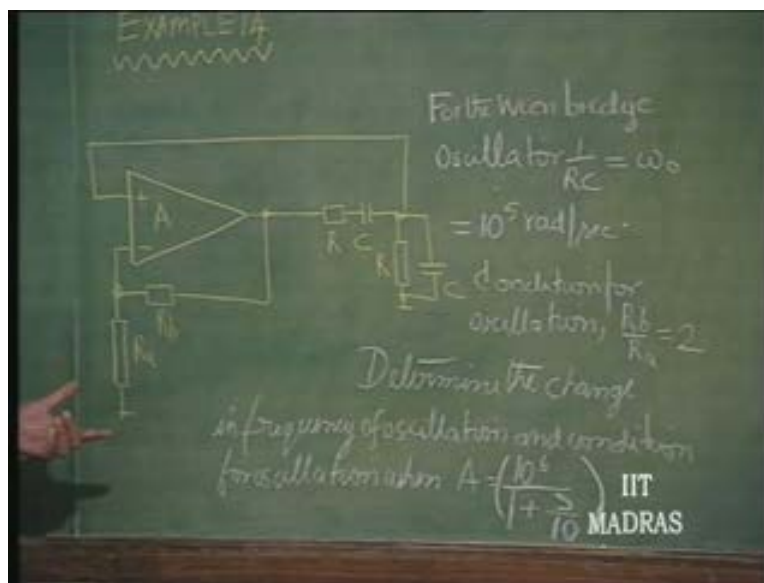


Electronics for Analog Signal Processing - II
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Lecture - 16
Frequency Compensation in Negative Feedback Amplifiers

So, we will take another Example, 14 in order to illustrate something about frequency stability or frequency instability in oscillators. Example 13 was L C oscillator and we saw that the frequency stability was very good if the Q of the L C network was high. We had chosen the frequency of oscillation of Example 13 to be the same as the current example, which is a Wien bridge oscillator. 10 to power 5 radians per second, Omega naught, because condition for oscillation is independent here. R b by R a is made equal to 2 so that the amplifier has a gain of 3. That we had derived in the earlier theory course.

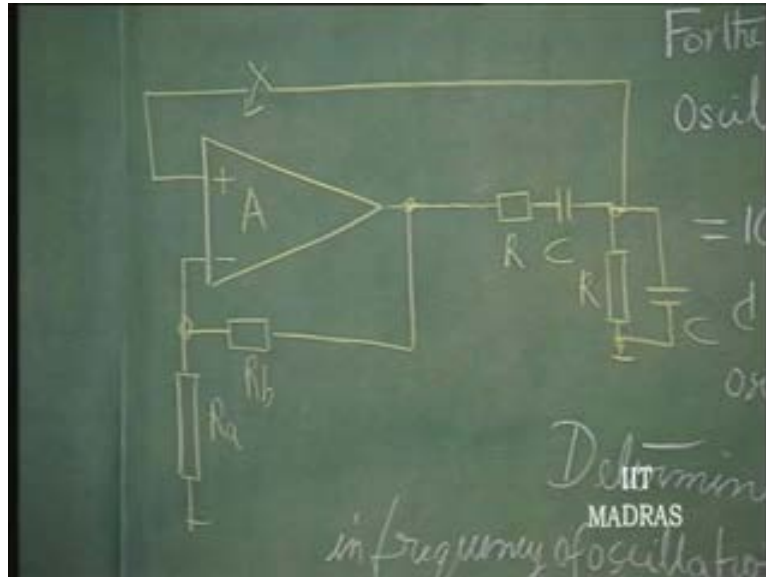
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Now determine the change in frequency of oscillation and condition for oscillation when A becomes, not infinity as it was assumed here; A is finite and frequency dependent; 10 to power 6 by 1 plus S by 10.

So, we would like to know how much change occurs in the frequency of oscillation. How do we solve this problem? We will again write this loop gain. This is to be closed. This is the loop.

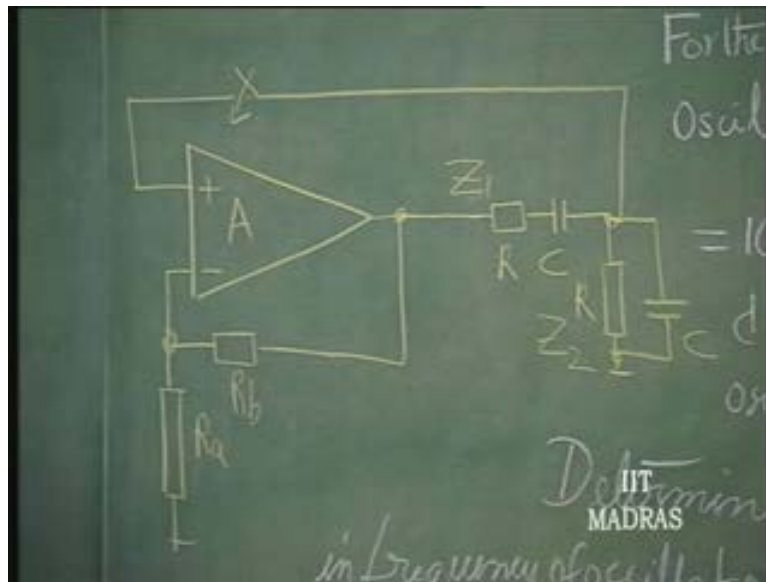
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So, $A \times \frac{R_2}{R_1 + R_2}$ is the gain from here to here, Beta being $\frac{R_1}{R_1 + R_2}$. That in this particular case is going to be nominally equal to $\frac{1}{3}$; R_1 by $R_1 + R_2$, because of that condition, is nominally equal to $\frac{1}{3}$. It might not change. It will become less than $\frac{1}{3}$ so that the gain is greater than 3, for making this get satisfied. That finite gain comes into picture and therefore you will need a slightly larger gain than 3, in this case. That, we will see later.

So, this is the transfer function from here to here; and from there to there, we have this as earlier represented, Z_1 , and this is Z_2 .

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Z_2 by Z_1 plus Z_2 which is therefore the loop gain. I will close this loop. That means this is going to be made equal to 1. So, that is the condition for making this go into oscillation.

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A hand-drawn equation on a chalkboard representing the loop gain condition for oscillation. The equation is written as:

$$\text{loop gain} = \left(\frac{A}{1 + A \frac{R_a}{R_a + R_b}} \right) \frac{Z_2}{Z_1 + Z_2} = 1$$

The text 'IIT MADRAS' is written at the bottom right of the slide.

So, this is equal to... We will re-write this. This gets $1 + R_b$ by R_a , which is actually the gain divided by $1 + \dots$ I am dividing by A_{β} throughout. So, whenever β comes here, $1 + 1$ over A_n to β ; that is R_a by $R_a + R_b$, which is written as A_{β} into $1 + S$ by W_d .

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$$= \frac{1 + \frac{R_b}{R_a}}{1 + \frac{(1 + \frac{S}{\omega_d})}{A_{\beta} \frac{R_a}{R_a + R_b}}}$$

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So, actually speaking, R_a by $R_a + R_b$ is equal to 1 over 3 nominally. So, we will put that value. This becomes 3 and A_{β} is 10 to power 6 and ω_d is 10 . So, this is the part that we have to now bother about.

Earlier, this was zero because A was infinity; only it was R_b by $1 + R_b$ by R_a . Now, this additional factor comes into picture and as is usual with the other factor, it is 1 by $1 + Z_1 Y_2$, which can be rewritten as, Z_1 is R plus... R plus 1 over $S c$. This is 1 over $R + S c$. So, this is the overall transfer function which has to be equal to 1 .

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$$\left(\frac{Z_2}{Z_1 + Z_2} \right) \frac{1}{1 + A \frac{R_b}{R_a + R_b}} = 1$$

$$= \frac{1 + \frac{R_b}{R_a}}{\left[1 + 3 \left(1 + \frac{s}{10^6} \right) \right] \left[1 + \left(R + \frac{1}{LC} \right) \left(\frac{1}{R} + sC \right) \right]}$$

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So, $1 + R_b / R_a$ divided by $1 + 3 \times 10^{-6} s$. We can neglect that; $3 \times 10^{-6} s$ to power 6 can be neglected compared to 1; plus $3s$ divided by 10^6 . This is the contribution of this part of the denominator. This is going to be represented as $1 + 1 + 3s / 10^6$.

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$$\frac{\left(1 + \frac{R_b}{R_a} \right)}{\left[1 + \frac{3s}{10^6} \right] \left[1 + 1 + 1 + sCR + \frac{1}{sCR} \right]}$$

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That means, actually speaking, this is 3 plus S C R plus 1 over S C R, which I can normalize now. S squared C squared R squared; this will be 1 and this will be 3 S C R.

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$$\frac{\left(1 + \frac{R_b}{R_a}\right)}{\left[1 + \frac{3s}{107}\right] \left[3sCR + s^2CR + 1\right]}$$

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And therefore, I will rewrite this as S squared C squared R squared. Of course, you get S C R in the numerator, plus 3 S C R plus 1. So, this is the loop gain.

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$$\text{loop gain} = \frac{\left(1 + \frac{R_b}{R_a}\right) sCR}{\left[1 + \frac{3s}{107}\right] \left[s^2CR + 3sCR + 1\right]}$$

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We will rewrite this. This is $1 + \frac{R_b}{R_a}$ divided by $1 + 3j\omega CR$ divided by 10^7 to power 7. Put S is equal to $j\omega CR$. And this is $1 - \omega^2 C^2 R^2$ squared plus $3\omega CR$, j ...

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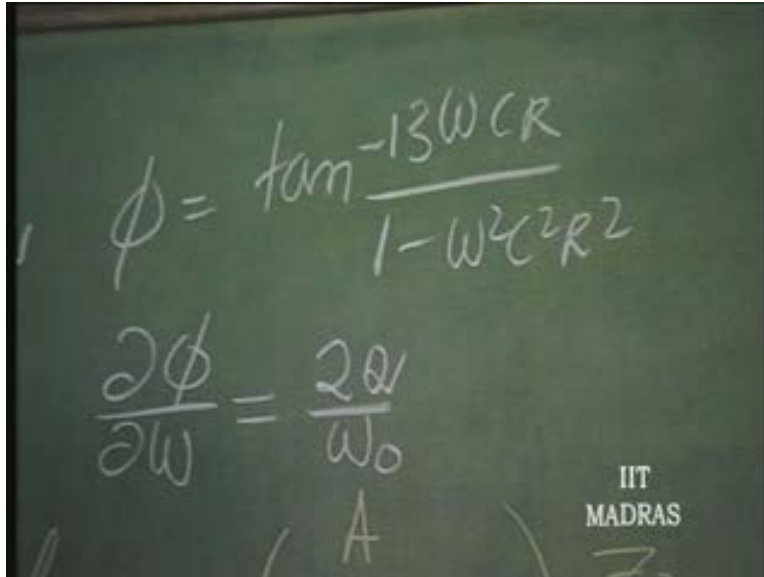
$$\text{loop gain} = \frac{\left(1 + \frac{R_b}{R_a}\right) j\omega CR}{\left[1 + \frac{3j\omega}{10^7}\right] \left[1 - (\omega CR)^2 + j3\omega CR\right]}$$

So, as you can see, the phase shift of this, this part; forget about this; this is the non-ideal part. And the ideal part, phase shift of this becomes equal to zero at ωCR equal to 1. Or, ω naught is equal to $1/CR$. The phase shift of this becomes equal to zero and this, if it is ideal, is not contributing to any phase.

So, this particular thing has a magnitude then of $1/3$ which has to be made equal to 1 by making this equal to 3. That is what we got as condition for oscillation. Now, because of the additional phase shift here, phase lag here, this phase shift cannot be equal to zero. This has to have a phase lead of the same amount by which this is having a phase lag. So, as far as this particular thing is concerned, we have already discussed it. At ω naught equal to $1/CR$, at ω naught equal to $1/CR$, the phase shift can be written as ϕ is equal to $\tan^{-1} \frac{\omega CR, 3\omega CR}{1 - \omega^2 C^2 R^2}$; and like in the Example 14, we can prove that. I am

not going to do it. Delta phi by Delta Omega is going to be equal to 2 Q by Omega naught.

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$$\phi = \tan^{-1} \frac{3\omega CR}{1 - \omega^2 C^2 R^2}$$
$$\frac{\partial \phi}{\partial \omega} = \frac{2Q}{\omega_0}$$

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This is what we had got earlier. We had derived this. Delta phi by Delta Omega is equal to 2 Q by Omega naught for this, the second order system. What is the Q here? You can notice that Q is... If I write this in this following fashion, Omega squared by Omega naught square, this is already normalized; this is Omega by Omega naught. Q is going to be equal to 1 over 3. This, co-efficient of this is 1 over Q. So, Q is going to be 1 over 3.

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$$\frac{j\omega CR}{- \left[\left(1 - \frac{\omega^2}{\omega_0^2} \right) + j3 \frac{\omega}{\omega_0} \right]}$$

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So, Q for this is 1 over 3 – pole Q; and therefore this is going to be 2...Omega naught by 3; whereas, in the previous example, Q was equal to 500.

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$$\phi = \tan^{-1} \frac{3\omega CR}{1 - \omega^2 C^2 R^2} \quad Q = \frac{1}{3}$$
$$\frac{\partial \phi}{\partial \omega} = \frac{2Q}{\omega_0} = \frac{2}{3\omega_0}$$

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So, you could see that by a small change in frequency, you could reach the required phase change. Here, you require considerable change in frequency in order to have a particular phase change. What is the phase change required?

Here, the phase change ϕ , $\Delta\phi$, required is straightaway this, when ϕ is small. So, we can write $\Delta\phi$ as straightaway equal to 3ω naught divided by 10 to power 7 . 3ω naught by 10 to power 7 is the phase lead required. So, 3ω naught by 10 to power 7 . If I therefore equate this to $\Delta\phi$... so, $\Delta\phi$ therefore is equal to... what is possible with this network is 2 by 3ω naught into $\Delta\omega$. So, $\Delta\phi$ is equal to now 3ω naught by 10 to power 7 .

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The image shows a chalkboard with handwritten mathematical derivations. At the top, there is a fraction $\frac{R_b}{R_a}$ and the term $j\omega CR$. Below this, a complex transfer function is written as $\frac{j\omega}{10^7} \left[\left(1 - \frac{\omega^2}{\omega_0^2} \right) + j3\frac{\omega}{\omega_0} \right]$. To the right, an equality is shown: $= \frac{1 + \frac{R_b}{R_a}}{1 + 3\frac{\omega}{\omega_0}}$. Below the main equation, two expressions for the phase change are given: $\Delta\phi = \frac{3\omega_0}{10^7}$ and $\frac{3\omega_0}{10^7} = \frac{2}{3\omega_0} \Delta\omega$. The IIT Madras logo is visible in the bottom right corner of the chalkboard image.

So, what do you get? $\Delta\omega$. $\Delta\omega$ therefore is going to be change in frequency of oscillation. The frequency of oscillation should so adjust itself that that much phase lead is going to be given by this network now, in order to compensate for the phase lag suffered by the amplifier. So, that $\Delta\omega$ is going to be 9ω naught square, ω naught being 10 to power 5 , ω naught square, divided by 2 into 10 to power 7 .

So, you can see that this is going to be equal to 4 point 5 Kilo radians per second. Compare this with the frequency of oscillation. Omega naught was 10 to power... That is actually 2, or 100 Kilo radians per second.

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The image shows a chalkboard with handwritten mathematical work. The main calculation is
$$= \frac{9 \times 10^{10}}{2 \times 10^7} = 4.5 \text{ Krad/sec}$$
 Below this, there is a term $(1 + \frac{R_b}{R_a})$ and a note $\omega_0 = 100 \text{ Krad/sec}$ with $\Delta \omega$ written below it. At the bottom, there is a partial equation $+ \frac{3S}{107} \left[\omega_0^2 R + 3 \Delta \omega \right]$ and the logo for IIT MADRAS.

So, when the frequency of oscillation was 100 Kilo radians per second, for using the same amplifier op amp in both the cases; in this case, the deviation due to active parameter sensitivity is 4 point 5 Kilo radians per second, it is about 4 point 5 percent error.

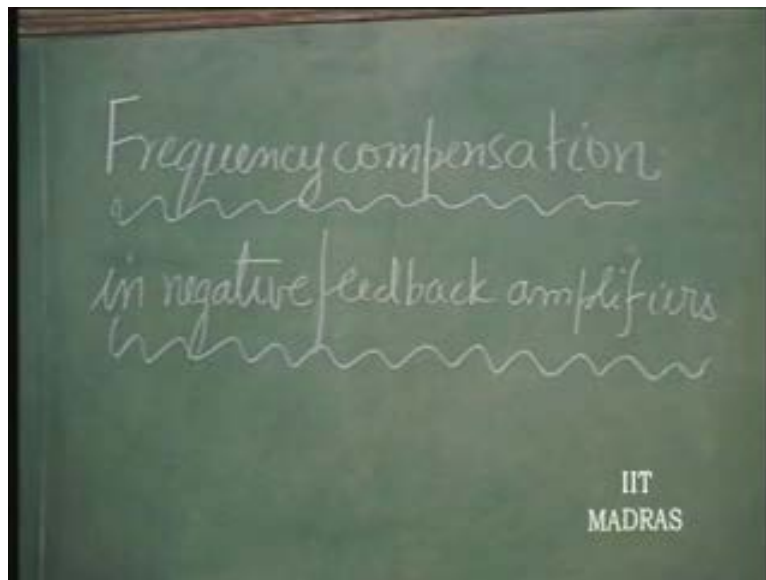
Now, see the other one. When the same 100 Kilo radians per second oscillator was designed, the other one had 2 radians per second as the frequency deviation; just 2 radians per second. Compare it to the change that has suffered because of the slow variation in phase with respect to frequency in the case of R C oscillators.

So, all RC oscillators like this; whether they are phase shift oscillators or Wien bridge oscillators, they suffer from this great disadvantage that the frequency stability is very poor. The active device that is used if it is phase, that is frequency dependent, then automatically, the compensation has to come for by the passive device; and the variation

required in frequency of oscillation is enormous for high frequency operation, of all these oscillators. Low frequency, the error, phase error itself is going to be very small. So, it can be only used for low frequency applications. For high frequency, it is better to use wideband amplifiers and it is better to use L C oscillators, for the purpose of frequency stability.

So now, we come to another important topic. We are ready for the discussion of this frequency compensation in negative feedback amplifiers. Now, what is this? I purposely postponed discussion of this even though I had discussed negative feedback earlier because I wanted to discuss about oscillators, sinusoidal oscillators and non-sinusoidal oscillators and then come over to frequency compensation.

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Let us understand. We can give negative feedback in order to make the amplifier go towards its ideality. Depending upon the type of control source, it can become better control source with negative feedback. This is the purpose of negative feedback. The positive feedback should not be attempted primarily because it will make the gain or transfer parameter more sensitive to active parameter. Positive feedback with regenerative action is adapted in order to make the circuit work only at 2 levels at the

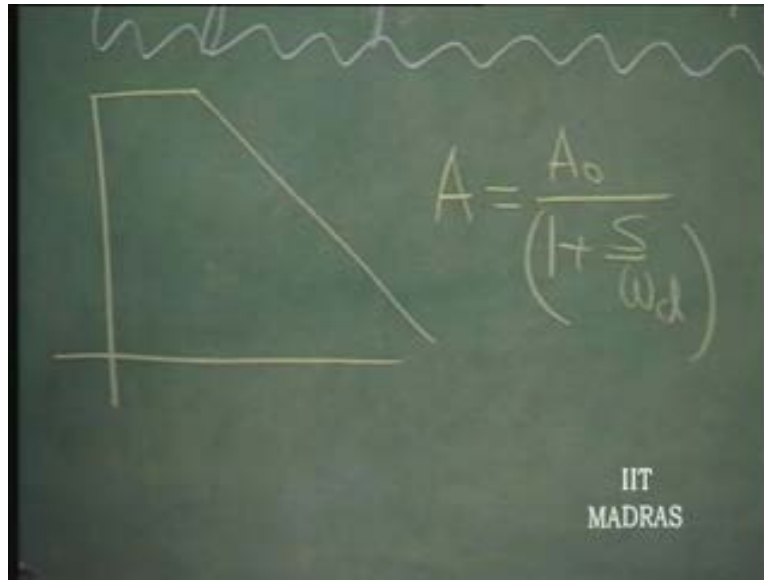
output. It is a non-linear function; it is going to be at high or low depending upon the... whether the input is increasing or decreasing; and that is a **memory event** and therefore we have discussed that as a separate application.

Now, further we discussed the aspect of making a sinusoidal oscillator by simulating a second order differential equation or making the poles get located on the imaginary axis. In all our filter functions, the poles must necessarily get located on the left half of the S plane. In the case of an oscillator, it can lie on the... at $j\Omega$ axis; or preferably, in order to make it go into buildup of oscillation, we make it purposely get located on the right half of the S plane and then bring it over to the imaginary axis slowly at the required amplitude of oscillation.

Now, when an amplifier is designed with negative feedback, what happens if the amplifier is frequency dependent? This, we have to understand. So, let us consider the amplifier.

Take that the amplifier is frequency dependent with one pole. That means, gain... I am talking of voltage control voltage source amplifier; A is equal to A_{naught} divided by $1 + S \text{ by } \Omega d$.

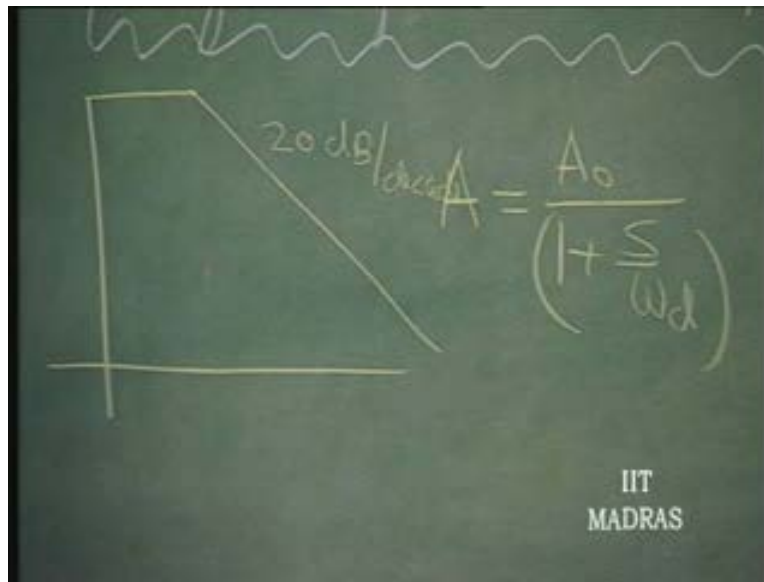
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I am considering this as Ωd because this is supposed to be the only dominant pole in the entire amplifier in its useful frequency range. That is why I call it Ωd . A naught is the D C gain at S is equal to $j \Omega$, Ω equal to zero; and it is going to fall off at let us say, 20 decibels per decade.

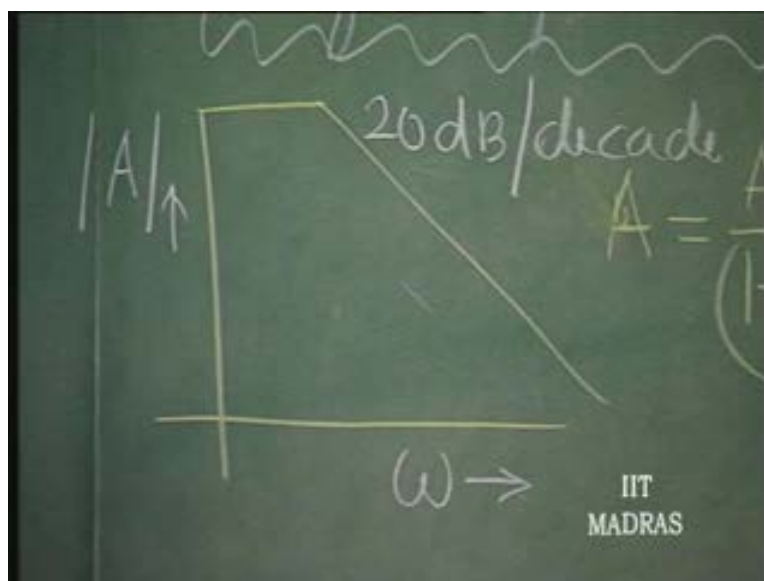
This is what is going to happen. As frequency increases, it is inversely proportional to frequency and from one frequency to another frequency, if the difference is a decade, that is a ratio of 10, it is $20 \log 10$, which is 20 decibels per decade. So, this is the rate of fall. This is what is called as Bode's plot.

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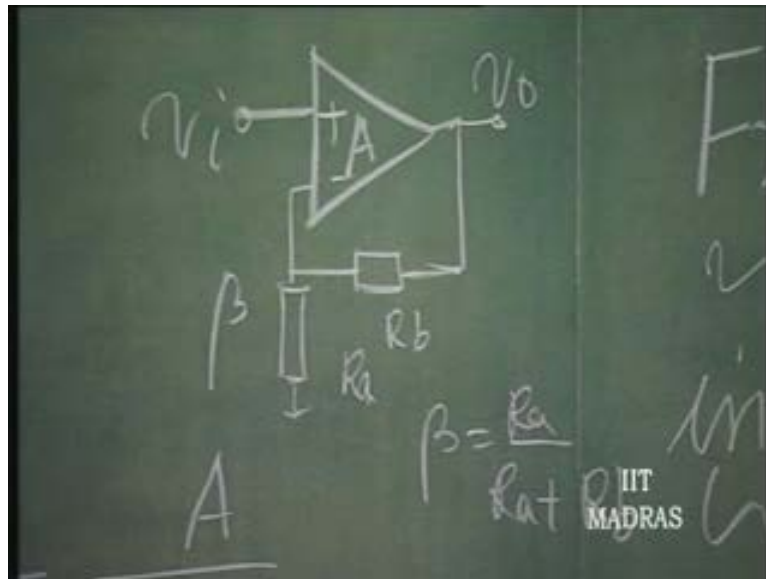
So, it is starting with a gain of A naught going at 20 decibels per decade. This is important. 20 decibels per decade because of single pole. If the second pole comes into picture, it will start dropping off at 40 decibels per decade. If there is a third pole coming into picture, it will drop off at 60 decibels per decade. That is why...this is Omega versus the magnitude of A . Now, if I now use feedback, negative feedback, what happens?

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We have seen that gain becomes A by $1 + A\beta$, let us say, for a voltage control voltage source with H feedback, because that is what it... what makes the voltage control voltage source go towards its idealization, H feedback; and this is β . This is corresponding to R_1 , let us say R_a , this is R_b . β is equal to R_a by $R_a + R_b$. All these things, we have earlier understood.

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So, the gain V_o over V_i is going to be... this also we have derived earlier. So, dividing by $A\beta$ throughout, this is 1 over β which is the ideal gain divided by $1 + 1$ over loop gain. This is important. The error is always 1 over $1 + 1$ by loop gain.

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$$\frac{V_o}{V_i} = \frac{A}{1 + A\beta}$$

$\beta = \frac{R_a}{R_a + R}$

$$= \frac{1/\beta}{\left[1 + \frac{1}{A\beta}\right]}$$

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So, this A naught... this becomes A naught, into 1 plus S by Omega d now, because of negative feedback. So, what happens? 1 over Beta divided by 1 plus... 1 over A naught Beta is going to be very small compared to 1. So, I ignore that. So, S divided by A naught Beta into Omega d.

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$$\frac{V_o}{V_i} = \frac{1}{1 + A\beta}$$

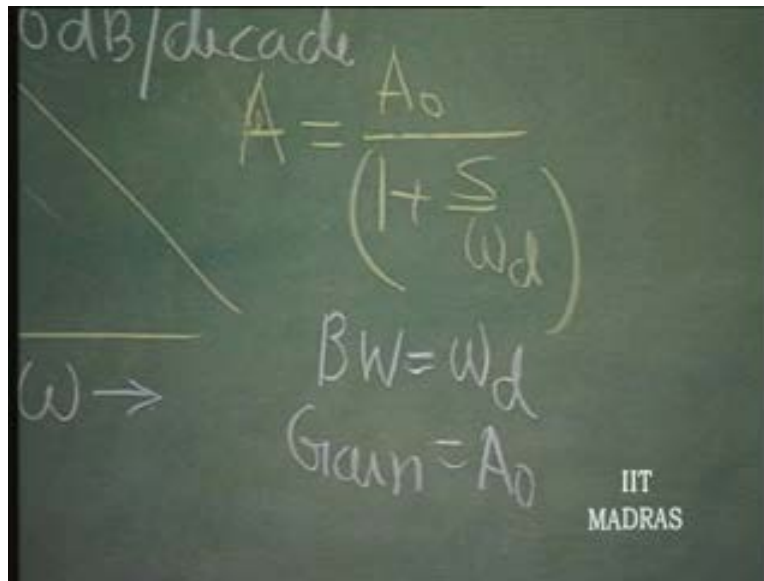
$$= \frac{1/\beta}{\left[1 + \frac{S}{A\beta\omega d}\right]}$$

$$= \frac{1/\beta}{\left[1 + \frac{1}{A\beta} \left(1 + \frac{S}{\omega d}\right)\right]}$$

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Let us consider. We started with an amplifier whose gain was A_{naught} and this is called the bandwidth. Ω_d is then the bandwidth. The point at which the gain falls to $1/\sqrt{2}$ times the maximum; or, it is also called minus 3 dB point. The gain falls to $1/\sqrt{2}$. So, this is the bandwidth. The gain of the open loop amplifier is A_{naught} .

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There is an important measure of how good the amplifier is. That is given by what is called gain into bandwidth. Gain into bandwidth. This is called gain bandwidth product. We will call it $G B$. That is equal to Ω_d into A_{naught} .

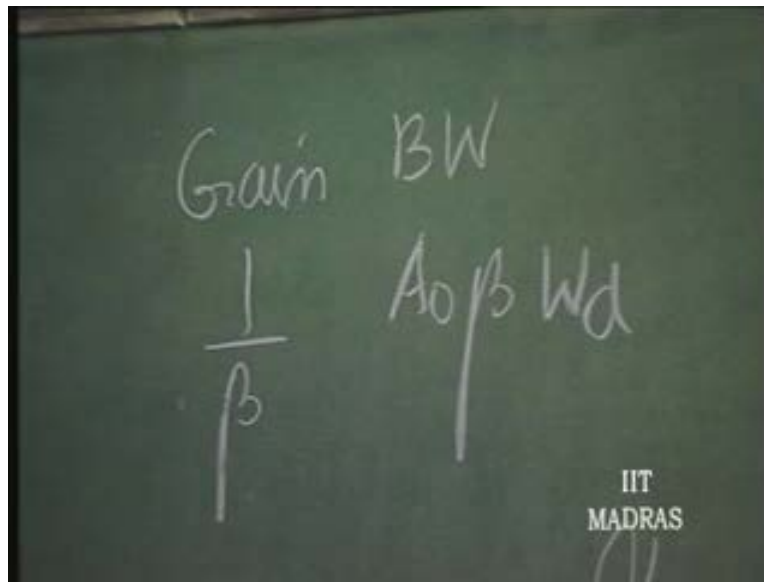
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The image shows a chalkboard with handwritten mathematical expressions. At the top left, the word "decade" is written. The main equation is $A = \frac{A_0}{(1 + \frac{s}{\omega_d})}$. To the right of this equation, it says "Gain BW product". Below the denominator, it says "= GB". Further down, it says "BW = ω_d " and "Gain = A_0 ". To the right of these, it says "= $\omega_d A_0$ ". In the bottom right corner, "IIT MADRAS" is written.

Just look at it. Come to this. Amplifier with feedback. This is the feedback gain. What is the gain? 1 over β ; gain is 1 over β ; and its bandwidth is A naught β into Ω d .

So, what it says is bandwidth of a feedback amplifier like this, negative feedback amplifier, improves because of negative feedback. By how much? – by loop gain, A naught into β . A naught into β is the loop gain. So, by that factor, it is improving and gain into bandwidth is a constant. Gain of the feedback amplifier is 1 over β . Bandwidth is A naught β into Ω d .

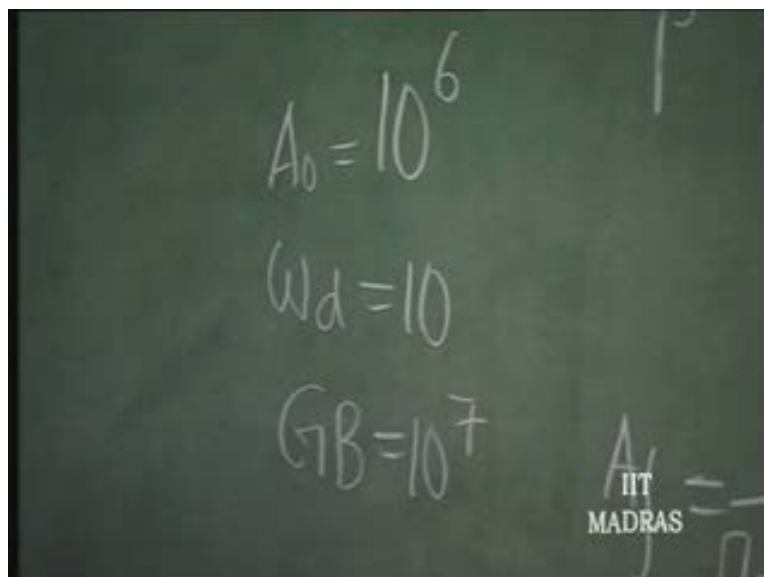
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Gain into bandwidth is constant, which is equal to Ωd into A naught. This is invariably true. If you use this amplifier in any negative feedback configuration like this, gain into bandwidth remains always a constant.

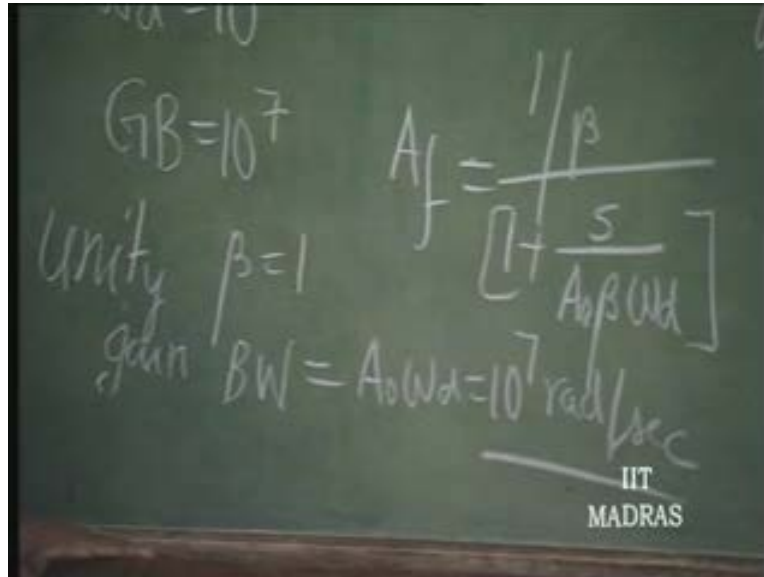
As an example, let us take that I am having gain of 10^6 ; Ωd equal to 10; gain into bandwidth is 10^7 .

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If I use it as a unity gain amplifier, then Beta has to be equal to 1. Let us say unity gain amplifier. Beta has to be 1. Then its bandwidth is going to be... Beta is 1; A naught into Omega d, which is 10 to power 7 radians per second.

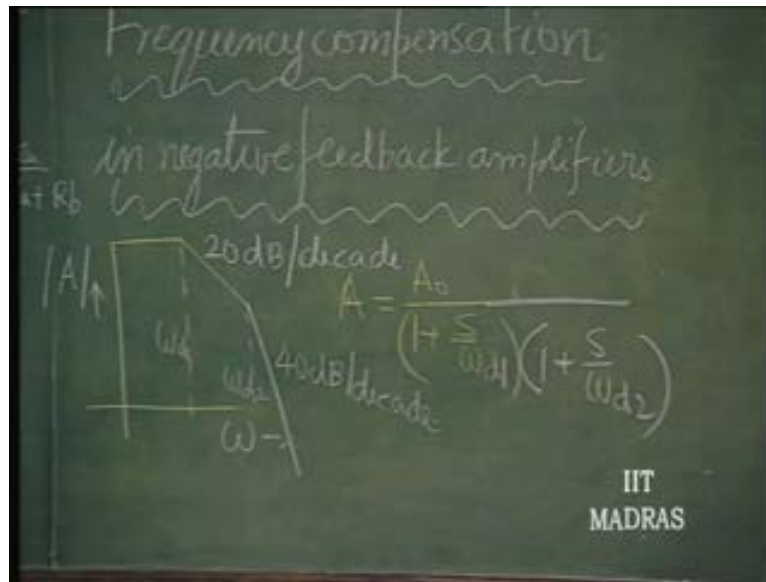
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So, as a unity gain amplifier, it can be used up to 10 to power 7 radians per second bandwidth. As a gain of 10, it can be used up to 10 to power 6 radians per second. As a gain of 100, it can be used up to 10 to power 5 radians per second. So, the gain into bandwidth of such a negative feedback amplifier is always a constant. This is applicable when we have this as a first order system; that the gain is predominantly determined by this pole in the entire range.

Now, let us consider a situation where this is not first order; but there is another pole. That is, then you call this system a second order system. The order of the system is always determined by the number of poles of the system here now. Such amplifiers... if it is one pole, it is first order; if it is no pole, it is zeroth order. One pole - first order; second order means there are 2 poles; one is Omega d 1 and the other one is, let us say, Omega d 2.

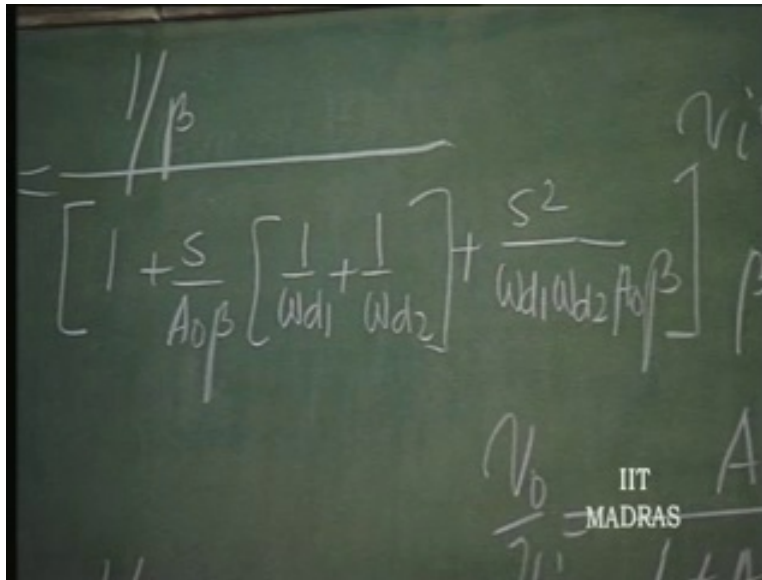
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What it means is that in the range of interest to us, before the gain falls to 1, we have this falling at 40 decibels per decade. This is Ω_{d1} and this is Ω_{d2} . So, this is a 2 pole system. If such system is used for negative feedback, then what happens? This is also fairly simple to analyze. So again, all these things are valid. Only thing is this expression is going to be valid even now.

Only thing is V_{naught} over V_i is now 1 over β divided by... not this.... $1 + 1$ over $A_{naught} \beta$ into $1 + s$ by Ω_{d1} , into $1 + s$ by Ω_{d2} . For A you substitute now. A_{naught} by $1 + s$ by Ω_{d1} into $1 + \dots$ That is all. So, it becomes a second order system.

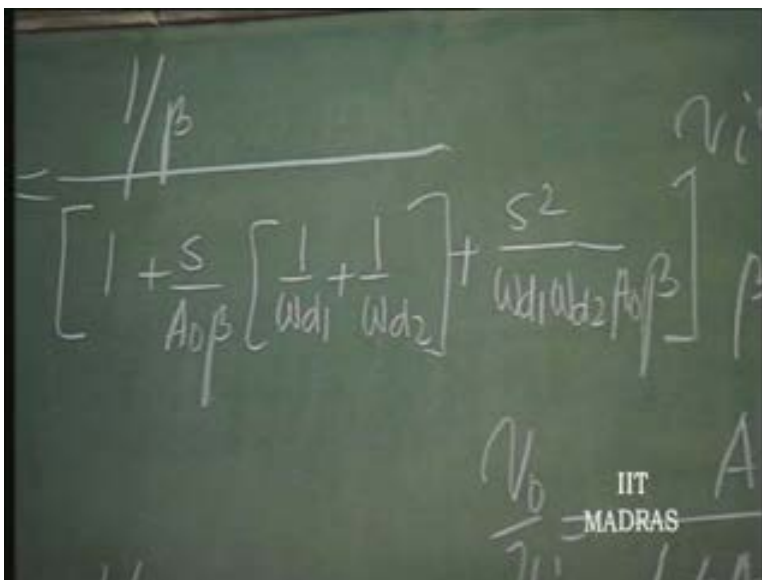
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The image shows a chalkboard with a handwritten mathematical expression. At the top left, there is a fraction $\frac{1}{\beta}$ with a horizontal line underneath it. Below this, the main expression is enclosed in large square brackets. Inside these brackets, there is a term $1 + \frac{s}{A_0\beta} \left[\frac{1}{\omega_{d1}} + \frac{1}{\omega_{d2}} \right]$ followed by a plus sign and another term $\frac{s^2}{\omega_{d1}\omega_{d2}A_0\beta}$. To the right of the main expression, there are some faint handwritten notes including v_i and v_o . In the bottom right corner of the chalkboard, the text "IIT MADRAS" is visible.

Once again, you expand this and this is 1 over Beta which is the gain of the amplifier with feedback, plus 1. We will ignore 1 over A naught Beta compared to 1; and then you have S into A naught Beta, 1 over Omega d 1 plus 1 over Omega d 2; plus S square into Omega d 1 Omega d 2 into A naught Beta.

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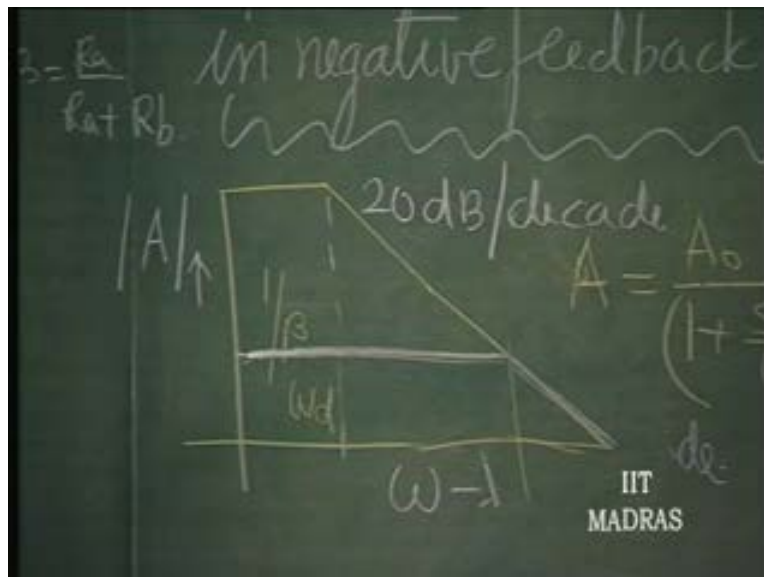


This image is identical to the one above, showing the same handwritten mathematical expression on a chalkboard. The expression is $\frac{1}{\beta} \left[1 + \frac{s}{A_0\beta} \left[\frac{1}{\omega_{d1}} + \frac{1}{\omega_{d2}} \right] + \frac{s^2}{\omega_{d1}\omega_{d2}A_0\beta} \right]$. The text "IIT MADRAS" is also visible in the bottom right corner.

So, this becomes a second order system with... What happens to this? Let us understand this. This can be rewritten as... Look at it. This is the original thing without feedback, open loop amplifier. In the case with only one dominant pole... Let me go back.

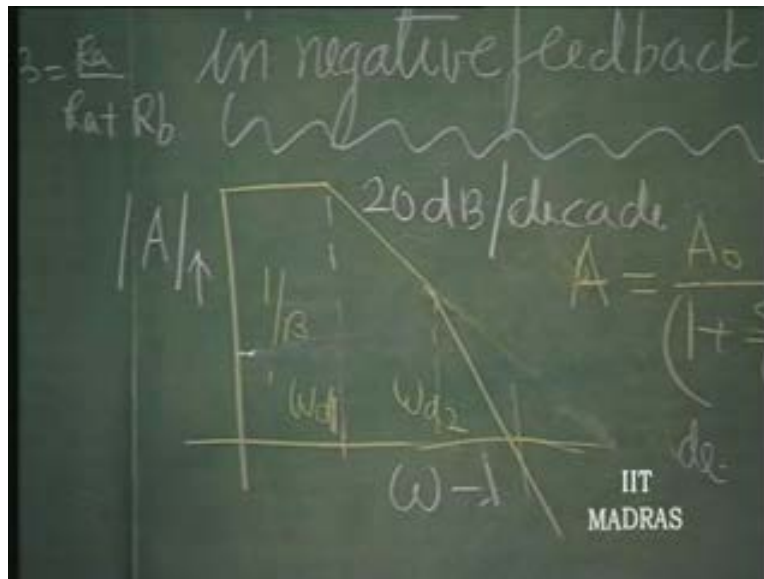
It was like this. This was ω_d . When you gave feedback, the gain got decreased to $1/\beta$ over β . But bandwidth got increased. This is the new bandwidth such that the gain into bandwidth remains a constant. This is what has happened. This is what has happened with the first order system. Its frequency response is this...this was the frequency response of the open loop amplifier and this is the frequency response.

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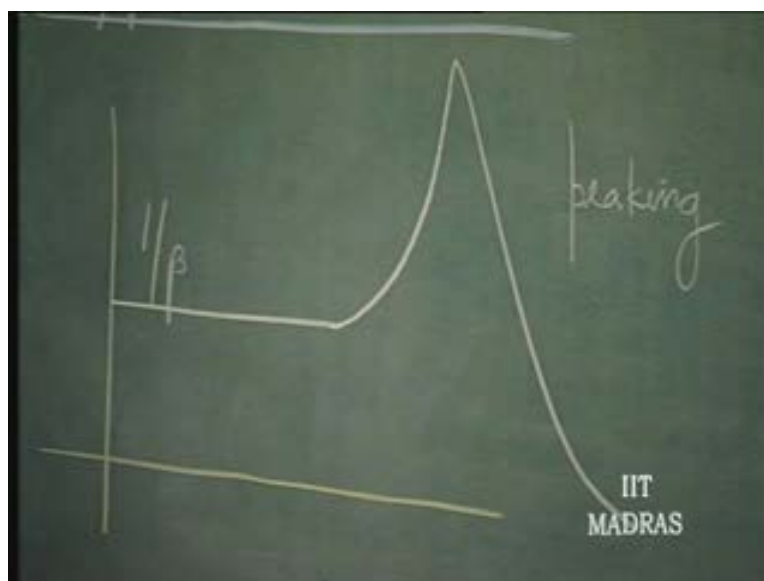
Now, when you changed it over to dominant poles, which are 2 in number. This is ω_d ; this is ω_d .

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I would like to know what happens to the transfer function. So, this is very interesting now. Draw this. If I draw this, the gain is going to be again $1/\beta$; just like the other one. At low frequencies, it is the same; same as this. But at high frequencies, what can now happen is that, that can be what is called peaking; because the poles of the system can now become complex conjugate pairs. Let us see what it is.

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Now, this can be rewritten as normalized frequency because it is already normalized. So, this can be...the whole thing now becomes a natural frequency, Omega naught square; just like in the case of a filter.

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The image shows a chalkboard with a handwritten transfer function. The function is:

$$= \frac{1/\beta}{\left[1 + \frac{s}{A_0\beta} \left[\frac{1}{\omega_{d1}} + \frac{1}{\omega_{d2}} \right] + \frac{s^2}{\omega_{d1}\omega_{d2}A_0\beta} \right]}$$

The term $\frac{s^2}{\omega_{d1}\omega_{d2}A_0\beta}$ is underlined and labeled as ω_0^2 . The text "IIT MADRAS" is visible in the bottom right corner of the chalkboard image.

So, Omega naught square is equal to Omega d 1 Omega d 2 into A naught Beta; or, Omega naught, natural frequency of this system, is root of Omega d 1 into Omega d 2 into A naught Beta, at a fairly high frequency; because A naught Beta may be pretty high even if Omega d 1 and Omega d 2 are pretty low. So, at a fairly high frequency, it has a natural frequency. It has a natural frequency like that. I am not bothered. Why should the peaking occur?

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Handwritten notes on a chalkboard:

- Top right: $\beta = \frac{R_a}{R_a + R_b}$
- Center: $\omega_0 = \sqrt{\omega_{d1} \omega_{d2} A_0 \beta}$
- Bottom left: "peaking"
- Bottom right: "IIT MADRAS" logo

This we will discuss here. This can be rewritten as again S by Ω naught and the rest of it is going to be Q there. So, we would like to see the pole Q of the system and show that the Q is going to be pretty high, if the loop gain is high. So, A naught β Ω naught is going to be root of this. So, I will divide this by Ω naught and multiply this by Ω naught so that S by Ω naught is taken out; the rest of the factor is nothing but what? – Q .

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Handwritten notes on a chalkboard showing the transfer function:

$$= \frac{1/\beta}{\left[1 + \frac{s \omega_0}{\omega_0 A_0 \beta} \left(\frac{1}{\omega_{d1}} + \frac{1}{\omega_{d2}} \right) \right] + \frac{s^2}{\omega_{d1} \omega_{d2}}}$$

Bottom right: "IIT MADRAS" logo

So, this Q is $A_{\text{no}} \beta$ by ω_n into $1 + 1/\omega_n d_1 + 1/\omega_n d_2$. If I divide by ω_n and multiply by ω_n , this factor is nothing but $1/\omega_n$. You have... habituated to writing this as a normalized component as $1 + S/\omega_n Q + S^2/\omega_n^2$.

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$$Q = \frac{A_0 \beta}{\omega_n \left[\frac{1}{\omega_{d1}} + \frac{1}{\omega_{d2}} \right]} \cdot \frac{1}{\beta}$$

$$= \frac{1/\beta}{\left[1 + \frac{S/\omega_n}{\omega_n A_0 \beta} \left[\frac{1}{\omega_{d1}} + \frac{1}{\omega_{d2}} \right] \right]} \cdot \frac{S/\omega_n}{\omega_n}$$

So, this is going to be equal to $A_{\text{no}} \beta$. You can see. Q is directly dependent upon $A_{\text{no}} \beta$, loop gain; and what is ω_n ? Square root of $\omega_{d1} \omega_{d2}$ into root of $A_{\text{no}} \beta$. That will bring about root of $A_{\text{no}} \beta$ in the numerator.

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The image shows a chalkboard with a handwritten equation. At the top, there is a partial equation: $\omega_{d1} = \omega_{d2}$. Below it, the main equation is:
$$= \frac{\sqrt{A_0 \beta}}{\sqrt{\omega_{d1} \omega_{d2}} \left[\frac{1}{\omega_{d1}} + \frac{1}{\omega_{d2}} \right]}$$
 To the right of this equation, there is a vertical line and the expression $1/\beta$. At the bottom right of the chalkboard, the text "IIT MADRAS" is visible.

So, this is root of A naught Beta divided by root of Omega d 2 by Omega d 1 plus Omega d 1 by Omega d 2.

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The image shows a chalkboard with a handwritten equation. At the top, there is a partial equation: $\sqrt{\omega_{d1} \omega_{d2}} \left[\frac{1}{\omega_{d1}} + \frac{1}{\omega_{d2}} \right]$. Below it, the main equation is:
$$= \frac{\sqrt{A_0 \beta}}{\left[\sqrt{\frac{\omega_{d2}}{\omega_{d1}}} + \sqrt{\frac{\omega_{d1}}{\omega_{d2}}} \right]}$$
 To the right of this equation, there is a vertical line and the expression $1/\beta$. At the bottom right of the chalkboard, the text "IIT MADRAS" is visible.

This is something like $x + 1/x$ and that is going to be maximum, if you want, when x is equal to 1; and this value can contribute to half only. At therefore Q of this system is directly proportional to root of A naught Beta. If the loop gain is high therefore,

the system will have poles located pretty close to the imaginary axis. Even though the amplifier had poles on the negative real axis; these are just poles on the negative real axis. The original amplifier had poles on the negative real axis. Because of negative feedback, for a second order system like this, the poles will now shift to...

If they are like this, they will go to complex conjugate pair. If $A_{\text{naught}} \beta$ is...that is why there is going to be peaking.

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There is resonance effect at a frequency which is very nearly equal to $\sqrt{\omega_n^2 - 2\zeta\omega_n}$. That we have understood. That means by the time the system has a second pole, the system is now going to be very nearly on the verge of instability; because, if we now have additional phase shift caused by additional pole, then this can shift to imaginary axis and system can become unstable.

So, this is the problem; that a negative feedback system is stable as long as the order of the system is second order. More than second order, it can become unstable. The poles can lie on the negative real... that is poles can lie on the imaginary axis; or they can lie on the positive half of the S plane. That means it can become unstable.

It will start oscillating at the natural frequency of the system, which is ω_n . If it can lie on the imaginary axis, if it is a third order system and the loop gain is high, it will definitely oscillate; and the poles will definitely lie on the imaginary axis; and therefore R on the right half of this plane... so it will go into oscillation.

So, this is the frequency instability in negative feedback amplifiers. That is primarily coming about because of the frequency dependence of amplifier. Another way of interpreting this is the loop gain becomes equal to 1 in magnitude when the phase shift of the loop gain becomes equal to 180 degree. What is negative feedback? The output is going to be developed in a manner that input voltage and the feedback voltage are opposing one another. If the...now, because of frequency dependence, the loop gain has additional phase shift of 180 degree; what is considered negative feedback becomes positive feedback.

At that point of time, if the loop gain is greater than 1, it will become unstable. There is regenerative positive feedback for that at that frequency and therefore it will sustain oscillation. There will be buildup of oscillation at that frequency and it will be a high frequency oscillation that you will see. So, an amplifier, negative feedback amplifier which is oscillating, is causing problem for us. So, we do not want this oscillation to take place. For that, we have to do what is called frequency compensation.

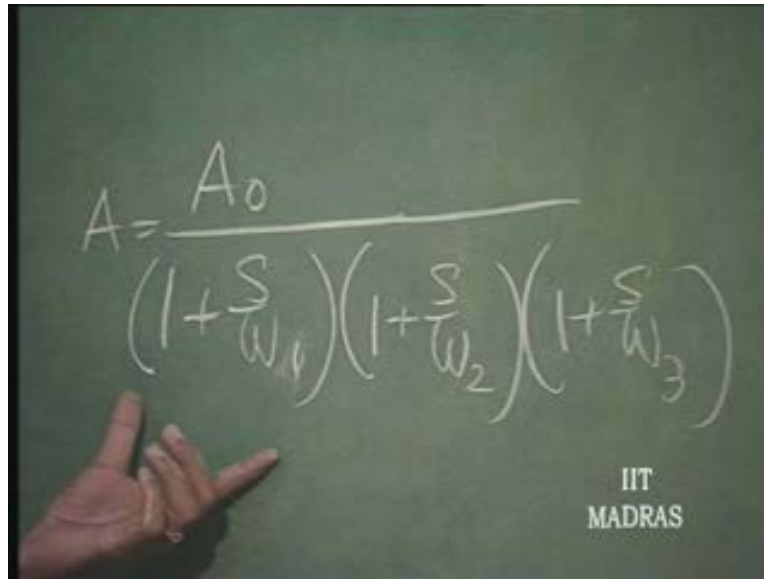
So now, let us consider the third order system put in the negative feedback. We have so far considered first order, second order. What happened in the first order was the bandwidth improved in such a manner that gain into bandwidth was a constant. What happened in the second order system was that the poles which were originally lying on the negative real axis got shifted to complex conjugate pairs and they could come very close to the imaginary axis. It can be interpreted in the following manner.

A was equal to A_n by $1 + S$ by ω_{c1} . Actually, we will call it as ω_{c1} now - first corner frequency; $1 + S$ by ω_{c2} - second corner frequency; $1 + S$ by ω_{c3} . Now, let us consider what happens. First we said, A is equal to A_n .

There was no problem with negative feedback. Now, A is equal to A_0 by $1 + S$ by Ωd . Phase shift of A can go at most to 90 degrees from zero, when it is first order.

When it is second order, the phase shift of A can go from zero to 180 degrees. That also, at infinity frequency. There is no finite frequency at which it can become 180 degrees; but, by the time frequency goes to infinity, the gain itself goes to less than 1; it goes to zero. So, there is no harm.

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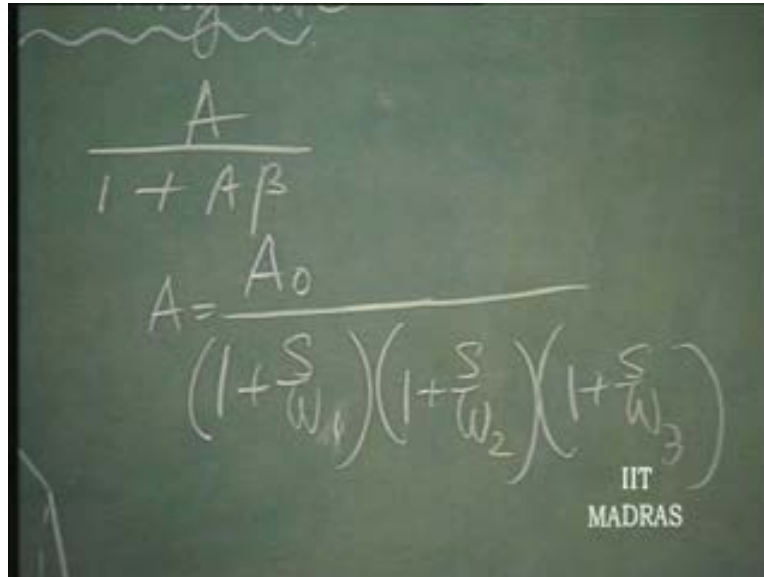

$$A = \frac{A_0}{\left(1 + \frac{S}{\omega_1}\right) \left(1 + \frac{S}{\omega_2}\right) \left(1 + \frac{S}{\omega_3}\right)}$$

But here in the third order system, A is equal to A_0 which is pretty high. The phase shift can now go from zero to 270 degrees; 90, 90, 90, at infinite frequency. That means there is obviously some frequency in between, at which phase shift can become equal to 180 degree. That means this A becomes equal to some minus, some value, negative. That means it becomes positive feedback.

Whatever you have earlier thought of as negative feedback now becomes...because the gain is going to be A by $1 -$... $1 + A\beta$, A becomes some negative value. That means it becomes positive feedback beyond that frequency; and if at that point...when it

is... becomes 1 minus A Beta, magnitude of that, we will take the magnitude of this and it has become negative. Then, if that magnitude A Beta is less than 1, there is no harm. It is still not going to give you any trouble.

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$$\frac{A}{1 + A\beta}$$
$$A = \frac{A_0}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)\left(1 + \frac{s}{\omega_3}\right)}$$

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If it is equal to 1, then it goes to infinity; gain goes to infinity. That means, at that frequency, it will start oscillating. For oscillator also, we had the same thing. Only difference was in the case of an oscillator, A Beta - loop gain, was becoming equal to 1 only at a single frequency; whereas here, beyond that frequency, if it is greater than 1, it can sustain oscillation at higher frequencies corresponding to that. And our oscillation may keep on building up if it is greater than 1 and it will be square wave instead of sine wave, at that particular frequency. The amplitude may be unlimited or it may get limited by the non-linearity of the amplifier.

So, this will invariably oscillate if magnitude of A Beta is greater than 1 when the phase shift of A Beta is equal to 180 degrees...this is an important statement...this is called Barkhausen criteria for stability or instability.

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The image shows handwritten notes on a chalkboard. At the top, the closed-loop transfer function is given as $\frac{1}{1 + A\beta}$. Below it, the open-loop transfer function is written as $A = \frac{A_0}{(1 + \frac{s}{\omega_1})(1 + \frac{s}{\omega_2})(1 + \frac{s}{\omega_3})}$. To the left, there is a partial Bode plot showing a corner frequency ω_3 . In the center, the stability condition is written as $|A\beta| > 1$ when $\angle A\beta = 180^\circ$. The IIT Madras logo is visible in the bottom right corner.

What is it? The loop gain in a negative feedback system... when the... when the phase shift of the loop gain becomes equal to 180 degrees, the magnitude of loop gain should be less than 1. Then only it is a stable system.

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The image shows handwritten notes on a chalkboard. At the top, it says "Darkhouse n" and "feedb". The word "Stable" is written and underlined. Below it, the condition $|loop\ gain| = 180^\circ$ is written. Further down, the condition $|loop\ gain| < 1$ is written. To the right, there is a diagram of a feedback loop with a block labeled A_0 and a feedback path. The IIT Madras logo is visible in the bottom right corner.

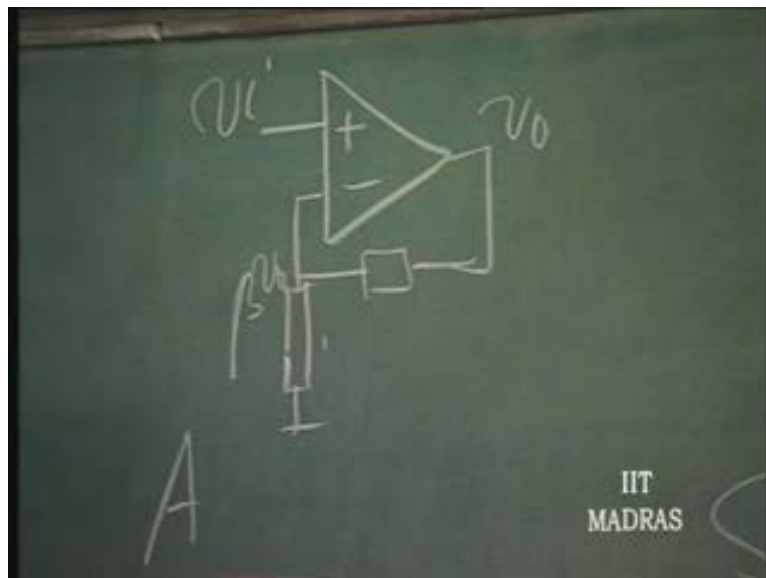
This is an important criteria. The...when the magnitude of loop gain becomes equal to 180 degrees, the loop gain magnitude should be less than 1. Then we do not have to worry about oscillation in amplifiers.

When do you have to worry about oscillation in negative feedback amplifiers? Only when the order of the system is greater than 2. Otherwise you do not bother. If the order of the system is 3 or higher, then you have to really worry about the stability problem and make sure that the loop gain, when it is equal to...when it has a phase shift of 180 degree, the magnitude of the loop gain is less than 1. Then also it is stable.

How do you do it? You have to have, obviously, low loop gain; and this should be spaced in such a manner that by the time the phase shift occurs of 180 degrees, the loop gain has gone down to less than 1.

So now, consider this. So, our amplifier has a... this is the same amplifier I am considering throughout.

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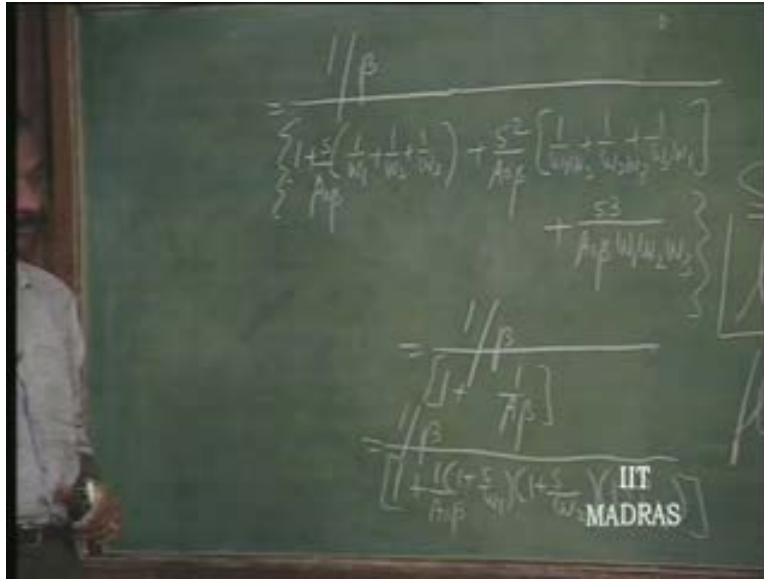
So, A by $1 + A\beta$; this is the amplifier gain with feedback; or this is equal to 1 over β divided by what? $1 + 1$ over $A\beta$. Just like last time. So, this is 1 over β divided by $1 + \dots$ 1 over $A\beta$, that will be neglected. And then you have, $1 + S$ by Ω_1 , $1 + S$ by Ω_2 , $1 + S$ by Ω_3 by $A\beta$.

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The image shows a chalkboard with handwritten mathematical expressions. The top part shows the derivation of the closed-loop gain:
$$= \frac{1/\beta}{1 + \frac{1}{A\beta}}$$
 Below this, the denominator is expanded to include three poles:
$$= \frac{1/\beta}{\left[1 + \frac{1}{A\beta} \left(1 + \frac{S}{\omega_1} \right) \left(1 + \frac{S}{\omega_2} \right) \left(1 + \frac{S}{\omega_3} \right) \right]}$$
 The bottom right corner of the chalkboard has the text "IIT MADRAS".

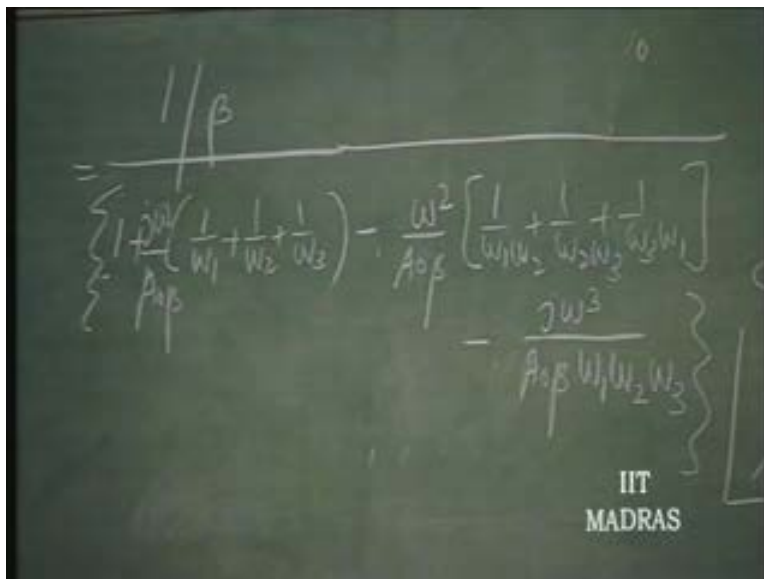
So, what happens here? 1 over β by 1 . Then, it will have a coefficient of S , which is S into $A\beta$, 1 by Ω_1 , plus 1 by Ω_2 , plus 1 by Ω_3 . This is one part. Next, plus S squared by $A\beta$. Then, coefficient of this will be 1 by $\Omega_1 \Omega_2$, 1 by $\Omega_2 \Omega_3$, 1 by $\Omega_3 \Omega_1$, then finally S cube $A\beta \Omega_1 \Omega_2 \Omega_3$. So, this is the third order system.

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What is the problem? The problem is if you put S is equal to j Omega, S is equal j Omega, then this becomes Omega squared. This becomes minus j Omega cube. So, you can see here, it was originally having no phase shift. Now, it can have a phase shift of 180 degrees. That is the loop gain. Then what happens if it has a phase shift of 180 degrees? Then it becomes purely real.

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When does it become purely real? – when the imaginary part goes to zero. So, you can now find out the frequency at which this becomes purely real. So, you can see here, this becomes equal to this. That is the frequency at which phase shift becomes equal to 180 degrees.

So, you can equate this. ω_0 by $A_{\text{mid}} \beta$ into $1 + \omega_0^2 \tau_1^2 + \omega_0^4 \tau_1 \tau_2 + \omega_0^6 \tau_1 \tau_2 \tau_3$. This becomes equal to ω_0^2 divided by $A_{\text{mid}} \beta \omega_0^2 \tau_1 \tau_2 \tau_3$. So, that is the frequency at which ω_0 square equal to... we can write this, equal to... What is that? $\omega_0^2 = \omega_0^2 \tau_1 \tau_2 \tau_3$, $\omega_0^2 = \omega_0^2 \tau_1 \tau_2 \tau_3$, $\omega_0^2 = \omega_0^2 \tau_1 \tau_2 \tau_3$...

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$$\frac{\omega_0}{A_{\text{mid}} \beta} \left[\frac{1}{\omega_1} + \frac{1}{\omega_2} + \frac{1}{\omega_3} \right] = \frac{\omega_0^2}{A_{\text{mid}} \beta (\omega_1 \omega_2 \omega_3)}$$

$$\omega_0^2 = \omega_1 \omega_2 + \omega_2 \omega_3 + \omega_3 \omega_1$$

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This is the frequency at which the phase shift becomes equal to 180 degree and at that point, the amplifier with feedback will have a gain going to infinity. How does it become infinity?

This particular thing becomes equal to this. That means this is already zero. This has gone. It has become purely real now; and now this can, at this frequency, ω_0 square, this can become equal to this, if $A_{\text{mid}} \beta$ is not adequately low. So, this can

become equal to this. So, if it becomes infinity, this can become equal to this. Beyond that frequency, what happens is that this will become negative and it will be positive feedback range.

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$$\frac{1}{\beta} - \frac{A_{OL}}{\frac{1}{\omega_1} + \frac{1}{\omega_2} + \frac{1}{\omega_3}}$$

So, you must have $A_{OL} \beta$ sufficiently low such that at that this frequency, this particular thing still remains positive. This 1 minus thing still remains positive. If it becomes equal to 1, there is a danger.

In the next class, we will work out an example to see how to limit this to reasonable value so that this is always less than 1. This should be always less than 1, when this happens. So, you can find out the value of β that you should use so that this condition is always satisfied.