Electronics for Analog Signal Processing - II
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Lecture Number - 32
Multipliers

In the last class, we discussed transconductance type material using what is called translinear principle. The voltage V x was converted into sort of linear differential current using a transconductor of this type.

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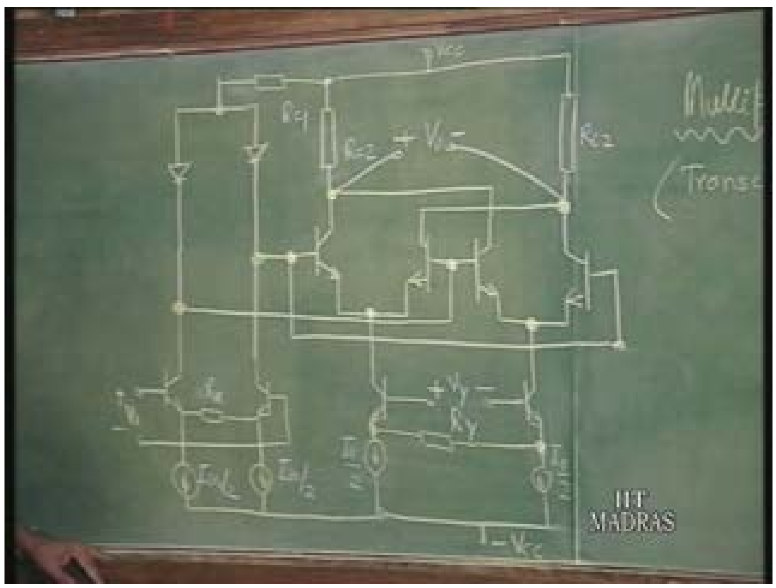

The current here was I naught naught by 2 plus V x by R x; and here, I naught naught 2 by...minus V x by R x. Differential current of this type was flowing here and that was the current in the diode; and we saw that the current in these transistors which we called as T 1, T 2, T 3, T 4, were respectively, let us say I... I... I c 1, I c 2, I c 3 and I c 4.

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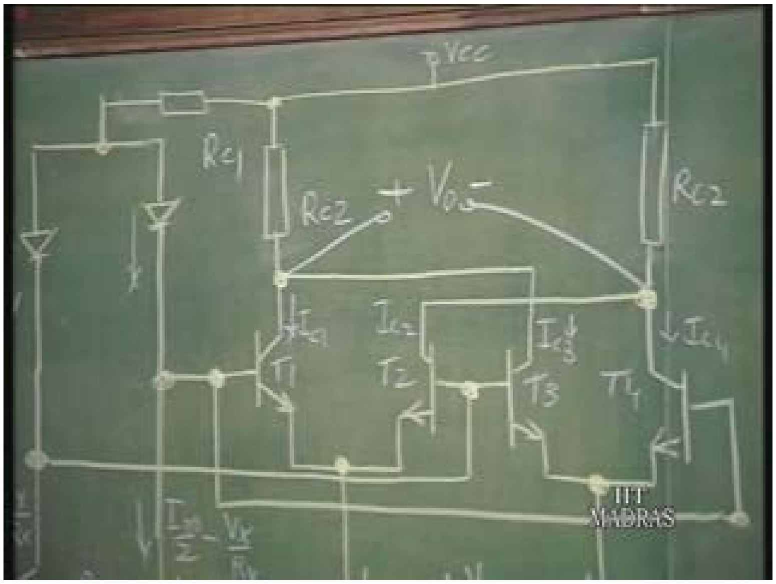

These are the currents and these currents are related to this in the following fashion that, this current into this current...because in the clockwise direction and anti-clockwise direction, the current products should be equated. This current which is I naught naught by 2 plus V x by R x divided by...into this current, that is I c 2, was equal to I naught naught by 2 minus V x by R x into I c 1.

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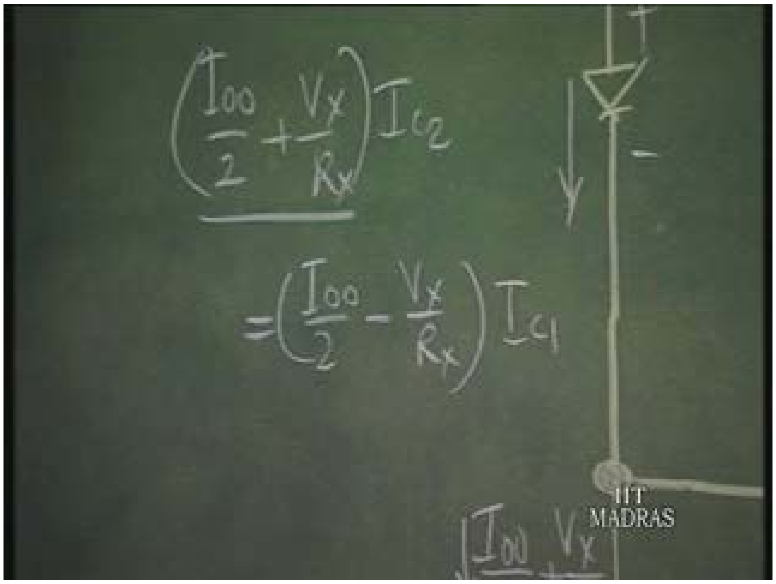

And since this...this was this voltage, these two voltages are adding. So, I naught naught by 2 minus V x by R x into I c 1; and the same voltage is also applied to another set; and therefore, what we get is that I naught naught by 2 by plus V x by R x into...this again is going to be coming like this; and then here. So...into I c 3 equals I naught naught 2 minus V x by R x into I c 4.

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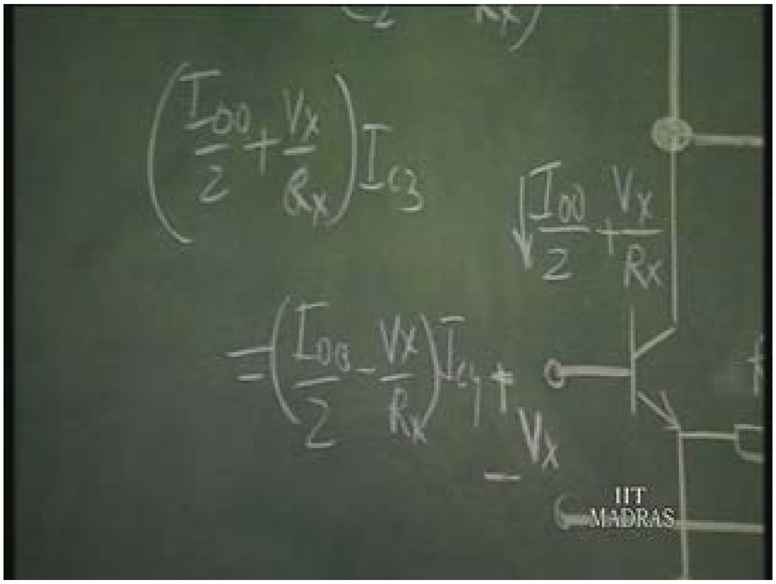

Using this relationship, and also the fact that I c 1 plus I c 2 equals I naught by 2 plus V y by R y. Here this is I naught by 2 plus V y by R y; and this is I naught by 2 minus V y by R y. Again, that means I c 3 plus I c 4 is equal to I naught by 2 minus V y by R y.

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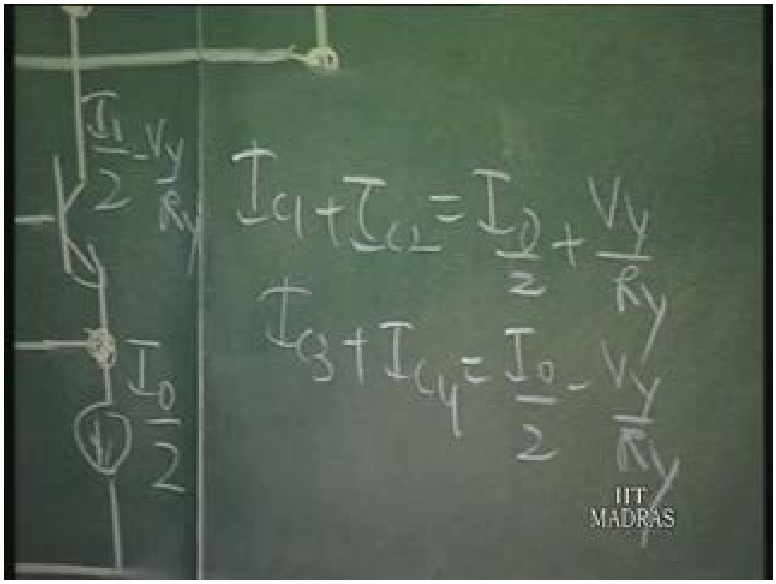

We could establish ultimately that V naught, the differential output voltage taken here, is going to be product V x V y divided by V R. So this, we had shown in the last class. This V R could be made equal to 10 volts by proper choice of R x, R y and R c 2 and I naught naught by 2 and I naught by 2.

So, we can make V R equal to 10 volts by proper...R x can be made equal to R y. R x can be made equal to R y and we have here...this I naught can be made equal to I naught naught, so that all these current sources can be derived out of the same current mirror principle.

So, I naught can be made equal to I naught naught. What is the expression in terms of R x R y can you give me? This V x V y, R x R y, I naught naught into R c 2; 4, 4 here. So, we can select R x equal to R y equal to 2 R c 2, equal to R. Then, what happens here is...this is equal to V x V y...R x equal to R y equal to 2 R c 2. 1R y gets cancelled with R c 2. So, we get R I naught naught by 2. So, this R I naught naught by 2 is made equal to 10 volts.

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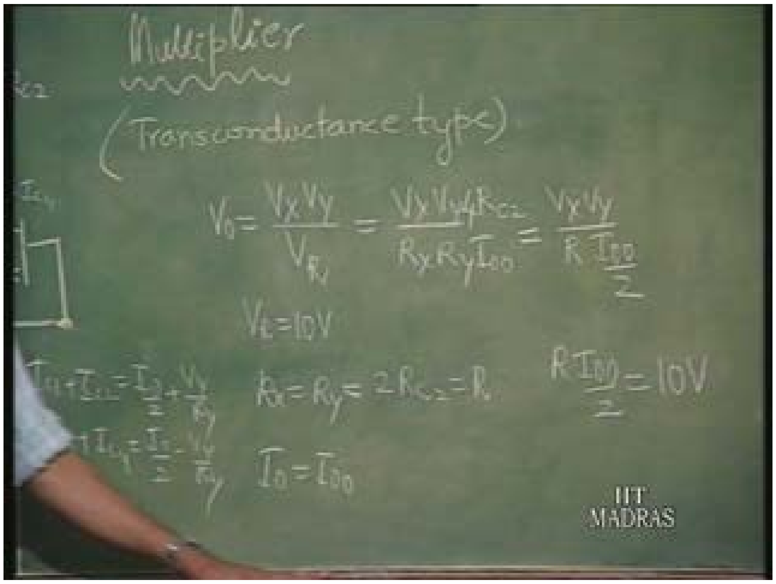

Therefore this becomes equal to V x V y by 10. Now, once we select this value, we can see here that this becomes equal to zero, under the worst condition, when V x max becomes equal to I naught naught by 2 into R x; and we have already chosen I naught naught by 2 into R x or R equal to 10 volts. So, this can now take care of V x and V y less than or equal to 10 volts as the dynamic range greater than or equal to minus 10 volts automatically.

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So, this design will see to it that this multiplier will function linearly in the range minus 10 volts to plus 10 volts automatically, the moment we select the components in the above manner. Is this clear?

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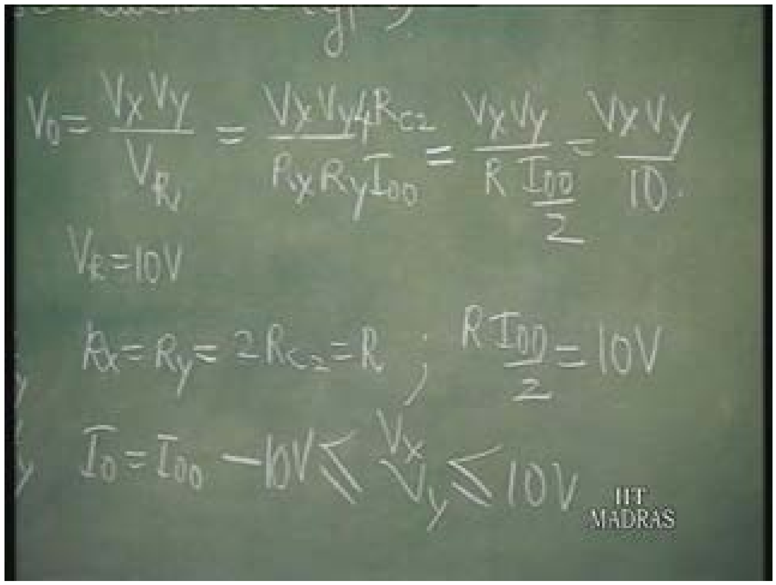

So, this multiplier therefore is useful for us within a dynamic range of minus 10 volts to plus 10 volts and V reference is 10 volts. So, it can give you at most plus minus 10 volts as the maximum output because V x equal to 10 volts, V y equal to 10 volts. Then, since V R is also 10 volts, V naught maximum also is plus minus 10 volts magnitude; it is going to be maximum of 10 volts. So, this transconductance type multiplier is a precision multiplier which can be basically used up to about 1 megahertz very easily.

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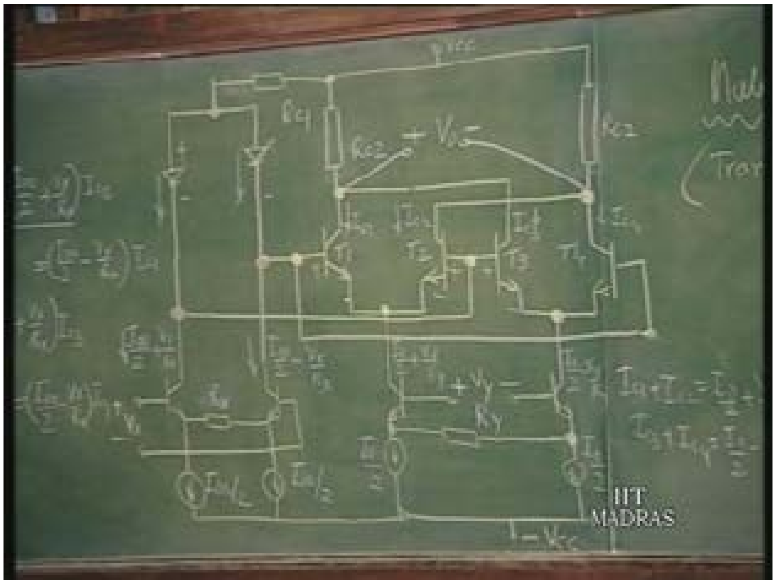

Now, this I C multiplier is readily available for use in a variety of applications. We will therefore go into application of precision multiplier and then go back to further types of multipliers at a later date.

Before we start discussing about other types of multipliers, we can digress and go to applications. It is obvious that the transconductance type multiplier is the one which is used in a very large number of communication applications and therefore it is time that we discussed some applications and then go back to discussion of other types of multipliers.

Now the multiplier, the ideal multiplier that we had designed, V x, V y and the output V naught let us say, is equal to V x V y by 10, with a dynamic range. With all the offset and feed through being adjusted to zero, this is the ideal multiplier.

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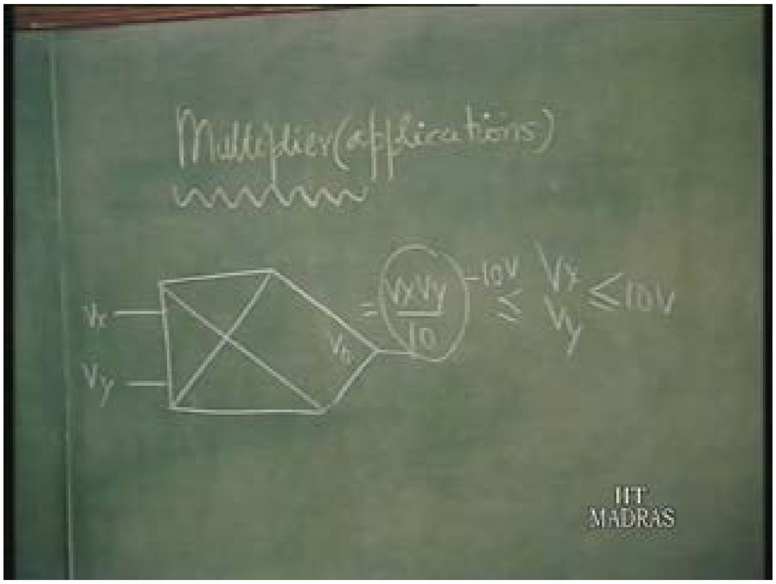

Let us therefore discuss the applications and see where and what are the areas this is of importance. First and foremost, let us consider V y is equal to a constant D C voltage V c, which is called the control voltage and V x is my regular input V i. Then, output V naught is going to be...this is the application ‘a’ as a voltage controlled amplifier.

Voltage controlled amplifiers find a variety of application, particularly in music synthesizers and A G C, A V C, etc. schemes; automatic gain control scheme, automatic volume control schemes, amplitude stabilization of oscillators,... In all these things, voltage controlled amplifiers are needed.

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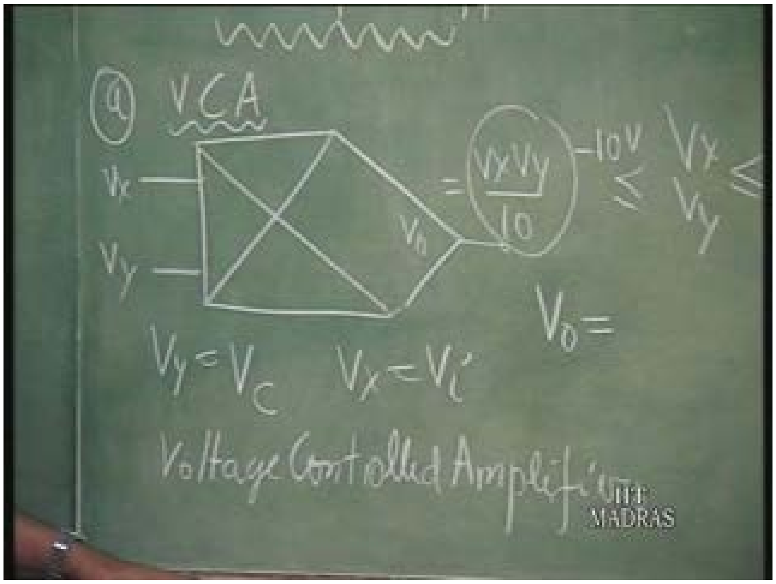

So, V naught then equals V c by 10 which is the gain, into V i. This is the gain of the amplifier; gain in the sense we know that it is not really voltage gain that we are talking of; it is power gain.

So, this still is an amplifier. V c is always less than 10 volts. So, the gain is always less than 1, in this case of ideal multiplier, please...But, V c can take on both positive and negative. So, it can become a non-inverting amplifier or inverting amplifier depending upon the sign of V c. So, this is a voltage controlled amplifier which can give you an inverting type of amplifier or non-inverting type of amplifier and the gain can be varied all the way from zero to 1, linearly.

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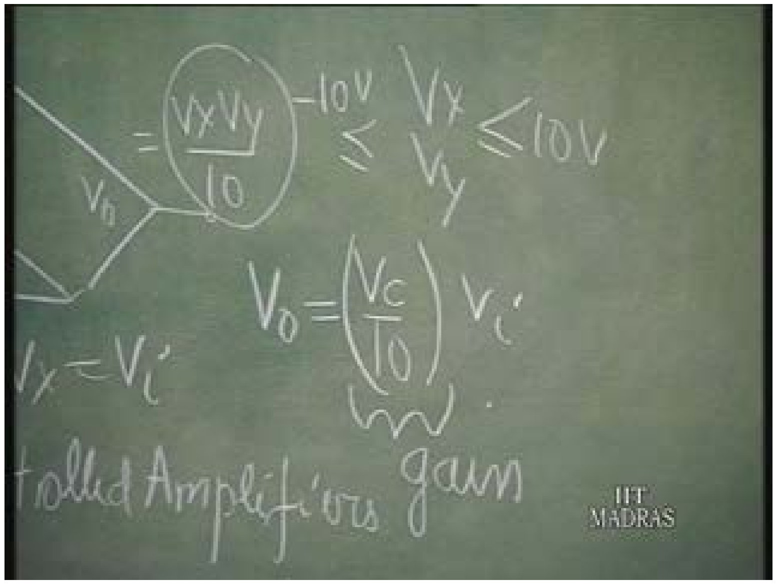

Next, there is such a voltage control amplifier is there and I use this in combination with, let us say an integrator. Let us see what happens. The normal integrator has the input here. But now, I am feeding the input through a multiplier. This is V i and this is V c. So obviously, this becomes V i V c by 10. So, output now becomes integral. Therefore, V i V c by 10 R c into s with a negative sign.

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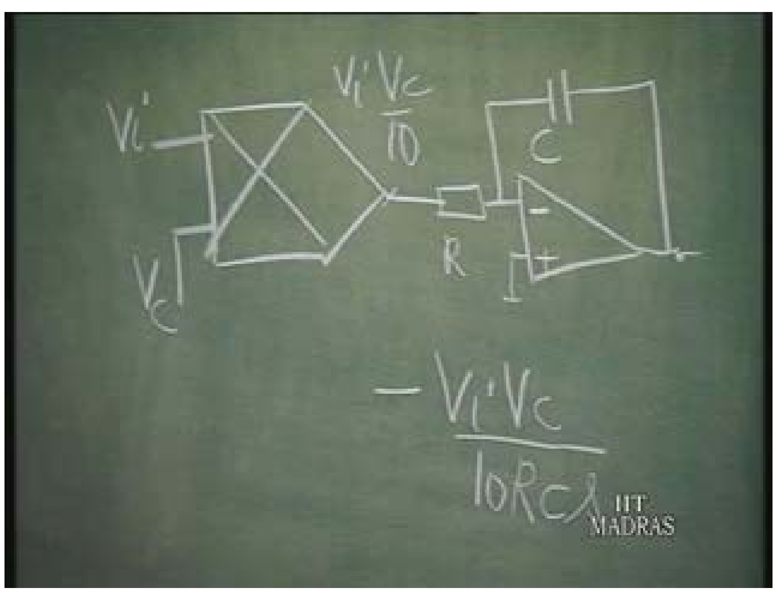

Normal integrator output is minus V i by S c r. So now, it becomes minus V i by 10 into V c by R c s. So, it is equivalent to saying that it is like earlier integrator, but with the time constant R C being replaced by R C into 10 by V c. So, the time constant is changed from the original R C into R C by... R C by V c into 10.

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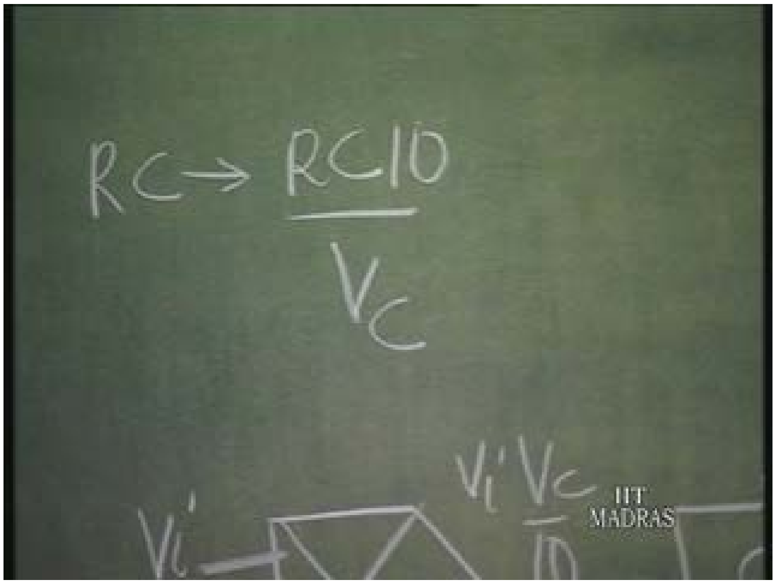

So, the time constant is inversely proportional to V c. If that is the case, if I design an oscillator, for that matter filter or oscillator, using this, then the frequency of oscillations or the pole frequency of such a system is going to be 1 over R C under normal case, going to be replaced by V c by 10 R C.

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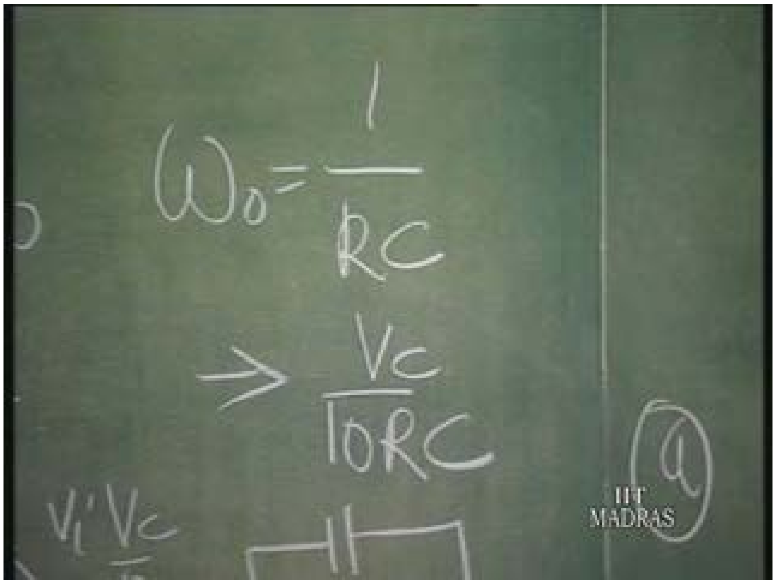

We had earlier discussed a K H N filter or universal active filter, which use two integrators. Both the integrators now, let us say, are replaced by such modified integrators, then the Omega naught or the pole frequency is going to change to V c by 10 R C. If you design a double integrator or quadrature oscillator, the frequency of oscillation or the natural frequency of the system also is going to be 1 over R C which is going to be replaced by V c by 10 R C. So, that means I can build voltage control filters, voltage controlled oscillators, linear; all this using such a replacement of the integrator.

So, wherever ordinary integrator is there, you replace it by a multiplier in combination with integrator. Then the R C time constants will be replaced by 10 R C by V c; and all such circuits become voltage controlled and they can be used again in music synthesizers, programmable filters and all these things. Voltage controlled oscillator, voltage controlled oscillator is again a circuit whose frequency of oscillation is directly proportional to control voltage and it is nothing but an F M generator because if I now feed a carrier frequency here plus a modulation frequency, if I feed this here as a voltage, then actually, modulating frequency is fed here. So, output will be a carrier frequency plus the quiescent frequency which is...quiescent frequency plus modulating frequency, which is nothing but an F M.

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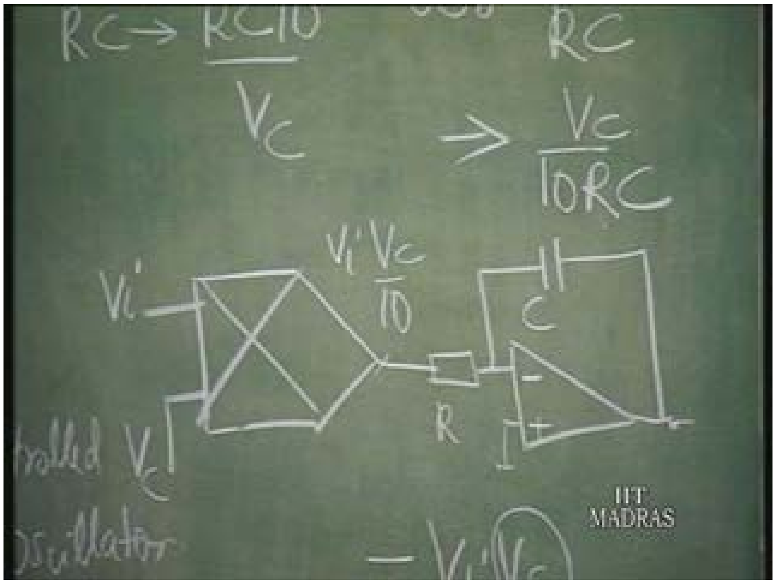

So, voltage controlled oscillators are nothing but F M generators. This is a linear thing and therefore it is a wide band F M generator. This is important that...because of its perfect linearity, it is possible to use it for wide band F M generation in things like, let us say, frequency shift keying, etcetera.

Such F M generation is F S K generation is possible; apply the digital ones and zeroes here, digital ones and zeros with additional D C. Then, output will be a quiescent frequency plus the frequency varying from certain value; let us say at 200 hertz, it is 1; at 100 hertz, it is zero; something like that. So, you can get an F S K output directly from this.

So, voltage controlled amplifier. How this is suitable for use in, let us say A G C and A B C, we will discuss slightly later, because it is going to be a controlled system like that of a voltage regulator. We will discuss slightly later.

Other applications like, now if V c...V x is equal to, let us say V p sin Omega 1 t, V y is equal to, let us say V p 1...V p 2 sin Omega 2 t; the inputs are now different.

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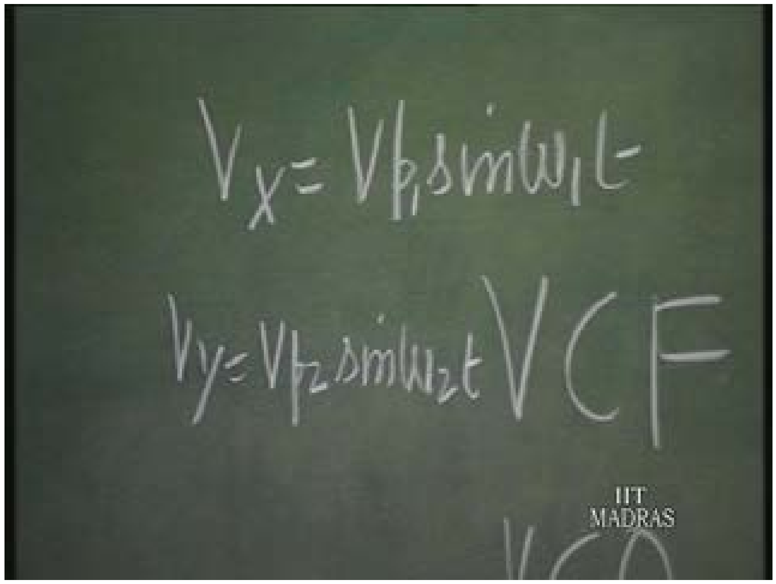

These are two different frequencies that are coming as inputs to the multiplier. So, what is going to be the output? This is of interest to us in communication.

So, output is going to be V p 1 sin Omega 1 t V p 2 sin Omega 2 t divided by 10, which is going to be V p 1 V p 2 by 20 times sin a sin b is cos; that is by 2 is there, factor of 2; cos a minus b which is Omega 1 minus Omega 2 t minus cos Omega 1 plus Omega 2.

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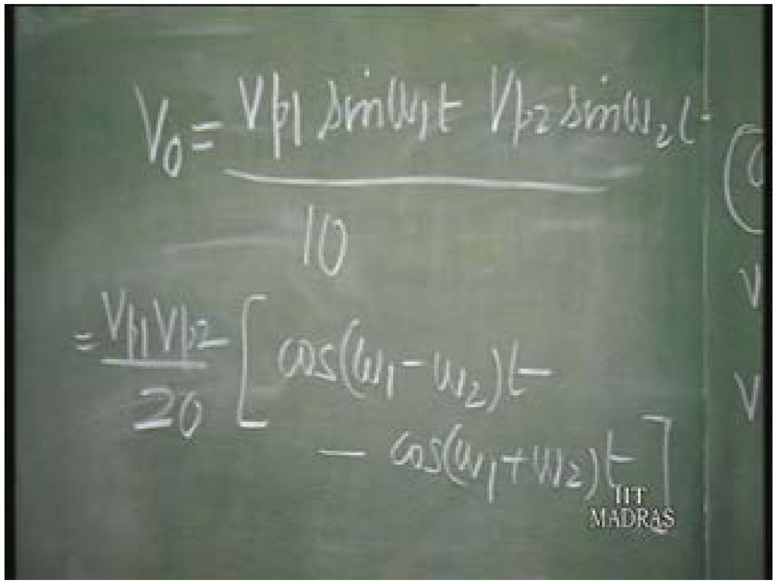

sin a sin b is equal to cos a minus b minus cos a plus b by 2. This is now... So, these are what are called side bands. If Omega 1 is the carrier and Omega 2 is the modulating frequency, Omega 1 is the carrier, Omega 2 is the modulating frequency, this output communication engineers called as double side band systems. Carrier is absent. This is called double side band. It will have Omega c minus Omega m and Omega c plus Omega m.

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So, in power efficient designs, we do not want to unnecessarily send the carrier. We just want only the side bands sent; and you can receive at the receiver end the modulated output and detect the output by putting an antenna whose bandwidth is sufficient to receive Omega c, plus or minus Omega M x. You can receive this information and then put it in what is called as a mixer. So, this is nothing but a D S B modulator. Multiplier is nothing but a D S B modulator.

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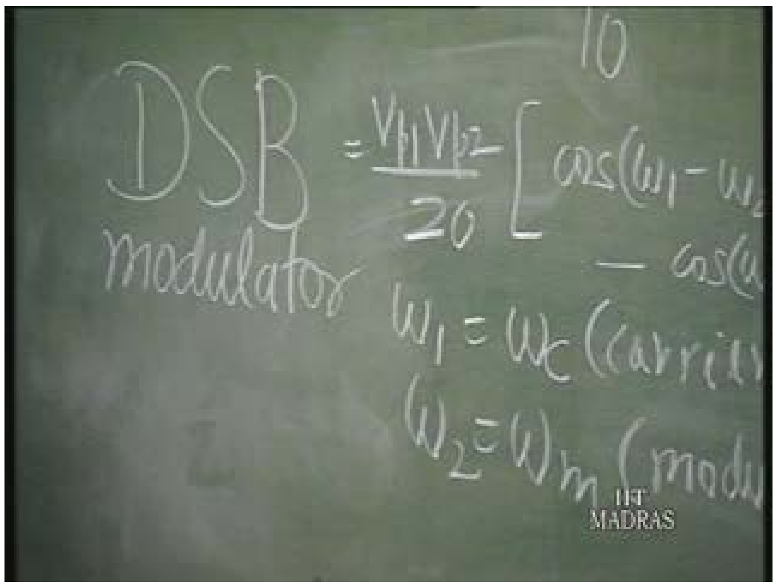

When V x and V y are respectively two sin waves, one is the carrier, another is the modulating frequency. Then it is called a D S B modulator. This is the second application that we are discussing.

Now, if this is received at the receiving antenna and let us say Omega 1 is Omega c plus or minus Omega m, that is one input is the D S B itself and Omega 2 is, let us say Omega c, let let us call this minus Omega I F, Omega c minus Omega I F, this is called local oscillator frequency because Omega c is a fixed frequency. Omega I F is another fixed frequency. What happens, let us see. Again, we get this plus this and this minus this.

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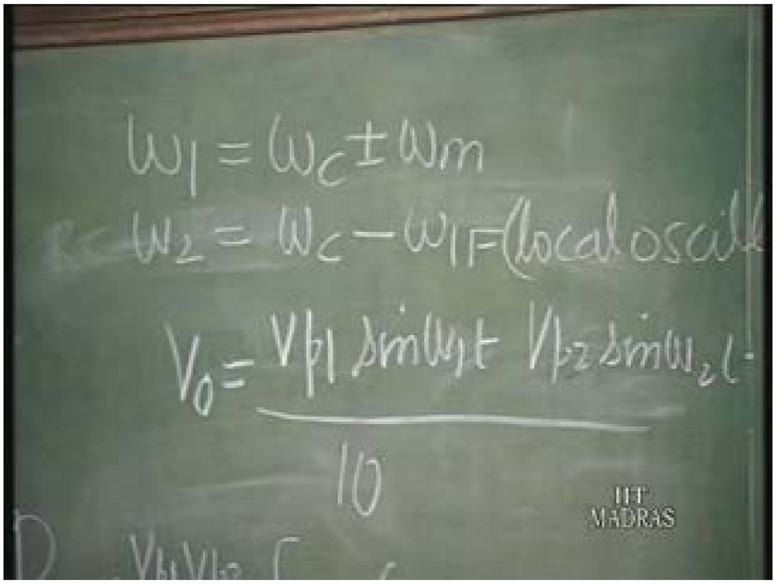

So, we will see that the output will correspond to 2 Omega c plus or minus Omega m minus Omega I F. This plus this and this minus this. Omega c will get cancelled. You get Omega I F plus or minus Omega m.

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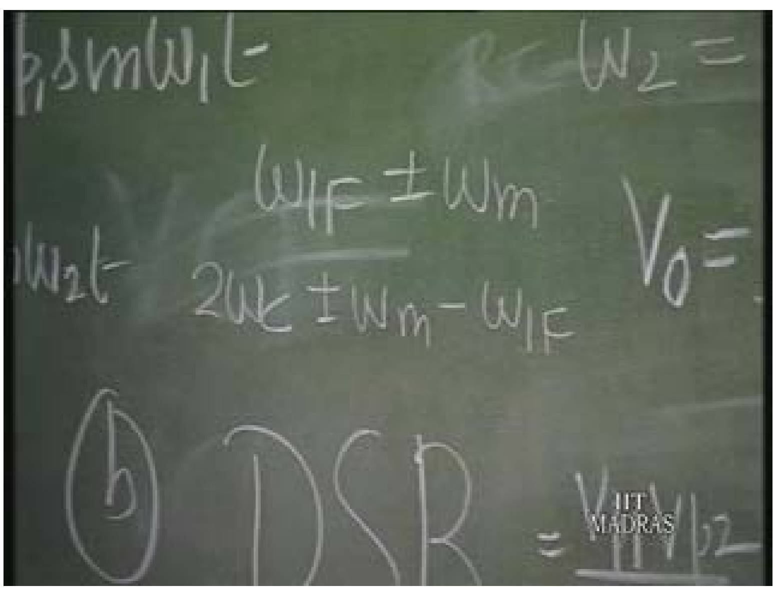

So, when you mix this, it is called a mixer. You are mixing these two frequency components; it is called a mixer. What is a mixer again? It is nothing but a multiplier.

Only thing is the input is different now. One is a D S B modulated input; another is the local oscillator which is having a frequency Omega c minus or plus Omega I F, it does not matter.

So, this local oscillator frequency is mixed with incoming frequency and you get Omega I F plus or minus Omega m and this. You have to isolate this from this. So, you put a band pass filter whose center frequency is Omega I F with a bandwidth sufficient for Omega I F plus or minus Omega m x. This gets eliminated. This is a very high frequency component.

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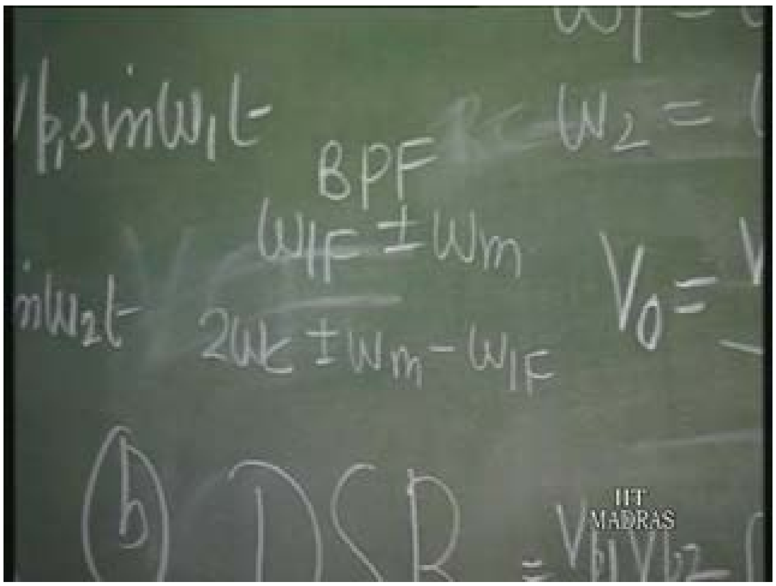

So, this gets eliminated and you have now transferred the wanted information which is plus minus Omega m from the carrier to I F, which is a lower frequency. This is what is done in both television as well as radio receivers so that the amplifier design becomes very simplified. Amplifier that is designed to amplify all the signals have to operate at a fixed frequency called intermediate frequency. Then, the amplifier design is very simple.

So, mixer is the name attributed to the same multiplier whose input is changed from the original carrier and modulating frequency to modulated frequency and local oscillator frequency. Now we will see how it also gets the name of, let us say detector. The same thing is going to be called modulator, mixer or detector simply based on inputs and output structure difference.

So, the multiplier is a basic component which is involved in communication again and again; as a mixer, as a modulator or as a demodulator. This is what...what this is. Now we have information in Omega I F plus or minus Omega m. I put a local oscillator whose frequency is Omega I F. So, output will be twice Omega I F plus minus Omega m and Omega m.

I can now eliminate Omega I F plus minus Omega m by using a simple low pass filter whose cut-off frequency is something higher than Omega m x. So, I just put a low pass filter. It will remove twice Omega I F plus or minus Omega m.

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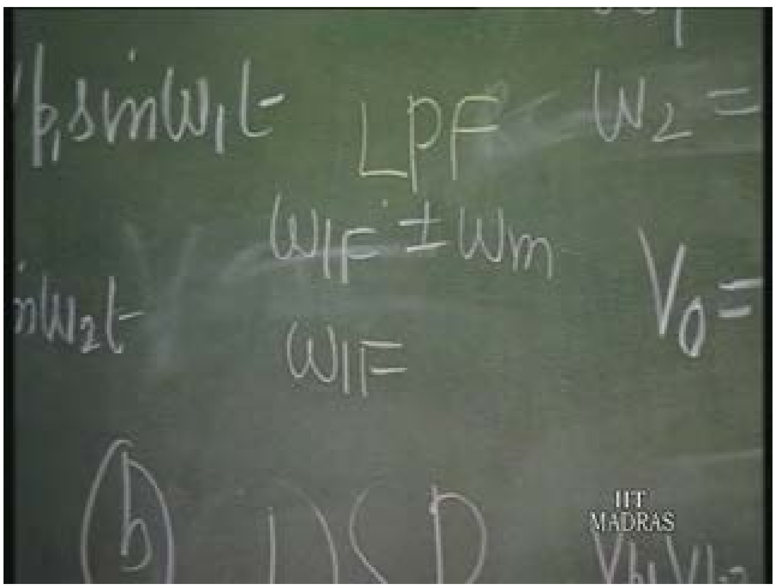

The input therefore is the I F output and an I F local oscillator frequency. Then it detects the modulated frequency. That is called a demodulator. This is also called balanced modulator and this is balanced demodulator. This kind of thing, receiver design, is possible only when at the receiving end, I can generate in the local oscillator, a frequency which is as stable as that has been transmitted. That is Omega c minus Omega I F should have the same stability. Not...it can be good or bad; but if the stability of transmitter is good, this could be as good or as bad. It should be tracking with that. Otherwise, there will be problems in receiving.

Same thing is true with this detector, that I should be able to generate a local oscillator frequency which is Omega I F with the same amount of stability as that of the Omega I F strip itself. So, with that requirement, the receiver design becomes pretty complicated. That is why it is not normally used in the common cheap radio receivers called super heterodyne receivers, which make use of a different principle all together in modulation. That is called amplitude modulation as against D S B.

What is an amplitude modulator? This we can learn from the fact that an amplitude modulator A M circuit is nothing but a multiplier. Let us say V x is V p sin Omega c t; V y on the other hand is a V d c plus V p 2 sin Omega m t ...so that when you multiply, there is apart from the product component which is Omega...sin Omega c t into sin Omega m t, you have a D C multiplied by the carrier, which will give you sufficient carrier to identify the signal properly.

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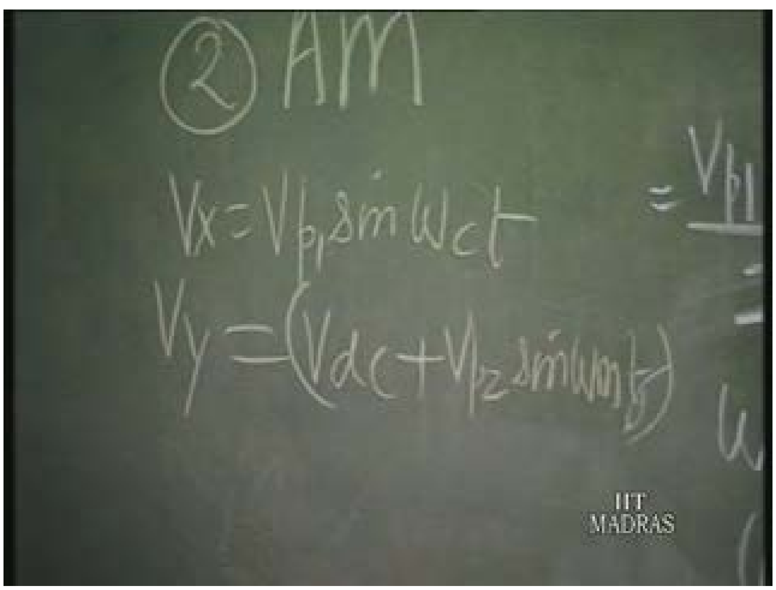

This is called...therefore this is nothing but a non-ideal multiplier where you have the x feed through component coming. See that this is nothing but a non-ideal multiplier where x feed through component is coming. It is not just the product component.

So, if you have a non-ideal multiplier, you can obtain an amplitude modulated signal. What is the advantage of amplitude modulated signal? As against the D S B, you can detect this simply by envelope detection, which is done simply by using a diode.

So, the amplitude modulated signal which looks like this, that is the envelop; and the envelop contains the full information. So therefore, chop this off using a diode and then put a filter so as to detect only the envelop. So, that...that detector is a very simple detector. It does not require generation at the local oscillator. That frequency, which is as stable as that, that is now transmitted.

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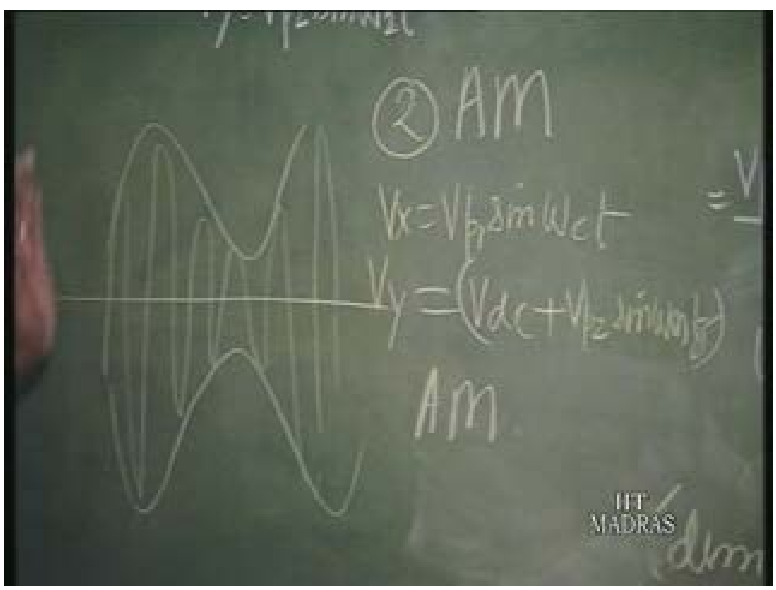

So, this is A M generation. That means A M generation is also possible with the same multiplier. Only thing is that multiplier need not be an ideal multiplier. It can be a non-ideal multiplier with x feed through or y feed through component.

Next, the communication application...we have another important application wherein the frequencies are the same, but the input can be now V p sin Omega t. This is V x and V y is V p 1, let us say V p 2, sin Omega t plus phi. Once the frequencies are the same, there can be a phase difference.

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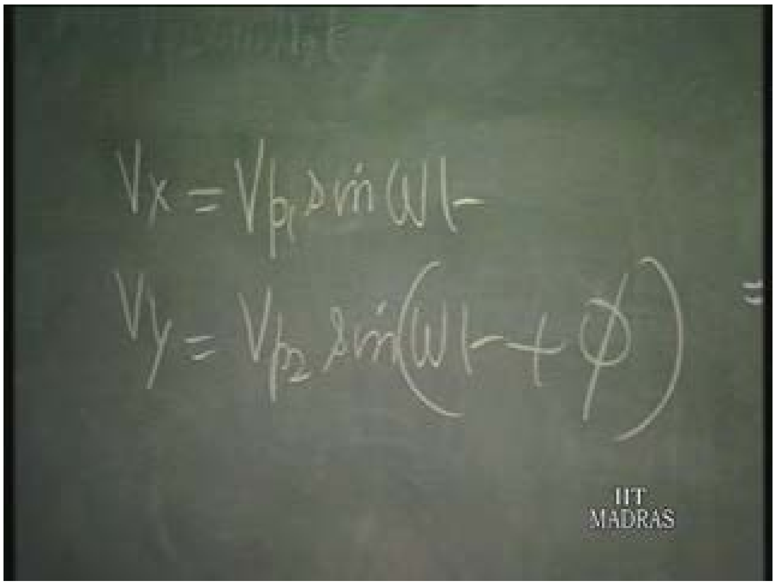

So, if you feed this as the input x and y, what will you get? Again, you will get a component which is V p 1 V p 2 by 20 cos Omega 1 minus Omega 2. Here, it is this minus that...that is cos phi minus this plus this. That means cos 2 Omega t plus phi. This is the output. You can now see that by putting a low pass filter, I can get rid of this and output is V p 1 V p 2 cos phi by 20.

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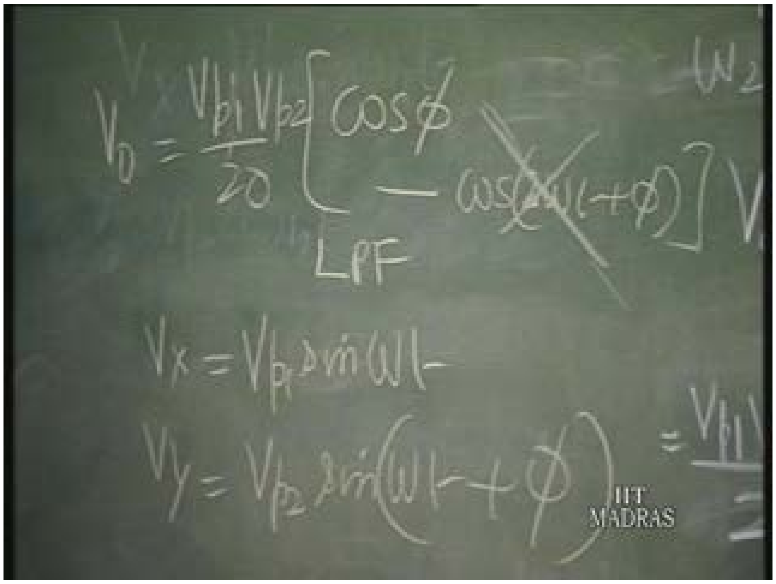

What is phi? phi is nothing but the phase difference. So, this is a phase detector, an important building block, in a variety of applications. Phase detector. It is not linear; it is non-linear, because output D C voltage is proportional to cos phi, not phi. You can make it linear by simply passing these waveforms through comparators or limiters, what are called as amplitude limiters.

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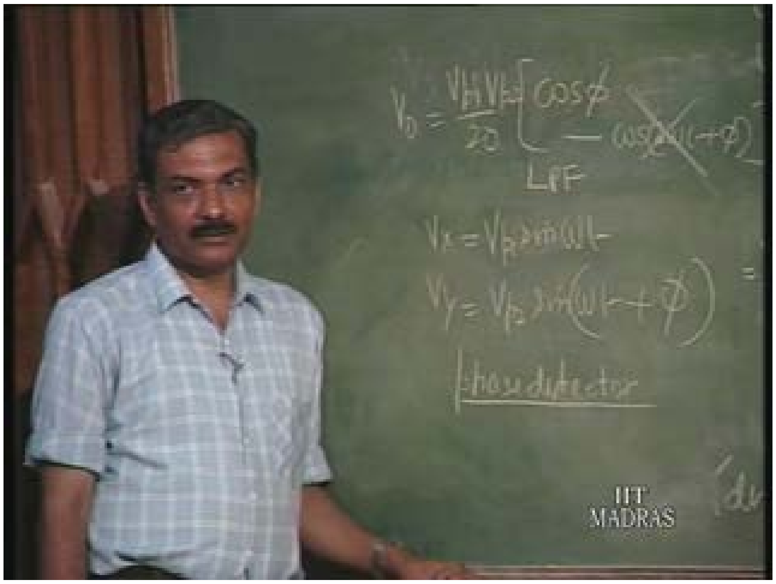

So if you, for example, pass it through amplitude limiters, output amplitude of V x is, let us say limited to some value. So, this is 2 pi, this is pi; and the other waveform is, let us say delayed from this waveform by a factor of pi. Let us say the amplitude to which it is limited is, plus V s and minus V s.

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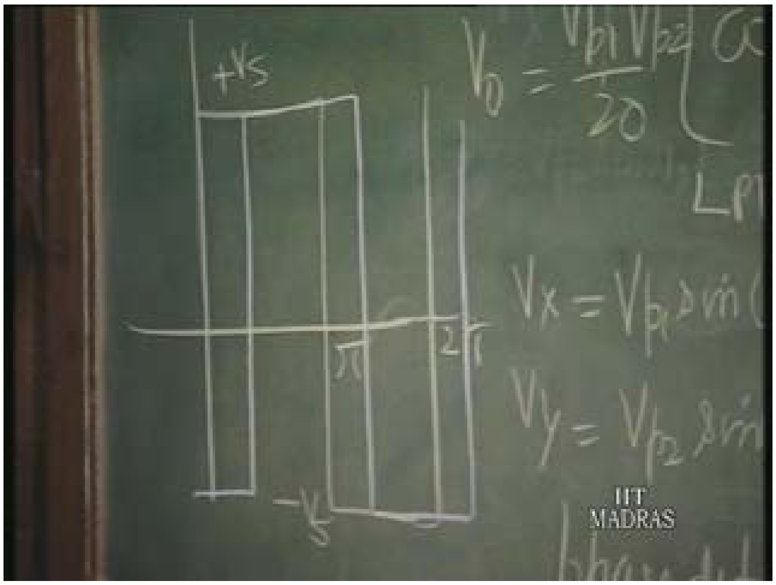

This can be done by making these voltages go through comparators. So, the amplitude will be limited to supply voltage plus V s and minus V s, both of them. So, V x and V y get fed through this and then to the multiplier.

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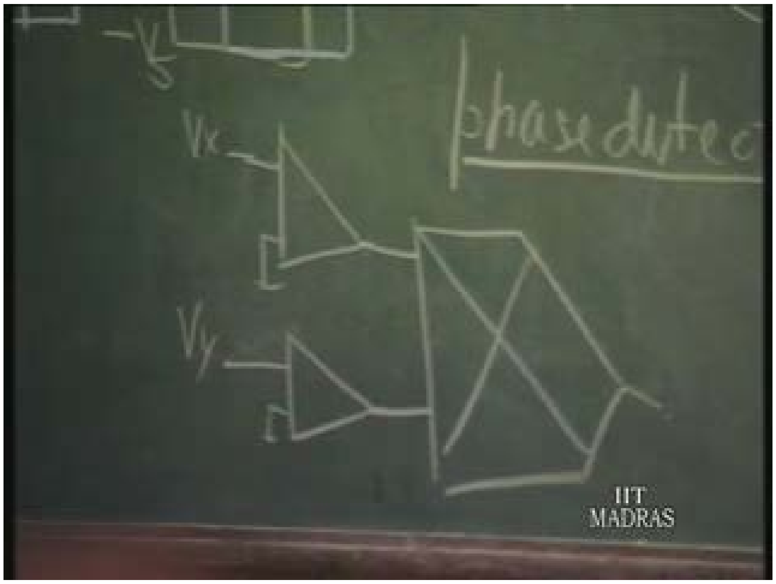

So, what will be the output? Here, when this is positive, this is negative. So, output is negative; and in this region it is V s square by 10, negative; and in this region, both are positive.

So, output is positive, V s square by 10. Again here it is, one is positive, another is negative. Again it is minus V s square by 10. Again here, both are negative. So, it is positive, V s square by 10. So, in one cycle of this waveform, this goes through two cycles.

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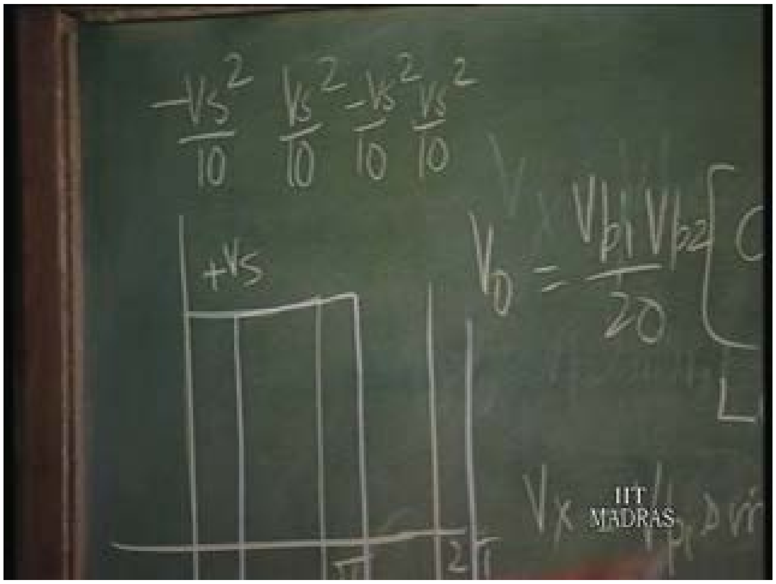

That is what it says in this multiplication also. There is a 2 Omega t component coming into picture. Apart from that, if you actually evaluate the average of this now... If this is, let us say phi, the average of this is going to be minus V s square by 10 into phi; this into minus V s square by 10. And this period is pi minus phi. So, plus V s square by 10 into pi minus phi; and since it is getting repeated in the same manner, it is enough if we take the average this way and divide it by total of pi itself.

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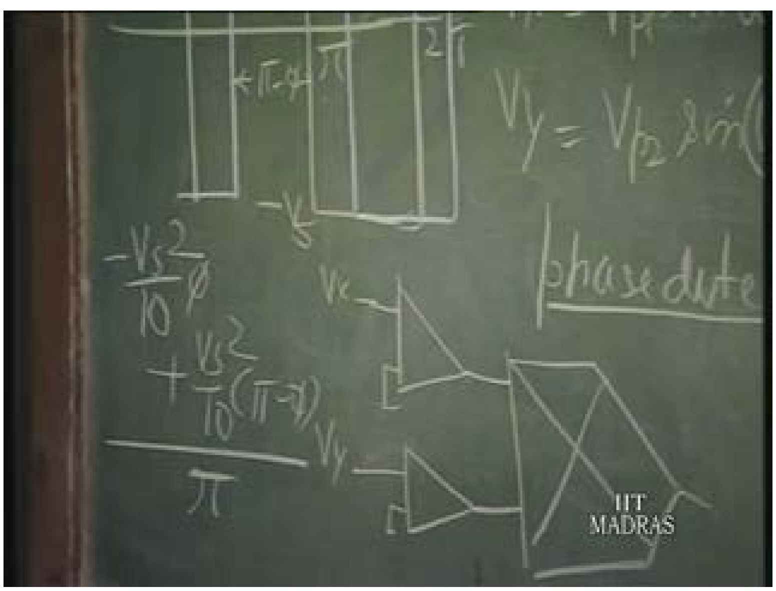

If you take the average for 2 pi also, it will be the same average. So, you get this here, V average as being equal to V s square by 10. This pi and pi get cancelled; 1 minus 2 phi by pi. You can see that the average D C component is directly proportional to phi. So, this becomes a linear phase detector.

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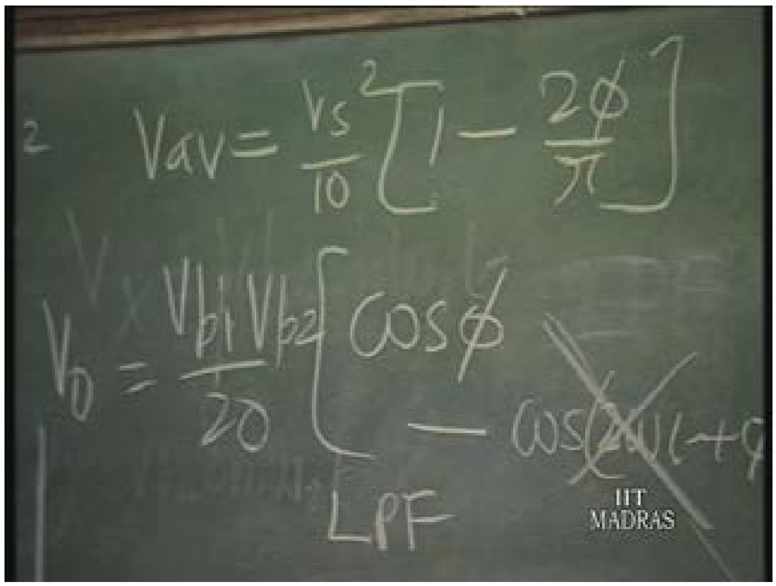

So, you can see that communication engineers call these amplifiers as limiters, not as comparators. You can limit the amplitude by using high gain amplifiers and apply this to a multiplier which is nothing but a modulator or mixer or anything; but if the two frequencies are the same, then the average component at the output will be directly proportional to phi. This is called a linear phase detector.

Therefore, we can use it as a linear phase detector in... We will see this is a basic building block of a phase lock loop also. Later on we will see this... Here, this point that it is proportional to cos phi can be exploited in using it for power measurement. If V x is proportional to line voltage, that means you put a voltage transformer there. Then, V y is proportional to line current. Then you put a current transformer; the output of the voltage transformer and current transformer, you apply directly to the multiplier; and you put a low pass filter, you will get the power factor directly. The output voltage will indicate the power factor.

If you really use the amplitude information, it is nothing but the power consumed in the line. You can use this information as a direct replacement of what is called two watt meter method of power measurements. The two watt meter method of power measurement in three phase circuit can be translated in terms of two such multipliers being used in order to evaluate the power of the three phase circuit. So, it can be used as a watt meter; phase detector, watt meter.

Now you can see that instead of taking this as the output, cos phi, this can be eliminated by using a high pass filter. Only this can be taken. What is this going to give us? You have put Omega as the input; you have got 2 Omega as the output. This is called frequency doubler. This output, when I select only the high frequency component, reject the D C, is called frequency doubler.

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This is very important in its application of generating stable, high frequency components from stable, low frequency components. Let us say it is very difficult to generate stable high frequency component. Then, you have to synthesize such frequencies by doubling or tripling or multiplying. So, for frequency multiplication, this can be used. If you can double, you can triple because this can be...the doubled frequency can be one input and again Omega can be the other input. 2 Omega and Omega, you will get 3 Omega. So, like that, you can multiply frequencies.

So, frequency synthesis...this can be used. If you can use it for frequency synthesis, I can get Omega 2, Omega 3, Omega etcetera, then I can generate the harmonic components and I can therefore obtain any periodic waveform using a periodic waveform as the input. A triangular waveform, for example, can be used as the input; triangular waveform is nothing but x or t. So, using this input, I can generate a sine waveform which is going to be t minus t cubed by factorial 3, so on... So, we can generate a periodic waveform of any shape using a triangular waveform, using this kind of frequency synthesis.

Other applications of the multiplier, like how to use it as a divider, we have seen that in the log-antilog multiplier. We can use it straightaway as a multiplier or divider; whereas, this transconductance multiplier, the output is V x V y by 10; straightaway a D C, which is generated using a D C current and a resistance. So, we cannot possibly use it as a divider straightaway. How do you convert a multiplier into a divider?

So, this multiplier can be put in the feedback path. This is always the case. I have been repeatedly telling you that if you have a certain function, the inverse of the function can be always obtained by putting that functional block within the feedback loop of an op-amp.

So, let us consider an op-amp of this type. So, this is, let us say V i; this is V naught. If this is V i, this should be V i, if the feedback is negative in the case of an op-amp; and that is straightaway equal to...let us say this is V i 1 and this is V i 2. So, this is V i 1. So, and that V i 1 is equal to V naught V i 2 by 10, V naught V i 2 by 10 because this is going to give you V x V y by 10; V naught V i 2 by 10.

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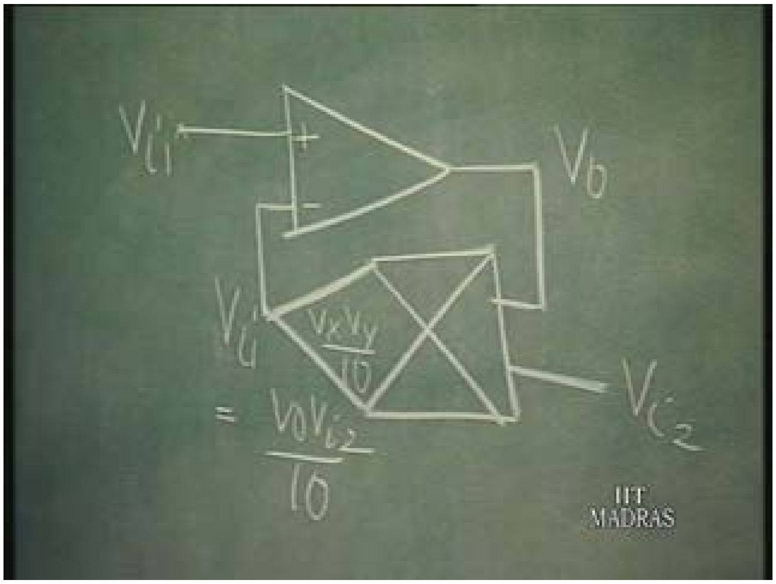

So, we see here that V naught is going to be equal to 10 V i 1 by V i 2 which is nothing but a divider. Now, a word of caution, because this is going to be a divider only when V i 2 is remaining positive; because if this control voltage is positive, this gain is positive; you remember.

If this control voltage is negative, the gain, it will be acting as an inverting amplifier, we have to say. Therefore, this feedback will become positive feedback, if this is negative. So, this is valid as along as V i 2 is positive. So, this particular thing is working satisfactorily for V i 2 greater than zero. Is that point understood?

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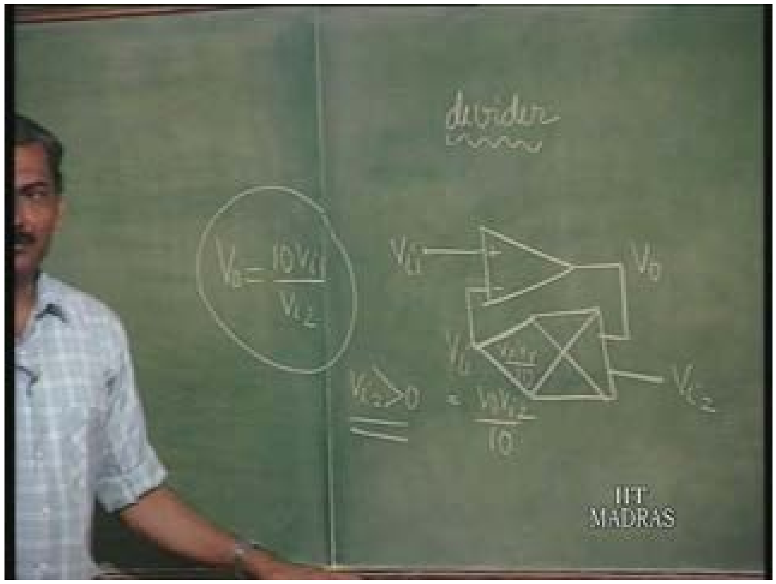

If this is positive, the phase...there is no phase inversion between this and this. If this is negative, there is a phase inversion. If there is a phase inversion, this...there is negative, already 180 degree. This is again 180 degree. There is a positive feedback; even this circuit will fail to function.

So, this is the limitation of this circuit with negative feedback. It cannot work for any polarity of a voltage that you are considering. Now, this also can be used for squaring. How do you do squaring? Squaring you do simply by connecting both the inputs to the same thing, V x and V x. So, output will be V x square by 10. But, we would like to do square rooting let us say, square rooting. How do you do? So, square rooting can be done by simply connecting, again this squaring circuit in the feedback path.

So, instead of...this still remains the same; V i 1, this is V i 1. This is now V x and V y; both are equal to V naught squared. So, V x and V y are equal. So, multiplication of this will result in V naught square; or we get now V naught square by 10 is equal to V i 1; or V naught is equal to root of 10 into V i. So, this is this.

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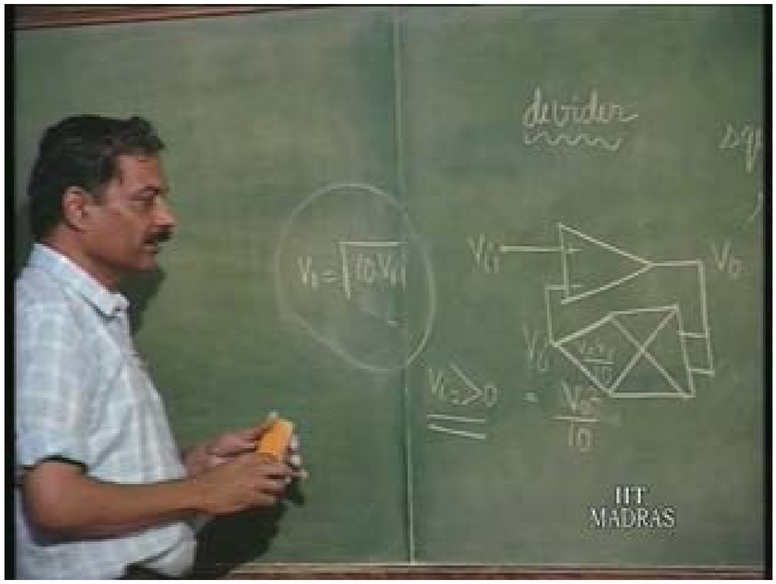

So here, the essential thing is once again, this V i 1 has to be always positive. If it is negative, once again, we cannot get another... What... what does that mean? That means, actually speaking, that V naught here has to be always positive. Otherwise, there will be the the sign change is not recognized here. This is because this is a squaring circuit; whether it is positive or negative, we will get the same output.

So, this is a square rooting circuit. If you have a square rooting circuit, you can do so many things, squaring and square rooting we know.

What is called as true root mean square value; root mean square value – R M S value - of a given waveform, periodic waveform. How do you...true R M S value, how do you find out that? By definition, you will take square. Suppose it is V; you take square of V and then mean...that means integrate and then root square. So, integrate over, let us say, say time period. That is the averaging. So, what do you do?

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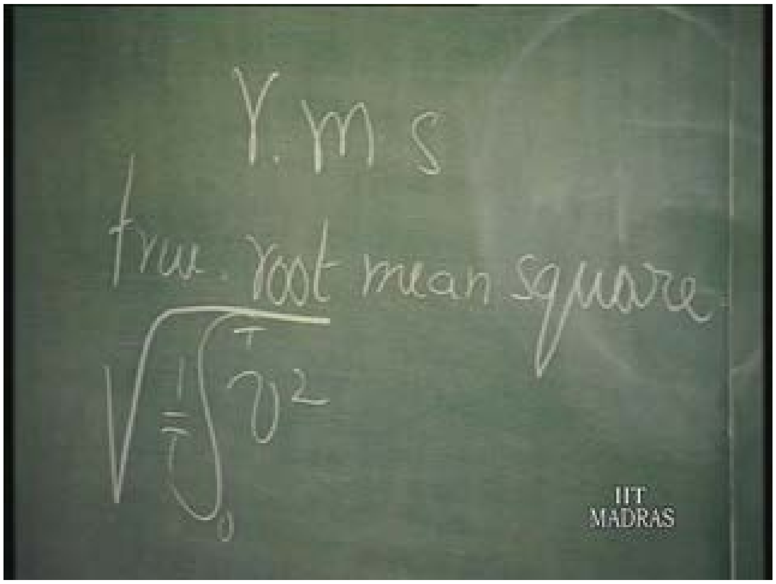

You put a squaring circuit which is nothing but a multiplier. This is V. So, you have got the square, then the averaging circuit. That means you will take the average. We can take R c time...this is nothing but a low pass filter. That will do the integration; averaging circuit and then square rooting. This averaging is a D C. So, you will now take the square rooting of that. So, this complete circuit is nothing but a very useful tool called true R M S, root mean square indicator.

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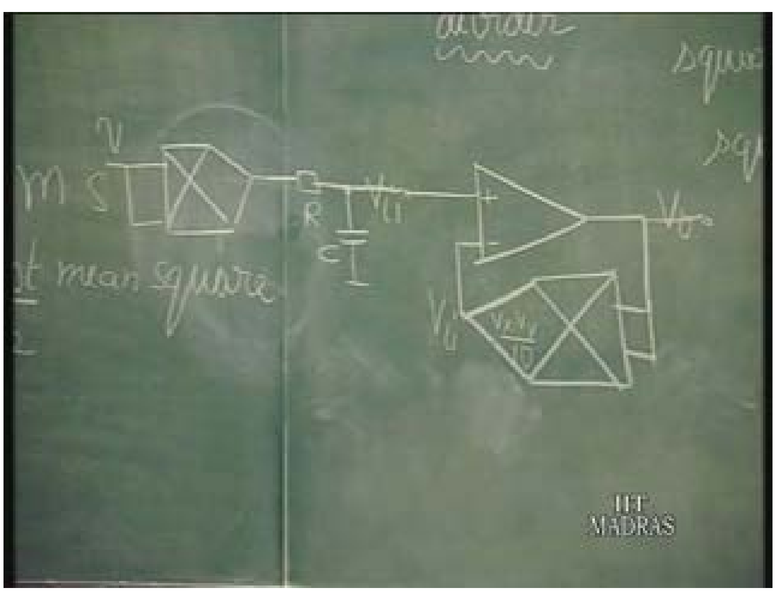

So, the entire thing can be built using multipliers. Such units are available in the market for finding out the true R M S value of a given wave form.