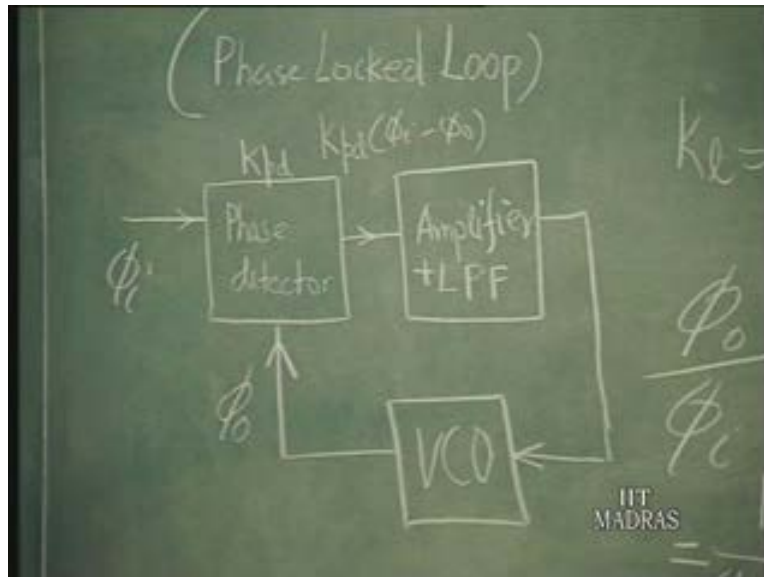


**Electronics for Analog Signal Processing - II**  
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**Department of Electrical Engineering**  
**Indian Institute of Technology – Madras**

**Lecture - 37**  
**PLL (PHASE LOCKED LOOP)**

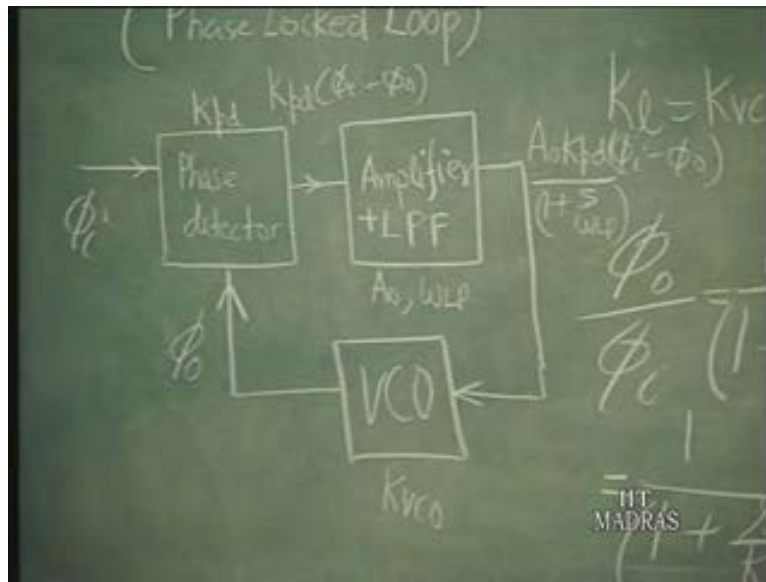
In the last class, just...we had seen how the phase locked loop works as a phase follower. I said, if  $\phi_i$  is the input phase,  $\phi_o$  is the output phase,  $\phi_i$  minus  $\phi_o$  being the phase detector input, then output of phase detector is going to be D C average which corresponds to  $K_{pd}$  into  $\phi_i$  minus  $\phi_o$ . So, this is  $K_{pd}$  into  $\phi_i$  minus  $\phi_o$ .

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This would respond to this amplifier and low pass filter. So,  $A_{naught} K_{pd} \phi_i$  minus  $\phi_o$  divided by  $1 + s \text{ by } \Omega L_p$ , that is the voltage here. That...this is going to have a sensitivity of  $A_{naught}$  and  $\Omega L_p$  is the low pass filter cut-off frequency. Here we have a sensitivity of  $K_{VCO}$ .

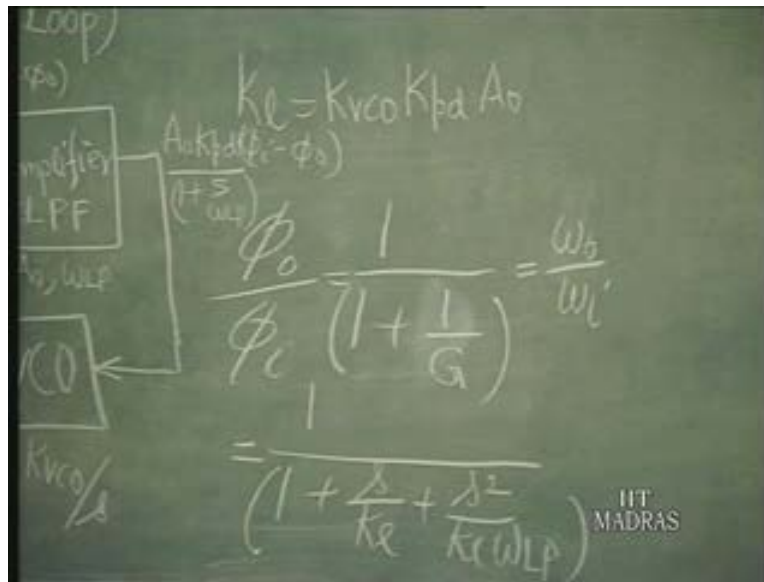
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So, this voltage is going to be multiplied by  $K_{VCO}$  divided by  $s$  because it is responding to phase; and therefore, that is itself equal to  $\phi$ . From that, we got  $\phi$  by  $\phi_i$  is  $1$  by  $1 + 1$  over  $G$ , where  $G$  is the loop gain and  $\phi$  by  $\phi_i$  is same as  $\Omega$  by  $\Omega$ ; that if it is a phase follower, it is also frequency follower because of the linear relationship between  $\phi$  and  $\Omega$ .

Now, this is equal to  $1$  by  $1 + s$  by  $K_L$  where  $K_L$  is nothing but the DC loop gain which is  $K_{VCO} K_{pd} A_o$ ; that is, entire sensitivity factors, product of sensitivity factors within this loop. So,  $1 + s$  by  $K_L$  plus  $s$  square by  $K_L \Omega_{LPF}$ .

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So, this is basically a phase locked loop; it is a second order system; second order system with natural frequency equal to root of  $K I \Omega L p$ ; or actually we have been writing natural frequency as  $\Omega_0$ ; and we can write down this as  $1$  by  $1$  plus  $s$  square by  $\Omega_0$  square. Then, this is  $s$  by  $\Omega_0$  into  $Q$ ; the normal way we write the second order system.

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$$\omega_0 = \sqrt{K_e \omega_{LPF}}$$

$$\frac{1}{1 + \frac{s}{\omega_0 Q} + \frac{s^2}{\omega_0^2}}$$
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So, Omega naught is quickly identified as root of K l into Omega L p. That means if K l is high, the loop gain is high. Omega naught is going to be high; but of course Omega L p may be low so that Omega naught is going to be much less than the...what is that? - K l itself.

Q on the other hand of this circuit is going to be... Omega naught Q is going to be equal to K l, by comparison; and therefore, Q is going to be K l by Omega naught which is root of K l Omega L p which means it is root of K l by Omega L p.

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The image shows a chalkboard with the following handwritten equations:

$$\frac{1}{1 + \frac{s}{\omega_0 Q} + \frac{s^2}{\omega_0^2}}$$

$$\omega_0 Q = K_l$$

$$Q = \frac{K_l}{\sqrt{K_l \omega_{LP}}} = \sqrt{\frac{K_l}{\omega_{LP}}}$$

The logo "IIT MADRAS" is visible in the bottom right corner of the chalkboard image.

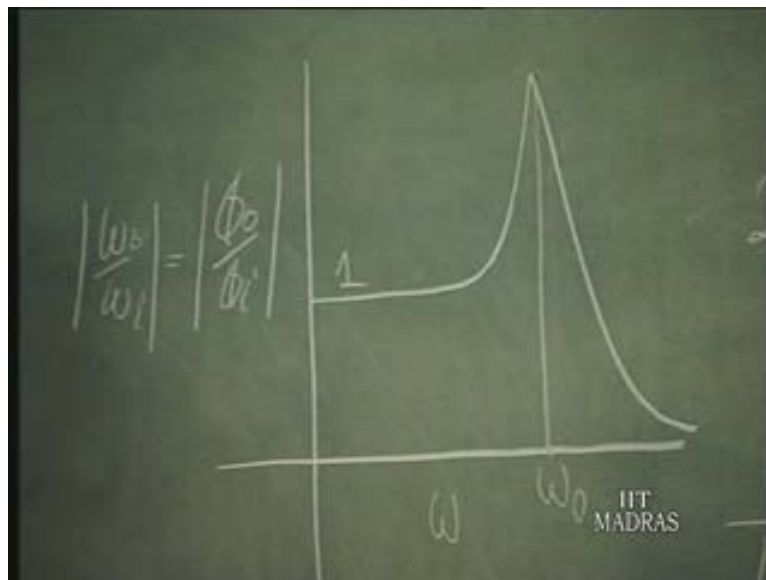
So, a good phase locked loop will invariably be a high Q circuit because K l is the loop gain and it has to be made in large in all these control systems. So, root of K l by Omega L p is normally going to be very high; and therefore, a good phase locked loop is a natural second order system whose poles are already very near the imaginary axis.

Why I am stressing this is because additional phase shift within the loop will make it become unstable and therefore we have to be extremely careful not to put anything other than a first order low pass within the loop. If you want to filter it better, you must not put it within the loop. You must put this always outside the loop.

So, this is something that we have to be always aware of in phase locked loop that, since it is a basic second order system with high pole Q, we will not put anything that will further increase the phase shift within this.

Now, this particular thing, if you obtain its response as a function of Omega...this...that is, either...let us say phi naught by phi i magnitude, it will be...where this is the natural frequency of the system which is nothing but root of K l Omega L p and this is one. That means, actually in a good phase locked loop...actually, this is also Omega naught by Omega i.

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In a good phase locked loop, we must always work much below this so that it exactly before the phase following. Output phase change is same as input phase change; or output change in frequency is same as input change in frequency. We should not work in this frequency range.

This is again important that this frequency that we are talking of is this frequency which has been permitted to pass through in the low pass filter; which means, it is the frequency with which this so called average is changing. So, that is something that we have to bear

in mind. Frequency with which the average of this is changing is the frequency that we are mentioning here; or, the frequency with which the phase is changing. So, that frequency is what we are talking of.

Now, we have now established that a phase follower is automatically a frequency follower. Now, let us see what exactly happens if this particular thing which is nothing but a multiplier is not given any input. So, there is no input at all. What happens? So, we are discussing the situation of phase locked loop under quiescent conditions that nothing is being fed to the phase detector input.

Of course, it is getting one input from VCO because it has 2 inputs. This is a multiplier. As I told you, if this is  $\Omega_1$  and this is  $\Omega_2$ , there will be a component  $\Omega_1 - \Omega_2$  and  $\Omega_1 + \Omega_2$  here. So because this is a multiplier we have, if this has  $\Omega_2$ , the component  $\Omega_2$  will be present here. That being a very high frequency component, nothing is permitted to come out of the amplifier plus low pass filter component; and therefore, the VCO remains at its quiescent state because this input does not change. Therefore, this continues to run at what is called as free running frequency of the phase locked loop.

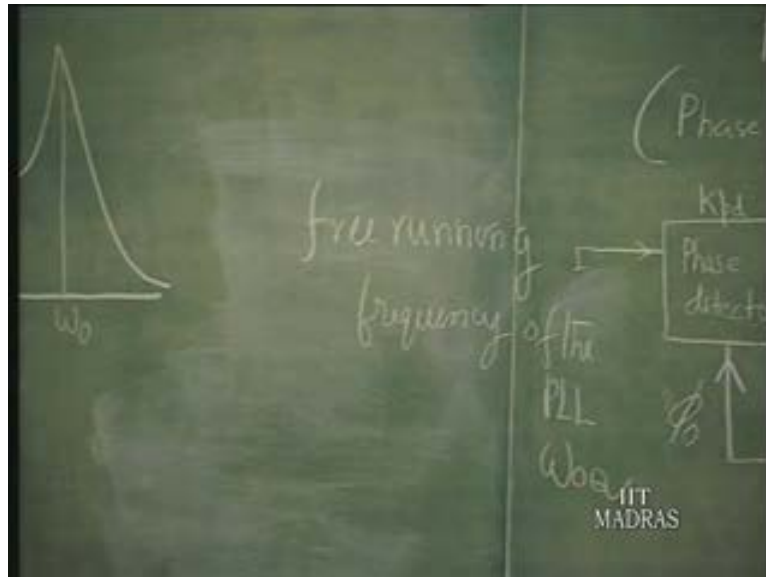
So, this continues to run at  $\Omega_2$ . So, that frequency at which the phase locked loop is running when no input is fed is called the free running frequency of the phase locked loop, or quiescent frequency of the phase locked loop.

Once again, let me go through... Phase detector is a multiplier. If there are 2 frequency components here, let us say  $\Omega_1$  and  $\Omega_2$ , this will give you  $\Omega_1 - \Omega_2$  and  $\Omega_1 + \Omega_2$ .

It is  $\Omega_1 - \Omega_2$  which might be pass through because it may become a low frequency, but  $\Omega_1 + \Omega_2$  may not be pass through. Here, since it is only  $\Omega_2$  that is existing, that  $\Omega_2$  may appear here and therefore it will not pass through a low pass filter and nothing happens to the VCO output. VCO continues to run

at the free running frequency and therefore that is called the free running frequency of the PLL. We will call it as  $\Omega_{naught}$  quiescent.

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Now, let us apply an input frequency, **apply an input frequency** - please follow me very carefully - which is equal to  $\Omega_{naught} Q$ . I am going to apply an input frequency which is the same as the free running frequency of the phase locked loop.

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We have already established earlier that output frequency has to be equal to the input frequency; but here, what I am doing is...free running frequency is the frequency that I am applying. I just want to see how the phase locked loop now functions. So, since the incoming frequency is already the same as the free running frequency, the output will be twice  $\Omega_{naught Q}$  at zero frequency, which means D C.

That means there must be a D C here and twice  $\Omega_{naught Q}$ . Twice  $\Omega_{naught Q}$  is not going to...already we said  $\Omega_{naught Q}$  is not permitted. Twice  $\Omega_{naught Q}$  is not permitted here. Therefore, this D C which is amplified by A  $\Omega_{naught Q}$  should change this D C, original D C here of the VCO, to some value. If that changes, then this frequency will change.

That means this D C that should occur here because of this should be zero; that is the only possibility. That means this D C is zero. That means this D C should be zero. That means when we have  $\Omega_{naught Q}$  as input and  $\Omega_{naught Q}$  as output, the D C average should be zero. In our phase detector, we have seen that such a phase occurs. That is, when the two incoming frequencies are of the same value, there can be a phase difference between the two wave forms.

That phase difference has to be equal to  $\pi/2$  in order to have the average equal to zero because in the multiplier it was  $V_p \sin \Omega_1 t$ ,  $V_p \sin \Omega_2 t$  divided by 10, which means it is going to respond to  $\Omega_1 - \Omega_2$  which is in this case,  $\pi/2$  because  $\Omega_1$  is equal to  $\Omega_2$ ; but there is a phase difference. So,  $\cos \phi$  is the average. If  $\cos \phi$  is zero,  $\phi$  has to be 90 degrees.

That means if the incoming frequency is the same as the free running frequency, the phase difference between these two will be automatically set by this loop to be 90 degree. That is the quiescent phase shift. Please understand that this is the quiescent phase shift at which the D C average here must be zero in order that this is not changing from its quiescent, in order that this should continue to free run at  $\Omega_{naught Q}$ .



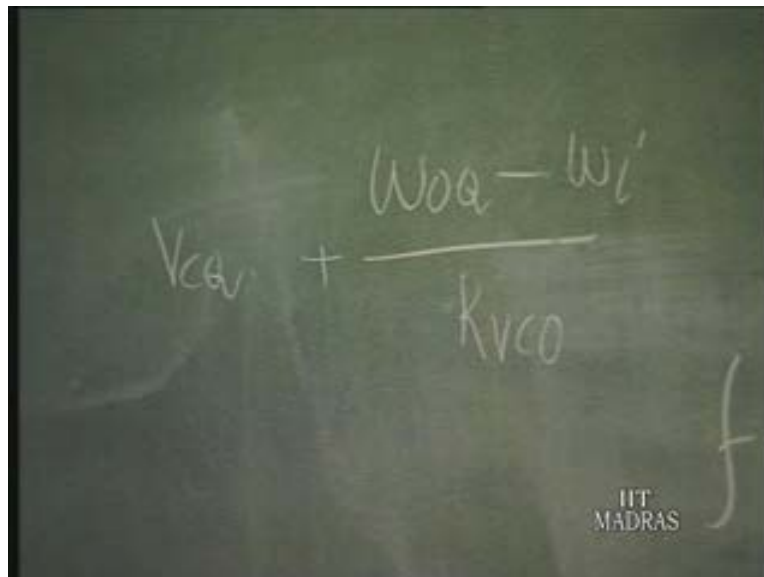


naught Q, what should happen? So, let us now consider the situation of Omega i being slowly changed around Omega naught Q.

Originally, it was at Omega naught Q. At that point, this particular voltage...let us say, this is called...let us say V c, control voltage. The control voltage was at a value V C Q for the VCO. As Omega i is changed to some value which is close to Omega naught Q, immediately this will follow. If this follows, what should be the V c? That will be nothing but Omega naught Q minus Omega i divided by KVCO because this is defined as the sensitivity. So, if this has gone over from Omega naught Q to Omega i which is different from Omega naught Q, then the change in frequency is Omega naught Q minus Omega i. That divided by KVCO should be the D C voltage here.

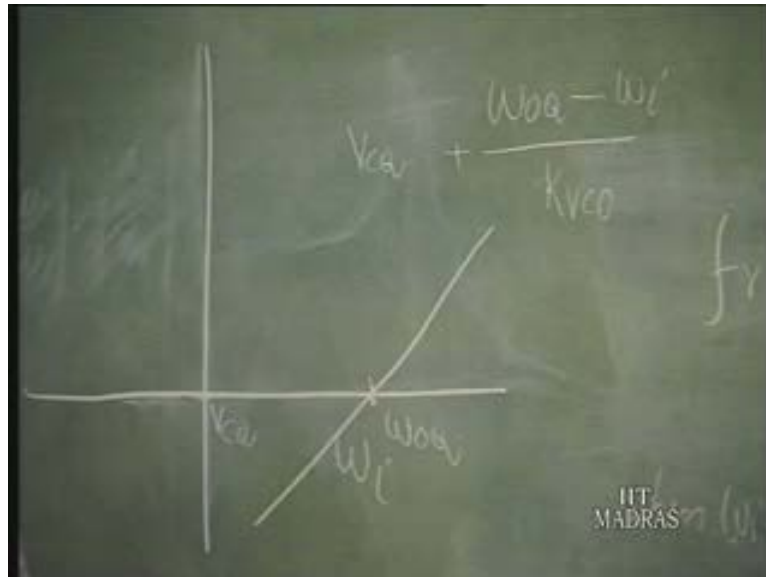
So, you can see that Omega naught Q minus Omega i divided by KVCO should be the D C here. So, that means you can see this. Apart from V C Q, when Omega i is equal to Omega naught Q, it was V C Q itself. There is an additional voltage that is coming here. If Omega i, for example, is less than Omega naught Q, this is higher. If it is greater than Omega naught Q, this voltage is lower.

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$$V_{c0} + \frac{\omega_{0Q} - \omega_i}{K_{VCO}}$$

So, there is going to be a sort of slope like this – linear, because this is...slope is minus 1 over KVCO.

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So, this  $V_c$  versus this frequency is going to be perfectly linear. That means, as I change  $\Omega_i$ ,  $\Omega_0$  is going to follow  $\Omega_i$  in a linear fashion like this such that  $V_c$  is a linear curve here.

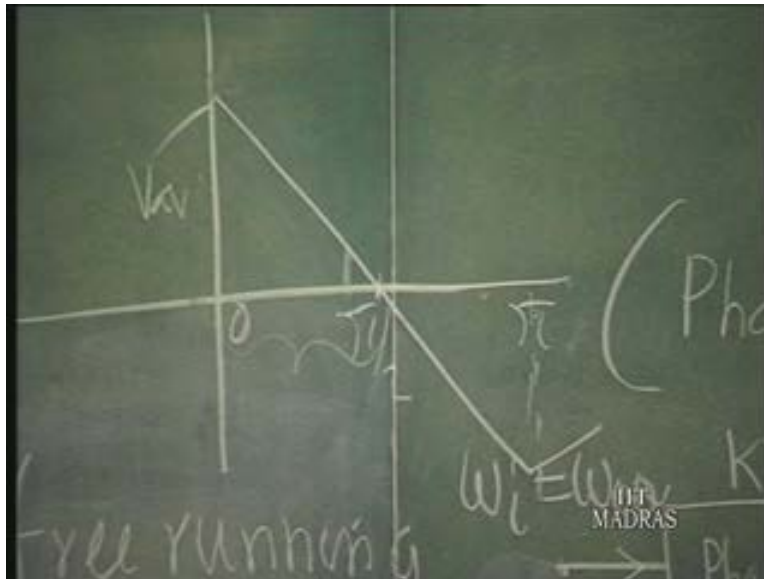
Now, once again let us come to this. This has changed over to a DC above  $V_c$  by an extent equal to  $\Omega_i$  minus  $\Omega_0$ ...that is  $\Omega_0 Q$  minus  $\Omega_i$  by  $K_{vc}$ . This has changed over to a DC which is that divided by  $A_{naught}$ . That DC is the average of the phase detector output. Frequencies are now again the same. So, there is an average component which is going to be equal to this DC divided by  $A_{naught}$ . That average is the result of a certain phase shift which is other than  $\pi/2$ . So, that means the phase shift now starts changing from  $\pi/2$  to some other value. This can go on until, let us say ours is a linear phase detector.

If you assume that ours is a linear phase detector with this kind of characteristic that we had earlier found out; may be something like this: at  $\pi/2$  it is zero; and this is at zero

and this is at  $\pi$ . So, this is at zero phase shift. So around  $\pi$  by 2, this is the average. So, this phase shift will now change by some amount.

It can go on changing until what happens? The phase shift becomes, ultimately at this point, zero. Thereafter again, our phase detector is not capable of detecting the phase. Again, this is not possible beyond  $\pi$ . Our phase detector is capable of detecting a phase change of  $\pi$  by 2 on either side of this  $\pi$  by 2. So, it can go on up to zero and it can go on up to  $\pi$ . What does it mean in terms of this?

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That means this DC is going to be this much above this value or this much below this value. Correspondingly, above we see, it will be A times that and below that, again A times that. So, this VCO is going to be varying around  $\Omega_n$  up to a certain limit on this side and up to another limit on this side.

Therefore, this range of frequencies is called the lock range of the phase locked loop. This range within which the phase locked loop is going to track the incoming frequency... As I vary the incoming frequency, it will follow the incoming frequency. This range - the limit from this value to this value is called the lock range.

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Now, how can we determine the lock range? It is very simple. If the phase detector phase output here changes to a maximum of  $\pi$  by 2, then what is the maximum voltage here? If this is  $K_{pd}$ , the slope of this is  $K_{pd}$ , this voltage is nothing but  $K_{pd}$  into  $\pi$  by 2. If this sensitivity is  $K_{pd}$  and it is linear, then the maximum voltage that is given by the phase detector corresponds to  $K_{pd}$  into  $\pi$  by 2. That is the maximum voltage. That into  $A$  naught is the maximum voltage that it can have here.

As long as the amplifier is not getting saturated before it reaches this voltage, this is the voltage it is likely to be reached. This into  $K_{VCO}$ , sensitivity of this, is the maximum frequency deviation from  $\Omega$  naught  $Q$  it can reach, on either side of  $\Omega$  naught  $Q$ .

So, this is the lock range from Omega naught Q on this side and this on this side. So, this we will call it as Delta Omega L.

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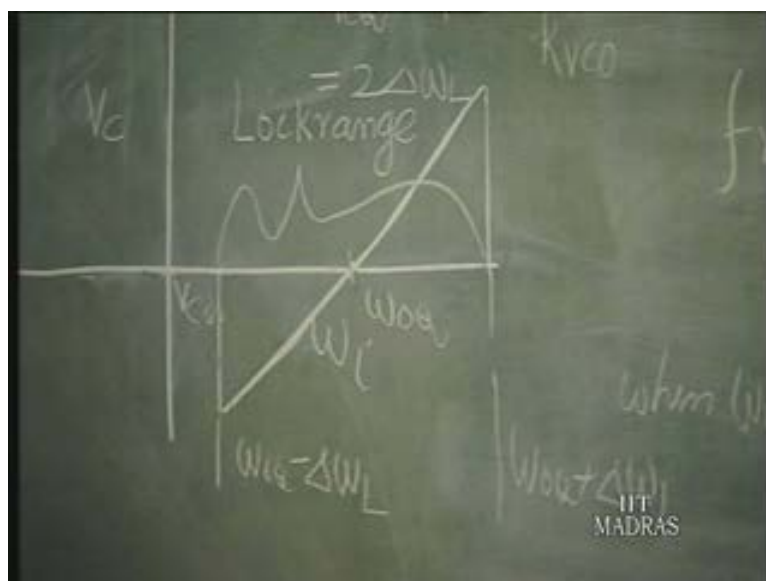
$$\Delta\omega_L = \frac{K_d J_T A_0 K_{VCO}}{2}$$

Lockrange

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So, this limit is Omega naught Q minus Delta Omega L. This is Omega naught Q plus Delta Omega L; or the lock range is 2 Delta Omega L.

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Delta Omega L being equal to  $K_{pd} A_{naught} K_{vco}$ , which we have already defined as being equal to  $K_1$ , the D C loop gain. So, it is nothing but  $K_1$  into  $\pi$  by 2. It is a very simple expression.

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The image shows a chalkboard with the following handwritten derivation:

$$\Delta\omega_L = \frac{K_{pd} A_{naught} K_{vco}}{2}$$

$$= \frac{K_1}{2} (\omega_{oa} - \omega_i)$$

$$K_{vco} + \frac{\omega_{oa} - \omega_i}{K_{vco}}$$

$$= 2\Delta\omega_L$$

The word "lockrange" is written below the equations. In the bottom right corner, there is a logo for "IIT MADRAS".

The lock range of a phase locked loop is nothing but the D C loop gain into  $\pi$  by 2. So, that is the Delta Omega L. That into, of course, 2 is the lock range. 2 Delta Omega on either side of Omega naught Q.

So this...only if...condition for this is, only if amplifier does not go to saturation before that, and VCO continues to free that up to that frequency without any problem; that means these rangers here of VCO and amplifier should not be limiting the lock range lower than this. Otherwise, these ones will limit the lock range.

This is the maximum possible lock range for this, if these two do not go to their limits. Otherwise, these will fix up the lock range. Let us say VCO is capable of going maximum of this frequency and minimum of this frequency. Then that will limit the lock range if it is lower than this.

So, this information about lock range is very important. This is like our limit for the voltage, output voltage for the op-amp, power supply. It can go almost up to the power supply... Like that, there is this dynamic range limitation for the face lock loop. It goes out of lock; beyond this, it is not going to be kept in lock.

If you have understood this, then we will also try to understand another important concept associated with the face lock loop. Once again, let us emphasize this fact. The face lock loop was free running at  $\Omega_{naught Q}$  when nothing was connected to it. Then I connected an incoming frequency which was equal to  $\Omega_{naught Q}$ . It is equivalent to...by saying that I want to take you to canteen; you are engaged busily; you are in the class listening to the lectures. So, I come to the class and call you Mr. Ramaswamy, let us go to the canteen. I go very close to him take his attention, draw his attention and then say we will go the canteen.

He says, I can come with you to any place because that is the whole thing. Now I have come close to him that I can now keep his attention all along to any place. Let us say it is...the limit is within the campus, IIT campus. So, I can take him anywhere around the campus without any problem. He is all the time along with me. That is the lock range.

Now obviously, you can now understand there is some problem. Suddenly, the attention is lost. Some disturbance occurs and we two are separated. Now, I have to capture the interest of Ramaswamy, before I can take him anywhere. This is important.

Therefore, if I say that it is free running at  $\Omega_{naught Q}$  and  $\Omega_i$  is not equal to  $\Omega_{naught Q}$  to start with, there is a problem of capture. The problem of capture does not arise if I have started very close to him straightaway. If  $\Omega_i$  is equal to  $\Omega_{naught Q}$ , I have already captured his interest and I can take him anywhere.

Suppose to start with,  $\Omega_i$  is not equal to  $\Omega_{naught Q}$ . It is far away from  $\Omega_{naught Q}$ . Obviously, I am far away. He is in this place. Until I am able to



somehow capture his interest...lots of things - he is listening to the lecture, it may be more important for him than the other activities.

So, it becomes very difficult for me to capture his interest. Another thing is - his hearing power, strength, my power of communication - all these things are important. That means strength of the signal is important factor. How close I am to him is important; whether I am able to capture his interest is another thing that is important.

That is, exactly similar thing happens in the case of phase locked loop. If  $\Omega_i$  is far away from  $\Omega_{lock}$ , this  $\Omega_i - \Omega_{lock}$  is the distance.

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What does it mean in terms of circuit? If  $\Omega_i$  is far away from  $\Omega_{lock}$ ,  $\Omega_i - \Omega_{lock}$ ,  $\Omega_i + \Omega_{lock}$ , both will be high frequency components. You understand this? If  $\Omega_i$  is far away from  $\Omega_{lock}$ ,  $\Omega_i - \Omega_{lock}$ ,  $\Omega_i + \Omega_{lock}$ , both are high frequencies which will be rejected by the low pass filter.

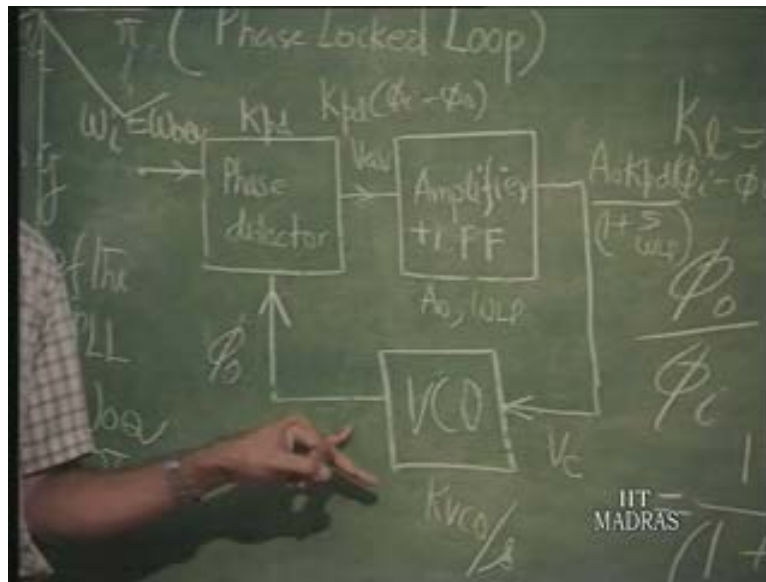
Nothing happens here. It will continue to do whatever he was doing earlier as though nothing has happened at the input. So, capture has not taken place. His attention has not been drawn. Until capture takes place, I cannot have lock.

So, the process of capture precedes the process of locking. So I come slowly; closer and closer I am coming. I can either come from here or come from there. That is, it is of no consequence. As long as I am far away from this place, whether I am coming from this side or that side, it does have no significance. Both will produce the same type of components.

Now, when  $\Omega_i$  is close to  $\Omega_{naught Q}$  such that the low pass filter cut-off frequency  $\Omega_{L p}$  is permitting something to appear here, then this will start responding to the input. But, if it just responds to the input, it is not sufficient. What has happened? I have become...I have come close to him; but he does not know exactly what I am telling. He is not able to hear. He is seeing me talking to him, but he is not able to hear.

So, he is sometimes listening to the lecture, sometimes looking to him trying to make out what he is trying to say. So, that means, this particular thing really does not know what to do. This has not elicited much response from here.

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Now obviously at this juncture, we have to talk of what is the probability of capture and when does it occur. When will really this  $\Omega_i$  start responding, starts getting response from output of the VCO in such a manner that this becomes ultimately equal to  $\Omega_i$ ?

Now for that, we have to again go for theory of probability. I can tell you about one test that they carried out on chimpanzee. Chimpanzee is supposed to be very intelligent, almost as intelligent or may be more intelligent than human beings.

So, they had put the chimpanzee in a cage. This is our chimpanzee. The goal is: will the chimpanzee think originally? There is...there was a rope hung from a sort of pulley here and a man was standing there trying to sort of swing this up and down. There is a lot of bananas tied to this.

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So, the chimpanzee is trying to catch these bananas. This is hung like this. Obviously, this is at a great height. So, chimpanzee is making an attempt. Now it knows that this particular thing, it must capture; but it has its own limitations. It can only jump up to a certain point.

Now, how does it know that it is going to capture or not? It will do it by trial and error. It takes a risk. It will jump and see. Obviously when this is at a great height, this cannot reach. That it will gradually come to know after jumping several times. That is what happens here.

$\Omega_i$  minus  $\Omega_{naught Q}$  has become low enough to elicit some response here and this is changing around its quiescent. That is, it is equivalent to the monkey jumping, but not reaching still. That is because the monkey is not able to jump up to this point. Now, in this experiment, these people did something. They started...obviously they do not want to frustrate the monkey. So, they started swinging this up and down like this. Meanwhile, this monkey has experimented upon - how to jump, at what time it should jump...

Now all of you know; I am sure you are better than monkeys in thinking about it. Once the swing is there, this monkey is likely to capture the banana when the maximum jump of this monkey is such that the minimum swing, the point of the swing comes to the minimum and there is an overlap.

So that means this is going to be higher than this. Then...also it need not capture because it depends upon the timing. The likelihood of capture is going to be good when the minimum of this here is just coinciding with the maximum of the jump of the monkey.

Now, let us translate it to something electrical here. Obviously, this quiescent frequency is going to change. Because, if this is a sine wave here, sine wave or some wave, which is changing on either side of the quiescent, the voltage here is going to change higher as well as lower. That means this frequency is going to change higher and lower.

So, this particular free running frequency is going to be changed around its value to a higher value and a lower value. If for example,  $\Omega_i$  is higher than  $\Omega_{naught Q}$ , the highest value this reaches when it is swinging, when it becomes equal to the incoming frequency, the probability of capture is good. Is that understood?

The whole phenomenon of capture is again non-linear because please understand, the moment this particular frequency changes, I cannot consider this any longer that it is changing around  $\Omega_{naught Q}$ . At every instant of time this will be changing, coming closer to  $\Omega_i$  and going away from  $\Omega_i$  also; and correspondingly, this frequency is not a single frequency. It is also a change in frequency. So, it is not possible to explain this until you accept the fact that this  $\Omega_{naught Q}$  is the quiescent frequency and around this point, it is going to change on either side equally.

All are assumptions which are not really valid strictly; but the probability of capture is going to be high only when the swing from the quiescent here is such that it reaches at some instant of time, the incoming frequency. So, that is roughly going to be the capture range on either side of  $\Omega_{naught Q}$ .

So, if this is the lock range...in fact, when I am coming from here, let us say at that frequency, this capture has just occurred; then immediately it will lock to the incoming frequency and it can go on like this. It will go out of lock here. Then, when I come from this side, I have to come pretty close to Omega naught Q, as much close to Omega naught Q as on the other side. So, it will get locked to this and it can go on remaining locked. So, this is the way...if you come from these two sides, the response is going to be.

Let us once again go through the thing. When I am coming from lower frequency end, it will get captured at a certain point and then it will remain captured or remain under lock up to the end of the lock range; goes out of lock. Then I come back here; it will again get captured at this point and it will remain captured up to this.

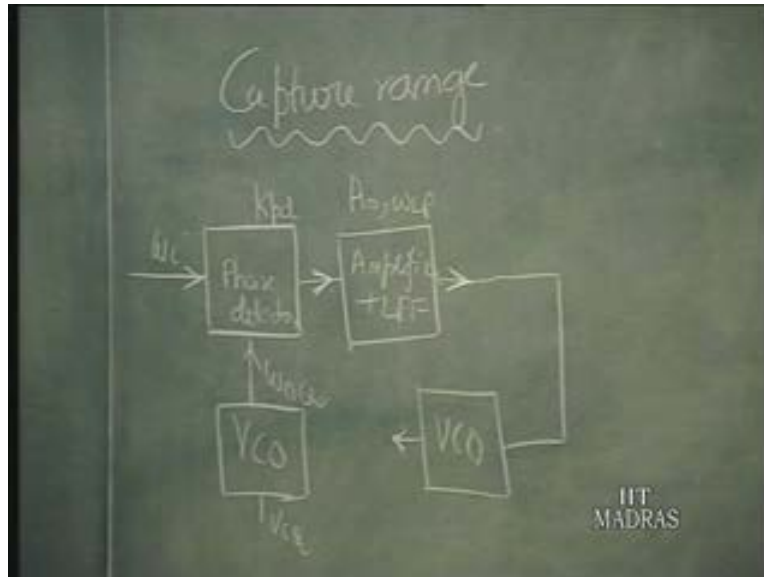
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Now, this phenomena being highly complex, there is one way we can obtain the capture range fairly approximately. That is this monkey technique. It is a fairly good approximation; almost to about 10 to 20 percent accuracy, you can predict the capture range by this and that is good enough for us. More exact versions are discussed very...in a very detailed manner by a book by Gardner on phase locked loop. So, let us see how capture range can be explained.

So, we were explaining about capture range and we can depict the whole situation in the following manner.

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$\omega_i$  is inputted to the phase detector and let us imagine that we have a VCO whose V C Q is kept constant at quiescent and it is having free running frequency as output frequency  $\omega_{nought Q}$ . Then, we can say that there is going to be an output here which is not going to be in this phase detector. Output is going to be now  $\omega_i$  minus  $\omega_{nought Q}$  and  $\omega_i$  plus  $\omega_{nought Q}$ .

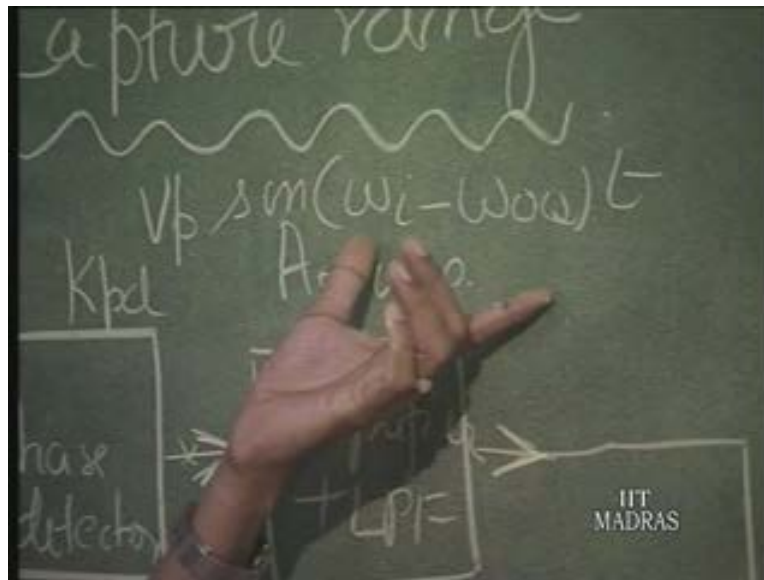
So, it is not a square wave or anything like that. It is a waveform whose frequency is changing; but out of that, we want to take only the lower frequency component -  $\omega_i$  minus  $\omega_{nought Q}$ . I am assuming for practical purposes that it is responding to only the sine wave part of the lower frequency which is also not correct strictly because it is an amplitude limited waveform.

So, let us assume the peak amplitude of this is  $V_p$  and it is a sine wave whose frequency is  $\omega_i$  minus  $\omega_{nought Q}$ . This is purely an assumption because otherwise it is

a fairly complex waveform which has to be split into its individual components and then see the output due to each component.

But we know that it is to this component that it is going to respond the maximum, the lowest frequency component. Other higher harmonic of this will not result in much response from the low pass filter.

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So, we will take only that. So, this is the assumption. If this is the output here, some peak voltage into sine  $\Omega_i - \Omega_0$  into  $t$ , then the low pass filter output will be  $V_p$  divided by  $1 + \Omega_i - \Omega_0$  whole squared.

What is the  $\Omega$ ?  $\Omega$  is this frequency -  $\Omega_i - \Omega_0$  whole squared under the root, sine  $\Omega_i - \Omega_0$  into  $t$  plus some  $\Psi$ , phase. We are not interested in that.

So, this is the output if sine wave input is given to a low pass filter with this kind of cut-off frequency; this into  $A_{naught}$ , amplifier gain.



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$$A_0 V_p \frac{\sin(\omega_i - \omega_{oq} t + \psi)}{\sqrt{1 + \left(\frac{\omega_i - \omega_{oq}}{\omega_{Lp}}\right)^2}}$$

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So, you can see that you get a sine wave here whose peak amplitude goes up to  $A_0 V_p$  divided by this, super imposed over what? -  $V_{oq}$ . So, this will be  $V_{oq}$  plus  $A_0 V_p$  divided by root of 1 plus  $\Omega_i$  minus  $\Omega_{oq}$  squared by  $\Omega_{Lp}$  square into sine  $\Omega_i$  minus  $\Omega_{oq}$  t plus  $\Psi$ .

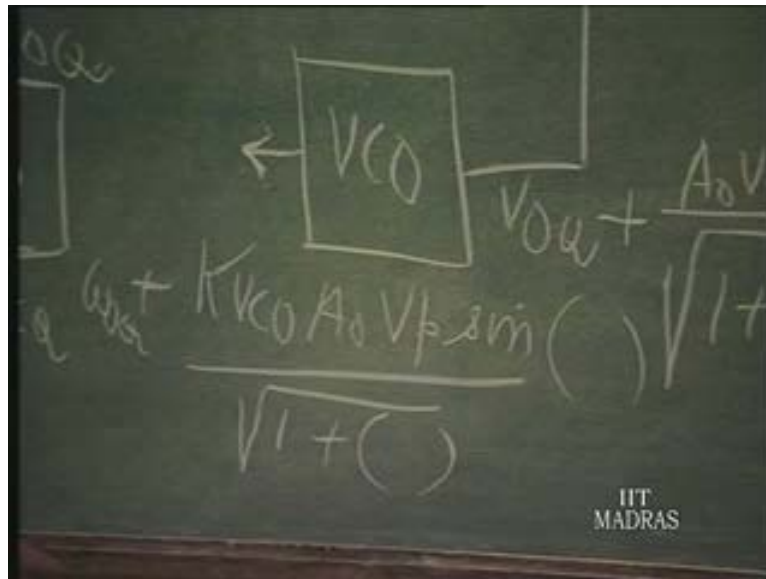
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$$V_{oq} + \frac{A_0 V_p \sin(\omega_i - \omega_{oq} t + \psi)}{\sqrt{1 + \left(\frac{\omega_i - \omega_{oq}}{\omega_{Lp}}\right)^2}}$$

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What does it mean? We are not again interested in all that. It might vary exactly sine wave wise or... The peak amplitude is going to change from  $V_{naught Q}$  plus this value and  $V_{naught Q}$  minus this value. What will this do? This will start swinging from this to  $KVCO \dots \Omega_{naught Q} \pm KVCO$  into  $A_{naught}$  into  $V_p$ . The same thing; sine of that divided by root of that.

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So that means,  $\Omega_{naught Q}$  plus this value and  $\Omega_{naught Q}$  minus this value, it will go. Once again, this sine factor is of no interest to us. It is just that  $\Omega_{naught Q}$  plus some value,  $\Omega_{naught}$ ...this is an important thing. That means...imagine that the moment  $\Omega_i$  is applied, this is now swinging. This has already started swinging around  $\Omega_{naught Q}$ ; on either side of  $\Omega_{naught Q}$ .

What is capture range? Capture range is that when this  $\Omega_{naught Q}$  plus this - if it is higher - becomes exactly equal to  $\Omega_i$ ; or,  $\Omega_{naught Q}$  minus this - if it is lower - becomes equal to  $\Omega_i$ . Is this clear?

So that means, when  $\Omega_i$  becomes equal to  $\Omega_{naught Q}$  plus or minus, it does... this value,  $KVCO A_{naught} V_p$  divided by root of  $1 + \Omega_i \text{ minus } \Omega_{naught}$

$Q$  squared by  $\Omega L p$  square. Is this understood? So this, you can call as  $\Omega$  i - limit capture frequency. This is then also the capture frequency  $\Omega$  i.

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The image shows a chalkboard with the following handwritten equations:

$$\omega_c = \omega_{oc} \pm \frac{k v_{co} A_p V_p}{\sqrt{1 + \frac{(\omega_c - \omega_{oc})^2}{\omega_{LP}^2}}}$$

The chalkboard also features the IIT MADRAS logo in the bottom right corner.

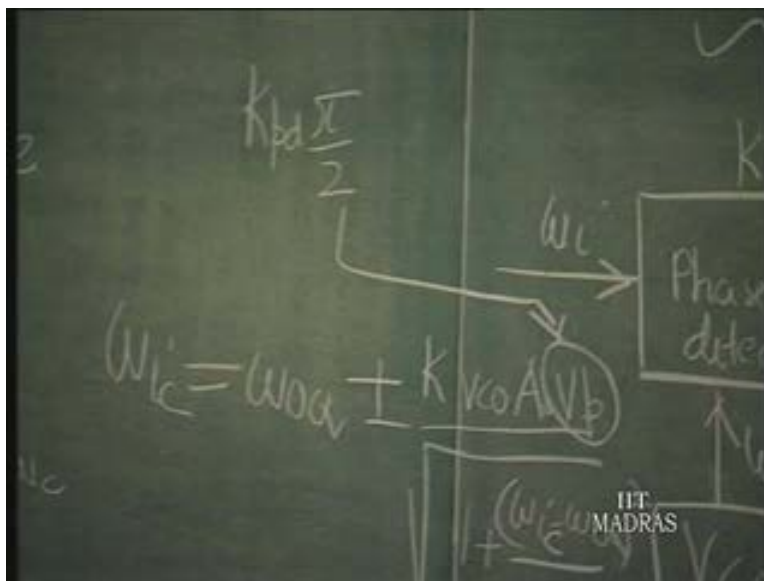
So in our curve here, we can say that around  $\Omega$  naught  $Q$ , it is this on this side and this on this side. This is  $\Omega$  naught  $Q$ ,  $\Omega$  naught  $Q$  plus...minus  $\Delta \Omega$   $C$ . This is  $\Omega$  naught  $Q$  plus  $\Delta \Omega$   $C$  and it is  $2 \Delta \Omega$   $C$ , which is the capture range and that is given roughly by an approximation as this.

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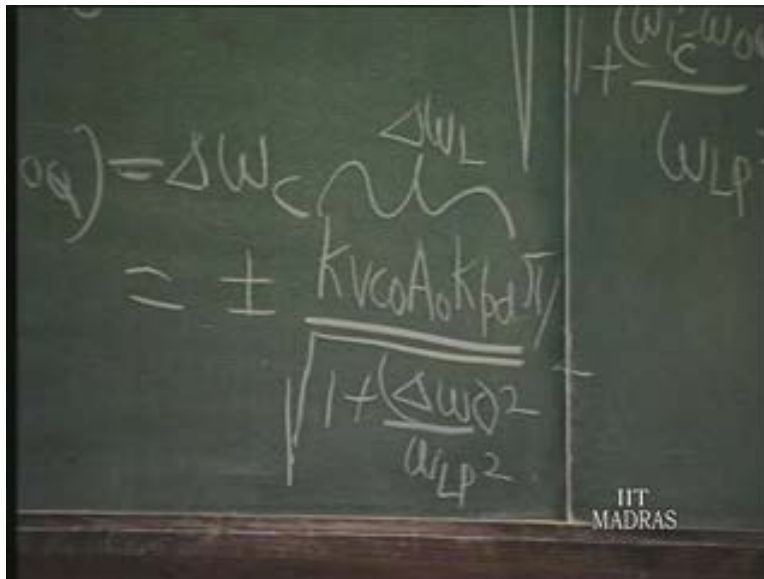
And what is  $V_p$ ?  $V_p$  is the peak voltage possible at the output of the phase detector which is also, if the phase detector is linear, nothing but  $K_{pd}$  into  $\pi$  by 2. The peak voltage possible, peak change possible at the output of the phase detector is nothing but  $K_{pd}$  into  $\pi$  by 2. So, this is nothing but  $K_{pd}$  into  $\pi$  by 2.

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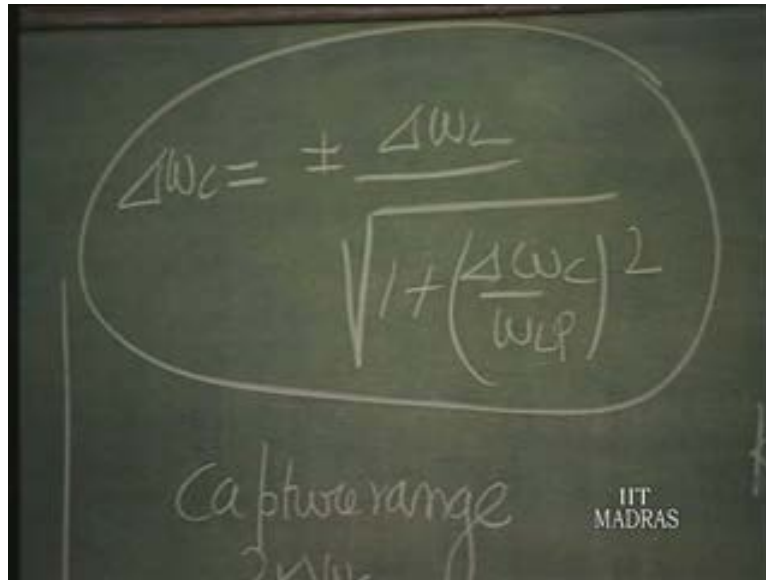
So, let us write it down.  $\omega_c - \omega_n$  - you take this to this side, which is called as  $\Delta\omega_c$  by us, is equal to  $\pm \frac{KVCO A_n K_{pd}}{2}$  into  $\pi$ . What is that?  $\frac{KVCO A_n K_{pd}}{2}$  is already defined by us, if you remember, as  $\Delta\omega_L$ . This whole thing is  $\Delta\omega_L$ . So, that divided by  $\sqrt{1 + \Delta\omega_c^2 / \omega_L^2}$ .

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Or, we will rewrite this.  $\Delta\omega_c$  equals  $\pm \Delta\omega_L$  divided by  $\sqrt{1 + \Delta\omega_c^2 / \omega_L^2}$ . This is the equation which you have to solve in order to obtain  $\Delta\omega_c$ .

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$$\Delta\omega_c = \pm \frac{\Delta\omega_L}{\sqrt{1 + \left(\frac{\Delta\omega_c}{\omega_{LP}}\right)^2}}$$

capture range

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Unknown is Delta Omega C. Delta Omega L is already known. Omega L p is known. So, you will get Delta Omega C in terms of Omega L p and Delta Omega L. Now you can imagine that the capture range is always less than the locked range. Let us consider the situation where capture range is much...Delta Omega C by L p is much greater than 1.

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$$\left(\frac{\Delta\omega_c}{\omega_{LP}}\right) \Rightarrow 1$$

Cap  
← 2

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Then, this one can be ignored and this becomes  $\Delta\omega_C = \Delta\omega_L$  divided by  $\Delta\omega_C \omega_{LP}$ ; or  $\Delta\omega_C$  equals root of  $\Delta\omega_L$  into  $\omega_{LP}$ . It becomes very simple. Otherwise, you have to solve a quadratic equation if this assumption is valid.

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$$\Delta\omega_C = \frac{\pm\Delta\omega_L \omega_{LP}}{\Delta\omega_C}$$

$$\Delta\omega_C = \sqrt{\Delta\omega_L \omega_{LP}}$$

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So the best thing is, assume that this is so. Find out  $\Delta\omega_C$ . Check whether this assumption is valid. If this assumption is not valid then, then you go and solve this. So, capture range is root of lock range into low pass filter cut-off frequency. So, you will see that this is an important aspect of phase locked loop design. The captured range can be made very narrow around  $\omega_{naught}$  so that it does not capture at all other frequencies around  $\omega_{naught}$ . But once it captures, it can keep itself under lock for a wide range of frequency.

Such a characteristic of the phase lock loop finds itself in variety of applications in communications and controls. This, we will discuss in the next class.