

**Electronics for Analog Signal Processing - II**  
**Prof. K. Radhakrishna Rao**  
**Department of Electrical Engineering**  
**Indian Institute of Technology – Madras**

**Lecture - 19**  
**Wideband Amplifiers (Contd.)**

So, in the last class, we started discussing about a wideband amplifier which is Cascode structure. Cascode you get, when you cascade common emitter amplifier to common base amplifier; or common source in the F E T case, to cascaded to common gate.

(Refer Slide Time: 01:53)



What is the virtue of this? How did we get the wide banding? Wide banding is always got in all these cascaded structures by maintaining the time constants low. Wherever time constants occur, we try to maintain these low.

How do the time constants occur? The real part of the impedance and capacitance, these cause time constants at the respective terminals. So this, in a Cascode structure, this is the input time constant caused by input resistance, source resistance and input capacitance. Intermediate time constant is caused by output resistance, input resistance parallel and

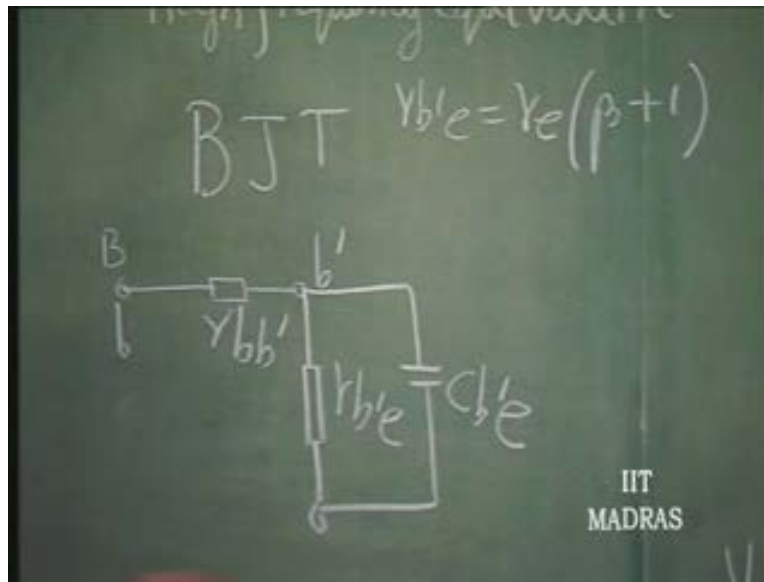
then capacitance at this junction; and this time constant is caused by the load, the output capacitor and the output resistance structure.

So obviously, capacitors have to be minimized; and we saw how Cascode structure reduces the input capacitor of the so-called common emitter amplifier. In that way, it retains the low frequency characteristic of the common emitter amplifier but can be used for wide band of frequencies.

So, let us put down the equivalent circuit. Now, as far as high frequency equivalent circuits are concerned, for the bipolar, you start with... in the case of common emitter high frequency equivalent circuit, you start with base. So, this is  $B$ . Then, you have a physical resistance called  $r_{bb}$ . This is the base resistance from the junction to the contact. This should be made as small as possible. This is of the order of few ohms, tens of ohms; and then we have the actual junction coming into picture. That is called  $r_{be}$ , because from the actual junction up to the base contact, there is a **non ohmic** resistance.

Then, junction comes into picture; and once we have the junction, we know that there is a resistance  $r_{be}$  which is, actually speaking...  $r_{be}$  is actually equal to your  $r_e$  which we have been calling as  $r_e = \frac{V_T}{I_E}$ , into  $\beta + 1$ . This is the normal impedance which we have been using for common emitter amplifier, except that there is some ohmic resistance of the order of tens of ohms added to that. Since this is of the order of Kilo ohms, in normal practice, this is always ignored; but at high frequencies, you will see there is a visible change in the characteristic, because the capacitance  $c_{be}$ , the junction capacitance comes directly across the junction. So, this is  $c_{be}$ .

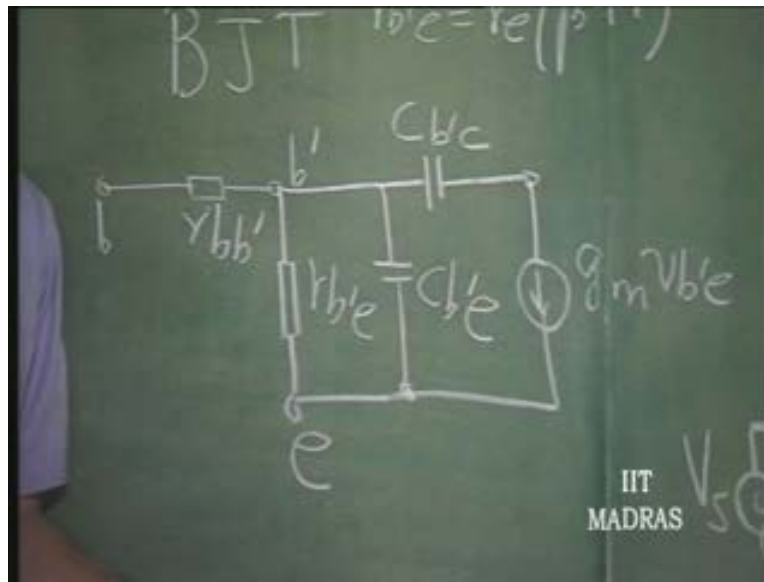
(Refer Slide Time: 05:38)



So, we have the emitter here. Then, from the base to the collector, there is a capacitance because it is a reverse biased junction. So, this is  $c_{bc}$ . This is going to be lower in order because this is a forward biased junction. This capacitance is called diffusion capacitance. This is going to be pretty high compared to  $c_{bc}$ .

So, this  $c_{bc}$  is the capacitance, reverse biased junction capacitance. This is called transition capacitance. This is the depletion layer capacitance. It depends upon the reverse biased voltage. As the reverse biased junction across the junction increases, this capacitance decreases. Then we have our usual low frequency structure which is  $g_m$  into  $v_{b'e}$ . This, we have been earlier calling just  $g_m$  into  $v_{b'e}$ ; this current source which we have earlier used;  $g_m$  into  $v_{b'e}$ .

(Refer Slide Time: 06:59)

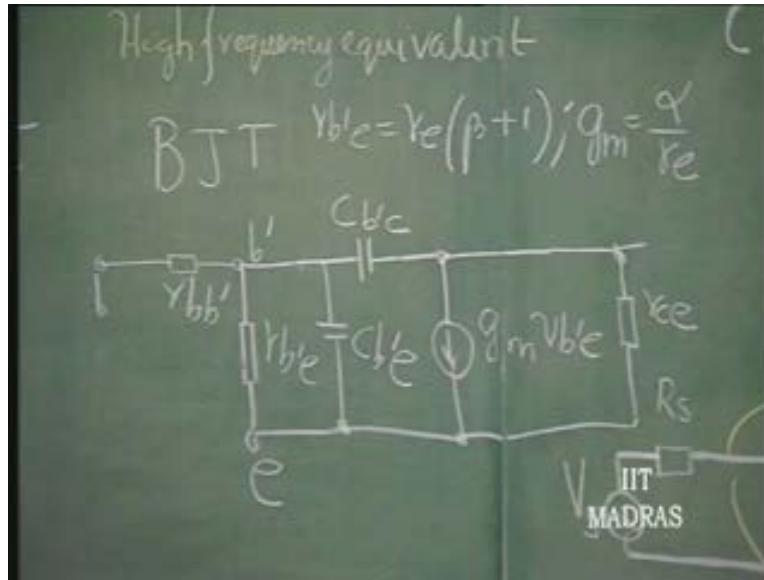


So then, actually,  $g_m$  is  $\beta$  divided by  $r_{b'e}$  according to us; low frequency. That is going to give you the low frequency collector current; and across the collector we have the so called  $r_{c'e}$ . That current source is not ideal current source. There is a shunt resistance  $r_{c'e}$  typically of the order of few mega ohms, in the case of integrator circuit transistors. So, this is what is called hybrid.

pi equivalent circuit for BJT. Hybrid because this **this** arm, this arm and this arm form a pi; but there is a T added to this, L added to this. So, it is called hybrid pi equivalent circuit, which is the most commonly used equivalence circuit for a common emitter transistor at high frequencies. In fact you can put a high resistance across this. This resistance is going to be of the order of hundreds of mega ohms. So, that is of no consequence for high frequencies.

So, these are the important parts of the equivalence circuit. So, when we discuss a wideband amplifier, we should put down this equivalent circuit in order to do the analysis.

(Refer Slide Time: 08:55)



In the case of field effect transistor, the same equivalent circuit is very simple because it starts with gate; and gate obviously is insulated from the source and the drain. So, there is only a capacitance, actually from gate, this is the collector terminal. So, gate to the drain, there is only a capacitance which is called  $C_{gd}$ , where the gate overlaps slightly the drain. If it does not overlap, this is zero; but it is very difficult to make it not overlap.

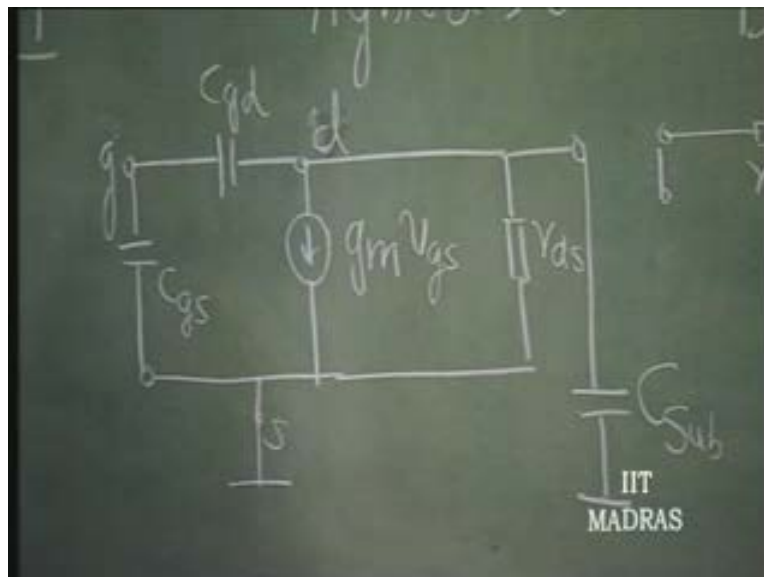
So, this  $C_{gd}$  is going to be very small; but it is there. Then similarly, we have similar to this capacitance,  $C_{gs}$  is going to the source. So, these capacitances are there. These are occurring here because of the junction. Here these capacitances occur because of the overlap between gate and drain, gate and source. With ion implantation technique, etcetera, this overlap has been made very nearly zero and therefore these capacitances can be made very low. Then, we have the same thing  $g_m$  into...in this case  $v_{gs}$ , the low frequency equivalent circuit that we have discussed earlier.

So, exactly same as that; and then across that, we have  $r_{ds}$ . In most of the cases, this is the high frequency equivalent circuit of the FET, JFET or MOSFET, it is the same equivalent circuit. Only difference is, if these are discrete circuits, this is the equivalent circuit. If they are integrate circuit components, then these components lie inside what is

called a well. A substrate has large number of wells, when... where these devices are put; and that each of these well is separated from the substrate by a junction which is reverse biased all the time.

So, that is how you separate one device from the other; and therefore there is what is called as substrate capacitance in each of these cases from this collector well. Or, **the** this well, we have a substrate capacitance. Normally, this capacitance is there both from the drain as well as the source here; but fortunately source is connected to substrate in most of the cases. So, this capacitance is not going to be of consequence.

(Refer Slide Time: 12:48)

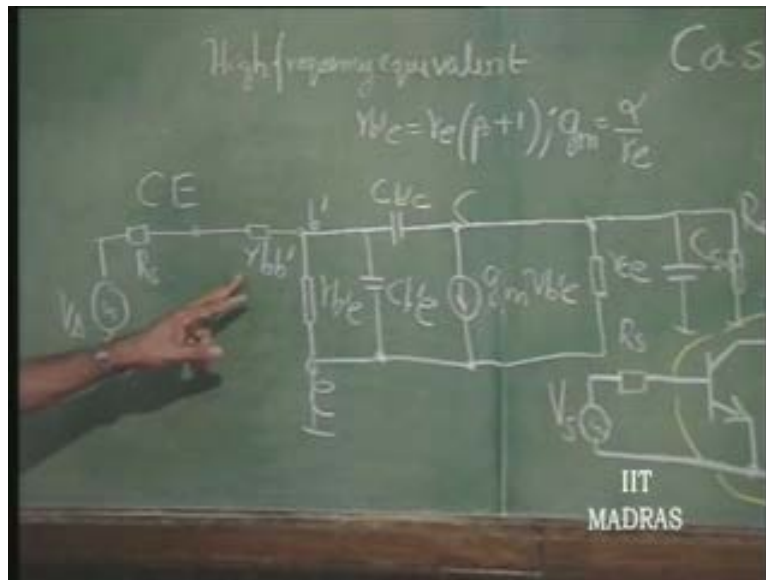


So, in the case of common source and common emitter amplifiers, this is going to be the equivalent circuit. That means this capacitance is going to be part of the output capacitance. So, we should have a complete picture of the equivalent circuit of the active device in order to understand and appreciate the limitations in band width.

Now let us take the case of Cascode and put this hybrid pi equivalent circuit and see what the time constants will be.

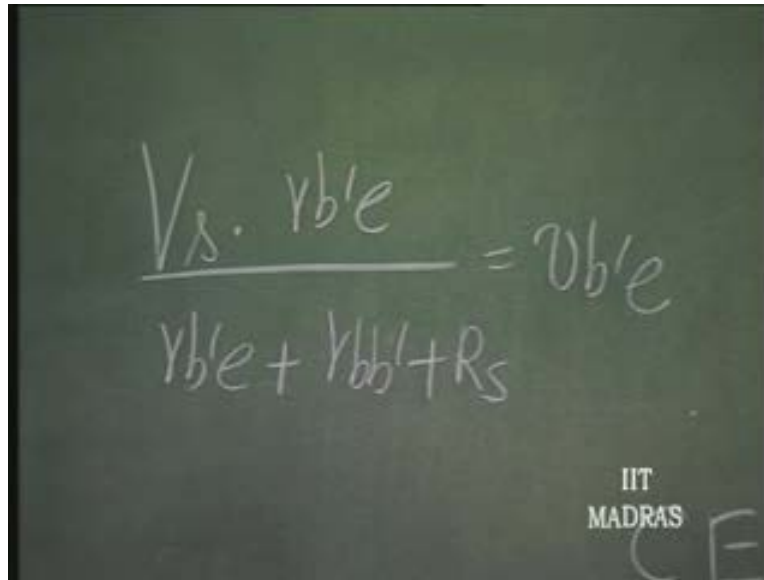
So, in a normal common emitter transistor, this collector is connected to a resistance  $R_c$ . So, in a sense we have  $R_s$ . This is for the common emitter high frequency equivalent circuit. So, if you forget these capacitors, let us look at only the low frequency equivalent;  $r_{be}$  comes in series with  $r_{bb}$ . So, it is essentially  $r_{be}$  itself totally and that comes in series with  $R_s$ . So, the gain gets reduced here to a certain extent by  $r_{be}$  divided by  $r_{be}$  plus  $R_s$  plus  $r_{bb}$ .

(Refer Slide Time: 14:14)



So, when voltage comes here,  $V_s$  into  $r_{be}$  divided by  $r_{be}$  plus  $r_{bb}$  plus  $R_s$  is nothing but  $V_{be}$ .

(Refer Slide Time: 14:53)


$$\frac{V_s \cdot y_{b'e}}{y_{b'e} + y_{b'b} + R_s} = v_{b'e}$$

IIT  
MADRAS  
CF

The  $v_{b'e}$  voltage here at low frequencies, this into  $g_m$  is the current in this.  $g_m$  into  $v_{b'e}$ . That current will flow into the parallel combination of  $r_{c'e}$  and  $R_c$ ; so this into  $r_{c'e} R_c$  divided by  $R_c$  plus  $r_{c'e}$ , that is with a negative sign because  $V_{naught}$ , we are going to define as this. So, this is going to be  $V_{naught}$  because current is flowing like this. It will develop plus minus as the polarity.

So, you see that this is the low frequency gain. What is it?  $g_m R_c$ . Actually, if it is  $r_{c'e}$  is very high, let us say, compared to  $R_c$ , it will be  $g_m R_c$ ; and here it is  $r_{b'e}$  divided by  $r_{b'e}$  plus  $r_{b'b}$  plus  $R_s$ . If this is very small, this again is going to be very nearly equal to 1 and essentially the gain is going to be minus  $g_m$  into  $R_c$ .

That is, if  $R_s$  plus  $r_{b'b}$  is much less than  $r_{b'e}$ , which is the case most of the time.



(Refer Slide Time: 16:56)

$$-\left(\frac{Y_{ce} R_c}{R_c + Y_{ce}}\right) g_m V_s \frac{r_{be}}{r_{be} + r_{bb'} + R_s} = V_o$$

$$1 - g_m R_c$$

$$Y R_s + r_{bb'} \ll r_{be}$$

When we are using a voltage source drive, the gain of the amplifier and then  $r_{ce}$  is much greater than  $R_c$ . This is the normal gain of a common emitter amplifier.

(Refer Slide Time: 17:10)

$$1 - g_m R_c$$

$$Y R_s + r_{bb'} \ll r_{be}$$

$$Y_{ce} \gg R_c$$

Now, what happens at high frequencies? So, I would like to obtain  $V_o$  over  $V_s$  at high frequencies. So, we will call this as our  $A_{mid}$ . That is the gain at low frequency DC indicating  $A_{mid}$ . So, at high frequencies,  $V_o$  over  $V_s$  becomes minus  $A$

naught divided by, let us see...  $1 + s \text{ by } \Omega_{p1}$ ,  $1 + s \text{ by } \Omega_{p2}$ , approximately.

(Refer Slide Time: 18:35)

The image shows a chalkboard with handwritten mathematical expressions and a circuit diagram. The main expression is the transfer function:

$$\frac{V_o}{V_s} = \frac{-A_o}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)}$$

Below the equation, there is a circuit diagram. It shows an input voltage source  $V_s$  connected to a feedback loop. The text "High freq" is written at the top left. The logo "IIT MADRAS" is visible at the bottom right of the chalkboard.

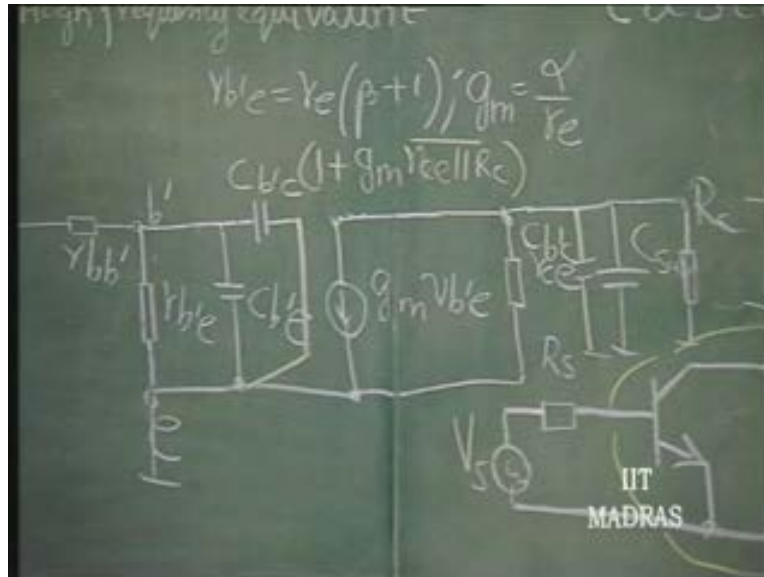
This we discussed in the last class; how it becomes. Let us see. The approximation is...  $C_{b-dash-e}$  is this capacitance and this  $C_{b-dash-c}$  gets amplified because this is the input and there is a gain of  $g_m \text{ into } R_c \text{ parallel } r_{c-e}$ . That is the low frequency gain.

So, it gets amplified by... this  $C_{b-dash-e}$  gets amplified by that factor  $1 + g_m \text{ into } R_c \text{ parallel } r_{c-e}$ . So, I can convert this capacitance. This is the approximation. Even though at that point I am not considering the frequency dependency of the gain, but it is going to give me more capacitance than what is actually going to exist because the gain is going to reduce any way.

So, I am actually over estimating the actual input capacitance. So, I am safe. So, this is going to simplify. This is going to be  $C_{b-dash-c} \text{ into } 1 + \dots$  This is the Miller effect which we discussed.  $r_{b-dash-c}$ ... there is  $r_{c-e} \text{ parallel } R_c$ . So,  $r_{c-e} \text{ parallel } R_c$  is very nearly equal to  $R_c$  itself. That we had earlier mentioned and across this now, we have a capacitance of  $C_{b-dash-c}$  coming into picture.

The Miller effect is, at the input it will be  $C_{bc} \text{ into } 1 + g_m r_{ce} \text{ parallel } R_c$  and at the output it is  $C_{bc}$  itself, very nearly.

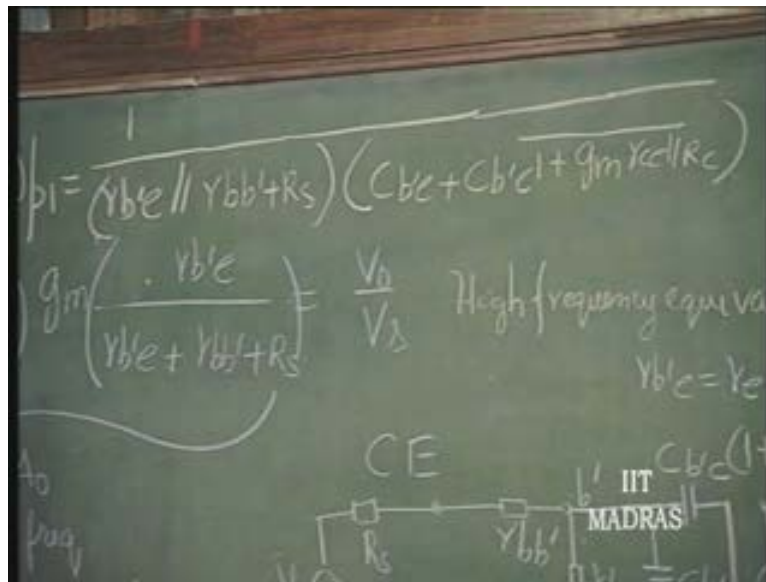
(Refer Slide Time: 20:26)



So now, we can indicate what  $\Omega_p 1$  is going to be.  $\Omega_p 1$  – it is going to be the time constant at the input. It is...it is determined by the time constant at the input. It is 1 over the time constant. So, 1 over... What is the net resistance across these capacitors?  $r_{be}$ ,  $r_{be}$  parallel  $r_{bb}$  plus  $R_s$ . This is important. If it is driven by a current source,  $R_s$  is infinity. Then, this will be just  $r_{be}$ . If it is driven by a voltage source, it will be very nearly zero. That means this time constant does not come into picture at all if it is a voltage source here.

Normally, even if  $R_s$  is zero,  $r_{bb}$  comes into picture. That is why the significance of  $r_{bb}$  at high frequency. Even if you are being able to make  $R_s$  equal to zero, there is a series resistance which is  $r_{bb}$ . So essentially, it will be  $r_{be}$  parallel  $r_{bb}$  plus  $R_s$  into  $C_{be}$  plus  $C_{bc}$  into  $1 + g_m r_{ce}$  parallel  $R_c$ . So, that is  $\Omega_p 1$ .

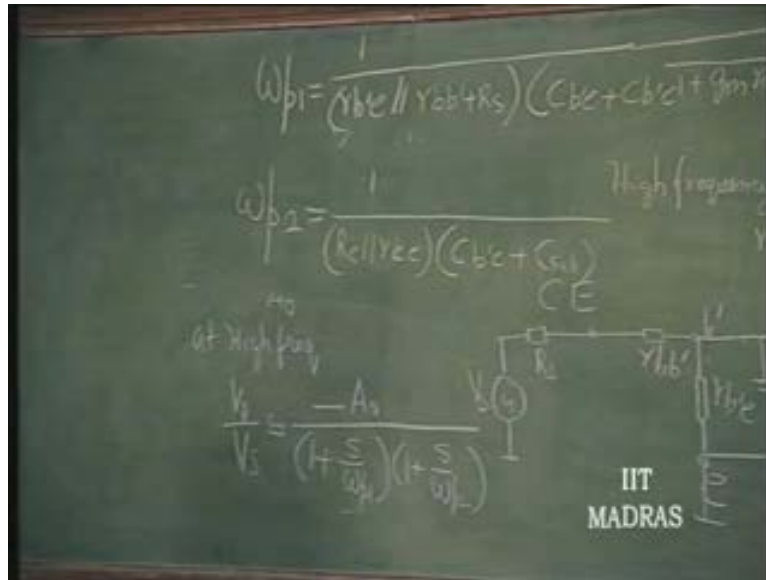
(Refer Slide Time: 22:14)



That can be written by observation. Always find out the net capacitance and the net resistance across the capacitance. That will determine the time constant at any point. So,  $C_{b'e} + C_{b'c} \parallel 1 + g_m r_c \parallel R_c$  is the net capacitance. Across that capacitance, we have a resistance of  $r_{b'e} \parallel r_{b'b} + R_s$ .

So, as far as  $\Omega p^2$  is concerned, this is going to be 1 over the time constant at the output. Let us see. Effective resistance is simply  $R_c \parallel r_{c'e}$ . Effective resistance  $R_c \parallel r_{c'e}$ ; and effective capacitance is  $C_{b'c} + C_{\text{substrate}}$ ; or, which is also called  $C_{\text{stray}}$ ; any stray capacitance. So, you can see that this capacitance is very small compared to this capacitance which depends upon the gain of the structure, which is likely to be high; and therefore this time constant normally dominates and that is the bandwidth.

(Refer Slide Time: 23:37)



How to improve the band width then? Then, that is why we had gone for a Cascode structure. What happens in a Cascode? All these things remain the same. Only thing is, now, there is no  $R_c$  here. Instead of this  $R_c$ , we have here just about  $r_e$ . That is a common base structure.

(Refer Slide Time: 24:17)

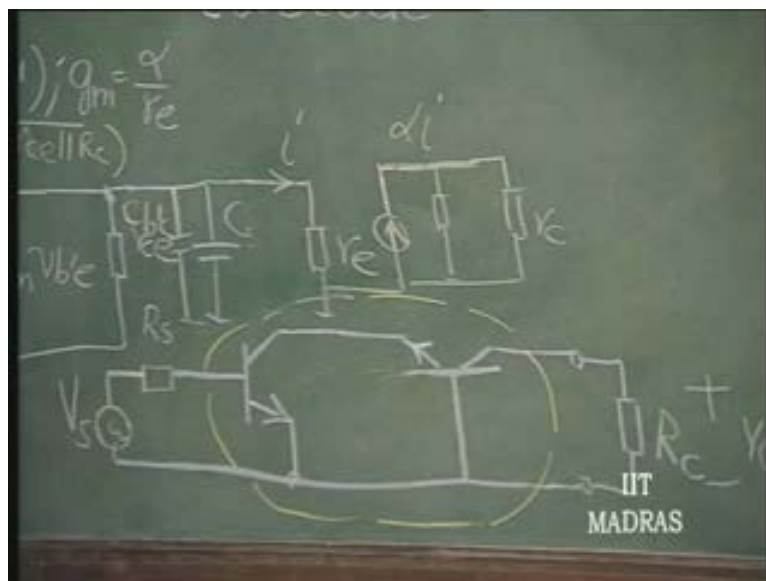


The resistance here is  $r_e$ ; very nearly... So, it is almost a short circuit at that point. And if this is the current  $i$ , then the current here is going to be... That is equivalent for the common base. At this frequency at which this is functioning, normally, common emitter, this common base normally need not be replaced by hybrid pi equivalent circuit. Even if you replace, you will know that ultimately the effect is only due to that, this common base structure, which is  $i$  and Alpha times  $i$ ; and Alpha cut-off of...Alpha cut-off frequency of this B J T is Beta times the Beta-cut off.

So, if common emitter is having a current gain which is Beta, its cut-off frequency let us say, is 100 megahertz. Then, if the Beta of the transistor is 200, this will be 200 times 100 megahertz. Since we are going to use the common emitter only in the range of 100 megahertz, there is no point in replacing this by its common emitter equivalent circuit and then finding out what the common base equivalent is. You can as well replace it by the common base equivalent which can be the low frequency equivalent.

So, Alpha times  $i$  and then  $r_c$  and this resistance, of course, is Beta plus 1 times  $r_c$ . This is for the common base. The output impedance is Beta plus 1 times the common emitter output impedance.

(Refer Slide Time: 26:13)



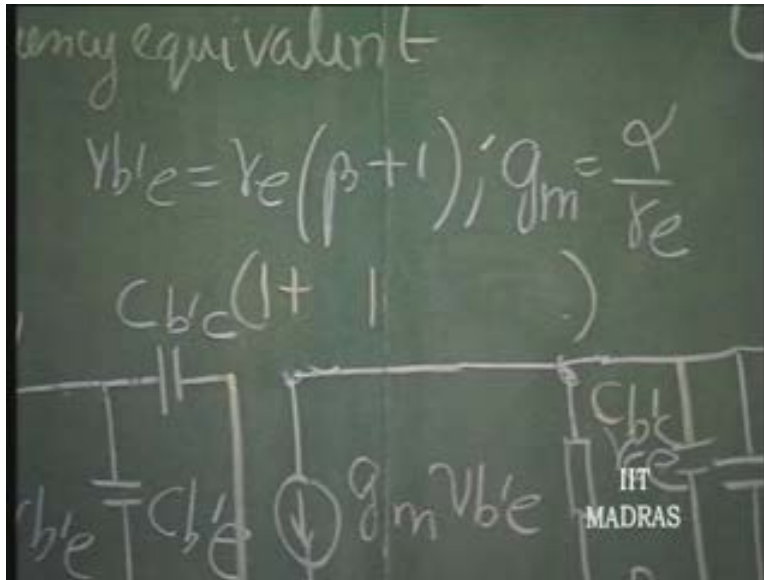
So, this resistance is going to be of the order of hundreds of megaohms. Anyway, that is what we had ignored in the equivalent circuit earlier also. It will come across  $C_{bc}$ . So, that resistance is normally ignored. So, this is the equivalent circuit; that is  $R_c$ . That is all.

(Refer Slide Time: 26:33)



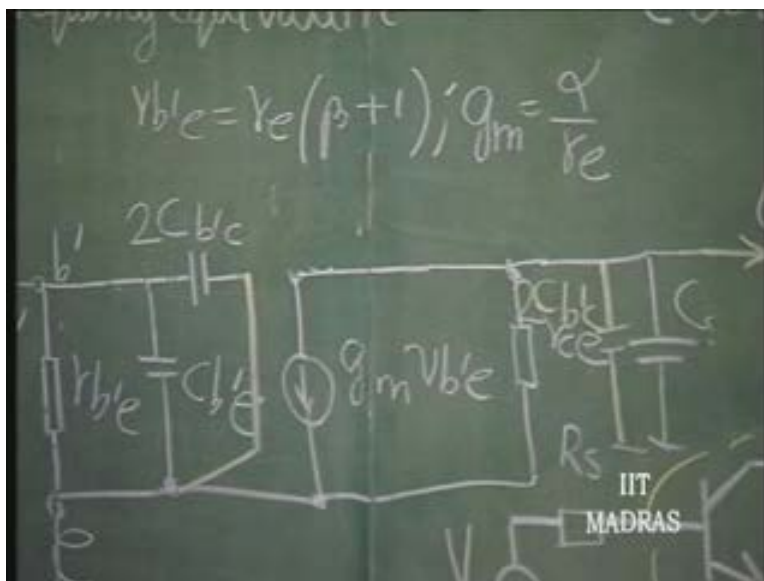
So, what this does is, this does not take part in deteriorating the high frequency performance of the structure. But, this part of the thing, as far as the common emitter is concerned, its high frequency performance is improved because Miller effect is now... just this, this is  $g_m$ ; and if both are operating at the same current, this will be  $1/g_m$  and that...this is going to be  $1 + 1$ .

(Refer Slide Time: 27:02)



And by the same token, this **this**  $C_{b'e}$  will be 1 plus 1, that is 2. That is an increase here because there, the gain is now not negligible. So, the output voltage and input voltage, they are of the same magnitude, but opposite in phase. So, we get  $2 C_{b'e}$  here at the output and  $2 C_{b'e}$  at the input. Is this clear? So,  $2 C_{b'e}$ ,  $2 C_{b'e}$  there.

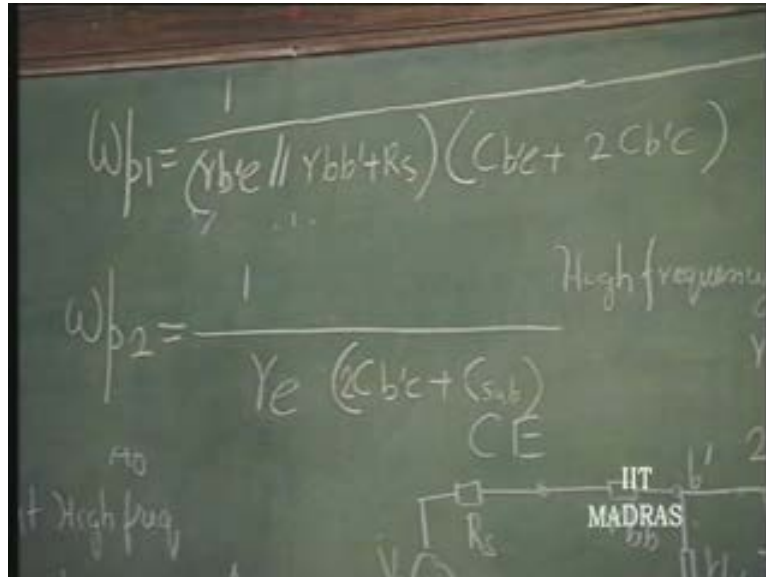
(Refer Slide Time: 27:38)





And therefore, this simply becomes... So, that is the thing. And as far as  $\omega_{p2}$  is concerned now, the resistive part is no longer  $R_c$  parallel  $r_{ce}$ ; It is  $r_e$  parallel  $r_{ce}$ , which is  $r_e$  itself, very nearly.

(Refer Slide Time: 28:12)

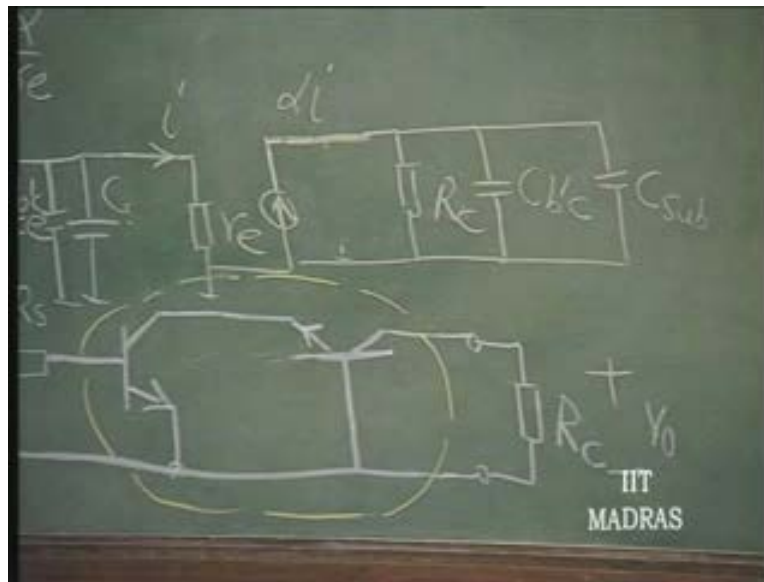


So, you can see, because not only this capacitance is low, now this particular resistance part is very low. Therefore this frequency is shifted to very high frequency. So, this goes very high. This is the intermediate pole. So, this is very high. This is still perhaps dominant because this resistance is not very small. This capacitance is quite small but compared to this, it is still large; but it has been reduced from the earlier value and ultimately now it has a third pole.

As far as the low frequency gain is concerned, it still remains  $A_{naught}$  because this current and this current are one and the same; and it still flows, flowing through  $R_c$ . So, the low frequency gain still remains  $g_m$  into  $R_c$ . In fact, it is just now,  $g_m$  into  $R_c$ ; no  $R_c$  parallel  $r_{ce}$ ; or also comes into picture. So, it is simply  $g_m$  into  $R_c$ .

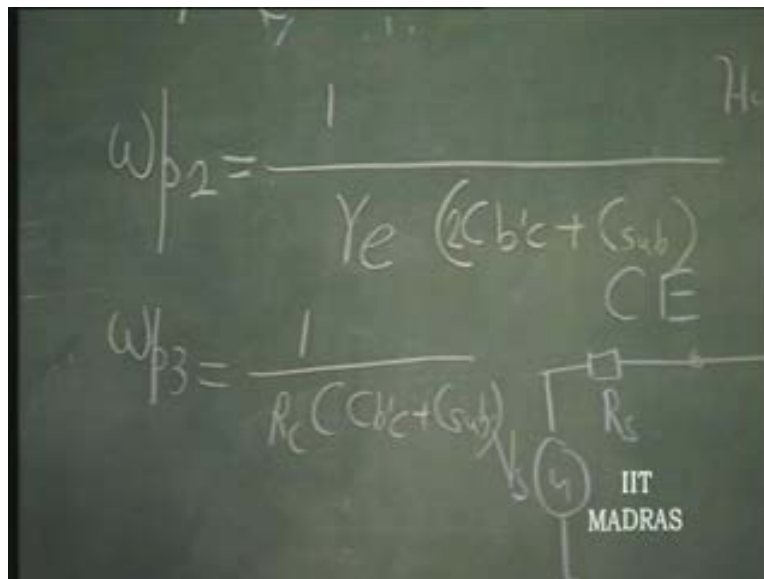
So,  $\omega_{p3}$  is the time constant here. Here we have a capacitance of course, which is nothing but  $C_{b'c}$ , the same reverse biased capacitance; and may be this  $C_{sub}$ .

(Refer Slide Time: 29:47)



So, we have  $1 \text{ by } R_c \text{ into } C_{b'c} \text{ plus } C_{\text{substrate}}$ .

(Refer Slide Time: 30:01)



So, you have 3 time constants coming into picture in determining the gain of the structure. So, what will be the gain? Therefore, for the cascode structure, I can write down the gain as,  $V_{\text{naught}} \text{ over } V_{\text{s as}}$ ,  $r_{b'e} \text{ divided by } r_{b'e} \text{ plus } r_{b'b} \text{ dash plus } R_s$ . That attenuation is always there; into  $g_m \text{ into } R_c$  with a negative sign.  $g_m \text{ into}$

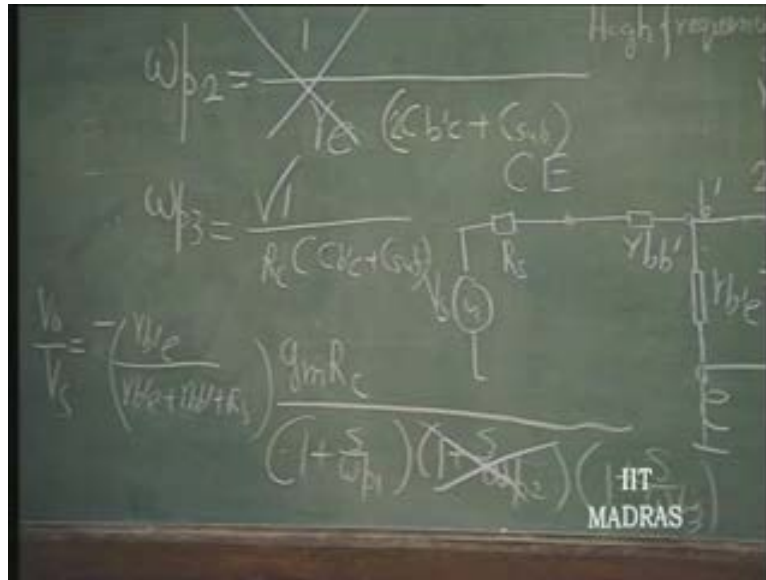
$R_c$  with a negative sign. Then, divided by  $1 + s \text{ by } \Omega_{p1}$ ,  $1 + s \text{ by } \Omega_{p2}$ ,  $1 + s \text{ by } \Omega_{p3}$ . So, this is the gain of the Cascode structure.

(Refer Slide Time: 31:30)



Once again, you see that it is very nearly same as the common emitter gain except that now the time constants here have become lower than that of the common emitter. So, the band width becomes higher. So, either this or this. This is normally very high. So, this does not come into picture in most of the cases. You can see because this is  $r_{e}$ ; so, almost nearly short circuit. So, this pole is of no consequence at all.

(Refer Slide Time: 31:53)



And, it is really speaking, a 2 pole system even now because this pole, intermediate pole, is at very high frequency at which point the gain might have fallen off to less than 1. So this, because of  $\Omega_{p1}$  and  $\Omega_{p3}$ , the gain fall off occurs. Which one is the one that is going to determine the cut-off frequency really now depends upon the magnitude of the source impedance as well as the magnitude of collector resistance.

Normally, the... if it is a voltage drive, the source impedance may be very small; in which case, it is this that fixes up the band width. Now you can see that this is very good because this is going to result in very large band width for your structure.

Take... typically  $R_c$  is, let us say, 1 Kilo ohm. If you have  $R_c$  of 1 Kilo ohm and  $g_m$ , let us say is 40 millisiemens,  $g_m$  into  $R_c$  is going to be 40; gain of 40. So now,  $\Omega_{p3}$  is going to be  $R_c$ , that is, 1 Kilo ohm into... let us take the  $C_{bc}$ , typical value. With integrated circuits, these will go to fractions of Pico farads. May be with individual distinct circuits, we can take typical value of 1 Pico farad, let us say. If it is 1 Pico farad... So, you get  $\Omega_{p3}$  which is 10 to power 9 radians per second Kilo mega radians per second; or in terms of this thing,  $f_{p3}$  is equal to 1000 by 2 pi megahertz, which is a very high frequency.

(Refer Slide Time: 34:37)

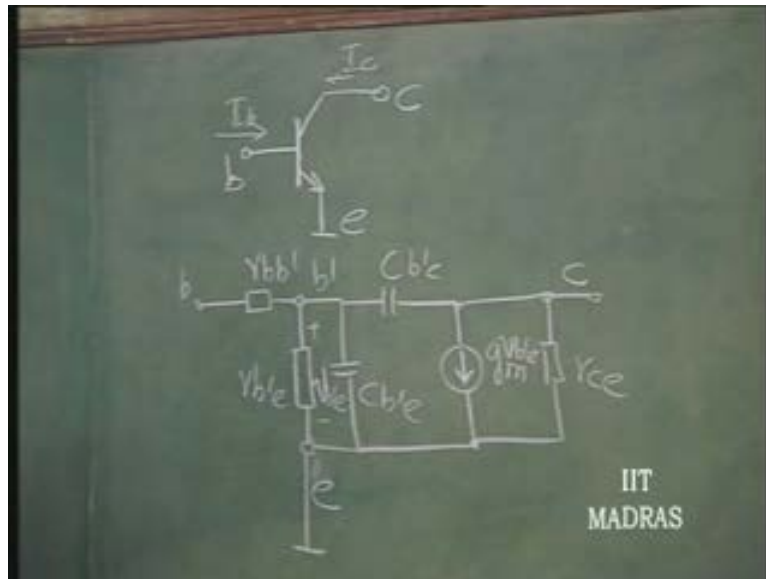
$$\omega_{p3} = \frac{1}{1000 \times 10^{-12}}$$
$$= 10^9 \text{ rad/sec}$$
$$f_{p3} = \frac{1000 \text{ MHz}}{2\pi V_0}$$
$$\omega_{p3} = \frac{\sqrt{1}}{R_c C}$$

IIT MADRAS

So, even if you consider this as few Pico farads, this is going to be definitely useable for wide band amplifier, that is video amplifier, which is of the order of few megahertz. So, this kind of structure is pretty common.

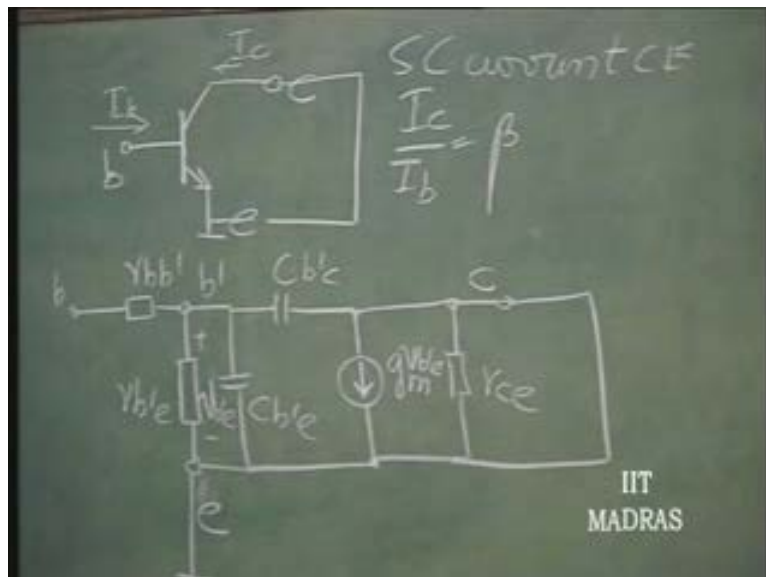
So, now that we have to use this hybrid pi equivalent circuit quite often in understanding video amplifiers, let us see how some of the low frequency parameters which we know are related to the high frequency equivalent.

(Refer Slide Time: 35:19)



This is the common emitter transistor. This is the hybrid pi equivalent. So, at low frequencies, these capacitors are not coming into picture. So, I know that I short circuit this, then  $I_c$  by  $I_b$  is called Beta by definition; is called short circuit current gain of a common emitter transistor. That means this is shorted.

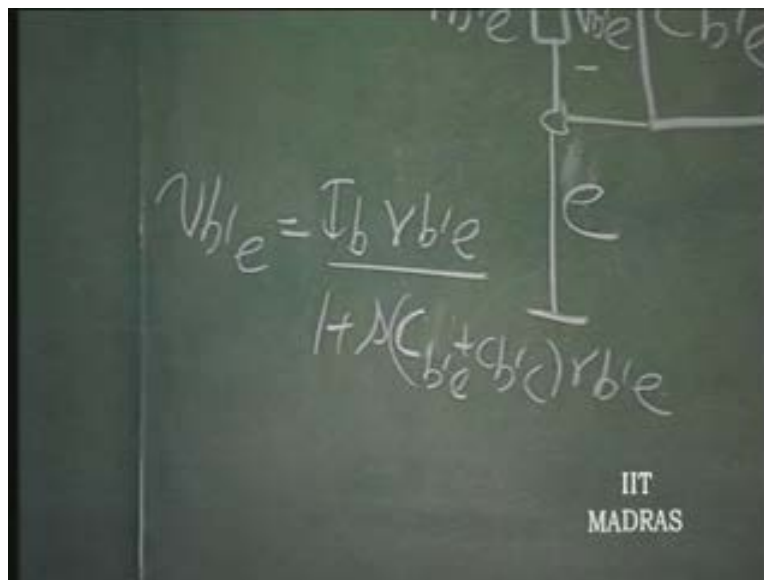
(Refer Slide Time: 35:56)



This is what you are going to do here; short the output and short circuit current gain is defined as Beta. At low frequency, it is Beta naught; independent of frequency. At high frequency, we would like to know what it is. So, the definition remains the same. At low frequency, this is called Beta naught. So, at high frequency, we will call that as Beta and see what that Beta is going to be from this. So, let us see here. We have  $r_{b'e}$  and  $r_{b'e}$ .

So, when I apply a base current  $I_b$  into this and output is shorted, this  $V_{b'e}$  at low frequencies or this  $V_{b'e}$  is going to be essentially equal to  $I_b$ ; it is independent of  $r_{b'e}$  because now, I am putting  $I_b$  as this current. So,  $r_{b'e}$  does not come into picture. This  $I_b$  will now flow through. This is shorted. So,  $C_{b'e}$  comes in parallel with  $C_{b'c}$ . So effectively, we have a resistance  $r_{b'e}$ . So,  $I_b r_{b'e} / (1 + s C_{b'e} r_{b'e} + C_{b'c} / C_{b'e})$ .

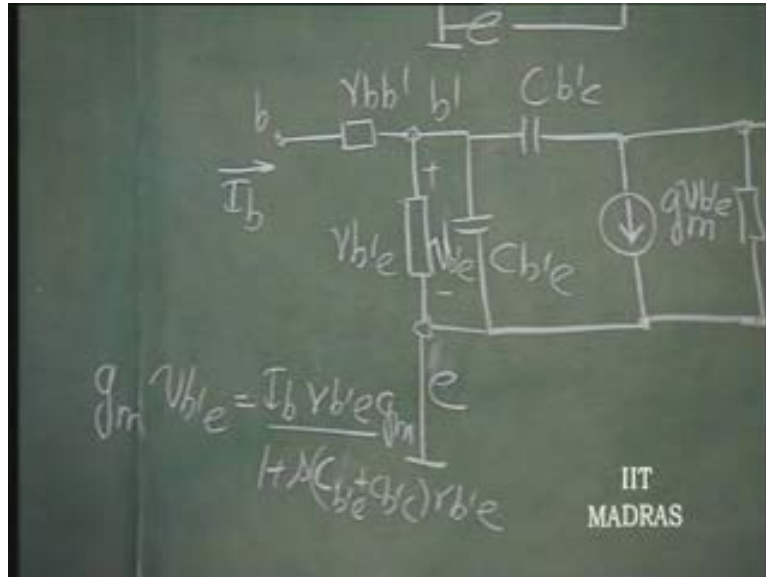
(Refer Slide Time: 37:45)



So, this is  $V_{b'e}$ .

This resistance comes in parallel with a combination  $C_{b'e}$  plus  $C_{b'c}$ . So, that is  $I_b$  into  $r_{b'e}$  divided by  $1 + s C_{b'e} r_{b'e} + C_{b'c} r_{b'e}$ . This is what  $V_{b'e}$  is. That into  $g_m$  is the current in this and that is the short circuit current.

(Refer Slide Time: 38:19)



So, this is the short circuit current  $I_c$ , short circuit current. So,  $I_c$  by  $I_b$  from this is nothing but, we will call this Beta, equals  $r_{b'e} g_m$  divided by  $1 + s r_{b'e} C_{b'e} + C_{b'c} r_{b'e}$ .



(Refer Slide Time: 39:12)

$$\frac{I_c}{I_b} = \beta = \frac{Y_{b'e} g_m}{[1 + s Y_{b'e} (C_{b'e} + C_{b'c})]}$$

Now, what is  $r_{b'e}$ ? We will expand that. This is also equal to  $r_e$  into Beta plus 1. That is  $r_{b'e}$ . And what is  $g_m$ ? Alpha by  $r_e$ . So, Alpha by  $r_e$ . And what is Alpha? Beta; Alpha is Beta; Alpha naught let us say, Alpha naught. Beta naught divided by Beta naught plus 1. So Alpha naught... So, Alpha naught is Beta naught.

(Refer Slide Time: 40:05)

$$g_m = \frac{\alpha_0}{r_e}$$

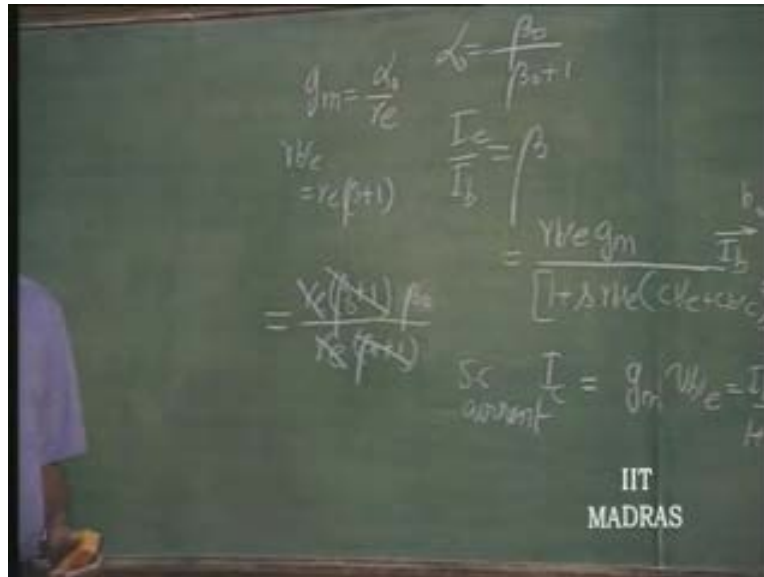
$$r_{b'e} = r_e (\beta_0 + 1)$$

$$\alpha = \frac{\beta_0}{\beta_0 + 1}$$

$$\frac{I_c}{I_b} = \beta = \frac{Y_{b'e} g_m}{[1 + s Y_{b'e} (C_{b'e} + C_{b'c})]}$$

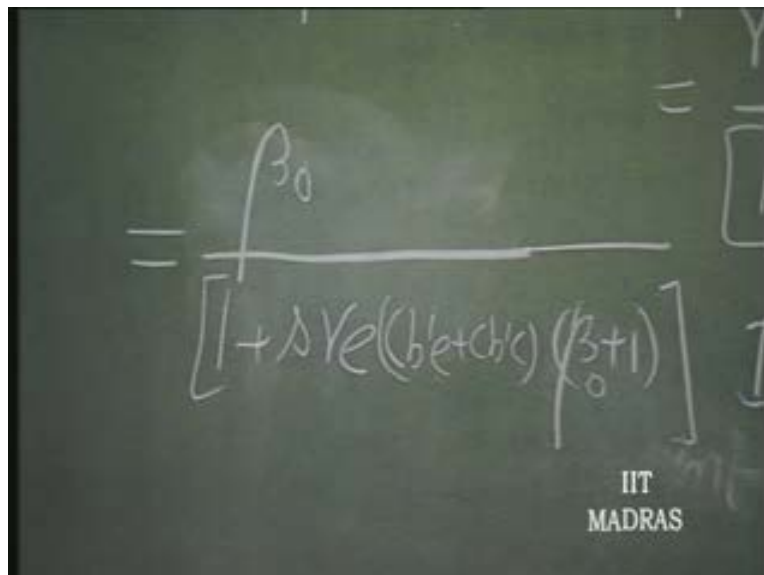
So, you can see here. This is, low frequency wise, nothing but Beta naught itself. This whole thing in the numerator is nothing but Beta naught.

(Refer Slide Time: 40:29)



So, we can just say that this is equal to Beta naught divided by 1 plus s into... now r b dash e is r e into Beta plus 1. So, we will put that C b dash e plus C b dash c into Beta plus 1, Beta naught plus 1.

(Refer Slide Time: 41:02)



So, you will see that Beta at low frequency falls off at 20 decibels per decade. This can be written therefore as... this is going to be Beta naught by 1 plus S divided by Omega Beta.

(Refer Slide Time: 41:29)

$$\beta = \frac{f_{\beta_0}}{[1 + s r_e (C_b + C_c) (\beta_0 + 1)]}$$

$$= \frac{f_{\beta_0}}{(1 + \frac{s}{\omega_{\beta}})}$$

IIT  
MADRAS

Omega Beta is called the cut-off frequency. Omega Beta in this case is nothing but 1 over  $r_e$  into  $C_b$  plus  $C_c$  into Beta naught plus 1. This is called the Beta cut-off frequency. So, this is determined by  $r_e$  into  $C_b$  plus  $C_c$  product; that into Beta naught plus 1.

(Refer Slide Time: 42:10)

$$\omega_p = \frac{1}{Y_e(C_{b1} + C_{b2})\beta_0 + 1}$$

IIT  
MADRAS

Let us now see what is Alpha? Alpha at high frequency is nothing but Beta at high frequency divided by Beta plus 1. This we have always known. So, this is equal to, let us write it as, let us say, 1 by 1 plus 1 over Beta. 1 over Beta here can be substituted from this as 1 plus S by Omega Beta divided by Beta naught.

(Refer Slide Time: 42:50)

$$\omega_p = \frac{1}{Y_e(C_{b1} + C_{b2})\beta_0 + 1}$$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{1}{1 + \frac{1}{\beta}}$$

$$\beta = \frac{\beta_0}{[1 + Y_e(C_{b1} + C_{b2})\beta_0 + 1]} = \frac{\beta_0}{(1 + \frac{S}{\omega_p})}$$

IIT  
MADRAS

So, we will rewrite this as Beta naught divided by Beta naught plus 1 plus S by Omega Beta into Beta naught; 1 plus 1 by Beta naught. That is, this Beta naught comes here. Beta naught... Sorry this is... just this. Beta naught here. Beta naught comes to the numerator. So, Beta naught by Beta naught plus 1 plus S by Omega Beta.

(Refer Slide Time: 43:33)

$$= \frac{1}{1 + \left[ \frac{1 + \frac{S}{\omega\beta}}{\beta_0} \right]}$$

$$= \frac{\beta_0}{\beta_0 + 1 + \frac{S}{\omega\beta}}$$

IIT  
MADRAS

So, I will take out Beta naught plus 1. So, this is equal to Beta naught divided by Beta naught plus 1 divided by 1 plus S by Omega Beta into Beta naught plus 1.

(Refer Slide Time: 43:46)

Handwritten mathematical derivation on a chalkboard:

$$\alpha = \frac{f}{\beta + 1}$$

$$= \frac{\frac{f\beta_0}{\beta_0 + 1}}{1 + \frac{S}{\omega_p(\beta_0 + 1)}}$$

$$= \frac{1}{1 + \left[ \frac{1 + \frac{S}{\omega_p}}{\beta_0} \right]}$$

$$= \frac{f\beta_0}{\beta_0 + 1 + \frac{S}{\omega_p}}$$

IIT MADRAS

What is Beta naught by Beta naught plus 1? Alpha naught. So, this is nothing but Alpha naught. So, Alpha is going to be Alpha naught by 1 plus S by Omega Beta into Beta naught plus 1.

(Refer Slide Time: 44:03)

Handwritten mathematical derivation on a chalkboard:

$$\alpha_0 = \frac{f_0}{\beta_0 + 1}$$

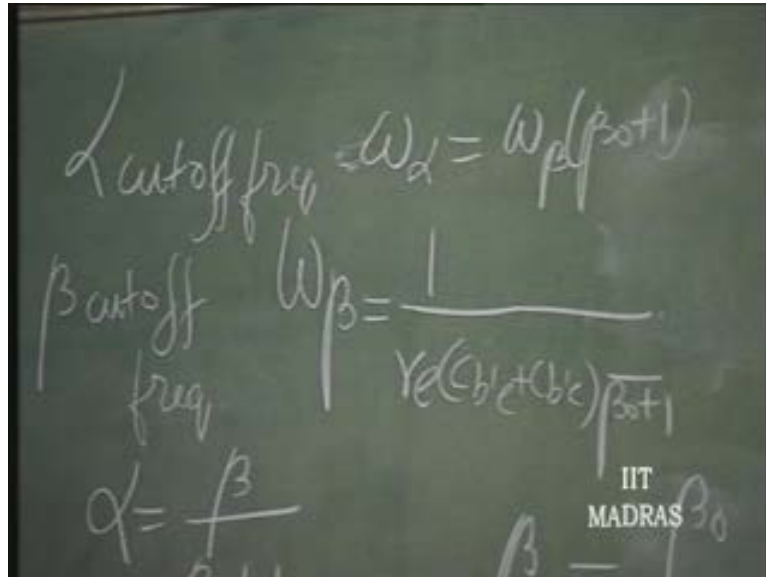
$$= \frac{1}{1 + \left[ \frac{1 + \frac{S}{\omega_p}}{\beta_0} \right]}$$

$$= \frac{f_0\beta_0}{\beta_0 + 1 + \frac{S}{\omega_p}}$$

IIT MADRAS

So Alpha cut-off frequency... This is Beta cut-off frequency. So, Alpha cut-off frequency is equal to Omega Alpha. We will call this as Omega Alpha; equal to Omega Beta into Beta naught plus 1, from this.

(Refer Slide Time: 44:32)



And we also know Omega Beta. Omega Beta into Beta naught plus 1 is nothing but 1 over r e into C b dash e plus C b dash c.

It is an important parameter. Alpha cut-off frequency. Beyond this, there is no worthwhile gain, current gain, for the transistor at all. So, this is also called f t, the transition frequency. What? It is that frequency...it is...f t is very nearly equal to... sorry, Omega t. We will call this, dimensionally, Omega t instead of f t.

(Refer Slide Time: 45:25)

Handwritten equations on a chalkboard:

$$\alpha_0 = \frac{\beta_0}{\beta_0 + 1}$$

$$\frac{1}{\beta_0 + 1} = \omega_t$$

$$\frac{1}{\beta_0 + 1} = \frac{y_{b'e} g_m}{C_{b'e} + C_{b'c}}$$

IIT MADRAS

$f_t$  is  $\Omega_t$  divided by  $2\pi$ . So, this is called transition frequency. What is transit? The gain actually, after this frequency, becomes less than 1. That is, Beta, if you consider, Beta itself is going to fall to magnitude which is less than 1.

And, or Alpha becomes point 707 times Alpha naught. So, it is the upper cut-off frequency of Alpha or where Beta falls to less than 1. That is the transition frequency and this is a parameter which is given by the manufacturer. That is why... So  $\Omega_t$ , if it is given,  $r_e$  can be evaluated depending upon the operating current.  $r_e$  is equal to  $V_T$  divided by  $I_{E Q}$ . Then you can find out what is  $C_{b'e} + C_{b'c}$ . So, the effective value of this capacitance can also be found out if  $\Omega_t$  is given. Manufacturer normally gives  $\Omega_t$ ; and therefore you can find out the value of  $C_{b'e} + C_{b'c}$  from that.

So, this transition frequency,  $f_t$ , is one of the important parameters, high frequency parameters, which is absolutely necessary to design any high frequency circuit. This is, really speaking,  $1/r_e$  is  $g_m$ ; normal gain is  $g_m$  into  $R_c$  and the band width is  $R_c$  into some capacitors  $C_{b'e} + C_{b'c}$ . So, gain band width product - it is a



measure which is called gain into band width product; current gain into current band width; gain band width product it is also called.

This is the same thing. Even in the case of a F E T, it is  $g_m$  of the F E T divided by  $C_{total}$ . So, this is... see, instead of  $C_{b\ e} + C_{b\ c}$ , there you will use  $C_{g\ s} + C_{g\ d}$ . So, this parameter is quite often used in evaluating the performance of, high frequency performance of, all the wide band amplifiers.