High Speed Devices and Circuits

Prof. K.N. Bhat

Department of Electrical Engineering

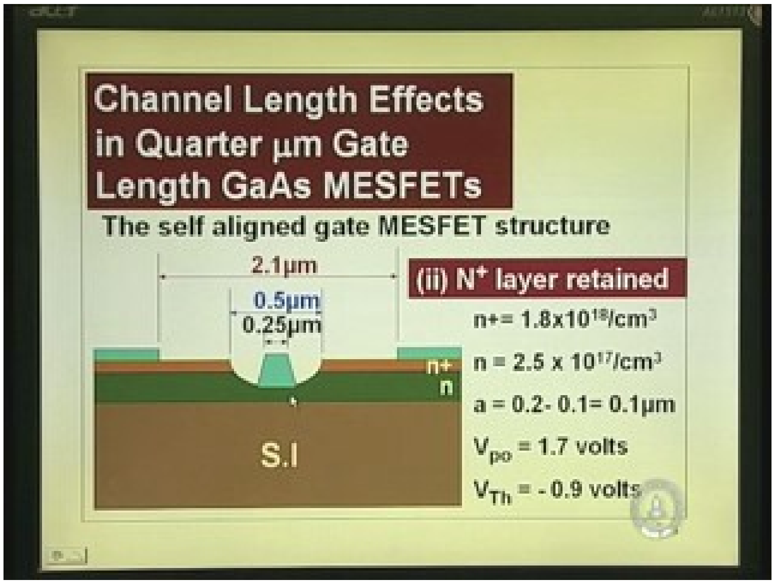
Indian Institute of Technology, Madras

Lecture – 26

MESFET: Effects of Velocity Saturation and Velocity Field Characteristics

We have been discussing the MESFET characteristics taking into account to the velocity saturation and we have analyzed it. Finally, we were comparing some of the experimental result that we have reported. Once the result that we saw was the reduction or the change in the gate length, there may be pinch of voltage changing unless you change the thickness.

(Refer Slide Time: 01:38)



Channel Length Effects in Quarter μm Gate Length GaAs MESFETs

The self aligned gate MESFET structure

(ii) N^+ layer retained

$n+= 1.8 \times 10^{18}/cm^3$

$n = 2.5 \times 10^{17}/cm^3$

$a = 0.2 - 0.1 = 0.1 \mu m$

$V_{po} = 1.7$ volts

$V_{Th} = -0.9$ volts

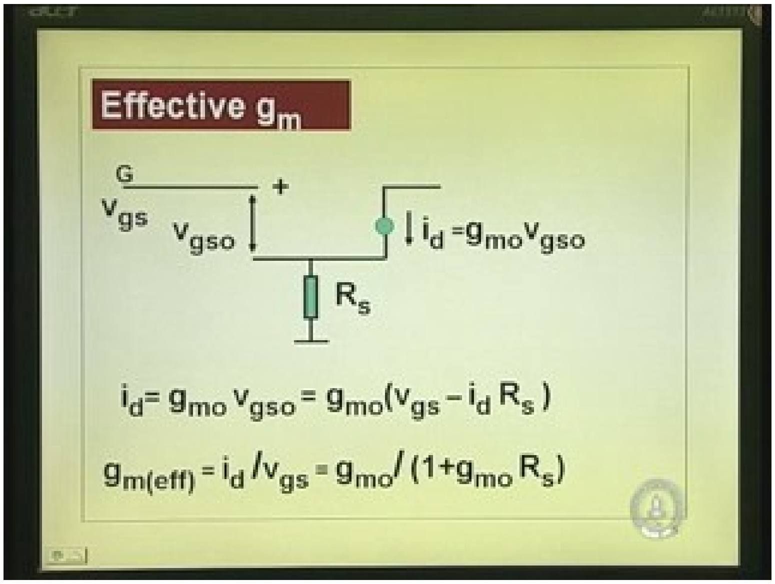
Then we took a look into the channel length effects. What we took was, we took effectively the result reported in the literature, we can say this is device 1 the n plus layer is not present here on the top. In a device two a layer is present on the top, so all the difference is this length, in this case this is the channel length gate length is 0.25 micron and channel length is 2.1 micron. All other things are remaining same you can just watch next one and previous one here that layer is absent. Now that layer is present so device two the N plus layer right up to this point. So, the channel length is actually 0.5 micron. It is as good as extending the metal right up to this point because this is heavy doped the metallic conduction. So, the difference is only in the channel length, gate length in both the cases is 0.25 microns.

(Refer Slide Time: 02:49)

Device No	Channel Structure	Gate Length µm	Channel Length µm	Observed g_m (eff) mA/V/mm
1	Without n^+	0.25	2.1	125
2	With n^+	0.25	0.5	215

Now, when they measure the results on the devices, the devices showed like this: device 1 does not have n plus layer on the top, I am just summarizing what we have listed in the previous lecture, gate length both same. Device 2 has n plus layer. So channel length in the gate one is 2.1 micron in the whole thing where n plus layer is present channel length is 0.5 micron. Now g_m delta I_D by delta I_{DS} by delta V_{GS} is higher in the second case where n plus layer is going all the way up to this point, this is the second device. If that is absent that is 125 but now the question is asked whether it is due to the, for example, if you have to remove that portion extra resistance is cog from this layer. Now, this resistance is not present whether that is playing a role in deciding this is making this difference.

(Refer Slide Time: 03:48)



Effective g_m

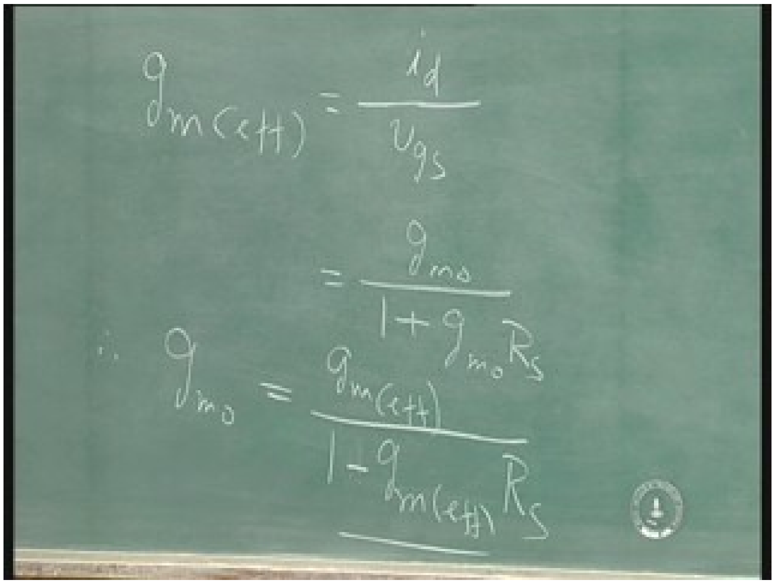
$$i_d = g_{mo} v_{gso} = g_{mo}(v_{gs} - i_d R_s)$$

$$g_{m(eff)} = i_d / v_{gs} = g_{mo} / (1 + g_{mo} R_s)$$

To do that let us see how it will effect. I did not discuss in the previous lecture the effect of that resistance whatever is present at the source. Source region between the source conduct and the gate there is a resistance. The two resistances are different in the two cases because when the n plus layer is present definitely that R_S is smaller when n plus layer is absent R_S is large whether that has made the difference that is how would make a difference. So, the definition of transconductance i_d is an equivalent circuit gate nothing there, at low frequency it is a capacitance its open when apply voltage that gives raise current i_d equal to g_{m0} which is the intrinsic transconductance which actually is a transconductance that you get if R_S is equal to 0. So, if R_S is equal to 0 i_d will be g_{m0} times the voltage between the gate and source that is V_{gs0}. Now that is what I have put here but, when the resistance is present the V_{gs0} is not equal to the applied voltage V_{gs} because V_{gs} between these two is shared between the V_{gs0} and R_S the voltage gets divided between the gate, the intrinsic source region and the resistance because this i_d flows through that; from the drain to the source through the resistance. So, that is why V_{gs0} is V_{gs} total voltage minus i_d times R_S. So, minus this term that is what is put here. Now, actually you what you do is you make rearrangements and i_d will be actually equal to i_d into 1 plus g_{m0} into R_S equal to this quantity g_m times V_{gs}.

Transconductance is actually in the real device is i_d divided by the applied voltage this is the small signal voltage not DC incremental quantities i_d divided by V_{gs}. So, V_{gs} is actually given by sum of these two so you get this quantity. So, what we are telling is g_m effective is g_{m0} intrinsic g_m divided by 1 plus g_{m0} into R_S.

(Refer Slide Time: 06:09)



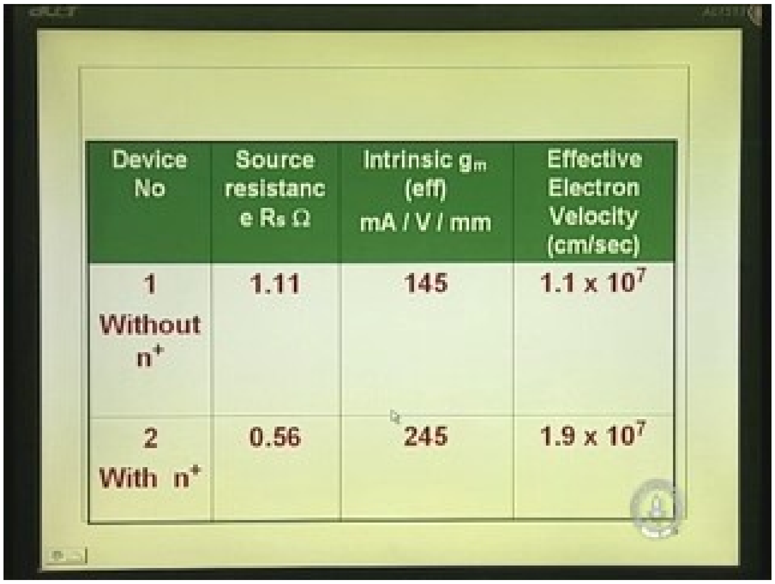
$$g_{m(eff)} = \frac{i_d}{v_{gs}}$$

$$= \frac{g_{m0}}{1 + g_{m0} R_S}$$

$$\therefore g_{m0} = \frac{g_{m(eff)}}{1 - g_{m(eff)} R_S}$$

That is effective g_m equals i_d drain current divided by i_d divided by 1 plus i_d divided by V_{gs} which actually is equal to g_{m0} divided by I am rewriting that 1 plus g_{m0} times R_S. What you measure is for this quantity, what you want to take a look at is the intrinsic g_{m0}. So, g_{m0} from this equation you can rearrange g_{m0} will be equal to $g_{m\ effective}$ divided by just rearranging this 1 plus $g_{m\ effective}$ 1 minus into R_S. I am writing in this form because $g_{m\ effective}$ is one that you measured delta i_d by delta V_{gs}. So, rearranging you get this equation. Once you know $g_{m\ effective}$ and once you know R_S which is another quantity you can measure all measure put together you can get the intrinsic g_m. Now, just this is the approach going back to that diagram. R_S can be measured by applying a voltage between the drain and the source and by measuring the voltage drop between this and this point. It is better seen in the previous diagram. See apply voltage between the drain and the source small voltage; we do not want velocity saturation you want the ohms law hold good. So apply voltage small current flows through that measure the voltage drop between the source and the gate. The whole thing is dropped here to the current flowing. So, you measure the voltage drop and the total current, drain current. Total voltage drop between these two divided by current gives the resistance between these two. That is the way they measured it and then measure the $g_{m\ effective}$ also use that equation g_{m0} is $g_{m\ effective}$ divide by 1 minus $g_{m\ effective}$ into R_S from this equivalent circuit and get the numbers.

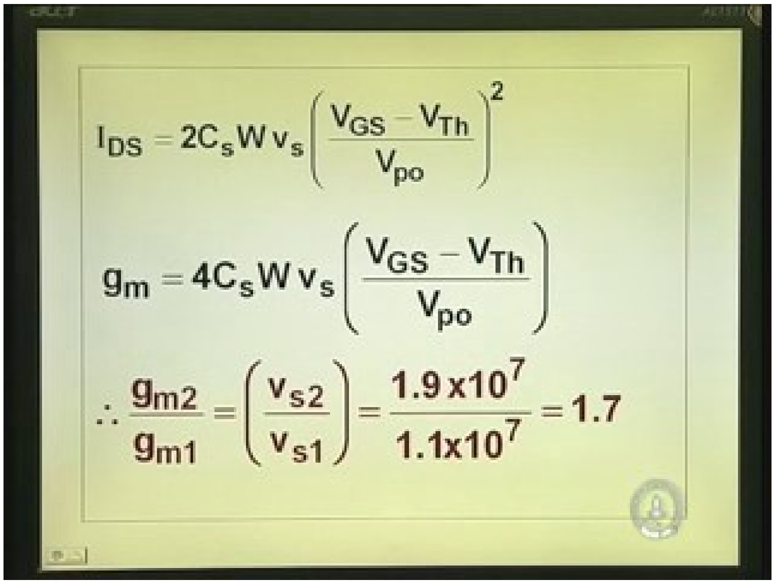
(Refer Slide Time: 09:13)



Device No	Source resistance R_s Ω	Intrinsic g_m (eff) mA / V / mm	Effective Electron Velocity (cm/sec)
1 Without n^+	1.11	145	1.1×10^7
2 With n^+	0.56	245	1.9×10^7

In the last lecture I just quickly went through this I will go through it again. Now, you can see without n plus corrected g_m g_{m0} is 145 milli amperes per volt per milli meter width. With n plus it is 245 what we are telling is the extrinsic g_m or effective g_m also higher also the intrinsic g_m is higher when n plus layer is present. So, it is not due to series resistance it is due to the property of the device at that particular point.

(Refer Slide Time: 10:00)



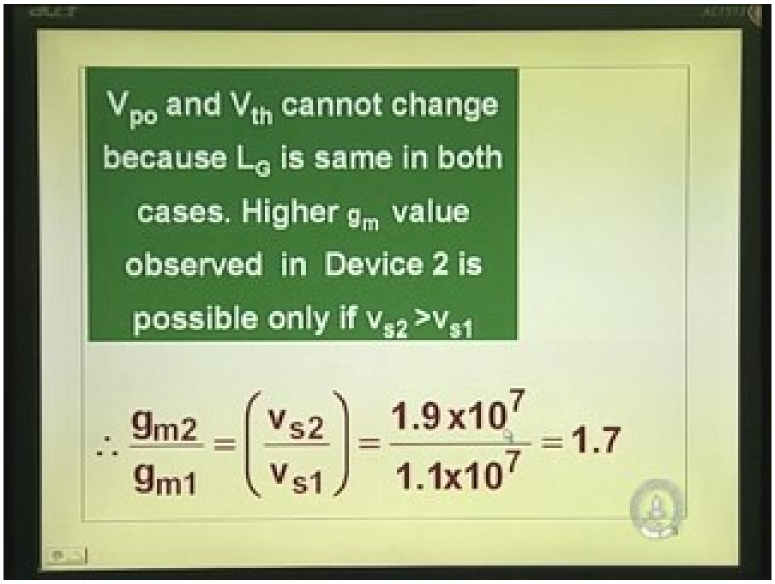
$$I_{DS} = 2C_s W v_s \left(\frac{V_{GS} - V_{Th}}{V_{po}} \right)^2$$

$$g_m = 4C_s W v_s \left(\frac{V_{GS} - V_{Th}}{V_{po}} \right)$$

$$\therefore \frac{g_{m2}}{g_{m1}} = \left(\frac{v_{s2}}{v_{s1}} \right) = \frac{1.9 \times 10^7}{1.1 \times 10^7} = 1.7$$

What are these quantities? Now, these quantities are given by this equation. This I_{DS} is given like that g_m is given by this. If I take the g_m in the two cases, the intrinsic g_m, C_S is the same because gate is the same thing; W is the same; V_{gs} is same 0 or whatever apply you apply $V_{threshold}$ voltage they cannot be changed because V_{bi} minus V_{p0} is $V_{threshold}$ voltage. V_{p0} will be affected only if the gate length if you change but gate length is same 0.25 micron. That will be the same so everything is same. Only possibility is this may be different, the two devices, we will see afterwards whether it is true. That is g_{m2} by g_{m1} is v_{s2} by v_{s1} so that you can actually compute v_s by putting all these values. When you substitute all these values you get 1.19 into 10 to the power 7 for second device. First device you get 1.1 into 10 to the power 7 so, ratio is 1.7. What you are telling is g_{m2} in the second case is higher than g_{m1} possibly because the velocity effective saturation velocity is more when you brought the n plus layer closed to the gate.

(Refer Slide Time: 11:20)

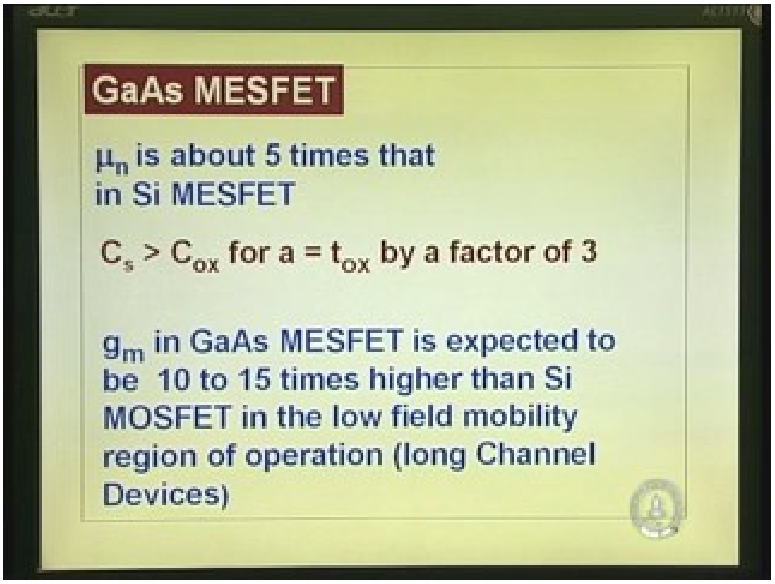


V_{po} and V_{th} cannot change because L_G is same in both cases. Higher g_m value observed in Device 2 is possible only if $v_{s2} > v_{s1}$

$$\therefore \frac{g_{m2}}{g_{m1}} = \left(\frac{v_{s2}}{v_{s1}}\right) = \frac{1.9 \times 10^7}{1.1 \times 10^7} = 1.7$$

Now, this is what we have summed up since V_{p0} $V_{threshold}$ voltage cannot change because L_G is the same in both cases, only change that you can see is the velocities. We have just now seen it is 1.7. Our next job is can this velocity be different and what could be the reason. We will go back a little bit into physics and see what models people have proposed for that.

(Refer Slide Time: 11:50)



GaAs MESFET

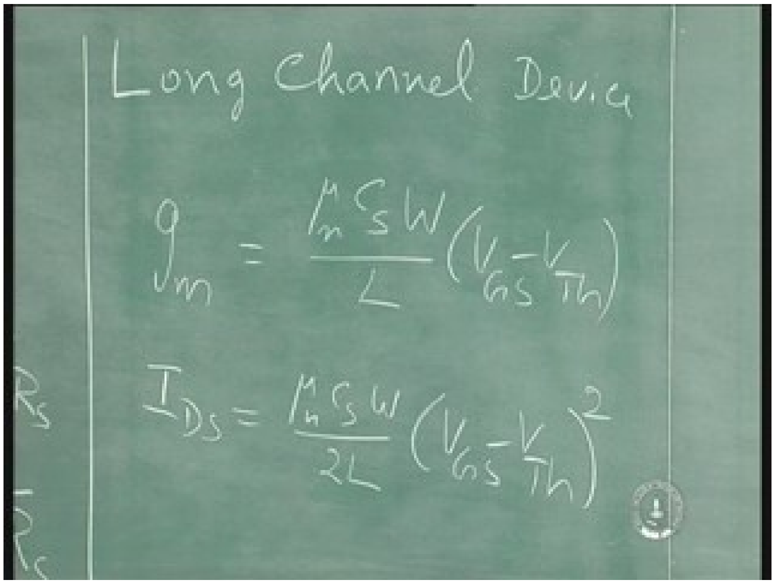
μ_n is about 5 times that in Si MESFET

$C_s > C_{ox}$ for $a = t_{ox}$ by a factor of 3

g_m in GaAs MESFET is expected to be 10 to 15 times higher than Si MOSFET in the low field mobility region of operation (long Channel Devices)

Before, we go into those models few things we have to see, just looking back whatever thing that you see n the MESFET. If you operate a linear region of if you operate in a long channel device there is no velocity saturation, it is only the square law with multiply factor mu$_n$ C$_S$ W by 2 L, mu$_n$ is the one which governs the drain current and the trans-conductance in those situations even it is four or five times larger than Gallium Arsenide than Silicon. What we are trying to compare is, is there any point in switching over to Gallium Arsenide, you get really the benefit and if you want to get the benefit what is to do. When the channel length is long no doubt you get the benefit because this is the situation.

(Refer Slide Time: 12:53)



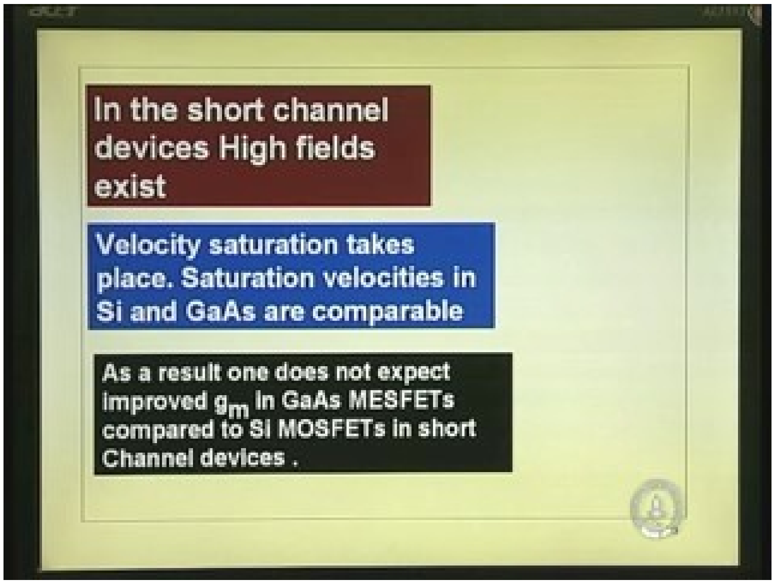
Long Channel Device

$$g_m = \frac{\mu_n C_S W}{L}(V_{GS} - V_{Th})$$

$$I_{DS} = \frac{\mu_n C_S W}{2L}(V_{GS} - V_{Th})^2$$

When the channel length is long whole thing is proportional to let me just write that down here because I do not have to write now there is long channels device, g$_m$ mu$_n$ C$_S$ W divide by L into V$_{GS}$ minus V$_{threshold}$ because I$_{DS}$ is equal to mu$_n$ C$_S$ W divided by 2 L into V$_{GS}$ minus V$_{threshold}$ whole square. Now, you can see if the channel length is long if you have a device or if you have a material whose mobility is higher five times than that of Silicon, you will definitely get g$_m$ five times. Added to that C$_s$ which actually depends upon epsilon$_s$ into epsilon$_0$ by a. If the thickness of the channel is same as that of oxide in MOS this will be larger than the factor of about three because epsilon$_s$ relative permittivity in Gallium Arsenide is 12.9 and relative permittivity in oxide is 4. About a factor of three this is factor of three higher command compare to C oxide in Silicon, this is factor of five compare to mobility in Silicon. About 10 or 15 times definitely, you get improvement in the trans-conductance. We are not worried about current so much but for trans-conductance because that gives how much change in current for change in voltage that gives the driving capability for the device. So, g_m is the most important thing whether you talk of bipolar MOS MESFET whatever it is comparison should be there. In a long channel device there is no problem, who wants long channel devices these days. You want to have when you go for VLSI. You compress make the devices smaller and smaller. You shrink the device size and increase the back end density, so you cannot sell a Gallium Arsenide MESFET better than Silicon MOSFET for a long channel device.

(Refer Slide Time: 15:26)



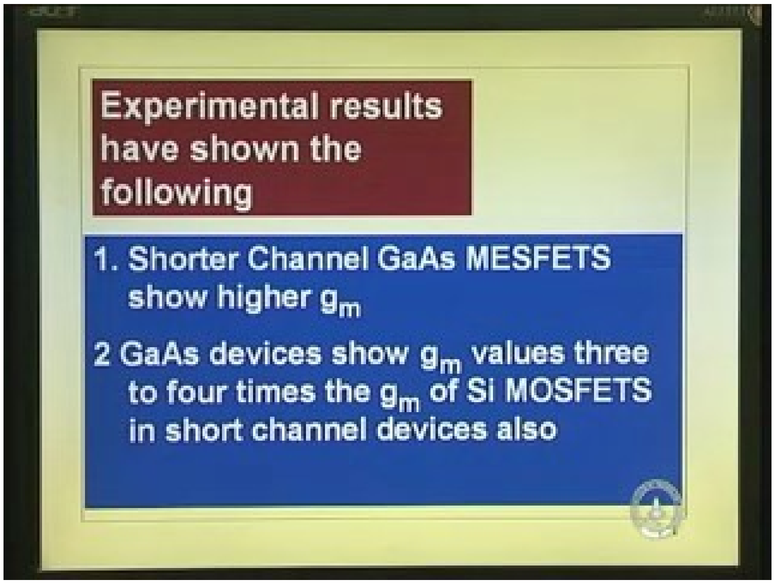
In the short channel devices High fields exist

Velocity saturation takes place. Saturation velocities in Si and GaAs are comparable

As a result one does not expect improved g_m in GaAs MESFETs compared to Si MOSFETs in short Channel devices .

The short channel devices high fields exist. For a given voltage drop across the channel the channel length is small fields are high. The moment fields are high you have the velocity saturation effects. It will come both in Silicon and Gallium arsenide it is not such preparatory Gallium Arsenide that field has velocity saturation, if you take a Silicon channel length is 1 micron or smaller very easily you can get velocity saturation effects. Now if the velocity is saturated qn into v_s is a current density. Velocity is v_s is same for Silicon Gallium Arsenide you would not get the benefit because currents are same trans-conductance will be the same thing, that is what we said. As a result, one does not expect improved g_m in the Gallium Arsenide MESFET compare to Silicon MOSFETS when these channel lengths are small. That is what is predicted or atleast believe by people who adore Silicon. In fact, all of us adore Silicon that as the lifeline of VLSI today but, if you want ultra high performance then only we look into other devices.

(Refer Slide Time: 16:49)

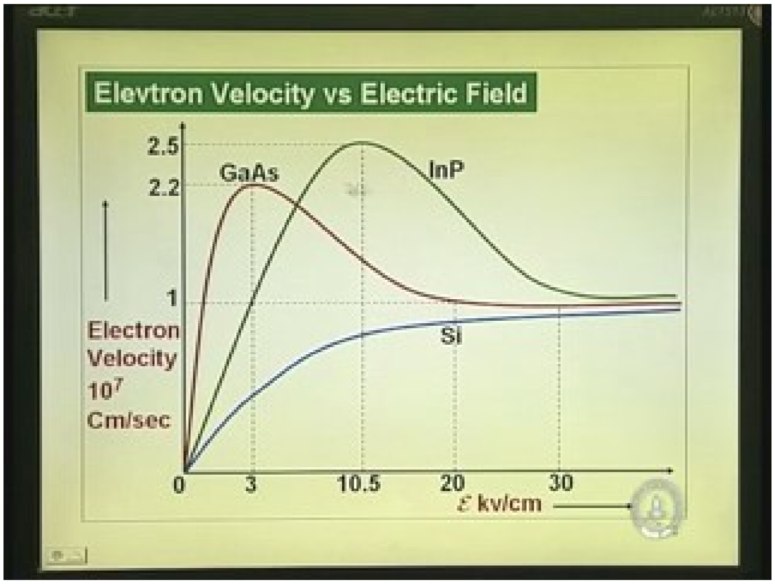


Experimental results have shown the following

1. Shorter Channel GaAs MESFETS show higher g_m
2 GaAs devices show g_m values three to four times the g_m of Si MOSFETS in short channel devices also

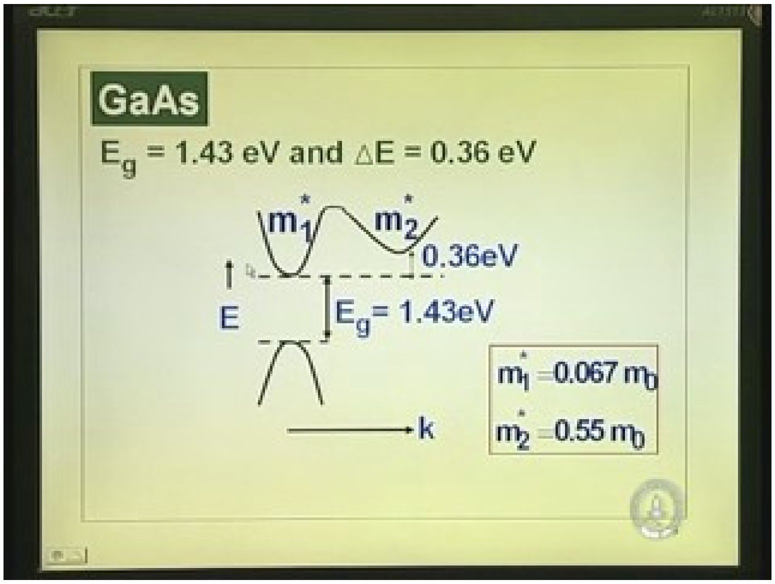
Let us see them. Experimental results however have showed the following: short channel Gallium Arsenide MESFETs show higher g_m just now we saw, a channel length is reduced by putting n plus layer you get g_m 1.7 times that for a particular case. Gallium Arsenide devices show g_m values this of course you take it. They made Gallium Arsenide devices short channel they made Silicon MOSFETs compared then said g_m is higher in the case of Gallium Arsenide three to four times compare to that of Silicon. In spite of velocity saturation taking place, if you have to understand that you are taking look into the velocity field characteristics.

(Refer Slide Time: 17:34)



We take look at the electron velocity versus electric field in Silicon and Gallium Arsenide and of course I put also Indium Phosphate let us not take look at it right now. We will take look at this curve Indium Phosphate curve afterwards. We will take look at the Silicon velocity of electron versus electric field in kilo volts per centimeter. Pipe turn about 30 kilo volts per centimeter is Silicon saturates. In fact what happens is the electrons remain much closed to the conduction band itself. They do not get transfer to any other region in the conduction band. We will see and understand what I am talking of very shortly when talking of Gallium Arsenide. The whole thing is because Silicon is an indirect band semiconductor and we will discuss that afterwards. As we keep on increasing the field the velocity keeps on increasing ultimately it saturated and saturation velocity is scattering limited velocity. These are actually drift velocities. After all there is the maximum velocity is thermal scattering random movement which is there, all the time the electrons are moving around. There is no velocity of an electron net velocity is 0 because hundreds or thousands of electrons are moving this way that way each one cancelled net velocity is 0. But, when you apply a field there is a directed movement n_1 particular direction and that is the net velocity in a particular direction. That is super imposed velocity over than the thermal velocity. So, maximum you can get is corresponding to thermal energy thermal velocity that is why scattering limited velocity. Now, Gallium Arsenide if it goes like this up then comes down ultimately going to the scattering limited velocity why should it go up like that. Now, you can see (Refer Slide Time: 19:32) this actually gives a scope. Supposing this is not people thought initially velocity characteristics if it is here if I am operating in region of this portion I have got velocity higher than this. What will never happen is it is very difficult to focus on to this when you go to high field, we will be there. Let us see what is happening here by understanding that, we can go back and see why you got when you made that n plus region present, why you got better velocities, better trans-conductance. Let us take look at this.

(Refer Slide Time: 20:07)



GaAs

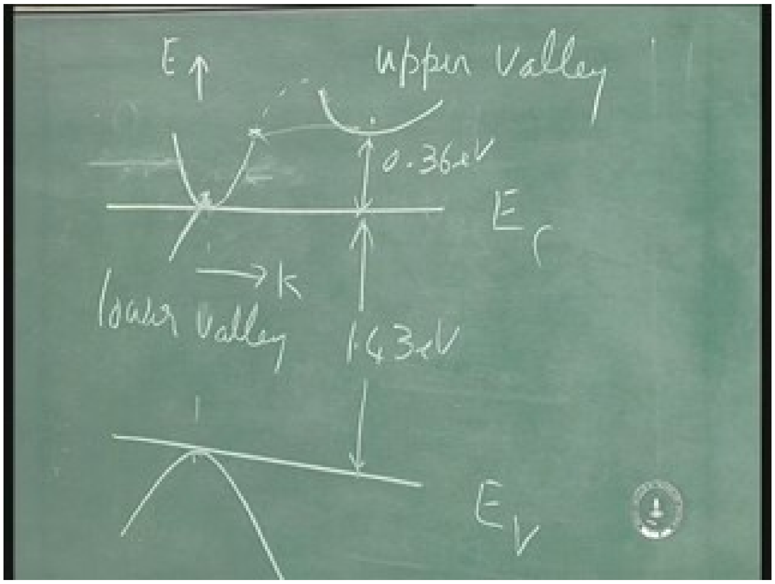
E_g = 1.43 eV and ΔE = 0.36 eV

$m_1^* = 0.067\ m_0$

$m_2^* = 0.55\ m_0$

Now, we are looking at some results which you borrow from physics. This diagram has projected earlier this region is the bottom of conduction band and this diagram here what you draw here E is the energy verse momentum energy upwards momentum k on the X axis. The energy versus momentum that if you plot in the conduction bands it is like this for Gallium Arsenide. Now, the valance band energy versus momentum is like this, I have discussed this and also the direct band gap and indirect band gap material, difference between the Gallium Arsenide and Silicon. Now, as a result this region is the bottom of the conduction band. I just want to make it clear by drawing the diagram.

(Refer Slide Time: 21:24)



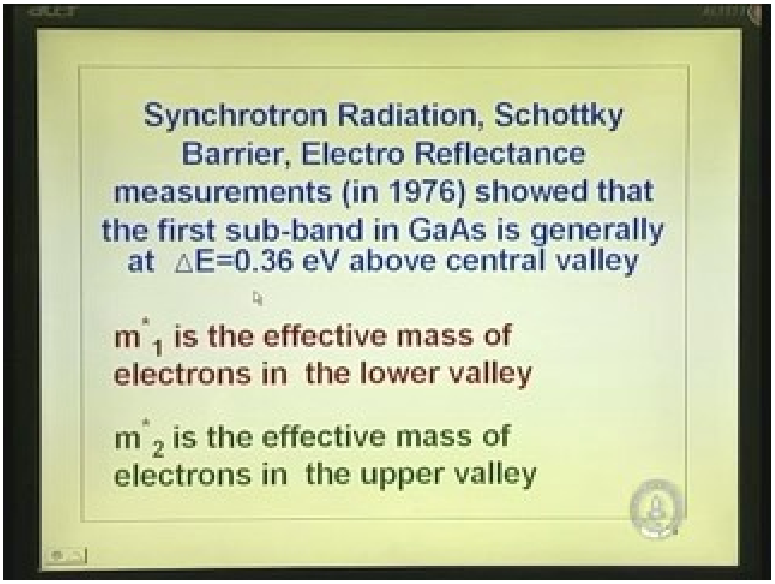
What we are talking of is E_C versus E_V we plot. We are drawing those diagrams there one is corresponding to this region. This is your distance but, if you plot for energy versus momentum this is the one value one distribution energy versus momentum. That is E versus k if I plot and then there is one more with goes like this, this is flatter and here directly below that we have got the valance band. If you have energy versus momentum in diagram like this, the implication is the electrons will move it will stay here that is the bottom of conduction band. So, electrons always occupy this position and holes will occupy this position. Now, if you give energy to this electron by applying field they acquire energy to move up. They move up. In fact, this is the continuous path this field has not shown. People are happy to see this edge and that edge this is called the upper valley, this is called valley because it looks like a valley no other reason. This is the central valley or the lower valley this is the lower valley, upper valley, lower valley. Now, the difference between these two valleys is- this is sharper and the sharpness actually is reflected in the effective mass that you have just borrowed and believe on that. This is flatter or if you do not want to think that that is why I have drawn this flatter more precisely the effective mass of electrons here is smaller compare to that. So, if the electron is in this region effective mass is smaller by some chance if the electron is here the effective mass is higher. So, if the electron is transferred from here to here suddenly it finds itself heavy. If it was here itself or if it had gone up here then, half of mu square M is the effective mass, if the mass is smaller velocity will be higher. If it is transferred on to this valley, mass is higher, velocity will be lower. Velocity is lower here because mass is higher half mu square. As long as it is here in this valley its velocity is higher. So, what happens is as you subject that electron to the electric field its velocity keeps on increasing because it gains the energy. Once it gains energy enough up to this (Refer Slide Time: 24:36) point it crosses on to this value because it does not go up here because electrons always has got the tendency to occupy the lowest energy level permissible. So, it is not moving up there from here it is scattered on to that because that is lower energy, it cannot go up there goes on to this. In other words it gains enough momentum, this is momentum difference, gains enough momentum it finds in that portion. As you keep on increasing the energy by increasing the electric field velocity keeps on increasing, I have talked about one electron getting transferred there that electron find itself having a lower velocity. All the hundred thousand electrons will not get transferred on to that simultaneously. This is sort of probability. So, many of them may get transferred as we keep on increasing energy. Finally, if all these electrons have transferred on to that the effective velocity of those would be smaller and the mobility here is almost closer to Silicon. The effective mass is closer to that of Silicon.

Once this is transferred here if increase further (Refer Slide Time: 25:53) the electric field it behaves like in the Silicon in the saturation velocity occurs here. The entire philosophy here, point that we are trying to point out is the transfer from here to here is impossible because this energy is 0.36 electron volts in Gallium Arsenide. That energy is that much high there is no chance for electrons to be transferred on to that. You would have got one velocity corresponding to this region because this is 0.36 and this actually is 1.43 electron volts. Once you understand this we can trip through an elevation of whole thing very easily that is 1.43. If this were 1.43 that gap between this upper valley and lower valley you would have break down means ultimately whole of them are leveraged. We would have interaction between these two because this is much smaller than that you have got this electrons transform on to that velocity drops that is the phenomena. In Silicon the difference has increased the field velocity goes on increasing because it absorbs energy but, as it is absorb energy when beyond effect of energy the electron gets transferred on to this upper valley. In fact what we can say physically is the mode of it transport slightly changes. The way it moves is slightly different. Physically, you can just visualize it this way.

I do not see it written in black and white anywhere but my own thinking in that is (Refer Slide Time: 27:56) hence between Silicon and Gallium Arsenide the difference is Silicon the electron goes through potentials due to the atoms. There is a potential variation because of their atoms as it goes to nucleus between the nuclei it goes through the potential peeks and down. That is the only thing that we have seen there, the electron which is moving that is the effective mass due to the potential changes. If there is no potential change if there is no field within the atoms the mass would have been the mass of free electron. The effective mass comes up because of their potential change due to the charge in the nucleus. In Gallium Arsenide there is additional field comes up due to net charge difference between the Gallium and Arsenide. There are between the two atoms there are additional forces coming up because of that you have the effective mass smaller that is physical concept. In fact energy band diagram this is sharper there. See the effective mass itself comes up because of additional forces which are present between the atoms in the Silicon in the crystal. Here you have got extra forces which are coming up because the charge difference between the Gallium and Arsenic atoms. That is why effective mass much smaller here compare to that Silicon that is physical understanding. Now you give enough energy that extra force that is experienced by the electric field due to the plus and minus charge between Gallium and Arsenic. It is very small source that is not any more effective it looks as if it is similar to that Silicon this is very over simplified thinking which sounds quite good from physical concentration point of view.

In terms of energy band diagram it is in fact you want to throw further jargons electrons get scattered from this value to that value why it is scattered is its mobility falls. Whenever mobility is changing we say it is scattered. This is called Inter value scattering. This is the jargon which was put up. All that happens is the mode of transport is slightly different because the energy acquired is so much that the extra field that comes up because of the plus charge in Gallium and minus charge in Arsenic that field is not no more effective because it is so much high here. If there is a plus charge the electron can tends to move in the direction like this but if the velocity is high it will just not see that effect of that. That is what a simple minded thinking on that.

(Refer Slide Time: 30:48)



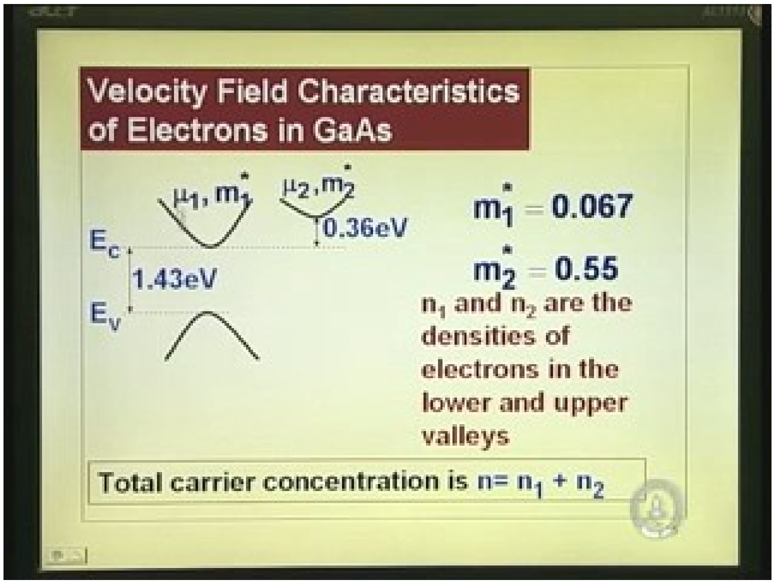
Synchrotron Radiation, Schottky Barrier, Electro Reflectance measurements (in 1976) showed that the first sub-band in GaAs is generally at $\Delta E = 0.36$ eV above central valley

m^*_1 is the effective mass of electrons in the lower valley

m^*_2 is the effective mass of electrons in the upper valley

Now, take a look at this particular analysis. Quickly run through this analysis. Now, this is all just I just summed up and put: Synchrotron radiation, Schottky barrier, Electro reflectance all those measurements have shown that delta E that is the gap between the upper valley and this lower valley is 0.36 electron volts because central valley or lower valley. That is the quantity we estimated upper valley lower valley or you can call central valley because it is not one valley that is present here central valley is there. If you go to three-dimensional, it can have four valleys like that all the round. Electrons can go transfer to that or transfer to that like that. It is a three-dimensional in the (31:32) zone if you take. So, m_1 star is the effective mass of electrons in the lower valley m_2 star is the effective mass f electrons in the upper valley. That is upper valley m_2 star m_1 star lower valley central valley.

(Refer Slide Time: 31:56)



Velocity Field Characteristics of Electrons in GaAs

$m_1^* = 0.067$

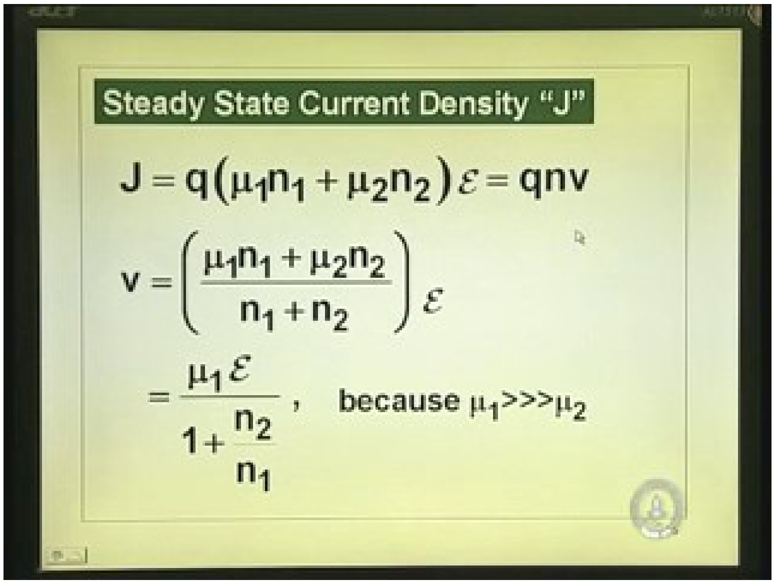
$m_2^* = 0.55$

n_1 and n_2 are the densities of electrons in the lower and upper valleys

Total carrier concentration is $n = n_1 + n_2$

I have put the diagram again so that if you missed something you can see here 1.43 is the band gap 0.36 is the delta E, m_1 is the effective mass here, m_2 is the effective mass upper valley. Correspondingly, mobility is here mu_1 mobility here is mu_2. This is about 8500 this is closed to that of Silicon here 1500 of that order. Now, total carrier concentration is electrons have remained here or here between these two locations that you provide for the electron. If you do not give an extra energy by applied electric field they will occupy here. They will be occupying lower valley or central valley, n is the total number of electrons and it is doping concentration and all of them will be here at 0 electric field. As you keep on increasing electric field some of them may get transferred to that. If you call it is n_2, n_1 plus n_2, n_1 here n_2 here sum of them will be equal to total. Here after if you understood the phenomena I can just go through three four slides very quickly. Now, m_1 star is 0.067 m_2 star is 0.55 this is closed to that of Silicon. That is why I said when you transfer here the implication is the extra field because of the net charge in Gallium and Arsenic that effect is not affecting the transport of carriers that is the meaning of that, n_1 and n_2 are the density of electrons in the lower and upper valleys.

(Refer Slide Time: 33:43)



Steady State Current Density "J"

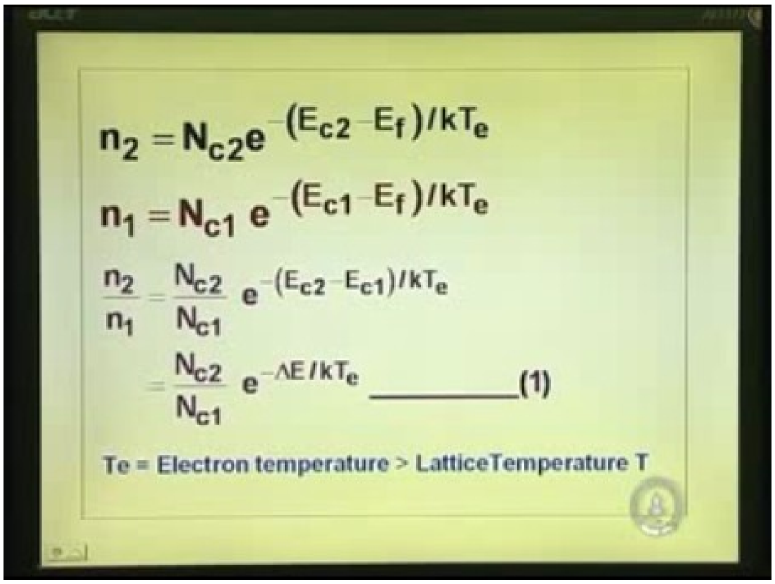
$$J = q\left(\mu_1 n_1 + \mu_2 n_2\right)\mathcal{E} = qnv$$

$$v = \left(\frac{\mu_1 n_1 + \mu_2 n_2}{n_1 + n_2}\right)\mathcal{E}$$

$$= \frac{\mu_1 \mathcal{E}}{1 + \frac{n_2}{n_1}}, \quad \text{because } \mu_1 >>> \mu_2$$

Let us go further. Here after wards what you do is how we find the velocity; velocity is related to the current. If there are n_1 electrons in lower valley and mobility is mu$_1$ and E is the electric field, total current due to the electron is actually J, current density q into mobility into carrier concentration in the lower valley into the electric field plus mobility in upper valley into number of electron in the upper valley into electric field. As far as outside world is concerned, you do not care whether the electron is here or there. What you see that is the current and net is what you see is electron concentration n and n is the n_1 plus n_2. We can still write as far as outside world is concerned, what to measure is the velocity. When you measured when you got velocity versus field it is that what you have seen here. The total current can be put as current density q into total electron concentration into the total velocity effective velocity combined velocity. Here, we have put this is the velocity all of them are moving in the same velocity there is a velocity. Now, what we do is therefore velocity is equal to J which is this quantity divided by qn, so q q get cancelled I get velocity is equal to mu$_1$ n_1 plus mu$_2$ n_2 into E, n is outside divided by n. Electric field is there so take this n n_1 plus n_2. Now, what we do is make little bit of simplification to get through this analytically, you may have to go through numerically otherwise you can see here. I am just neglecting this quantity compare to that. We can do that, we can do neglect this quantity because of two reasons one: mu$_2$ is small compare to mu$_1$ mu$_1$ n_1 and mu$_2$ n_2 if you take, you are neglecting mu$_2$ into compare to mu_1 and n_1. Reason is mu_2 is smaller than mu_1 n_2 will be smaller than n_1 at least till certain field is there because all the electrons do not get transferred immediately. Up to certain electric field n_1 will be large atleast when all the electrons are here that is nothing here initially till the electrons get an energy corresponding to that n_2 is actually 0. So, most of the portion, you can state that second term is small. What I am doing here is the derivation is available in the over book of J (36:48) of device physics. Literally, I followed that the derivation is available there. I am just going through that physics of semi conductor devices by J. That I neglect and divide by you get this. Now, velocity versus electric field I can find out. If I find out n_2 by n_1 ratio that is electron concentration upper valley divided by electrons concentration in lower valley.

(Refer Slide Time: 37:23)



$$n_2 = N_{c2} e^{-(E_{c2} - E_f)/kT_e}$$

$$n_1 = N_{c1}\, e^{-(E_{c1} - E_f)/kT_e}$$

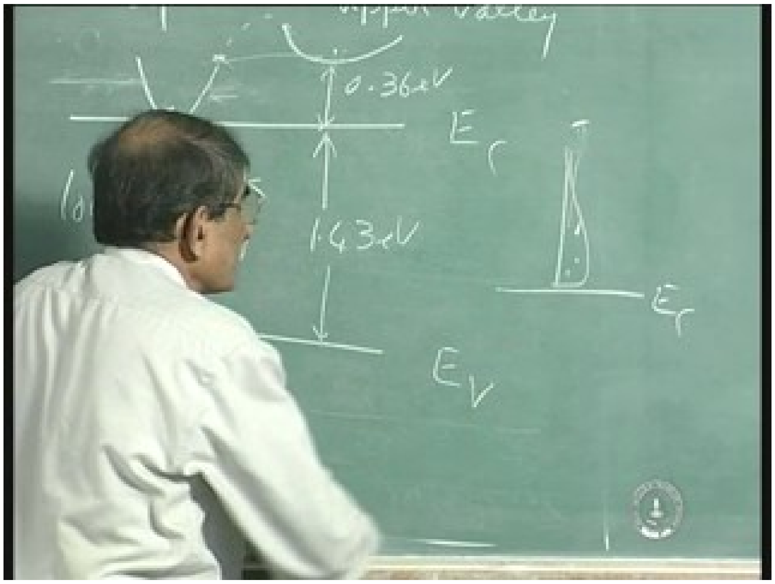
$$\frac{n_2}{n_1} = \frac{N_{c2}}{N_{c1}}\, e^{-(E_{c2} - E_{c1})/kT_e}$$

$$= \frac{N_{c2}}{N_{c1}}\, e^{-\Delta E/kT_e} \quad \underline{\hspace{3cm}}(1)$$

Te = Electron temperature > LatticeTemperature T

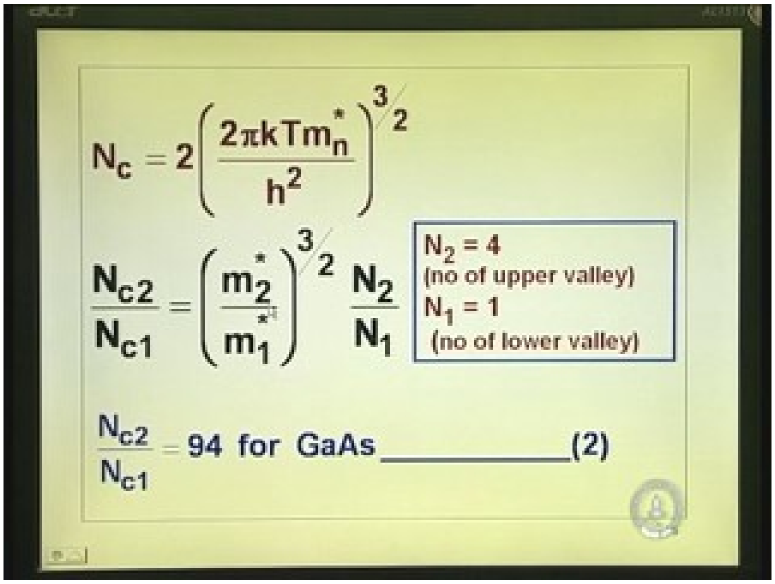
Electron concentration is related to density of states at that edge. For example: we write electrons concentration at this conduction band edge here, n_1 is equal to N_c e to the power of E_c minus E_f by kTe. I call this as E_{c1}; I call this as E_{c2}, E_{c2} minus E_{c1} is actually equal to delta E. The difference between the two conductions, I can talk this also the conduction band as for this, n_1 is N_{c1} is e to the power of E_{c1} minus E_f, n_2 electron concentration here it depends upon N_{c2} e to the power E_{c2} minus E_f. Apparently, it is smaller because E_{c2} minus E_f is larger than compare to that. Almost all the electrons will be here but notice one thing. Now, let us go back to further n_2 by n_1 I am just putting that T_e there. In all the cases I have put T_e there because if T_e is equal to T, that T_e is the lattice temperature. You will find the n_2 equals to 0 let me go through things and see. Let us say that T remain as T_e we will examine it again. n_1 by n_2 is actually given by N_{c2} by N_{c1} n_2 by n_1 N_{c2} by N_{c1} into e to the power of minus E_{c2} minus E_{c1} by kT_e which is actually equal to density of states here divided by density of state here, multiplied by e to the power of minus delta E by kT_e. Now, I have replaced T by T_e, T_e is actually the electron temperature and T is the lattice temperature. Usually, we use T_e is equal to T temperature is lattice temperature. We do not talk of hot electrons when I say T_e the temperature of electron is higher than that of lattice temperature means it is not in thermal equilibrium in the lattice. It has acquired so much energy and it has gone to the different total mode of operation, it has gone into the upper valley. That is actually, you are talking of hot electron like with Shockley barrier, we talked of electron being injected into the metal having higher energy than that of the metal that is hot electron. Here also you are talking of device who match the electrons are hot, when you say electrons are hot, I repeat whatever I said earlier, the implication is the electrons have energy in excess of kT corresponding to kT temperature T where, T is lattice temperature. If it is T lattice temperature it would reveal here, it is around that distributed up to certain height but, they are taken up into higher value.

(Refer Slide Time: 40:59)



See the implication is supposing I draw a conduction band you get electrons emission like that from E_c here. The meaning of that is thermal equilibrium situation. Supposing the raise the temperature of the electrons these electrons move up. The electron if it is here that does not correspond to the situation where, the electrons are in thermal equilibrium in the lattice. This electron is the hot electron which does not obey this law corresponding room temperature but, still you can write those equations taking a temperature which is corresponding to that. If you distribute the whole thing up there you will get a distribution which goes up like this. You will get a distribution which goes something like that. Beyond thermal equilibrium that is then you have to change T to T_e that is what you have done. You write the equation corresponding to that temperature of the electron that is higher energy. The most important thing now is n_2 by n_1 is N_{c2} by N_{c1} into e to the power minus delta E by kT_e out of these you know delta E 0.36 electron volts.

(Refer Slide Time: 42:20)



$$N_c = 2\left(\frac{2\pi k T m_n^*}{h^2}\right)^{3/2}$$

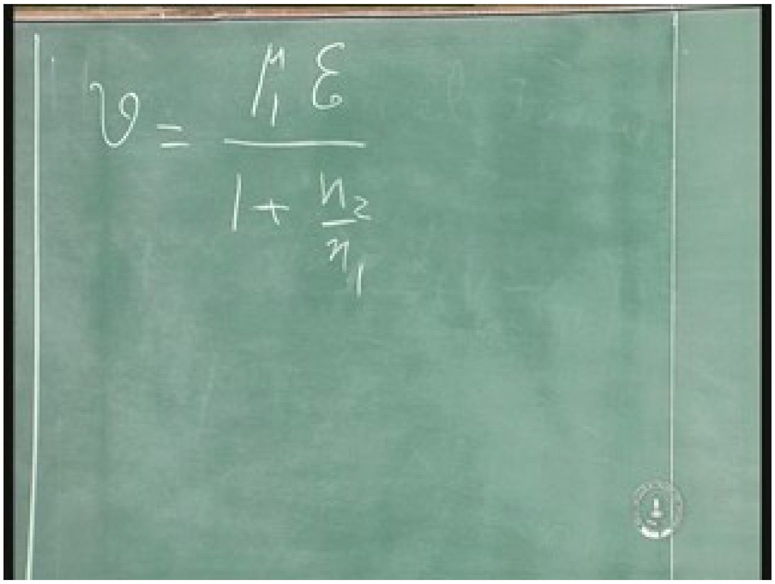
$$\frac{N_{c2}}{N_{c1}} = \left(\frac{m_2^*}{m_1^*}\right)^{3/2}\frac{N_2}{N_1}$$

$N_2 = 4$
(no of upper valley)
$N_1 = 1$
(no of lower valley)

$\frac{N_{c2}}{N_{c1}} = 94$ for GaAs ____________(2)

N_{c2} by n_1 N_c is that quantity T_e by T of course and N_{c1} N_{c2} by n_1 will be equal to taking the same temperature for the electrons N_{c2} will be m_n star to m_2 star. N_{c1} will be this will have m_1 star m_2 star by m_1 star to the power 3 by 2. What is another N_2 by N_1 coming up there? Number of total density of state in upper valley, total density of state in the lower valley it would have been m_2 star by m_1 star to the power 3 by 2 ratio if there is one valley here and one valley here. Now, what is happening is of course the three-dimensional picture central value is 1 there are four such values that n_2 that I have put there n_2 by n_1 that is on the right hand side there. That quantity this is number of values upper valleys there are four, lower valley there are there is 1 N_2 is four N_1 is 1. This is slightly getting in to little bit of physics but, we borrow those results and believing those resistance and say that is all right, comes on the prelone zone analysis. Now, m_2 star is 0.55 into m_0 m_1 star is 0.67 by m_0 and N_2 by N_1 is four the plug in all that this is 94 for Gallium Arsenide for different material that will be different. So, that is 94.

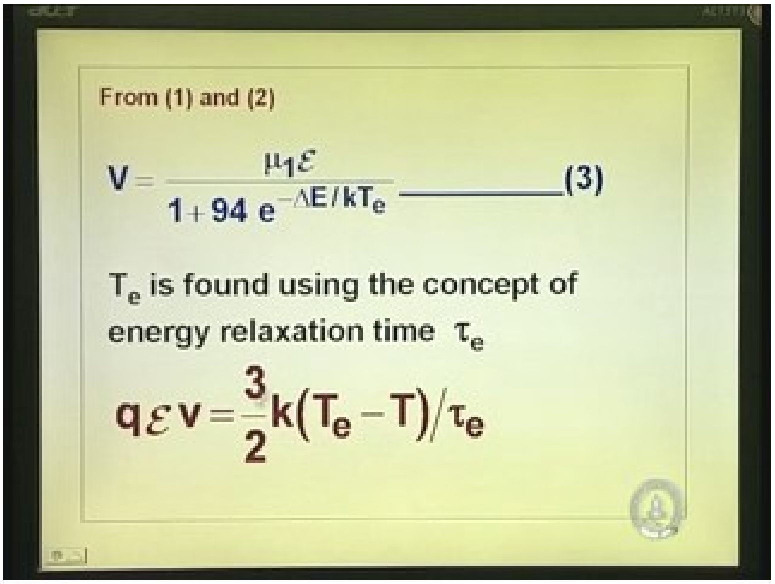
(Refer Slide Time: 44:29)



$$v = \frac{\mu_1 \mathcal{E}}{1 + \frac{n_2}{n_1}}$$

Now, you see if you recall, what we did was. I will retain that equation velocity is mu_n into electric field divide by 1 plus n_2 by n_1 that is what we did and n_2 by n_1 is N_{c2} by n_1 into e to the power minus delta E by kT_e N_{c2} by n_1 just now we saw it is 94.

(Refer Slide Time: 44:52)



From (1) and (2)

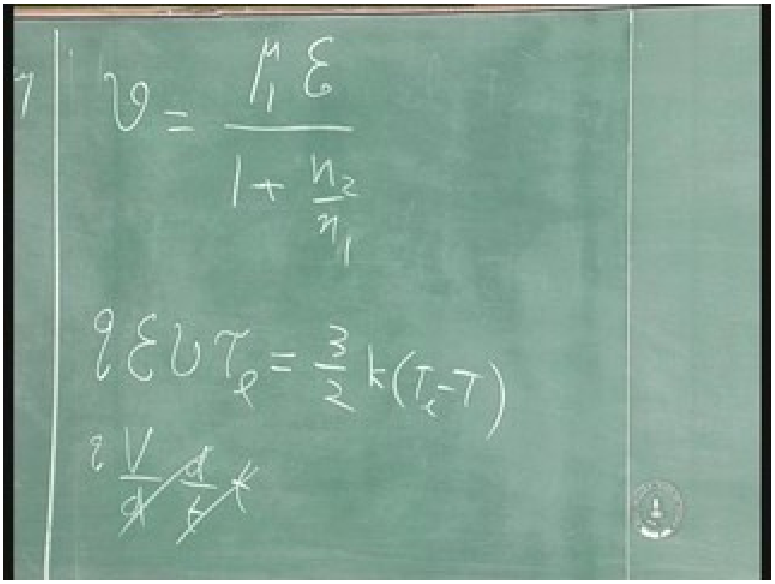
$$V = \frac{\mu_1 \mathcal{E}}{1 + 94\, e^{-\Delta E/kT_e}} \qquad \text{\_\_\_\_\_\_\_\_\_\_}(3)$$

T_e is found using the concept of energy relaxation time τ_e

$$q\mathcal{E}v = \frac{3}{2}k(T_e - T)/\tau_e$$

Remember that T is not T it is T_e all through, all these equations are valid for the hot electrons to have the energy above the lattice temperature. Now I can find the velocity versus electric field from this equation. This equation that I must use T is the unknown quantity you find out T_e by using this equation. This quantity 3 by 2 kT_e minus T is actually is an energy of the electron over and above thermal energy that is the energy acquired from the electric field. What is this quantity q epsilon v tau$_e$? This is the velocity. What is that quantity tau$_e$? tau$_e$ is the energy relaxation time meaning of that is you are applied an electric field entire distribution of electrons versus energy has got effected they have got excited to higher energy state. Now you remove the electric field there will drop back to this original state thermal equilibrium state the average time required for the electrons to get back to that thermal equilibrium state that is the relaxation time. You excite a fellow give the time for the fellow to relax it depends upon the particular particle. Here, electrons have a relaxation time of the order of pico seconds what you are telling is you excite it and by the time it has relaxed that, that is the energy that is lost.

(Refer Slide Time: 47:25)



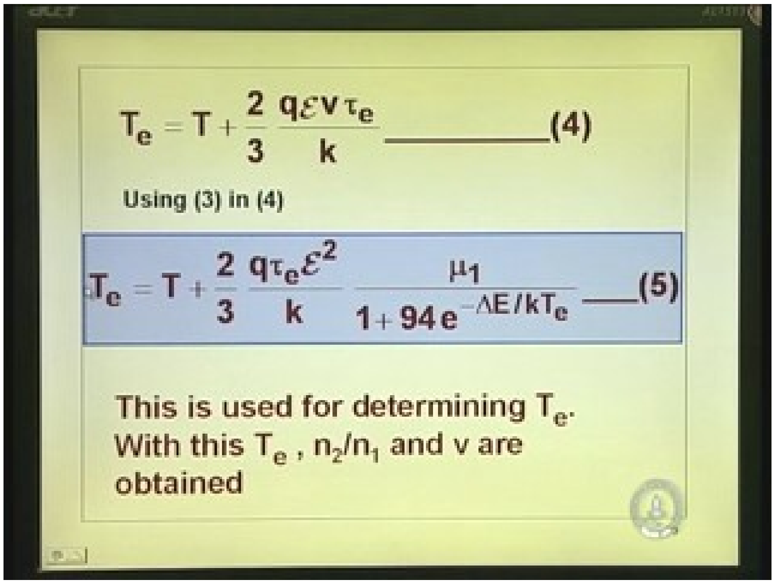
$$\vartheta = \frac{\mu_1 \mathcal{E}}{1+\frac{n_2}{n_1}}$$

$$q\mathcal{E} v \tau_e = \frac{3}{2}k(T_e-T)$$

$$q\frac{V}{\cancel{d}}\frac{\cancel{d}}{\cancel{t}}\cancel{t}$$

This quantity here is the energy that will be lost when it relaxes. Right hand side what you wrote there q into v is energy q into electron volts this is actually charge into voltage by distance. This is the energy charge into voltage by distance this is distance by time and this is time. It gets cancelled, that whole thing is dimension of an energy. I am just flowing through that when you write like this, this is the energy that has the last in that relaxation time tau$_e$ that is in that time whatever energy was there it has last and come in to thermal equilibrium. This quantity you put it as 3 by 2 k T$_e$ minus T, this is the energy of the electron in the relaxation time it will loose that much of energy. You make use of this concept and rewrite that. I am almost to the end of the derivation.

(Refer Slide Time: 48:50)



$$T_e = T + \frac{2}{3}\frac{q\mathcal{E}v\tau_e}{k} \text{ \_\_\_\_\_\_\_\_ } (4)$$

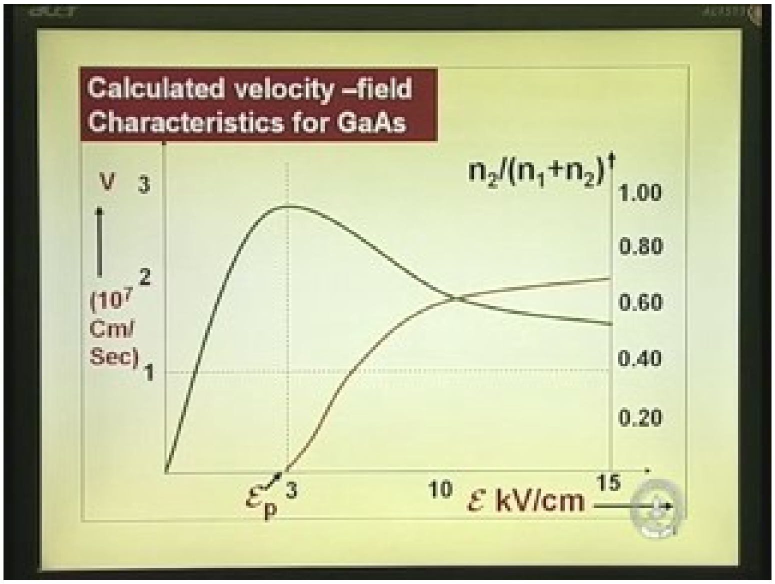
Using (3) in (4)

$$T_e = T + \frac{2}{3}\frac{q\tau_e\mathcal{E}^2}{k}\frac{\mu_1}{1+94e^{-\Delta E/kT_e}} \text{ \_\_\_\_ } (5)$$

This is used for determining T_e.
With this T_e , n_2/n_1 and v are obtained

I will just rewrite that same equation this equation I get it as T_e in terms of T. Whatever equation we have written here, previous equation rewritten here, what we are trying to find out is T_e. Once you know T_e you know N_2 by N_1 ratio 94 into e to the power minus delta E by kT_e. Here velocity itself is the function of T so plug in for velocity you get T_e is equal to T plus two thirds of q tau_e electric field by k and v is equal to mu_1 into E that is become E squared divide by 1 plus n_2 by n_1 n_2 by n_1 is nothing but 94 e to the power minus delta E by kT_e. All that you have done is you have used that electron temperature and that unknown quantity can be found out from this equation. Tau_e is a relaxation time we known E is what electric field that you are talking of what you are trying to find out now is with electric field how hot the electron becomes. There are two on both sides you have got $T_{e.}$ It is a transcendental equation. You have to solve it numerically finally. Once you solve this you get the temperature and once you get the temperature you substitute in n_2 by n_1 substitute in velocity. Velocity is mu_1 divided by 1 plus 94 into e to the power minus delta E by kT_e T we got form this particular transcendental equation.

(Refer Slide Time: 50:31)

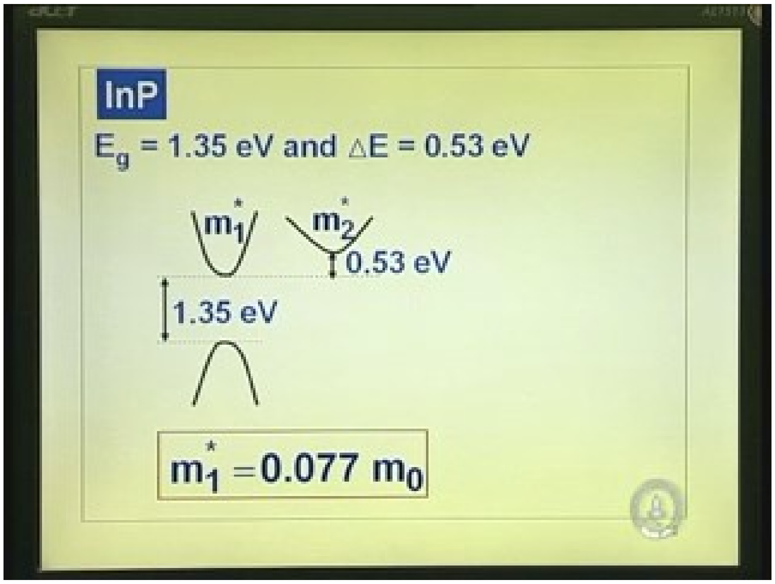


Calculated velocity –field Characteristics for GaAs

Once you do that, you plot that you find out the temperature and substitute in that equation you get n_2 by n_1 n_2 by n_1 is this. In fact, this is n_2 by n_1 plus n_2 on the right hand side as if it is 0.2, 0.4, 0.6 etcetera n_2 divide by total electron concentration. You can see corresponding to E_p is about 3 kilo volts this is calculated. This is what I am plotting. At 3 kilo volts per centimeter that is the point at which n_2 star begins to increase till 3 kV per centimeter. n_2 is equal to 0 what does it mean, till apply an electro field is equal to 3 kV per centimeter all the electrons are here. All the electrons keep moving with the velocity corresponding to higher mobility. All these electrons are moving up to that mobility, they are not subjecting into velocity saturation or anything. They are actually keep on increasing. That is why it shoots up and once you reach the 3 kV per centimeter the transfer by the electron in upper valley keeps on increasing. As more and more increase qn into v part of them have a lower velocity the effective velocity falls. If you look into the equation k is equal to qn into v where n is n_1 plus n_2 mu_1 n_1 plus mu_2 plus n_2 that n_2 keeps on increasing previously it was mu_1 into n_1 it is n_1 falls n_2 increases. So, the effective velocity falls down. Transition point is this 3 kV and you can see if you have a different material whose delta E is different. Suppose delta E is higher what will happen to the transition voltage or field. It should increase because you need more energy to transfer electrons from here to here. That is what is happening in material in Indium Phosphate.

This is the whole thing that you have here. This is the steady state characteristics not yet come to the transient conditions. We will discuss that in our next lecture.

(Refer Slide Time: 52:54)



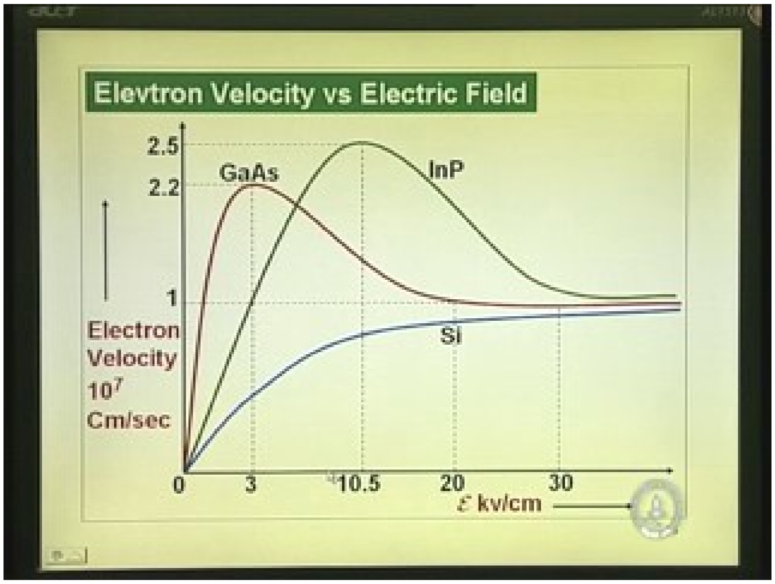
InP

$E_g = 1.35$ eV and $\Delta E = 0.53$ eV

$\boxed{m_1^* = 0.077\ m_0}$

But, right now we will take look at the Indium Phosphate. Indium Phosphate also it is a three five semiconductor. I am putting this term because it is completing with Gallium Arsenide for high speed devices. Reason you will see when you see the plot after velocity field characteristics. Now, here you can see that is 1.35 that is 0.53. In case of Gallium Arsenide is 1.43 and delta E is actually 0.36. Immediately you have to transfer four electrons from here to here you would need much more energy much more electric field. Now, that means by that time it is transferred from here to here you would have acquired much more energy much more velocity. Now, the effective mass here is not as much as that Gallium Arsenide as slow slightly has 0.077. That gives the mobility of electrons in Indium Phosphate it is slightly lower than that of Gallium Arsenide over 4500 or so. This is smaller over a 5000. Here, this is almost comparable.

(Refer Slide Time: 54:12)



Elevtron Velocity vs Electric Field

Now, let us see what happens because of that Gallium Arsenide, you can see it gets isolated fast. See the slope is higher here notice this three curves Silicon Gallium Arsenide indium Phosphate mobility of electron is much higher than that of Silicon. E is equal to mu into E deeper compare to Silicon. Mobility of electron in Indium Phosphate is smaller than that of Gallium Arsenide. Slope is slightly smaller v is mu into E here, now the transfer of electron takes place here at below 3 kV per centimeter; whereas, the delta E in the case of Indium Phosphate is almost, not double at least this is 0.36 that is 0.53. The transfer of electrons to the upper valley takes place that is transfer of electron from here to here takes place at much higher energy that is much higher field 10.5 kV per centimeter. By the time it has acquired energy the velocity is gone up beyond that and it takes much longer to transfer all the electrons to the upper valley. See to get in to this position almost all the electrons go into the portion. That is why what you get is a velocity versus characteristics it looks like this from the lower field mobility point of view that is Gallium Arsenide is the best among the three devices. From the velocity saturation if you only take a look at there is no choice all of them seems to behave almost same. In fact, I have seen some people degrading Gallium Arsenide putting below this quantity. I do not believe that it is almost same there. After all, the mechanism of transfer is same. Indium Phosphate has velocity higher than that of Silicon over a wide range of field. Gallium Arsenide has got higher the velocity is higher than that of Silicon over certain field range. These things make quite a bit of difference in performance. We will see that transcend situation what you have seen now is if you keep increasing the electric field keep it at certain value 3 kV per centimeter that is the steady state velocity. But, now if I launch an electron from cold to that high field suddenly, what will it feel, it is not here, it is here from there to there it has moved. There is a transient condition we will see what is known as a velocity over shoot effect. That is whole cost for success for Gallium Arsenide. Indium Phosphate, we will see it in next lecture. It is very interesting, do not miss.