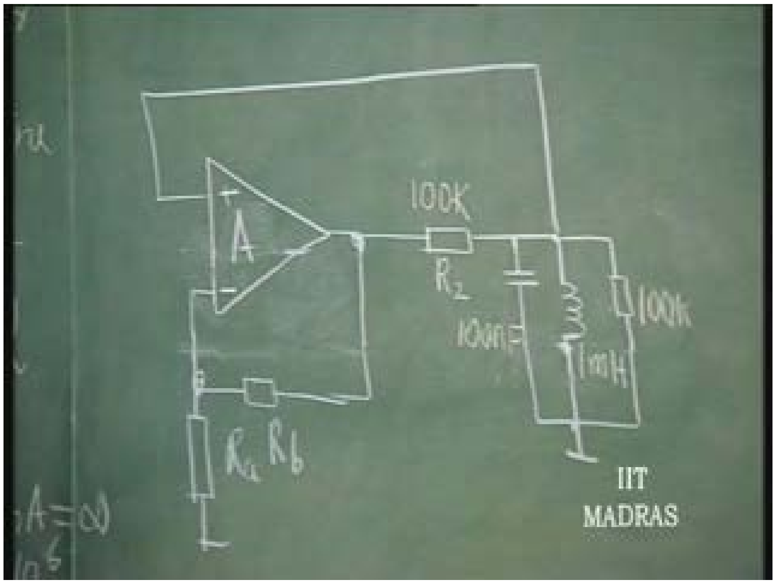

Ultimately at resonance, this L C network will act as a pure resistance 100 K, and that frequency Omega naught is going to be 1 over root L into C; L is 10 to power minus 3. This is the value of L; and C is 10 to power minus 7, 100 nanofarads. So, that is C. That is going to be therefore equal to 10 to power 5 radians per second. That is the resonance frequency which is equal to 15 point 9 Kilohertz.

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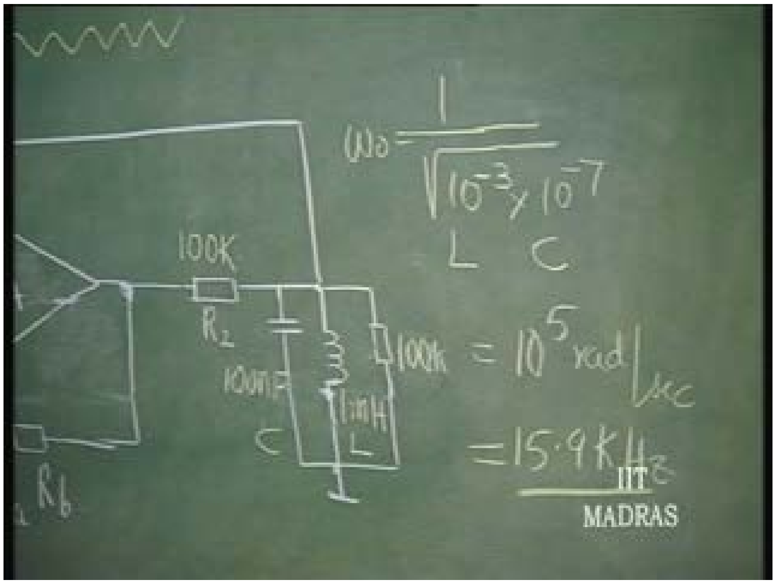

That is the frequency of oscillation I say, because at that frequency of resonance, this whole thing resonates and this will act as a 100 K resistance; and from here to here, the attenuation is half. 100 K divided by 100 K plus 100 K; so, half. So, output voltage will be same as input voltage.

If I make R b equal to R a equal to 1, R b by R a equal to 1, then the gain is going to be 2 from here. So attenuation, half. So, this is the condition for oscillation. This gain is 1 plus R b by R a; and this into the effective attenuation here is nothing but Z divided by Z plus 100 K. This is the effective attenuation.

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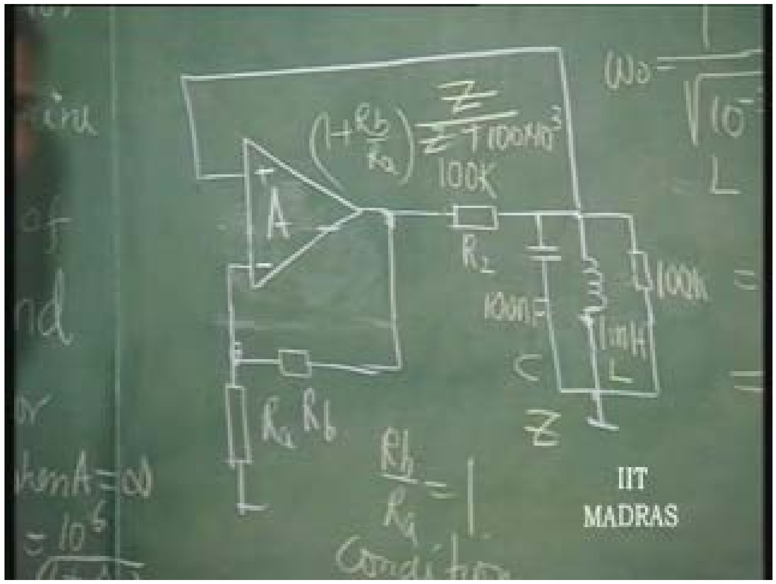

This impedance, we will call it as Z; and that is the loop gain. From here to here, 1 plus R b by R a into Z by Z plus 100 K is the loop gain, if I break the loop here. So, this is the loop gain. So, this has to be... I close this. That means it is made equal to 1.

So, 1 plus R b by R a into 1 by 1 plus 100 K, that is 10 to power 5, into Y.

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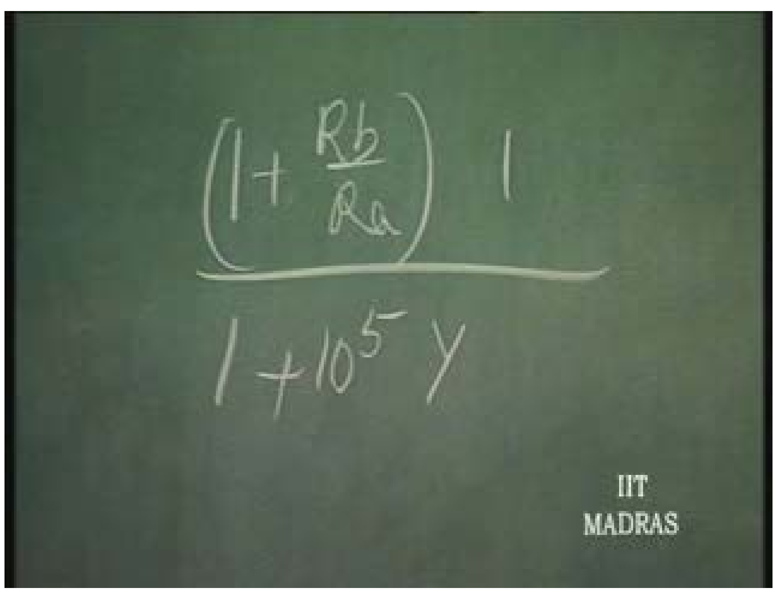

Y is the inverse of Z, 1 over Z. That is nothing but 1 over 10 to power 5. This is the resistance. 1 over 10 to power 5, conductance of that, plus s c, plus s c. C is really equal to 10 to power minus 9; plus 1 over s l; l is 10 to power minus 3. So, that is the transfer function, composite transfer function. So, we will take this inside where... This s c is 10 to power minus 7. It is 100 nanofarads. Yes, thank you. So, we take this inside. This is going to be loop gain.

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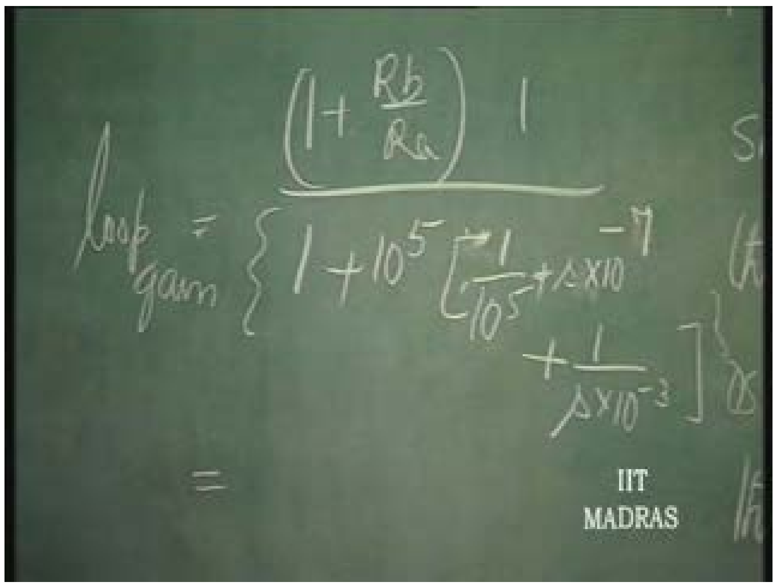

Now, we would like to write it in general for for finite value of A. That means 1 plus R b by R a gets modified. We have seen in the lecture that it will get modified as 1 by 1 plus 1 over loop gain. That is 1 over A into Beta, Beta being equal to R a by R a plus R b. So this, if A is infinity, this will be zero. Otherwise, this is the composite loop gain. This has to be equal to 1 for oscillation to take place.

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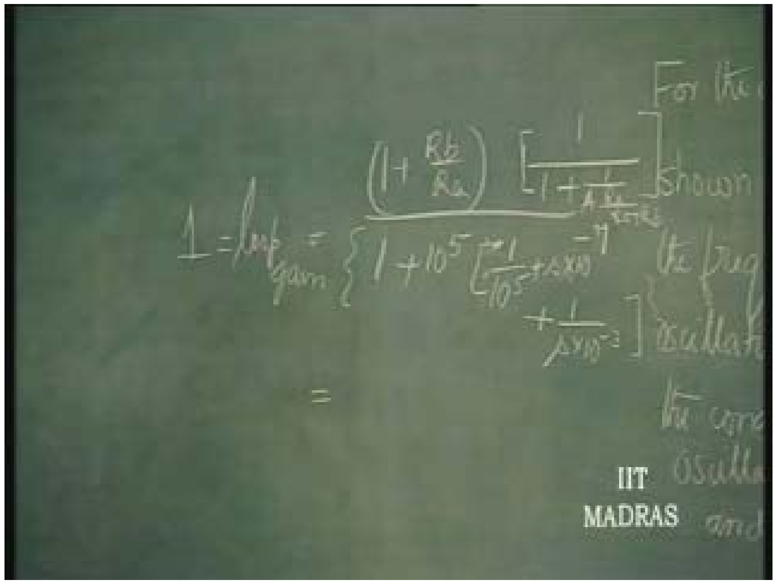

Now what it means is 1 plus R b by R a divided by 1 plus A. A is 1 over A naught plus s by Omega d. That is what... See, A naught by 1 plus s by Omega d. That is how we have taken. So, A naught is in this case, 10 to power 6 and Omega d is 10. That into R a by R a plus R b. We can take the nominal value here. R a by R a plus R b is very close to half, we have chosen; because this is going to be still very close to half. So, we will take this nominal value in this case and this is going to be how much? Half. That means this factor will be 2.

This is R a by R a plus R b is half, nominally. It is not exactly now. It is exactly half only when A is infinity. Now it will be less than half so that the gain, overall gain, is slightly greater than 2.

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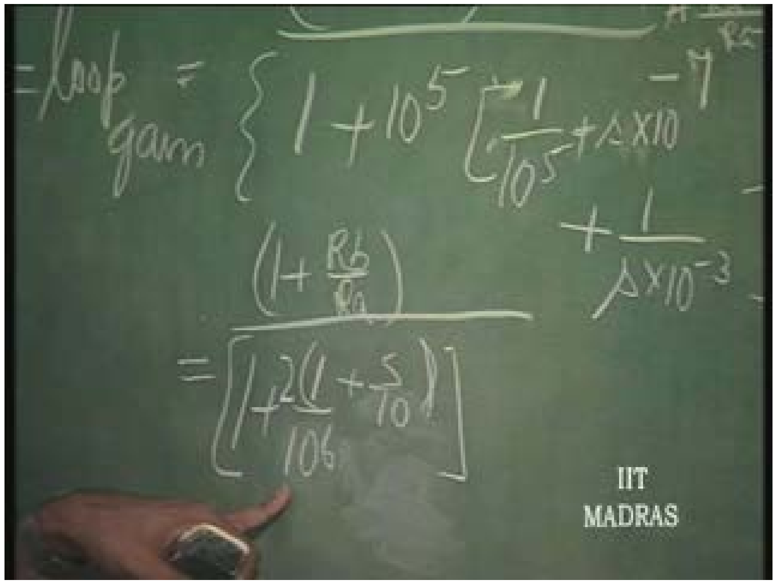

So, to that extent, what value of R b by R a we should get? We will come to know from this expression. So, this is an approximation to a certain extent, into 1 plus 10 to power 5 by 10 to power 5. That is 1 plus s into 10 to power minus 2 plus 1 over s into 10 to power 8.

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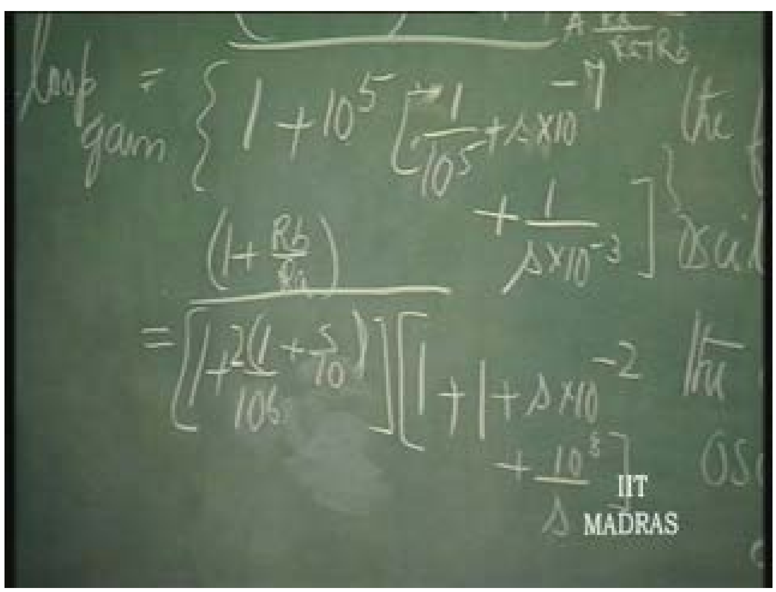

So, this is equal to 1 plus R b by R a. Once again, I say that R b by R a, we have taken as equal to 1. Only in the non-ideality it will be close to 1, but will be different. That value is going to be fixed by this.

So, this is a factor of 2. That factor 2 you take out. This 2. So, it gets normalized now. 1 plus s into 10 to power minus 2 divided by 2. We have taken out 2 here. This is nothing but 2; that 2 we have taken out; 10 to power 8 by 2 s. So, this is the expression. Now you can see that this loop gain has to become equal to 1. Now, actually speaking, I think I removed this. Where is that? Where is this factor? This into 1 plus...what is that? 2 into 1 plus s by 10 divided by 10 to power 6.

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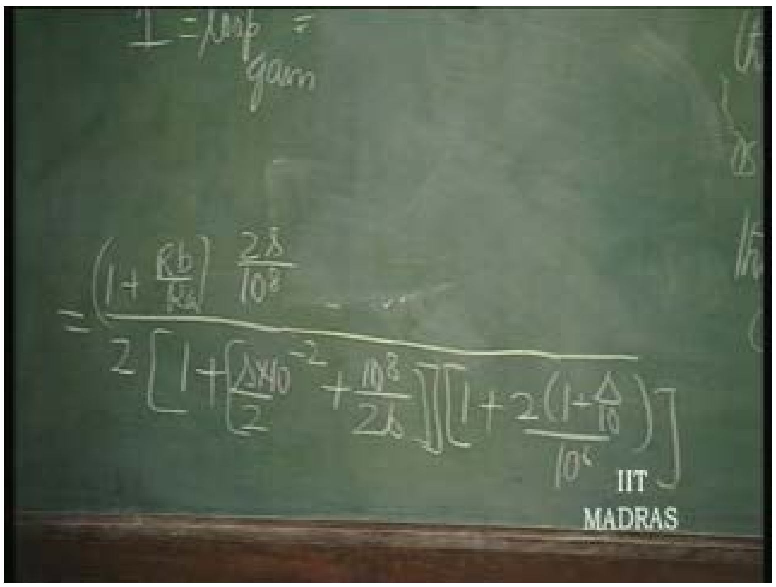

So, now you can see that the phase shift is governed by this factor here. This is the active parameter phase shift.

This is contributing to active parameter phase shift and that is going to be very little because there is a 10 to power 6 factor coming here; and therefore, we can evaluate this. Now, you can see that this is not contributing to anything; R b by R a should be made equal to 1 so that this 2, this 2, get cancelled. So, the loop gain becomes the magnitude equal to 1. Frequency is determined by this factor. This is j Omega and this is minus j by Omega. So, these factors equated will give you the frequency Omega naught as 10 to power 5 radians. Is this clear?

So, we will see that it really depends upon a Q factor coming here. I have earlier, in the case of filters, explained to you how to normalize this whole thing so that we get this as... Now, I can multiply this as a... There is a 2 s factor here. This factor has to be made equal to 1. That means I multiply the whole thing by 2 s by 10 to power 8.

So, that means this factor becomes equal to 1; this factor becomes equal to 2 s by 10 to power 8; and this 2 s means s square by 10 to power 8. Is it correct? 2 s. So, 2, 2, gets cancelled and 10 to power 8... Here you get this as 10 to power minus 10. So, this whole thing simplifies to in the denominator.

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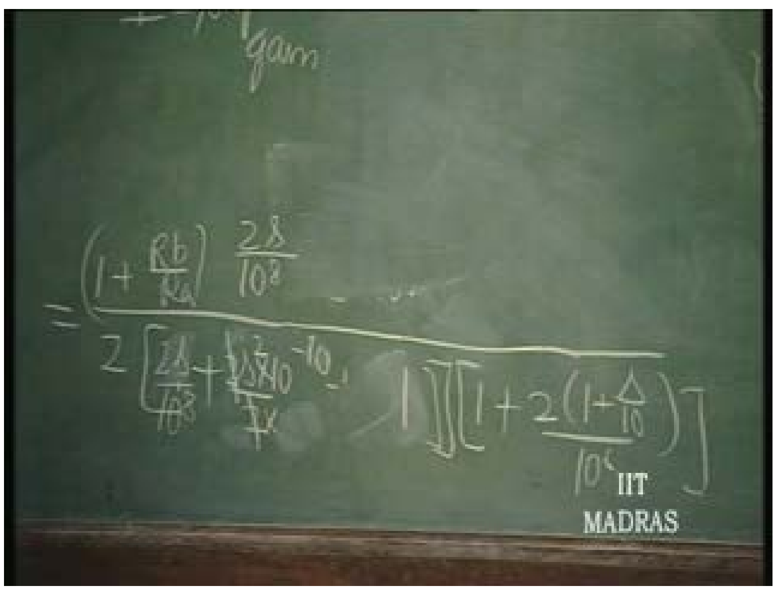

This is going to be neatly written as 1 plus R b by R a into 2 s by 10 to power 8 divided by 2; 1 plus 2 s by 10 to power 8 plus s square into 10 to power minus 10.

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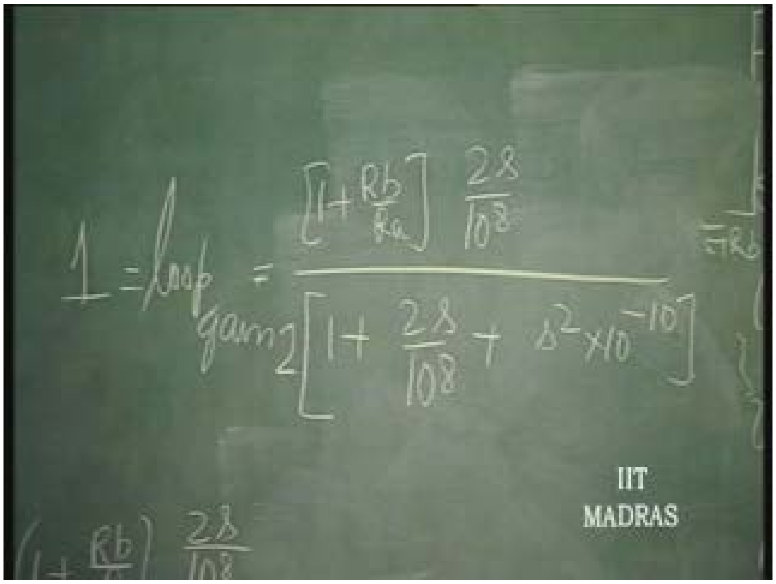

And we said the coefficient of s square here is going to be 1 over Omega naught square, which is called the resonance frequency, in the case of filters. So, you can see that 1 over Omega naught square is 10 to power minus 10. So, 1 over Omega naught square is equal to 10 to power minus 10 or Omega naught is equal to 10 to power 5, which we have got. So, we will write it as s square by 10 to power 5 square. This is s squared by Omega naught square.

So here, we have to have s by Omega naught for normalizing. s by Omega naught means s by 10 to power 5. Apart from that, we will have 10 to power 3 and this factor, 2 by 1000 is going to be written as 1 by 500 which we have earlier defined in our filters as Q factor of the pole; pole Q, if you remember.

So here, the pole Q is equal to 500. So, pole Q is 500; resonance frequency is 10 to power 5; and this is the normalization. So, this L C network has a pole Q of 100, 500; so pretty high value.

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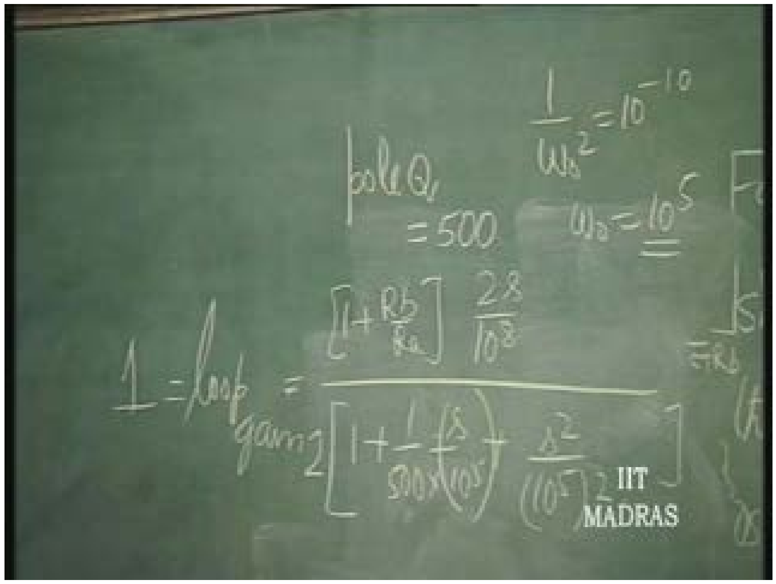

That means the variation in phase here with respect to frequency, Delta phi by Delta Omega in this case, is going to be around... Omega equal to Omega naught, is directly proportional to 500. It is a huge quantity. How do I really determine that?

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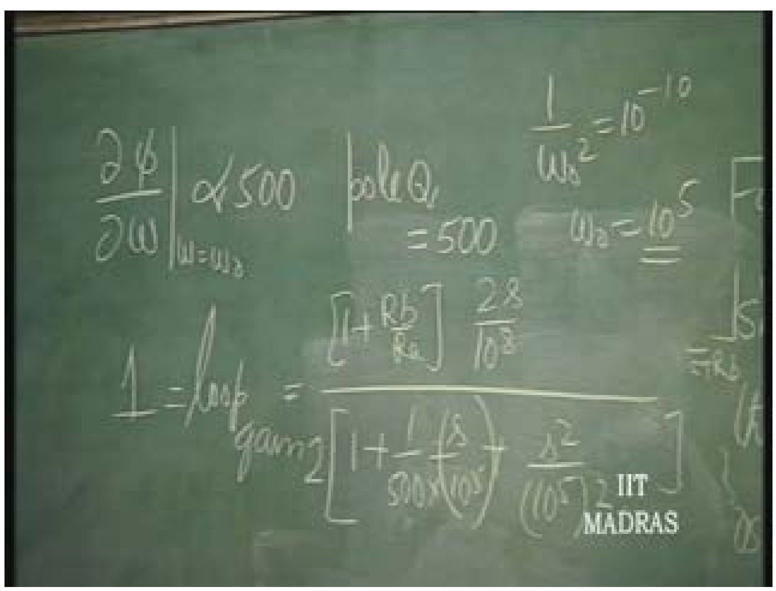

You can now find out the phase of this network, phase variation here. This contributes to a constant phase of 90 degree here.

This only contributes to a phase. This is going to be 1 by 500 into Omega by 10 to power 5 divided by 1 minus Omega square by 10 to power 5 square. This is tan inverse of this. This is the phase contribution due to the pole, imaginary part, divided by the real part. Imaginary part, put s is equal to j Omega.

So, Omega by 10 to power 5 into 1 over 500. That is the j Omega part; one without j omega. Real part is 1 minus Omega square by 10 to power 10; 1 minus Omega square by 10 to power 10. So, the real part divided by imaginary part; tan inverse of that is the phase. Is this clear?

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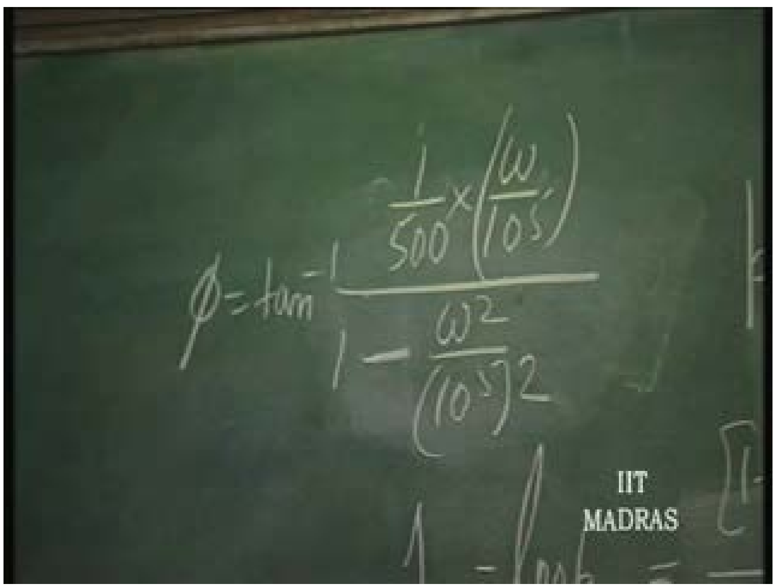

So now, I am going to make an approximation here and show... I want to establish only that at very close to Omega equal to Omega naught; very close to Omega equal to Omega naught, I would like to find out the slope. So, instead of differentiating, we will adopt this procedure. At Omega very close to Omega naught, Omega minus Omega naught, I will take it as change from Omega naught and Omega plus Omega naught is going to be twice

Omega naught. This is the approximation. At Omega very close to Omega naught, Omega minus Omega naught is Delta Omega; that change. Omega plus Omega naught is twice Omega naught. If you do that, this 1 minus x square is 1 plus x into 1 minus x. So, this denominator can be put as 1 minus Omega by 10 to power 5 by 1 plus Omega by 10 to power 5.

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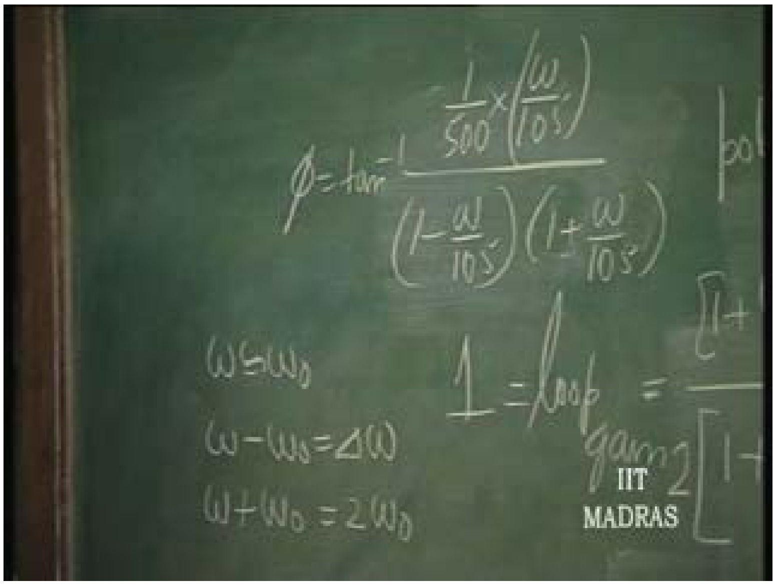

So, this quantity is equal to 2. At Omega very close to Omega naught, this quantity is equal to 2. This quantity is 1 minus Omega plus Omega naught; or, Omega minus Omega naught. Omega plus Delta Omega, we can put. Omega plus Delta Omega or Omega minus Delta Omega.

That means this is Delta Omega by 10 to power 5. So, this 10 to power 5 gets cancelled with this 10 to power 5. We get this phi as tan inverse Omega by 1000. This is important. You can see that the Q factor will come in the denominator of this; 1 by 1000 Omega by Delta Omega. Is this clear?

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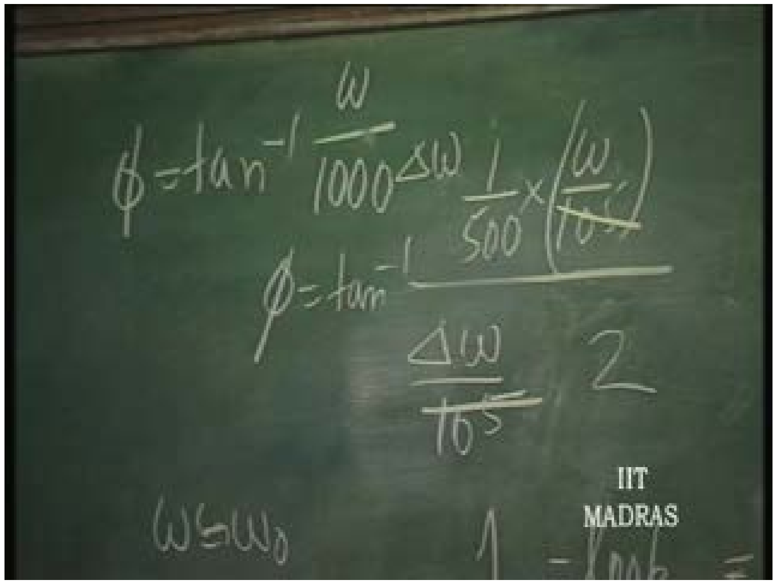

From this, we can see that Delta Omega tending towards zero. That Omega equal to Omega naught. This is going towards infinity. That means phi is equal to pi by 2. That is the phase shift contributed by the pole; phase shift contributed by the zero remains constant at pi by 2. So, overall phase shift is zero. So, phase shift contributed by the pole alone is pi by 2 at Omega equal to Omega naught. As Omega changes from Omega naught, it keeps changing. We would like to know how much it is changing from pi by 2.

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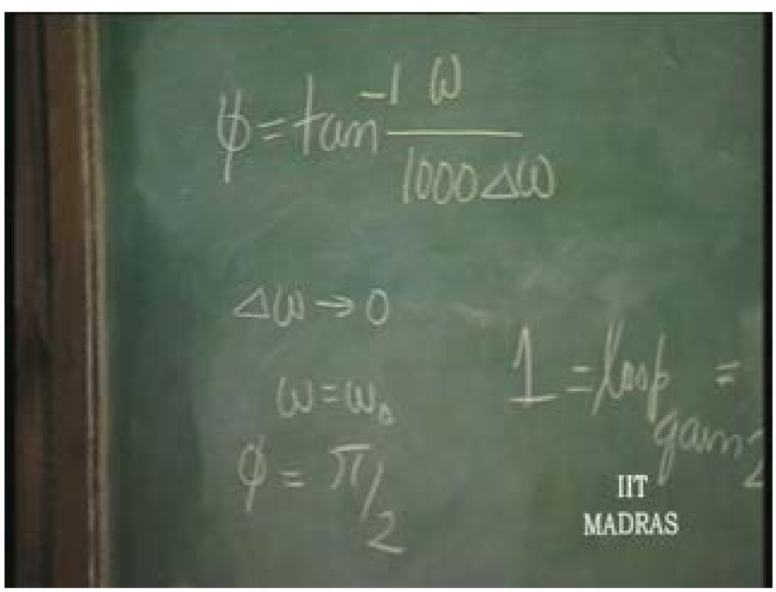

So, we will put this as Delta Theta plus pi by 2. It is going to change to some phi. So, this phi is going to be put as around pi by 2. So, how much it is going to be different from pi by 2 is going to be given by this Delta Theta. The phase is going to change from pi by 2. At Omega equal to Omega naught, it is pi by 2.

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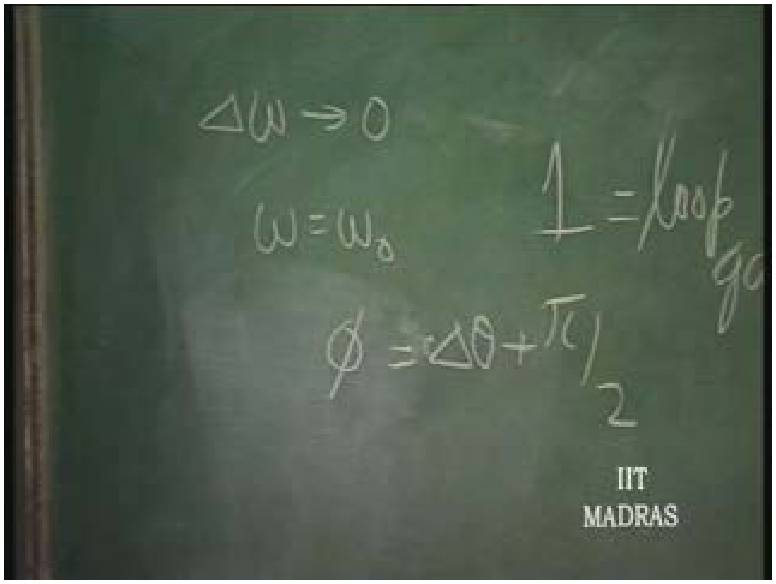

So, if I put that this is going to be Delta Theta plus pi by 2, so I take tan.

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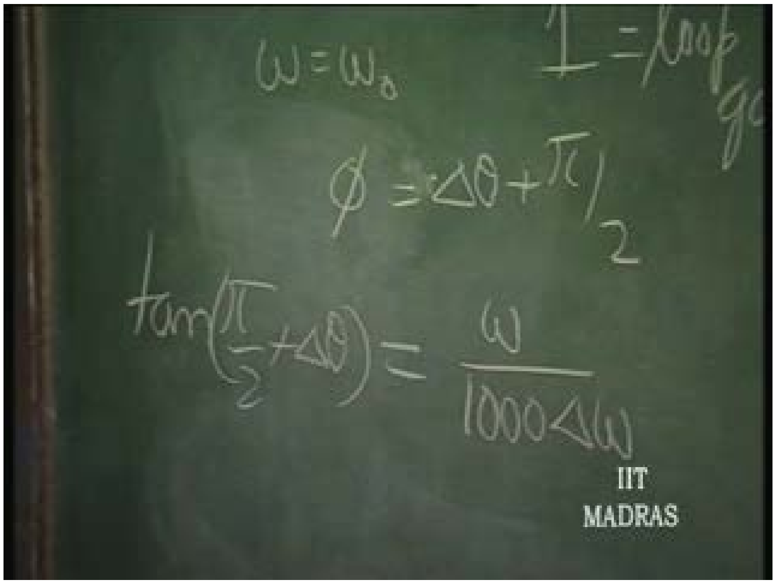

So, tan pi by 2 plus Delta Theta equals Omega divided by 1000 Delta Omega. So, Delta Theta being very small, we are close to Omega naught.

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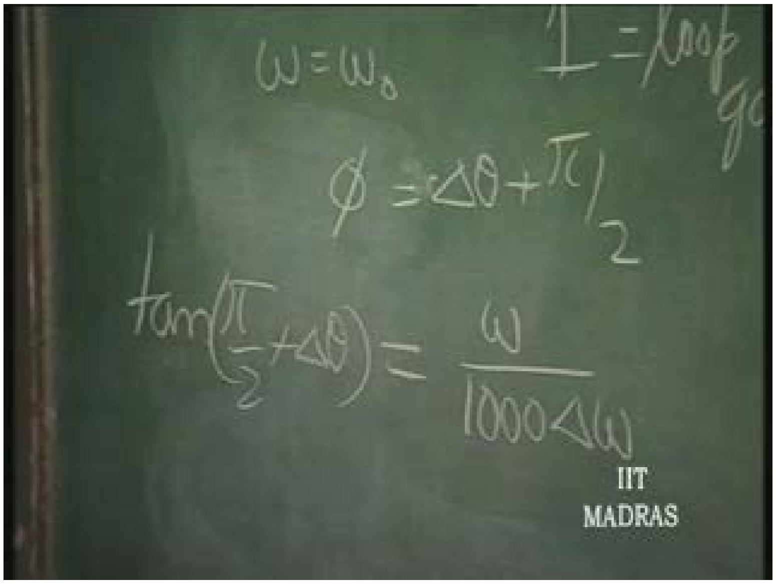

So, this is equal to cot, cotangent Delta Theta which is cos Delta Theta divided by sine Delta Theta. Delta Theta is very small and therefore cos Delta Theta is 1 and sine Delta Theta is Delta Theta itself.

So, from this expression, you get Delta Theta. The change in phase for a change in frequency around Omega naught. Omega naught, this is important, equals 1000 divided by Omega. This is important. In this 1000, Q of 500 is there. So, it is really equal to 2 into 500 by Omega naught because we are substituting Omega equal to Omega naught. So, this is nothing but 2 Q by Omega naught. This is an important expression.

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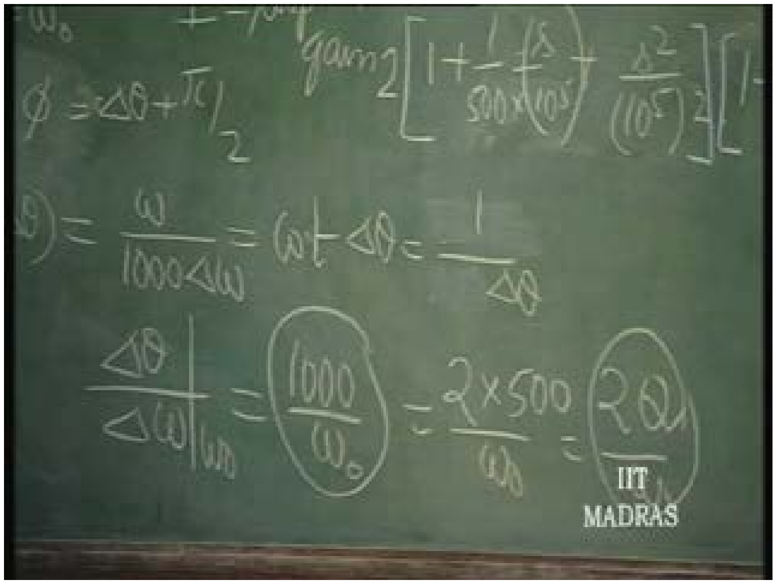

For any such circuit with pole Q equal to Q, this change in phase with respect to frequency is equal to 2 Q by Omega naught. This can be done by regular mathematics by finding out the Delta phi by Delta Omega straightaway. Instead, by the approximation, you can do it this way also; 2 Q by Omega naught.

So, this is very steep variation of frequency with Omega naught. Now, if you consider the whole expression at Omega equal to Omega naught, this expression is going to contribute to zero phase shift. But, there is going to be a phase lag contributed by this gain, finite gain; and therefore this is 1 plus 2 divided by 10 to power 6. So, that 1 plus 2 by 10 to power 6 can be ignored. So, this can be ignored compared to this. So, it is 2 s by 10 to power 7 that is going to remain here.

So, if you put s is equal to Omega, j Omega, this is going to contribute to a phase, additional phase lag of... At that frequency, the additional phase lag will be 2 Omega naught by 10 to power 7. That being very small angle, you can say that tan inverse of that is that angle itself. So, 2 Omega naught by 10 to power 7 is the additional phase error.

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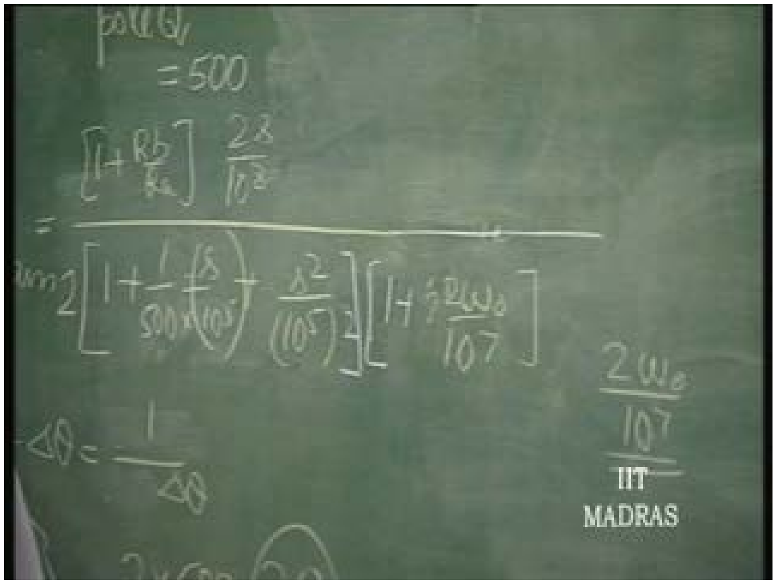

So, to that extent, this will have to contribute and that can be done by that changing very little from Omega naught. So, this phase error can be compensated by this slope here. The slope is directly proportional to Q and therefore, because Q is very high, the Omega has to change very little from Omega naught, in order to accommodate this phase lag. Is this clear?

So, you will just equate this to this phase error; and therefore, you will see that the actual phase frequency that is going to be different is going to be different from Omega naught by a factor which is determined by Q; 1 over Q of this factor. So, this is the phase error. This has to be compensated for by Delta phi, by Delta Omega, by Delta Omega changing accordingly from Omega naught.

Delta phi by Delta Omega is 2 Q by Omega naught. That means this into Delta Omega. So, Delta phi is 2 Omega naught divided by 10 to power 7. That is the phase error and what should be the Delta Omega? That Delta Omega is given by 2 Omega naught square by 10 to power 7 divided by 2 Q. So, you will see that Q always comes in the denominator. So, the Delta Omega which has to be different now becomes very very small.

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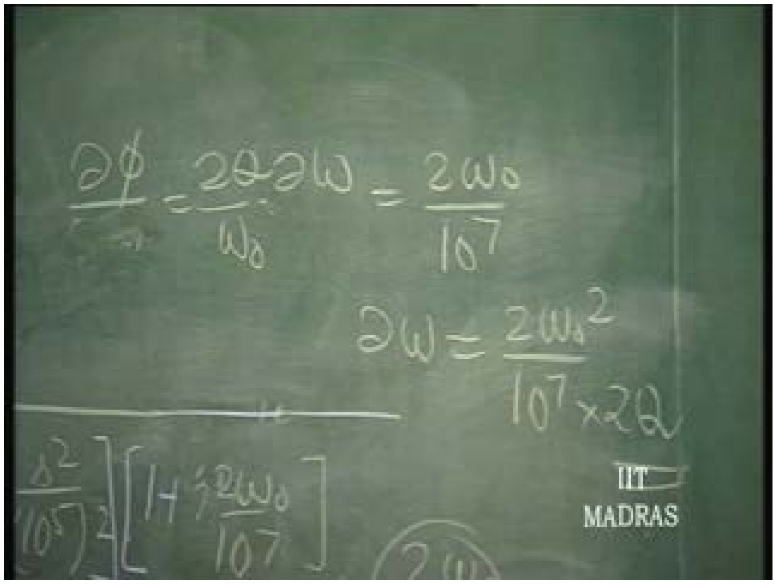

In this particular case, you can now substitute what? Omega naught and C. 2 into 10 to power 10 divided by 10 to power 7 into 1000; 10 to power 3, is it? 1000, is it? So, this is the 2... What is that? Radians per second.

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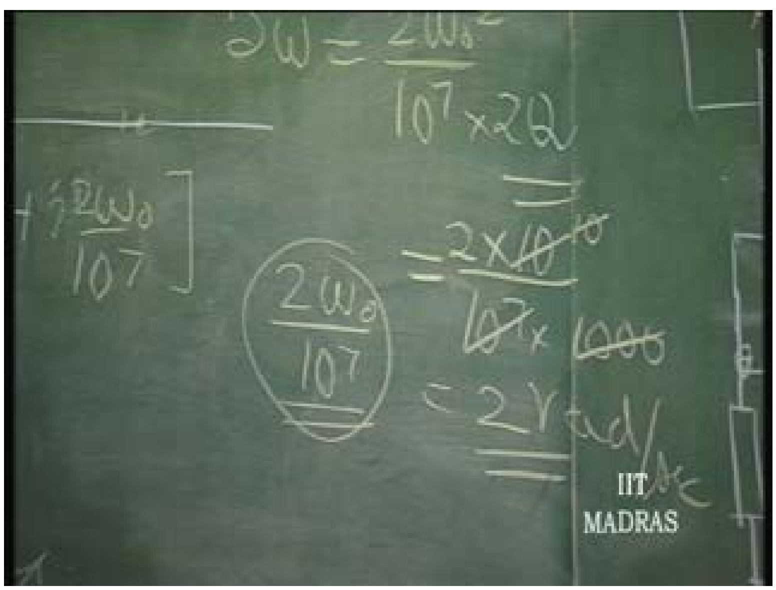

Omega naught is 10 to power 5 radians per second. So, it...this much deviation can be accommodated by frequency of oscillation, changing only by 2 radians per second. So, we have solved the problem as well as made you understand what is frequency stability.