High Speed Devices and Circuits

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Lecture – 21

MESFET I – V Characteristics Shockley's Model

We have been discussing the MESFET.

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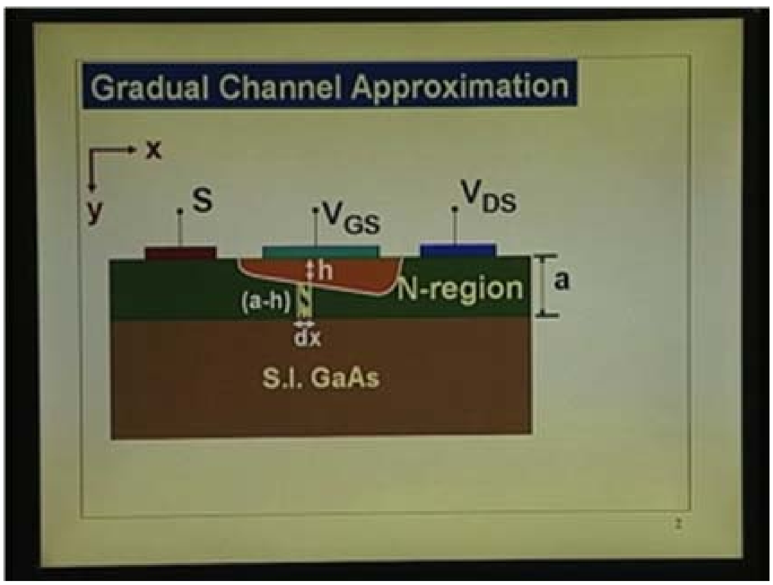

We started with the principle of operation. What we said is the operation of the MESFET is based on the variation of the resistance of the region below the gate. That variation in resistance is brought about by applying voltage to the gate. We have used terminologies V_{GS} gate to source potential. V_{GS} has a positive value, if it is forward biased; V_{GS} has a negative value, if it is a reverse biased. We also saw yesterday or in the previous discussion that when the V_{DS}, drain to source voltage is small, the drop from here to here is small that is along the channel length is small, this region is called the channel. So, drop in the channel region is small and then voltage across the depletion layer is constant from here to that end, source to drain end. But, as V_{DS} become larger, current become larger.

It is after all we saw, i_d is proportional to V_{DS} and it is linear initially. How long will it be linear? That is what we want to see and what happens if keeps on increasing V_{DS}. As V_{DS} increases, the current increases and the drop also increases; as drop increases, the potential across the depletion layer changes. The drop here is appearing across the gate and this channel region. This is plus and that is minus. It is appearing as we apply extra reverse bias across the junction. When we apply extra reverse bias across the junction, the depletion layer widens. The extract reverse bias across the junction due to the drop in the channel increases as we move from the source to the drain. At this point, for example, near the source end of the gate, the potential across the depletion layer is V_{bi} minus V_{GS}. I am saying minus, that is the polarity that we are taking. If we put V_{GS} negative, it adds on, reverse biasing. If V_{bi} is 0.9 volts and V_{GS} is reverse bias minus 1 volt, it is 1.9 volt across that. When we say V_{bi} minus V_{GS}, we are very clear what we are talking of. As we go from here, 1.9 volts we said just now and if the drop is 1 volt then the potential across the depletion layer at this stage will be 0.9 and 1, 1.9 plus 1 volt, 2.9 volts. The depletion layer voltage keeps on increasing from source to drain. How much is this increase?

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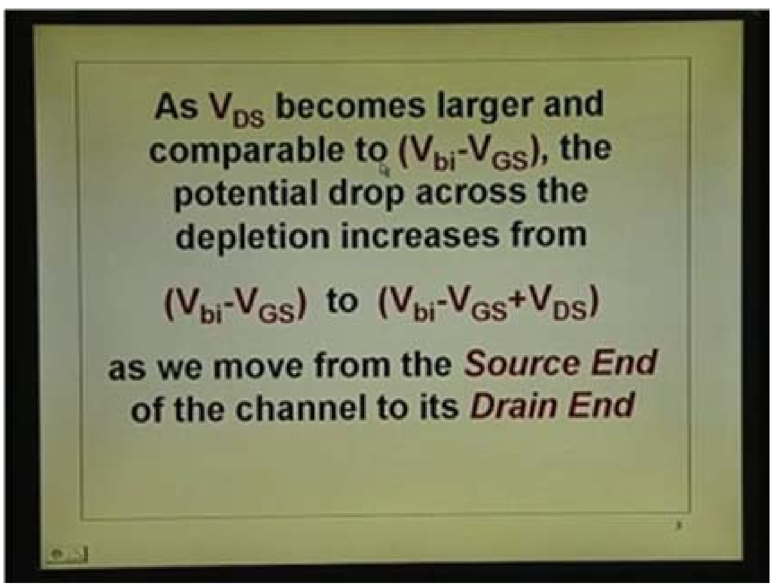

As V_{DS} becomes larger and comparable to V_{bi} minus V_{GS}. If the V_{DS} is small compared to V_{bi} minus V_{GS}, the drop across the channel is minimum, you will have the depletion layer width constant and it will be a linear characteristics. It becomes larger. The potential drop across the depletion layer increases from V_{bi} minus V_{GS} to V_{bi} minus V_{GS} plus V_{DS}, as we move from source to drain.

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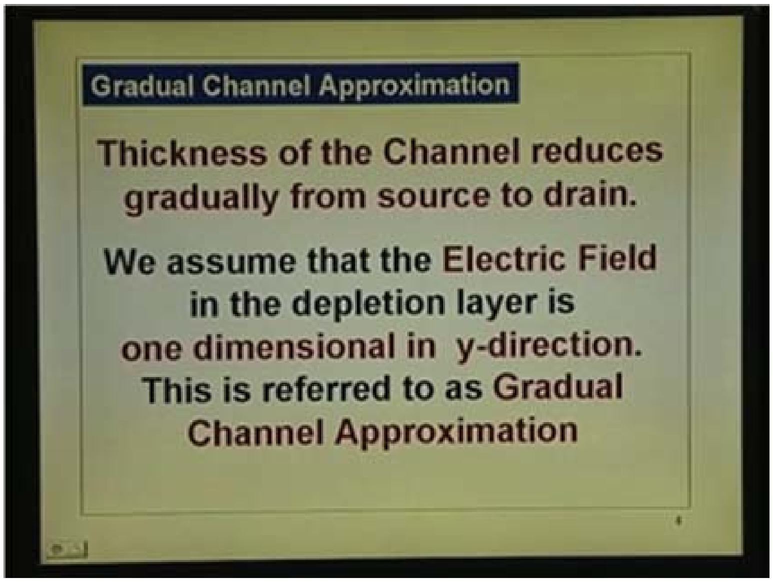

But we are telling that is, due to that the thickness of the channel reduces gradually from source to drain. Go back to that curve, (Refer Slide Time: 05:03) the depletion layer voltage keeps on increasing, depletion layer keeps on widening; the channel thickness keeps on reducing gradually from source to drain. We also mentioned in the previous discussion, we assume that, the electric field in the depletion layer is in the vertical direction, y-direction; perpendicular to the channel and along the channel direction, the electric field is there because V_{DE} is there. There is a drop along the thing. There is a field, but that field we are neglecting compared to the vertical field, so that, we can apply one-dimensional equation for finding out the depletion layer width. After all, the voltage across the depletion layer is equal to pN_D into W square by twice of $epsilon_r$ $epsilon_0$. That is the implication of that. This is referred to as the gradual channel approximation. You can just say that the channel is gradually changing. The correct thing is the field is only in the vertical direction. That is the assumption which holds good when the channel lengths are long.

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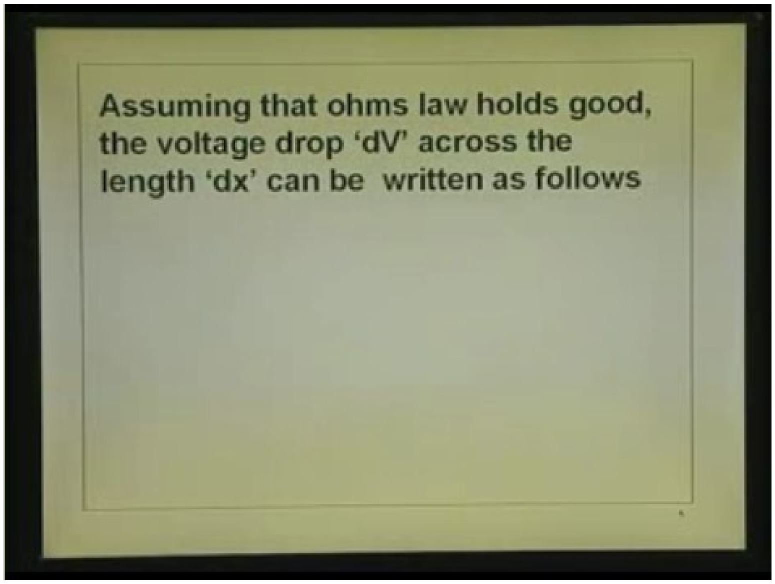

Assuming that, ohms law holds good, the voltage drop dV across the length dx. I think instead of keeping on going I think go back to the diagram otherwise We will draw the diagram here and keep it.

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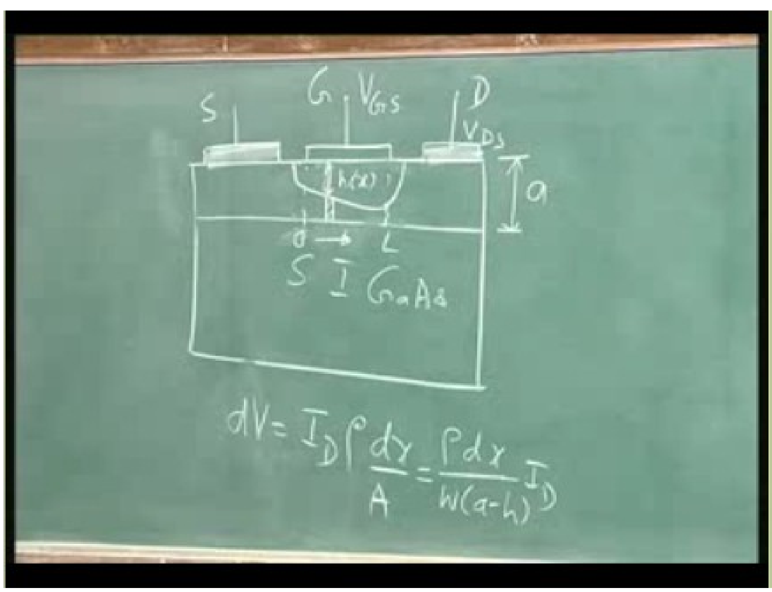

That is the source and that is the drain. You have got this semi-insulating gallium arsenide. It can be silicon. If it is silicon, this will be sapphire; silicon and sapphire, totally insulated. I have a gate here and I am applying V_{GS}, polarity we have to decide whether it is plus or minus. If it is plus, then it is forward bias; if it is minus, it is reverse bias. Reiterating it; before that, there is no confusion about that V_{GS}. So here, if the drop V_{DS} is, we are taking across this, neglecting the drop here for whatever happens is within this region, the gate has control only in this region and gate does not have any control in this region. If the drop is more there, if it is 0 drop, nothing applied the depletion layer is like that. Because of the drop in this region, it keeps on widening that is what we are talking of.So, depletion layer keeps on widening: that is the depletion layer. The depletion layer does not contribute to current. It is only this region that contributes to that. If that is a and if this is h at any x, please note, we are talking of 0 here and x, and it is L here at the centre at the drain end; 0 is at this end from that end. If we go beyond that point, we are just talking of this point and that point. Whatever happens within that region, we are not taking into any two-dimensional efforts.

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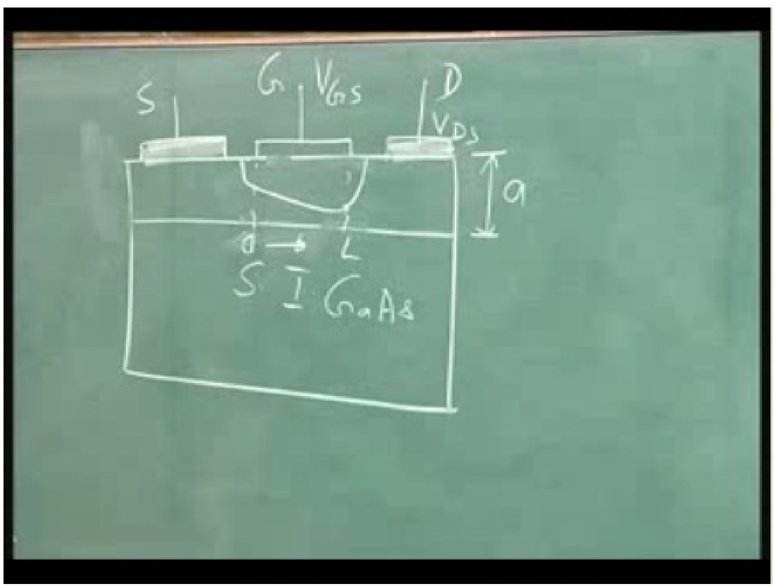

Within this region, as we move, that is L, that is 0 and you got x here. What we are telling is we have to find out the voltage drop across this region and relate it to the current, same law, ohms law: v is equal to i into r. We have find out the resistance. What we do is, take a chip here of width dx. Find out what is the drop across that, due to current I_D to the current is constant right through. There is no current accumulation. Find out the drop, I_D

into rho dx divided by area of cross section that is all what we are doing. In the previous case is, this area of cross section was constant, as you move from that end to this end. Now because the channel thickness keeps on reducing, it is not a constant resistor, it is a varying resistance. You can see what happens because of that. This is dx and what is this quantity (Refer Slide Time: 10:21) at this point x? We call it as h of x. We are keeping this diagram because we do not want to go back to the slide again and again. That is h of x and this point is nothing but a minus h. The area of cross section to the current flow is a minus h into depth, W. The current I_D or the voltage drop dV will be equal to I_D into rho into dx divided by A, where A is area. A is nothing but rho dx divided by w a minus h into I_D. So that is what we are going to compute. All that we are going to compute in the entire analysis today is compute that voltage drop due to current I_D and when we go to the drain end that is equal to the applied voltage V_{DS}, total drop.

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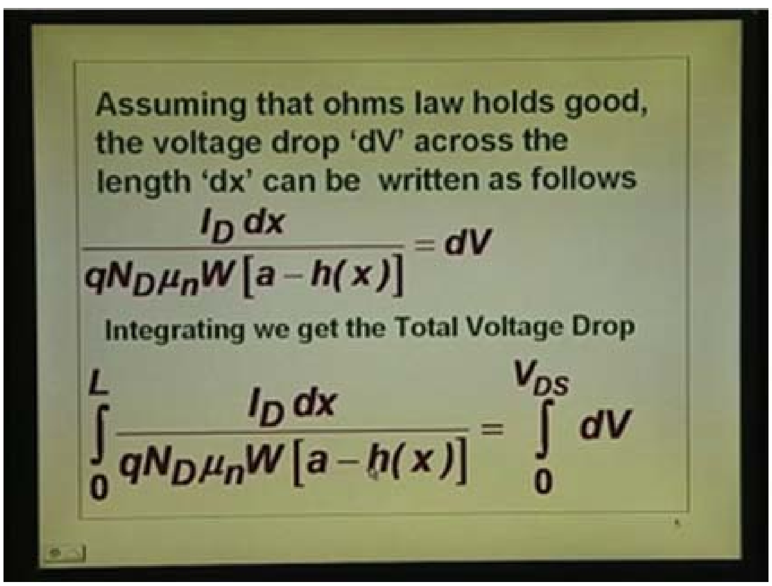

Assuming that ohms law holds good, the voltage drop dV across the length dx that we are projected there can be written as follows; I have rewritten that there dV is equal to I_D into dx into rho divided by a minus h into W. We are putting it there.

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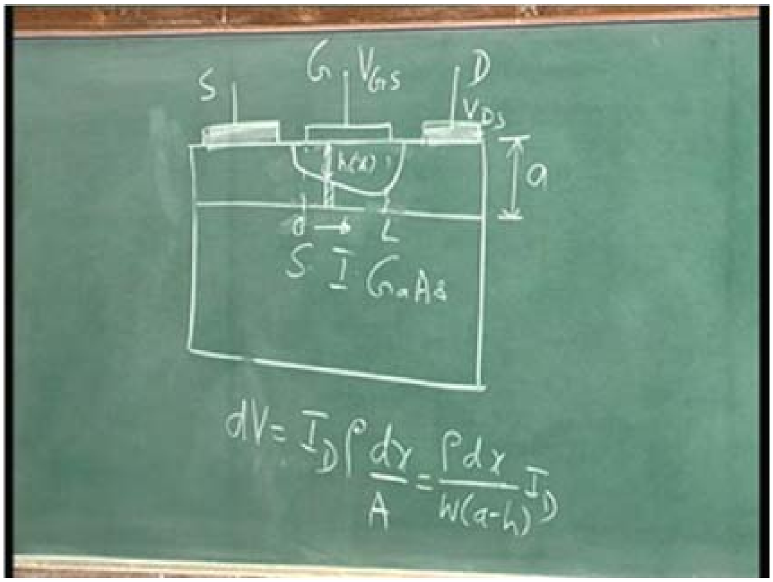

Now I am substituting for rho in that equation, I am substituting for rho which is nothing but 1 by q N_D into mu. Let us go back to this slide, (Refer Slide Time: 12:18) I_D into dx and rho is 1 by q N_D mu_n and W, this is the area and h, I have put h as h of x because it is function of x. Now integrating, we want to find out what is V_{DS} is; integrating we get the total voltage drop. All I have done is I used the integral term. Left hand side: x is equal to 0 to L because L is the other end of the drain, x equal to L. So channel is varying from x equal to 0 to L and very increasing it from 0 to L, integrating it. So this is the term, all that we have worked on is, see this particular term h of x, h of x is a function of x no doubt but actually, it is a function of the voltage; more is the depletion layer voltage, more is h. This is the voltage dependent term, this particular term. What is the do(13:25) is, take the entire denominator to the right hand side; put this sum within the integral because it is a voltage dependent term and on left hand side, retain just I_D into dx.

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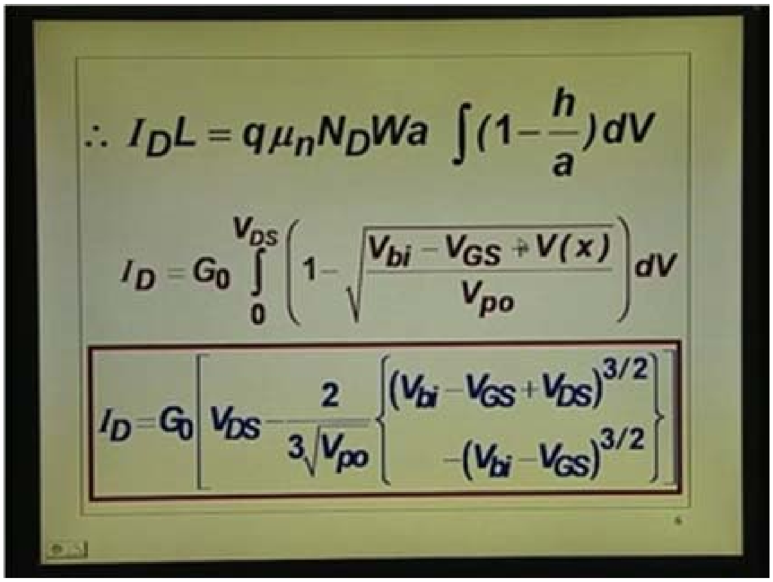

Left hand side, when we integrate from 0 to L, it is I_D into L; I_D into dx, integral is I_D into L and I have taken the entire denominator to the right hand side. Just see it here. We have taken this entire thing to right hand side, pull this a outside, (Refer Slide Time: 13:55) we get 1 minus h by a. That is what we have done. This is the equation and integral will be of course 0 to V_{DS}. The limits, we have not put there. We have put it again. Now, let us see what is this term. What we have done is, instead of keeping on writing q mu_n N_D Wa divided by L. We have removed this L from this side and taken to the right hand side. Now, I_D will be that particular term, a constant of the device q mu_n N_D Wa divided by L, we call it as G_0 Inside, this is particular term. What is this G_0? G_0 is actually the channel conductance. Let us go back to that. G_0 is sigma, this is sigma into W into a, if the area of the channel, if no depletion layer is present,full channel open. Entire channel is open, (Refer Slide Time: 15:11) a is the thickness, a into W 0 cross section, l is the length. G_0 is the conductance of the channel, which is fully opened. This is easy to remember in that particular way. We do not have worry about what is the thing. In fact it is 1 over r channel resistance when the channel is fully opened. Rho l by a, I put it as, l sigma into a by l. Now integral, 1 minus h. What is h? h and V_{p0}. V_{p0}, we have defined in the last lecture, is the pinch of voltage, that is the potential that must be present across the depletion layer so the depletion layer width is a. Just recapitulate your memory.

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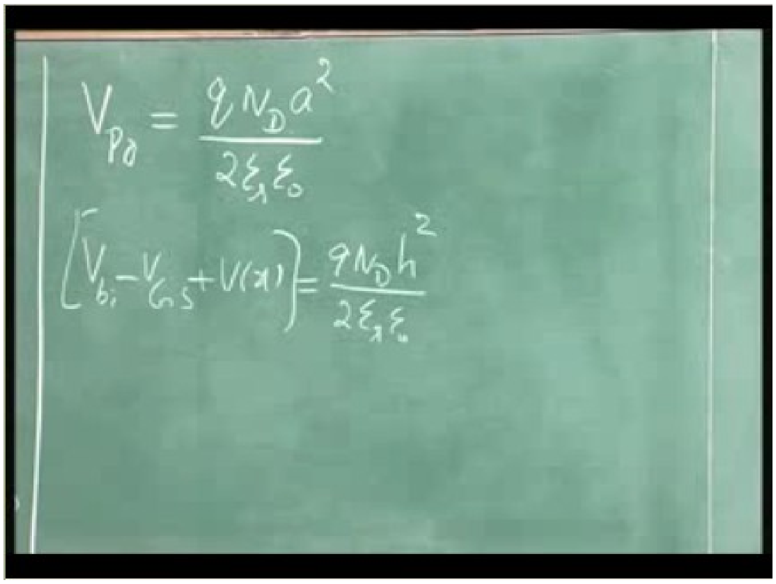

V_{p0}: q N_D into a square divided by 2 epsilon_r epsilon_0 and V_{bi} minus V_{GS} plus V of x. What is that? V_{bi} minus V_{GS} is the potential here, plus V of x is the potential here; (Refer Slide Time: 16:34) the potential across that is related to the h. So, that is equal to h square divided by twice epsilon_r epsilon_0. Now, we know how to write that h by a, Notice now, hereafter we do not have rewrite this. The depletion layer widths are proportional to square root of voltage across that; a is proportional to square root of V_{p0}, h is proportional to the square root of that voltage there. Divide, h divided by a, you get square root of this divided by square root of that. So, that is what we have done. What we have done is, we have divided h by a here (Refer Slide Time: 17:19) and h is proportional to V_{bi} minus V_{GS} minus h of x square root of that, a is proportional to square root of V_{p0}. Hereafter, whenever I write ratios of the depletion layer widths, we will write square root of the voltage across that. For example, the width of the depletion layer at the drain end will be, if we have to write h, V_{bi} minus V_{GS} plus V_{DS} square root of that, drain to source voltage.

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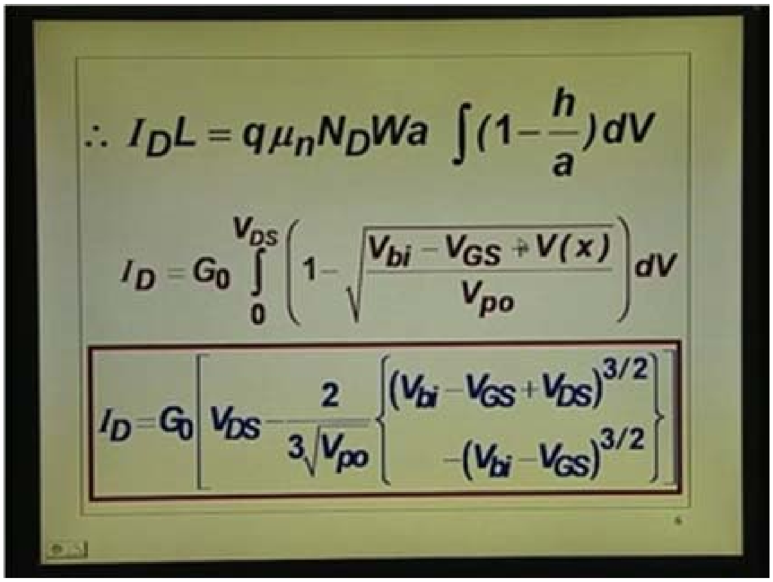

Now integrate that. Not difficult, looks cumbersome, but it is a simple expression. A simple integral, y to the power of n, integral is y to the power n plus 1 by n plus 1 that is what we have done. G_0, integral of 1, evaluated from 0 to V_{DS} is V_{DS}. Square root of V_{p0}, pull it out if it is a constant. All that we have to integrate is, V_{bi} minus V_{GS} plus V of x to the power of half. That integral is whole thing to the power 3 by 2, divided by 3 by 2. That divided by 3 by 2 is here. Now, this integral is V_{bi} minus V_{GS} plus V of x to the power of 3 by 2 because, it is root of half. Within the limits V of x is equal to 0 to V_{DS}. You can see, V_{bi} minus V_{GS} plus, V of x become V_{DS} at the drain end, to power of 3 by 2. At the source end V of x equal to 0, so it is V_{bi} minus V_{GS} to the power 3 by 2. All that we have done, we integrated this term V_{bi} minus V_{GS} plus V of x to the power of half put the limit, V_x is equal to V_{DS} at x equal to L and V of x equal to 0 at x equal to 0. That is what we have done. This term will be haunting us for some time this particular thing. We will have to deal with this and modify it to bring it to the shape that we usually see. Let us do that. First thing, how will this particular term will be that I_D versus particular this thing. How will that be? Let us see that. When V_{bi} minus V_{GS} is large compared to V_{DS} that is V_{DS} is small, that case we have obtain from the first principles, from the previous lectures by taking the width to be constant. Now, this must be hold good and we have to get that equation, if what you are saying is correct. Let us see that. We are going back to that linear region. We will see whether we get the linear region or not. V_{bi} minus V_{GS} is much lesser than V_{DS}. Using binomial expansion we get the linear region. So, what you have to do is, I think I just quickly do that. So, that you are not left with any handout (20:37) on that.

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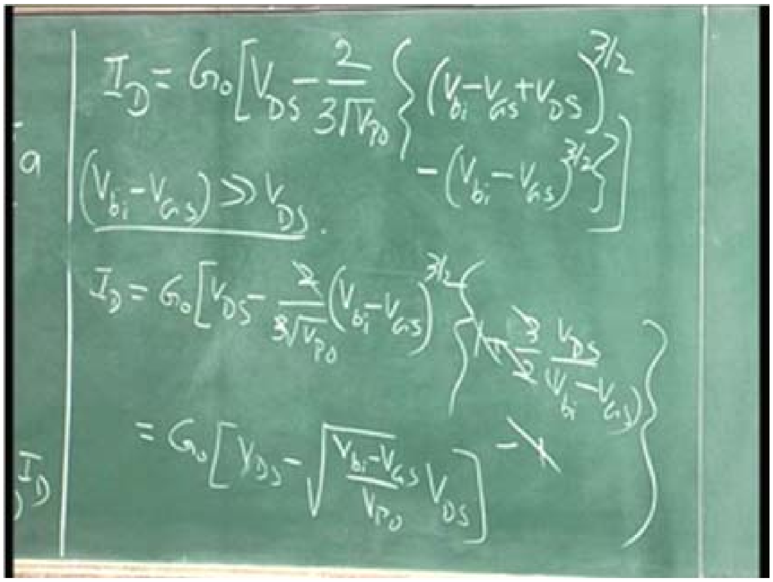

What we have is I_D is equal to G_0 into V_{DS} minus 2 by 3 root of V_{p0} into V_{bi} minus V_{GS} plus V_{DS} to the power of 3 by 2 minus V_{bi} minus V_{GS} to the power of 3 by 2. What we are trying to see is, when this is small compared to that; V_{bi} minus V_{GS}, it will much large compared to V_{DS} that is the case one that we are discussing. Expand that, I_D will be equal to G_0 into V_{DS} minus 2 by 3 root of V_{p0}, we will pull out that V_{bi} minus V_{GS} to the power 3 by 2 into what we get now? 1 plus V_{DS} divided by that quantity. 1 plus x which is x is small 1 plus x to the power of 3 by 2 or 1 plus x to the power n, binomial expansion is 1 plus n x taking one term. 1 plus V_{DS} by V_{bi} minus V_{GS} into 3 by 2, n of x, we will write it as binomial expansion using that, 1 plus 3 by 2 V_{DS} divided by V_{bi} minus V_{GS}. Correct? What about this term? I have taken that out already, so minus 1, so these two cancelled. These two cancelled, these two cancelled and we will be left with, I_D equal to G_0 into V_{DS}, all cancelled; this 3 by 2 cancels with that and we will get V_{bi} minus V_{GS} divided by V_{p0} into V_{DS}. What we have done is, just expanded this particular term, binomial expansion, pulling this V_{bi} minus V_{GS} out, we get 1 plus V_{DS} divided by V_{bi} minus V_{GS} to the power 3 by 2 that we write it as, 1 plus 3 by 2 V_{DS} is that. The second term, we pulled out so minus 1 these two cancel out. It is very simple. Now, what we have written there is, we have pulled out this V_{DS} from this inside the bracket, you get it as G_0 into 1 minus V_{bi} minus V_{GS} by V_{p0} square root into the whole thing into V_{DS}. This is the linear relationship. So, whatever we have discussed now summed up and put here. That is the same expression that we have written there, which is actually G_0 into 1 minus V_{bi} minus V_{GS} by V_{p0} square root into V_{DS}.

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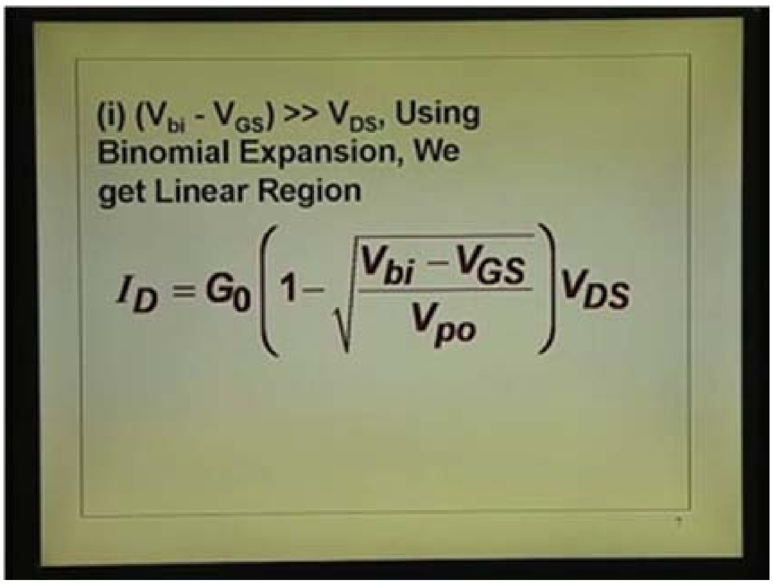

These are what we have got in the previous lecture, by taking that uniform thing, this is linear characteristics. Now, let us just go back and stop for a while before we write that the further equations. We have got the integral, now physically what do we expect?

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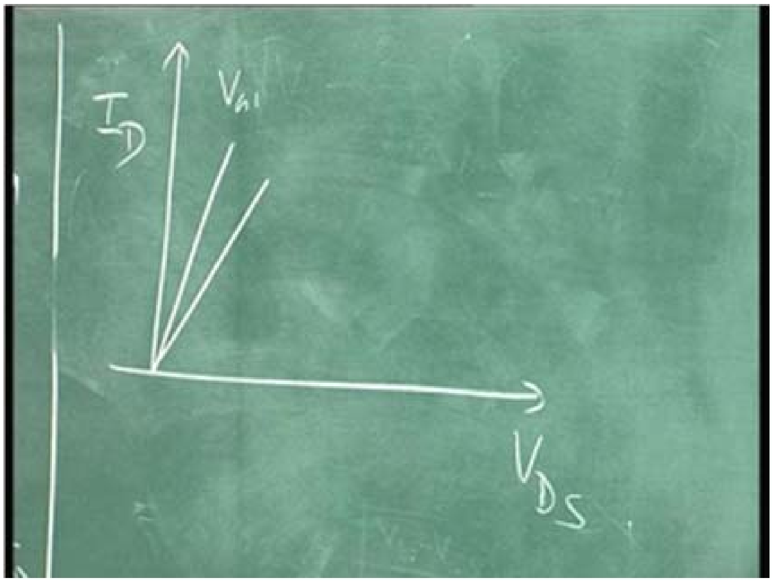

What we are said now is, you get a linear characteristics. From that expression what we have got there, if V_{GS} is 0, you may get a characteristic like that. Linear region means it is flat, very small depletion layer width, area of cross section large, for current is more for a given V_{DS}. If this is V_{GS1} and you get increase it negatively, depletion layer widens, we get linear. Let me not draw number of curves, let me draw only one curve for one particular V_{GS} and it goes on. What we have seen is, if we keep on increasing V_{DS}, the resistance which was having a same area is no longer maintained. The area is not maintained constant. The area keeps on falling. How much the area keeps on falling depends upon how much is the V_{DS}.

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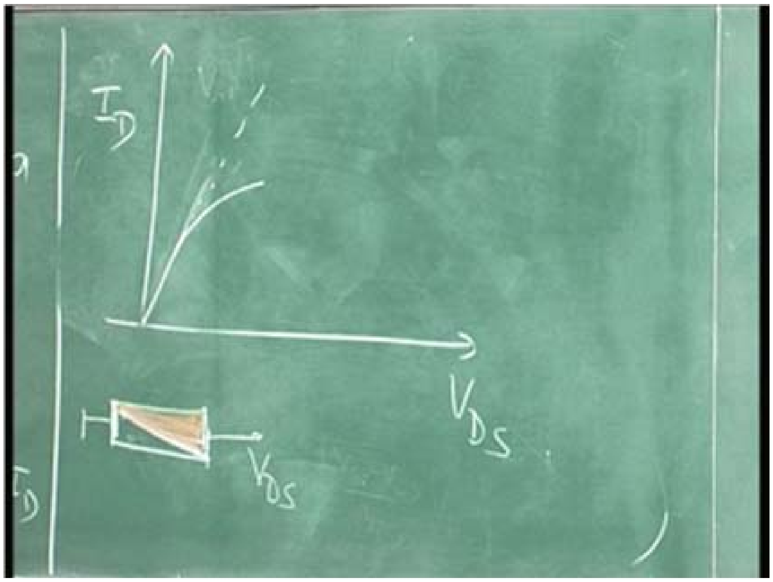

You have resistor now, originally, you had a resistance like this with a contact like this. You can see that, it is so simple. We are talking of the resistance r there, with the channel length thickness equal to whatever was there. Now, when you keep on increasing this voltage, this remains same thing, channel that falls, not exactly linearly, it falls in some fashion depending upon the voltage there, like that, like that. Each increase in voltage has move down here, each increase in drain voltage will make that region at this point I have the channel thickness varying like that. What happens to the resistance? If I take a resistor which is constant, and if I keep on reducing this cross section in that fashion, the resistance actually increases. If the resistance does not increase, I_D would increase linearly with voltage. If the resistance increases, I_D will not increase as much. It will continue to increase, but nonlinearly. What happens now is, you have the resistance instead of increasing like that it keeps into, increase linearly but now it does not increase that much. Nonlinearly it goes because this resistance goes on increasing. Till what time? What stage it will increase? Till this region comes in and merges with this h. Once it merges with this h, what happens? Supposing, it closes the channel completely, what would happen? If it closes the channel completely, current will fall down to 0. If the current falls down to 0, voltage falls down to 0 but you have the voltage there. Now, you have the situation there which is very tricky, qualitatively we can understand it. First, qualitatively what we would say is, when it closes here, the current tends to fall. If the current tends to fall, the voltage drop tends to fall. If the voltage drop tends to fall, this tends to open up. If the voltage drop falls, that voltage reduced, that will fall, it will open up. In other words what we have is a dynamic equilibrium situation. There will be a small opening left; hitting equilibrium between the voltage drop and the channel opening. We will see more to it more than what I needs is right now. Right now we will say that a dynamic equilibrium is there where the depletion layer slightly leaves the channel open to let in the current flow, and equilibrium between the current and the voltage both should be satisfied that is what is done here. At that point, once this happens here, thus resistance shape is like this and there is a voltage drop here. What is the voltage drop there? The voltage drop there is such that, the depletion layer width is almost close to that. The depletion layer width should be such that it is almost closing but still there is a small opening is there. Let us say, that is the delta.

There is a small opening here, the depletion layer width you can say almost equal to a. What is the voltage across that? If the depletion layer width is a, it is pinched off at the drain edge. You say that, the channel is pinched off. The voltage drop across the depletion layer when this happens is V_{p0}, pinch off voltage that is V_{p0} and this voltage is equal to, it cannot change beyond that point because if it falls, it will open up, but it has reached an equilibrium situation where that is held constant now. So, this is just about to close. Understand very carefully, you have situation here where this voltage now remains constant such that, this is V_{p0}. What is this voltage now? That is $V_{DS(sat)}$, beyond that it is not increased. If this has saturated, what happens to current? We are still working on the theory of the resistor. If this has reached constant and this shape does not change, resistance does not change. The current is V_{DS} divided by the resistance. Still we are holding that law valid. $V_{DS(sat)}$, the voltage saturates since resistance is also saturated now, current saturates beyond that point. This is actually $V_{DS(sat)}$. That is the entire concept involved in the I-V characteristics of the MESFET and the model that we have discussed is the Shockley's Model. The Shockley was the one, the noble laureate. The first time he gave it way back to in 1952 or so. That model still holds good very well so long as the channel lengths are not too short. We do not deal with short channel lengths that very well holds good. Now, what we want to know is, what is this voltage? How much is the $V_{DS(sat)}$? We said that V_D has saturated, the whole thing is that whether it is correct or not we have to examine later; it is quite correct for most of the devices where it fails and we will examine it afterwards. $V_{DS(sat)}$ is a voltage at which has pinched off at the drain edge, this has closed on this path. This has come all know it cannot close totally but almost closing down; the opening is very small finite. The $V_{DS(sat)}$, how do you find out? That plus V_{bi} minus V_{GS} is that voltage plus $V_{DS(sat)}$ is actually equal to? We are talking of situation where, let me that redraw this now. At saturation, it is almost like that, this is almost equal to a, this drop is $V_{DS(sat)}$ and this drop is actually equal to V_{bi} minus V_{GS}. So, V_{bi} minus V_{GS} plus $V_{DS(sat)}$ equal to V_{p0}, pinch off voltage. What we are saying now is, we can look into the analysis.

The characteristics become non linear because the resistance become large, no longer same, resistance value changes where, V_{DS} is increased, I_D does not increase correspondingly, it increases less, you have resistances increases and ultimately it reach a saturation, the point at which the channel pinches off at that drain end. Suppose it is pinched off at the source end; what will happen? You have no chance, it is closed here and the current flow is 0. To make the current flow 0, you must flow at the source end; the moment you open at this here, it will let go through this small opening here that is the idea. Turn off the device you must apply V_{GS} such that the source end we just blocked that totally. Once in letting the current flow, it has to come out through the drain end and it will give as the small opening. Even though it is pinched off at the drain edge, the current is there, saturation current.

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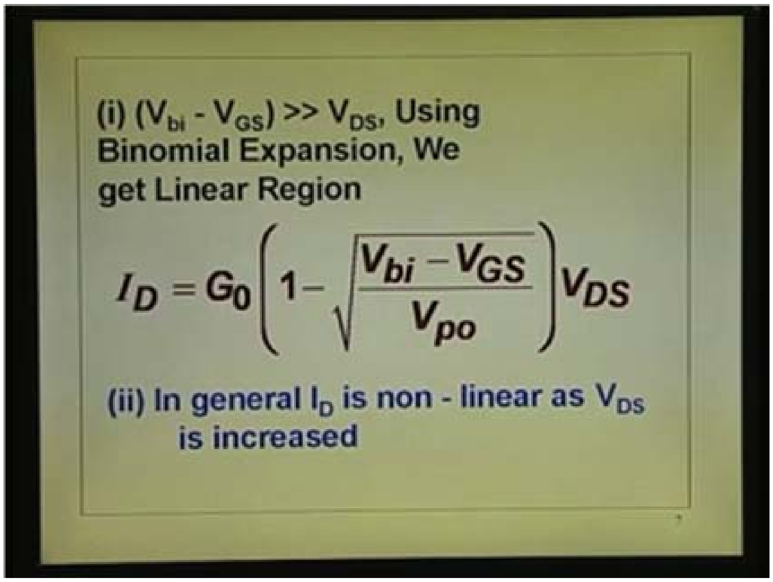

We said, I_D is non linear.

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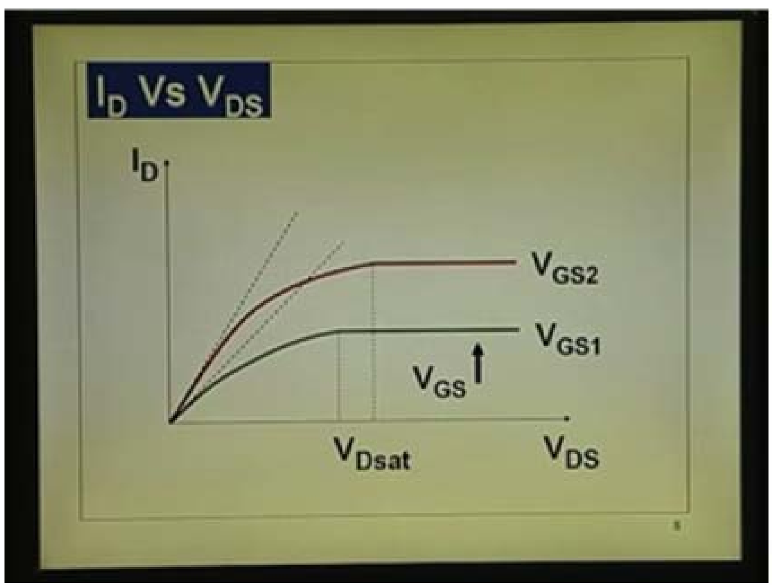

Now, we have plotted that linear, nonlinear saturation. Now V_{GS1}, V_{GS2}; two voltages are there. What is the voltage at which it saturates? That is what you are trying to find out

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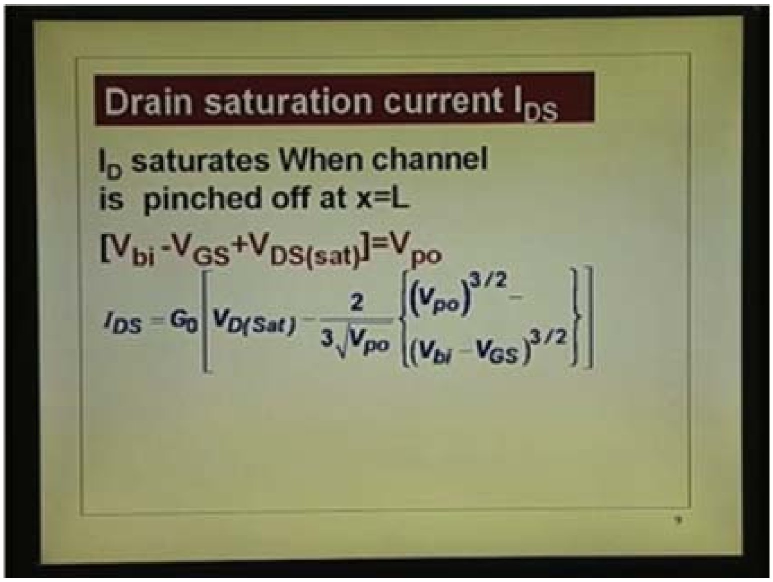

Drain saturation current I_{DS} that is what you have trying to find out is how much is that? How much is this? If we know what is $V_{DS(sat)}$, we know the current because in that equation that you have derived just now, there are only two unknowns: V_D and I_D. Once you fixed $V_{DS(sat)}$ by this condition that V_D saturates when the channel pinches off, we can find out $I_{DS\ (sat)}$. Now, what we have discussing so far we have put here by I_D saturates the channel is pinched off at x equal to L, drain end. We have reiterated it more than once. What is that voltage V_{p0} in terms of $V_{DS(sat)}$? At the source end, it is V_{bi} minus V_{GS}; at the drain end, it is $V_{DS(sat)}$ and at that some of the voltage should be equal to V_{p0} that is what you have said. This voltage must be V_{p0} and this voltage is equal to this voltage that is V_{bi} minus V_{GS} plus $V_{DS(sat)}$. Now, we are writing that equation, we are back into that equation. I think I better write that equation once on the board that it is all the time available to you, that is the general equation. I will just write it and keep it because, otherwise we have to go back and forth I_D is equal to G_0, this equation holds good up to the pinch of point. Once it pinches off, you find I_D and $V_{DS(sat)}$ that is it. Let me write down these formulae on the board so that we can just see substitute for these values. We know now that the current saturates when the channel is pinched off at the drain end.

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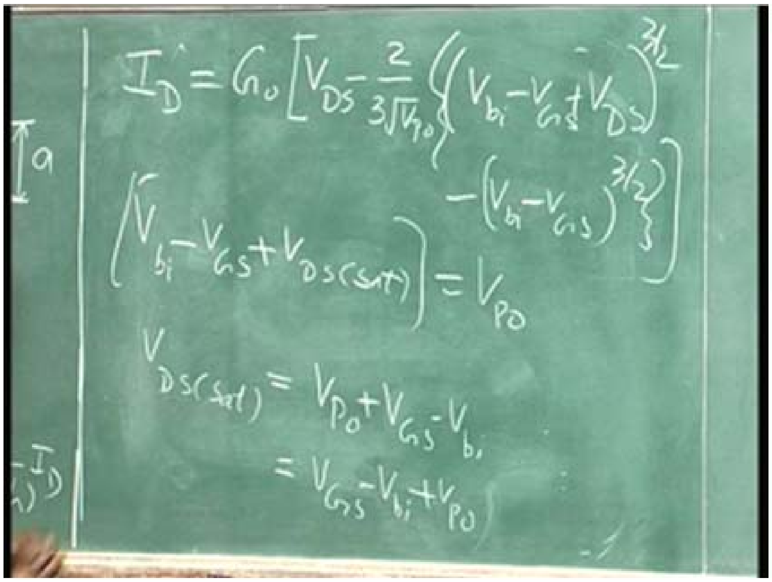

We will just write it for the time being keep it on, so that we can use it all the time. G_0 into V_{DS} minus 2 by 3 of root of V_{p0} into (Refer Slide Time: 38:37). So, what we say is, we want to find out the drain current when the drain voltage saturates, drain to source voltage saturates that is the voltage across the channel saturates. That happens when V_{bi} minus V_{GS} plus V_{DS} saturation voltage is equal to V_{p0}. All that we have to find out is, substitute for $V_{DS(sat)}$ equals V_{p0} plus V_{GS} minus V_{bi} or we can put V_{DS} equal to V_{GS} minus V_{bi} plus V_{p0}, I will leave it at that. What you want to see is how this equation gets modified? Ultimately, we want to correlate it to terms like threshold voltage that we will see. Now, let us go back to this equation. Here, we substitute (Refer Slide Time: 40:44) for this quantity V_{bi} minus V_{GS} plus V_{DS} and the V_{DS} is saturated that is V_{p0} that is what we are doing. We keep this as $V_{D(sat)}$, what we do is that now that is $V_{D(sat)}$ and this whole thing what was there (Refer Slide Time: 41:09) that is V_{bi} minus V_{GS} plus $V_{DS(sat)}$ is V_{p0} and this quantity. Now, what we do further is just manipulation of terms. I_{DS} equal to G_0 into once again we are substituting $V_{D(sat)}$, what we are doing is, we are substituting for $V_{D(sat)}$ there as (Refer Slide Time: 41:31) this quantity V_{p0} plus V_{GS} minus V_{bi} that is what we are doing there. That everything we want in terms of these quantities. Now, notice that we are able to eliminate $V_{D(sat)}$ from that, we are trying to eliminate that put it everything in terms of V_{bi}, V_{GS} and V_{p0}. This is the particular term that we get now. How do we do it further? This is saturation current and we want to simplify this.

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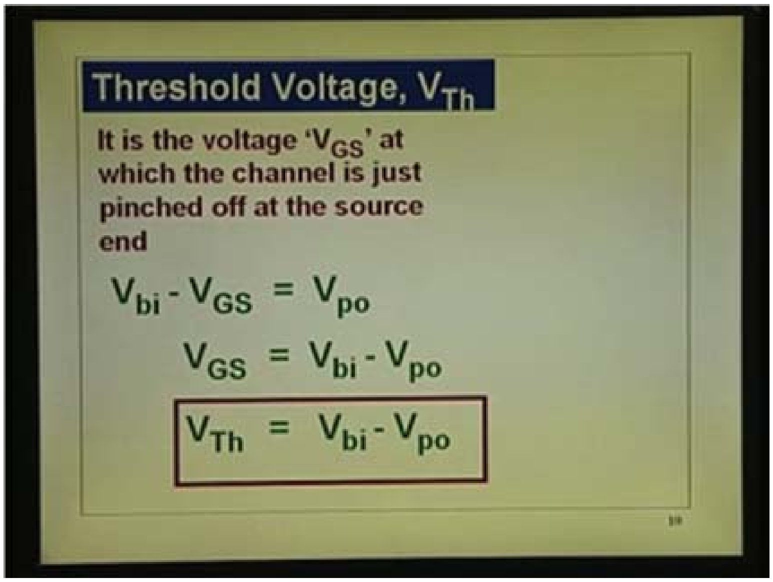

What we do is we use a term called threshold voltage. Threshold voltage is the V_{GS} at which the channel is just pinched off at the source end. Suppose, I apply voltage V_{GS} that is what we mentioned some time back. You can prevent the current flow that is just to prevent make the current equal to 0 or just allow the current just start flowing when this is closed here. This is closed here when the potential drop from the gate to this end is equal to V_{p0} and what is that quantity? V_{bi} minus V_{GS}, should be equal to V_{p0}. When this is V_{p0}, the voltage across, there should no drop in the y direction, x direction. What you have is, V_{bi} minus V_{GS} is equal to V_{p0} and that particular V_{GS} is actually a threshold voltage. Threshold voltage is a voltage that you must applied to the gate so that potential drop across the source at the source end is V_{p0}. Simple remember like this, total potential drop should be equal to V_{p0} and instead of writing V_{GS}, I have rewritten this equation as V_{GS} equal to V_{bi} minus V_{p0}. We put it that side and V_{p0} that side and this voltage is the threshold voltage that is V_{bi} minus V_{p0}. Now, you would appreciate why that term might put there $V_{D(sat)}$ equal to V_{GS} minus that quantity. Let us just go back to this and see.

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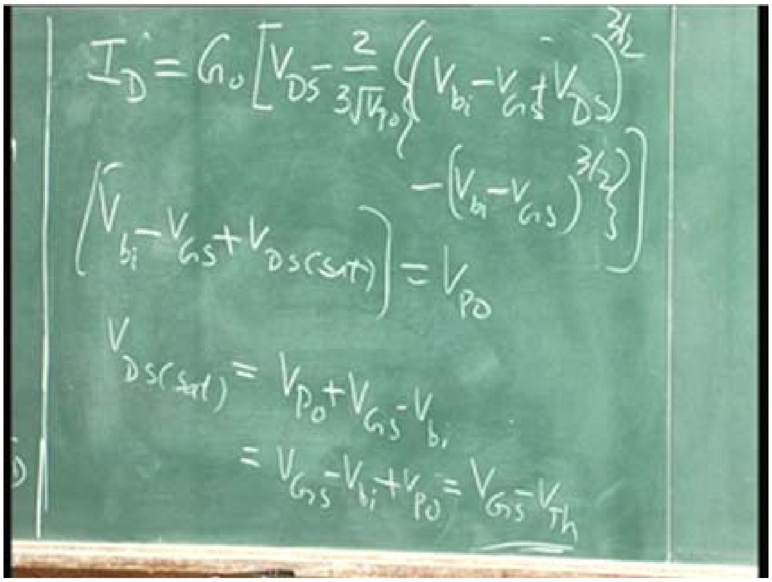

We have just now said, from there threshold voltage is V_{bi} minus V_{p0} which is the whole quantity is actually equal to V_{GS} minus $V_{threshold}$ that is something already we written there, that is equal to V_{GS} minus $V_{threshold}$. This is exactly the same as that of MOSFET. The MOSFET, the current saturates V_{DS} equal to V_{GS} minus $V_{threshold}$ and V_{GS} minus $V_{threshold}$ is a channel potential, potential drop across the channel. Here also, that is the potential across the channel. Saturation voltage $V_{D(sat)}$ is V_{GS} minus $V_{threshold}$. Once we do that, all that we do is substitute for these quantities. We have written there V_{DS} equal to this quantity, saturation voltage that if you see the previous equation that we written, we have put it as that, let us go back and show. All that we do is meddle with it, V_{p0} plus V_{GS} minus V_{bi}. The same thing we have written that is $V_{D(sat)}$ minus two thirds of (Refer Slide Time: 45:44) this quantity. What we have done is just rewritten those terms. Let me just see here, the first term is that (Refer Slide Time: 46:06) within bracket, you have got V_{p0} to the power 3 by 2 and this is that V_i that is what you have written here. We have just gone back to the threshold that you are not changed anything and then we did this quantity and then we are just substituting the same equation, the V_{p0} and this is minus this quantity.

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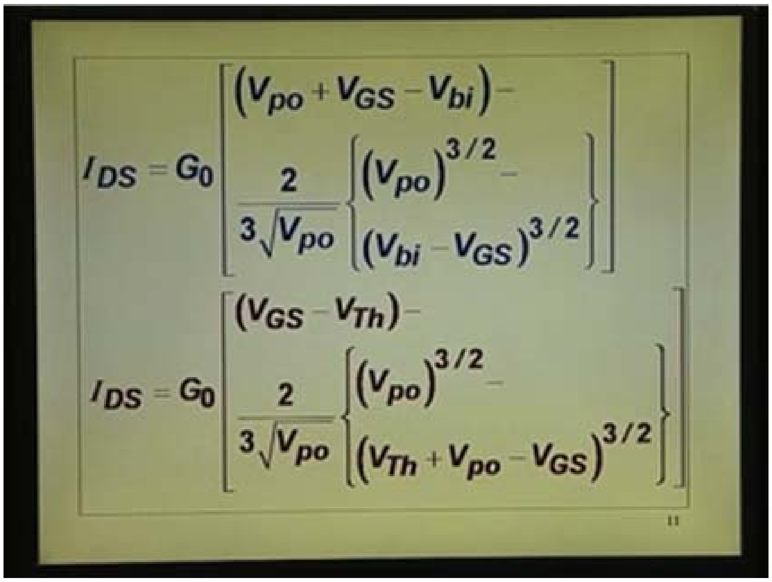

All that we have done is we have substituted in that equation that we have been writing so far that V_{bi} minus V_{p0} is $V_{threshold}$ that is what we have written here. Here, you can see V_{GS} minus $V_{threshold}$ that is V_{bi} minus V_{p0} that comes here, minus the same term V_{p0} to the power 3 by 2 and I do not want to put it as V_{bi}, we want put in terms of V_{GS} and $V_{threshold}$. That is why; we just rewrite for V_{bi} here $V_{threshold}$ for V_{p0}, V_{bi} minus V_{p0} is $V_{threshold}$. That is why; we are able to write V_{bi} is equal to $V_{threshold}$ plus V_{bi} that is all what we did. Now, we have V_{p0}, V_{GS} and $V_{threshold}$. Let us see how it changes. What we are doing is just, there is no more concept involved, all the concepts have been crystallized and all that we are doing is just manipulation of this equation to bring into the form that we usually see in the case of MOSFET. Rewriting this, again substitute for V_{p0} we get V_{GS} minus $V_{threshold}$ that quantity first term.

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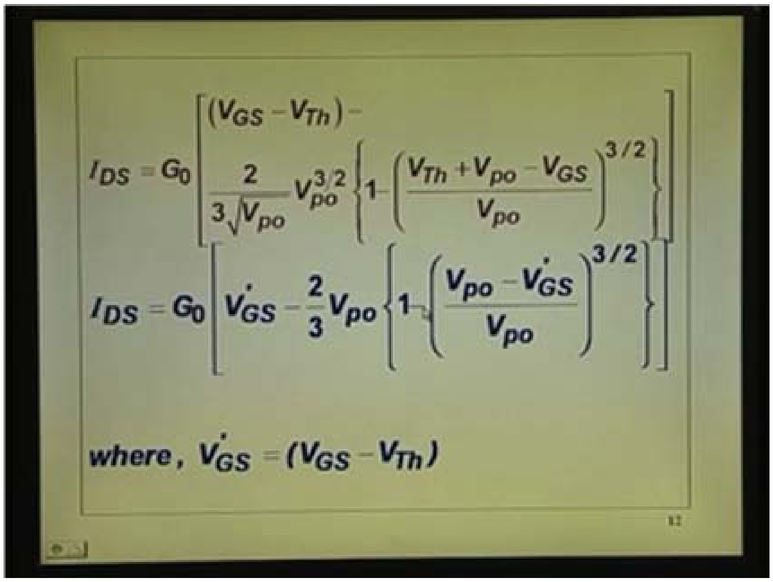

Second term is, it is in the same thing just we have to go back and forth once. First term is same second term 2 by 3 root of V_{p0}. What we do is, we pulled out and then it is become 2 by 3 root of V_{p0} into V_{p0} to the power 3 by 2 into 1 plus 1 minus whatever is there. Once again, we will go back and come back; I have pull this out so I will get 1 minus this whole thing divided by V_{p0}. Now, let us say how this is works out. You have got here two thirds of into V_{p0}. What we will do now is to instead of writing keeping on writing V_{GS} minus V_{Th}, we call it as V_{GS} dash. Then, we write V_{GS} dash that is V_{GS} minus V_{Th} and this term is V_{p0} to the power 3 by 2 by root of V_{p0} that is that. Inside this, now using the whole thing becomes smaller now equation. 1 minus V_{GS} dash is what? V_{GS} minus V_{Th}. So, minus V_{GS} dash is V_{GS} minus V_{Th} and V_{p0} (49:33) I have not made out changes all that have done is put V_{GS} dash is equal to V_{GS} minus V_{Th}. Now, we get this quantity, how do we simplify this? Slightly not pleasant equation to see it and to look at but we will see further. I have rewritten this equation G_0 into V_{GS} dash minus two thirds of V_{p0} plus minus two thirds of V_{p0} minus, minus- plus two thirds of V_{p0} into that quantity. That quantity putting it as 1 minus V_{GS} dash V_{p0}, simple manipulation and nothing much done; once more, we have look at it, this is pulled out we will have two thirds of V_{p0} into this quantity.

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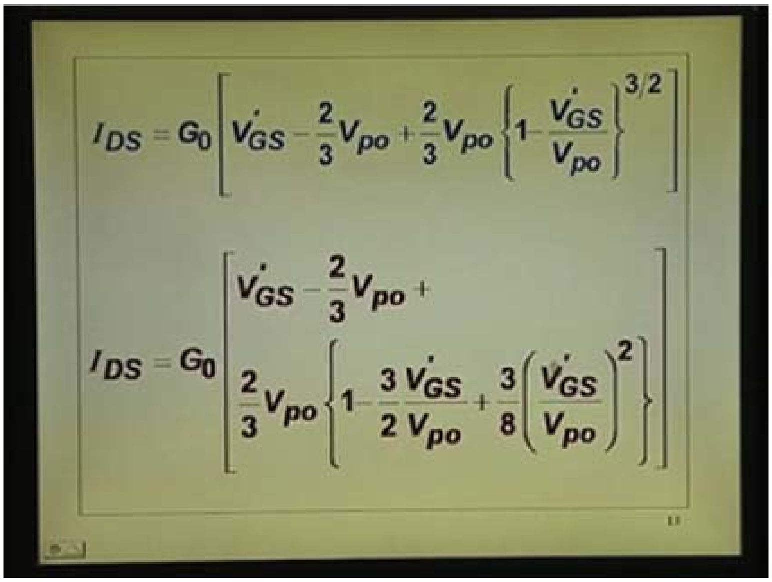

That we are taking into inside two third of V_{p0} into 1 minus V_{GS} dash by that. Now pulling out that V_{p0} from that side and now it looks more bearable this equation. In the sense, you can see that, you can simplify it under certain condition. When V_{GS} dash is small compared to V_{p0},, I can use binomial expansion for this. Now, if you use binomial expansion once only 1 minus x to the power of n is equal to 1 minus n of x. You just use that, you see if you get, we get 0. Everything cancels out. I use a second term also that is 1 minus n of x plus n x square by factorial 2. That we will do , V_{GS} dash minus two thirds have retaining plus 2 by 3 V_{p0} into 1 minus 3 by 2, 1 minus n of x, x is V_{GS} dash by V_{p0} plus square term 3 by 8. Now, you can see all other terms cancel out. V_{GS} dash cancelled, these two terms cancel out. You will see all these terms cancel out and you left with that quantity alone. That is actually, you have got ultimately last term, everything goes off. You will have this canceling with that; no nothing wrong, V_{GS} dash canceling with this term and this canceling with that one, you left with this term three, three goes off 2 by 8, 4 gives 1 by 4.

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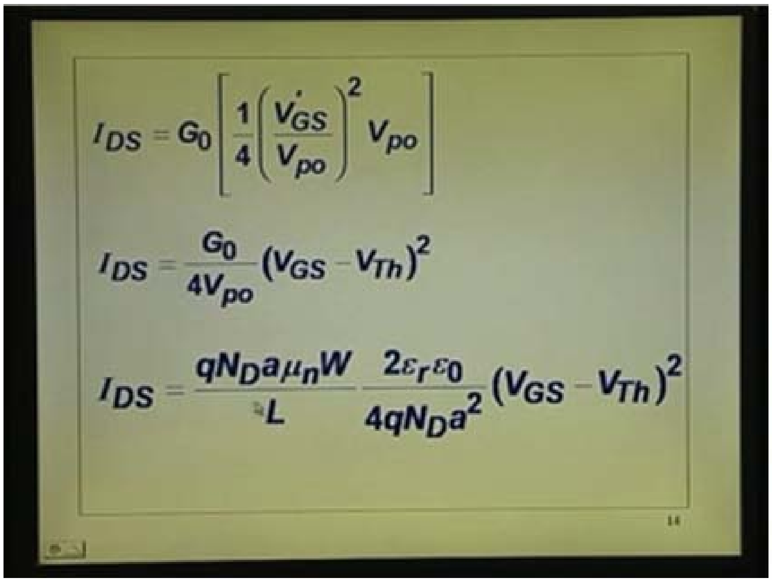

V_{p0} into G_0 into 1 by 4 V_{p0} into V_{GS} dash square term, now, see it is more pleasant thing to see that please remember this is valid when V_{GS} minus dash is small compared to V_{p0}, very frequently V_{p0} is not valid . Now, quickly finishing it off; I_{DS} is therefore expanding that G_0 by 4 V_{p0}. I have got V_{p0} pulled out from there at V_{p0}, 4 is pull out and V_{GS} minus V_{Th} whole square. Now, it is even more pleasant it looks like our familiar equation for the MOSFET what is this quantity? Expand that. G_0, we write it is a channel conductance which is actually q N_D a whole by sigma a by L, sigma is that quantity area is W into a by L, that is G_0. I have written this term and 4 is also there. V_{p0} is q and the denominator q N_D a square divided by twice epsilon$_r$ epsilon$_0$. It was here manipulation power that is all nothing more than that. Now, here you see q N_D cancel q N_D and 1 a will cancel with 1 a and 2 will cancel with 4 become 2. We are left with mu$_n$ W epsilon$_r$ epsilon$_0$ divided by a and 2 L.

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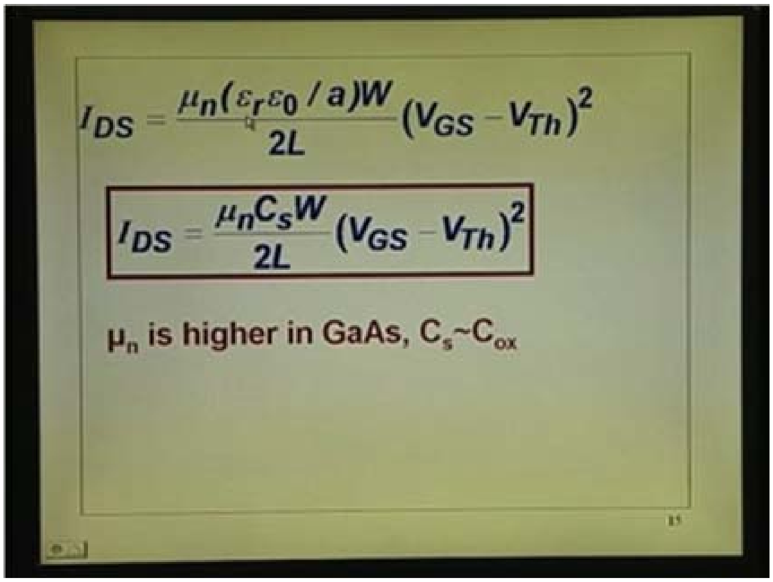

Canceling out there; we have that is mu_n into this quantity, I just look back, I will see that this is a same equation just canceling out all these terms, we are left with only that mu_n into $epsilon_r$ $epsilon_0$ by a into W by 2 L. What is this equation? The very familiar equation of the MOSFET mu_n C_S W by 2 L V_{GS} minus V_{Th} whole square MESFET is not different to the MOSFET. Only that you use the same equations to find the saturation current and all that you do is replace the C oxide per centimeter square per C_S, that is all. That makes the world of difference because now, we are talking gallium arsenide mu_n will be higher. You are talking of C_S instead of C oxide, C_S is $epsilon_r$. I think, we will continue on this starting from the equation. This is the good point for us to start in our discussion because now we can compare this MESFET to MOSFET, Gallium arsenide MESFET with the silicon MOSFET where do we stand all those things we can bring in by comparing these numbers. When you take the trans conductance these two will go. We will see that in the next lecture.