Networks and systems
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Lecture-7
Discontinuous Signals – Step, Ramp

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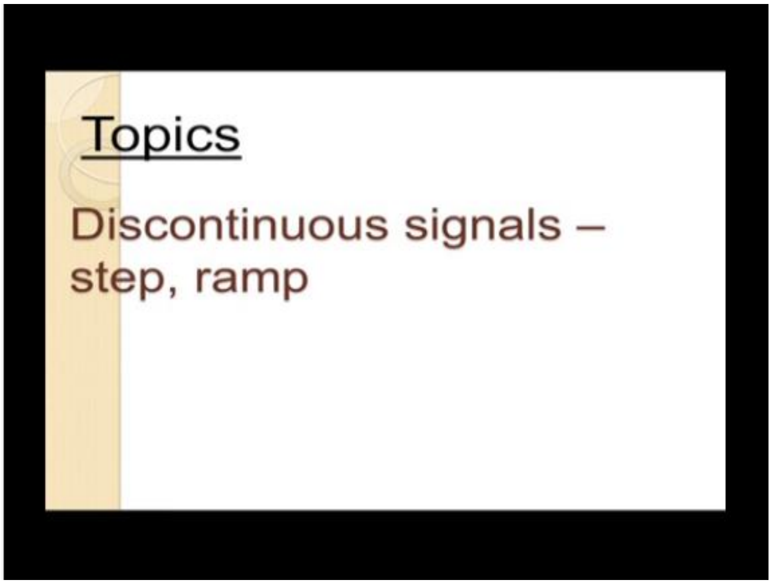

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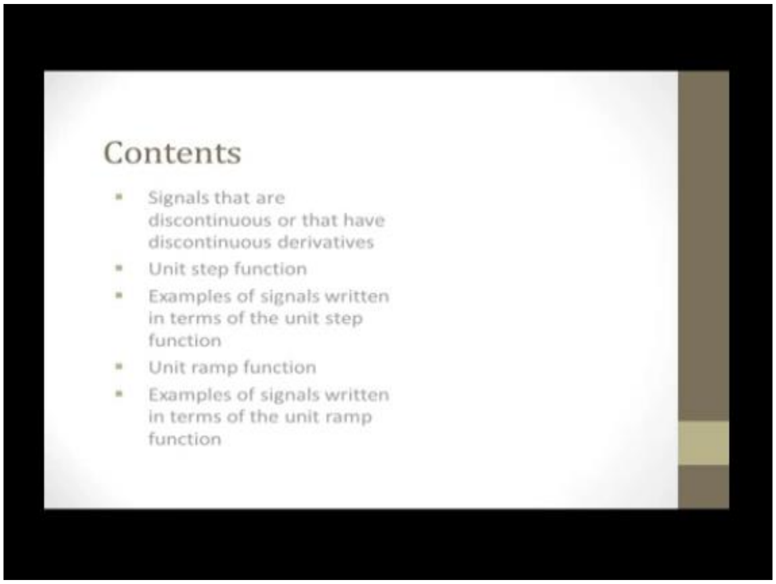

Now, all these signals as I mentioned are those which are smooth which are continuous by themselves and also have continuous derivatives.

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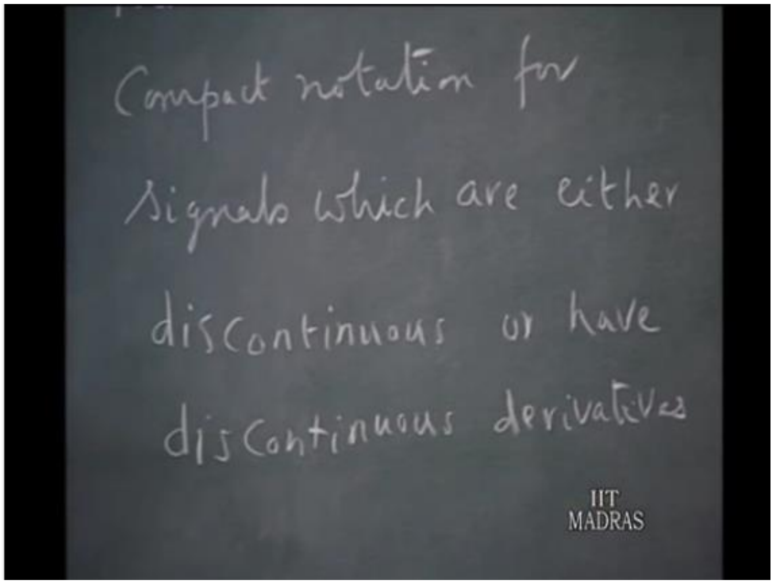

There is a need to have the compact notation for signals which are either discontinuous or have discontinuous derivatives.

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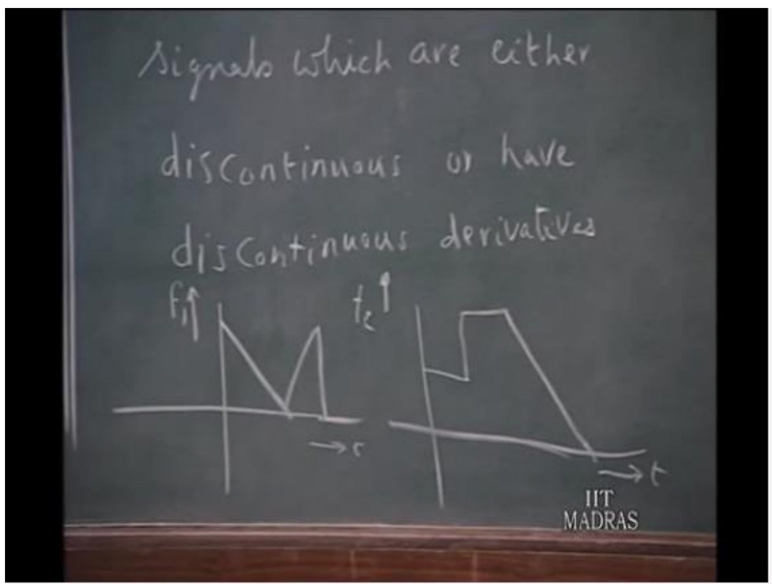

Take for example, this particular function of time or for that matter another 1 like this. Suppose these are the 2 functions of time that are given to you f2 and t. It is difficult to describe this elliptically because we have to say that function is 0 for negative values of time and from this point to this point it has got such you give an elliptical expression for this. From this point to this point you give another elliptical expression for this and beyond that once again you say it to 0.

Similarly, if you want to describe this f of 2 elliptically we have to say it is 0 for negative values of time. It is a constant up to for t in this range another constant for t in this range, another constant for t in this range and it has got a certain straight line relation in this range and beyond that it is 0. That means, you have to describe give separate expressions for 5 different regions as far as this function f 2 is concerned.

So, it becomes little cumbersome, but 1 would like to see if we can express this by means of some notational expression, some notations just like we have said this is A e to the power of s t if it possible for us to arrive at some kind of expression for these signals f1 and f2 where we do not have to qualify further as saying this is valid for this interval of time and this expression is valid for a different interval of time.

And this leads us to the topic of what is called singularity functions which are used to describe functions of this type which are discontinuous by themselves or have discontinuous derivatives. And it is the singularity functions that we will take up for discussion later and 1 of the most important singularity functions in the step functions which I will describe in a moment.

This is originally introduced into the literature by oliver heaviside a british engineer who made signal contributions to communication theory and operational methods of analysis of networks and systems. He was a controversial figure and most of his work was not founded on rigorous mathematics.

So, he got into trouble with mathematicians of his day, but he introduced the step function among other things and the operational calculus methods and as a retort to the mathematicians who were not happy with his work he would say do I stop eating, because I do not exactly understand the process of digestion. That was his retort to those people who were critical of his work. Because he introduced some methods when they were working and.

So, he said why should you poke further into this as long as it produces results we should be happy with them. That is, just a brief historical note on Oliver Heaviside who introduced the unit step function which we will take up now. We shall now take a look at 3 important singularity functions. These are called singularity functions because, either they have discontinued this or the derivatives have discontinued this and therefore, in the regular classical sense of mathematics we do not have derivatives.

If derivative fails to be continuous the classical mathematics you say it does not have a derivative. So, it is singular in that point that is why these are called singularity functions.

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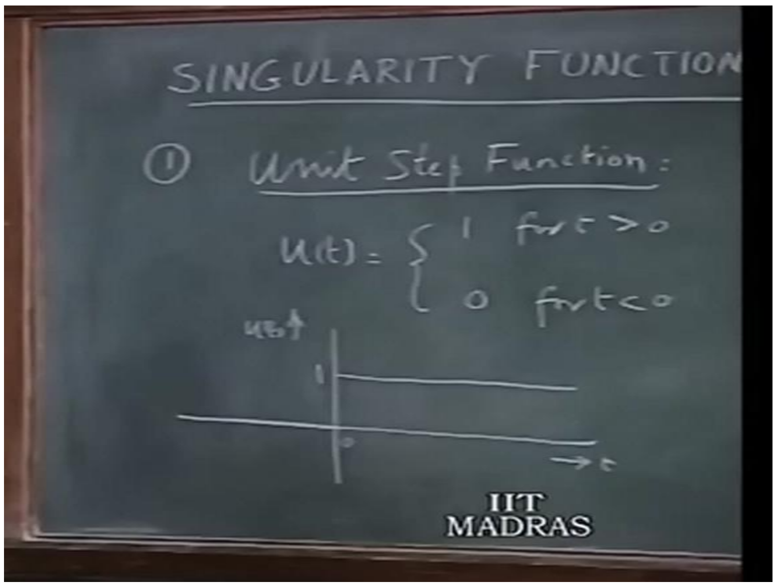

The most important perhaps is the unit step function. And this is what i said was introduced by Oliver Heaviside. This is indicated as u of t symbol symbolically it is represented as u of t and this is defined as 1 for t greater than 0 and 0 for t less than 0.

So, the wave form for this would be it is 0 up to this point t this time axis t 0 and then from this point onwards has a value 1. So, as a function of time it has got a waveform like this. 0 for negative values of t and a constant 1 for positive values of t at t equals 0 itself, at this point we don’t really care what it is it can be left undefined you can take it as 1 if you wish 0 if you like or half whatever it is.

Normally we are not really concerned in most of the work. It does not matter how you take it. So, we will leave it undefined, but if really it becomes necessary you take it as half. Because, it is the average of the right and left extreme limits and in some limiting processes it will converse to that value half. So, you may take it as half, but in any case the important thing here is that it is a discontinuous function of time it is the left hand limit is 0 and then suddenly it jumps by a value 1 and stays constant from that point onwards.

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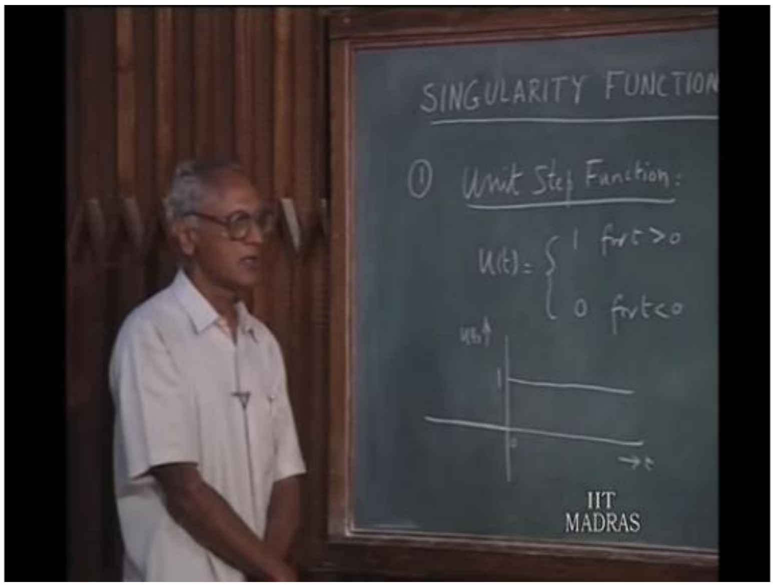

So, it resembles a step here at this point a steep rise and that is why it is called a step function and since this height of the step is 1 it is called a unit step function u of t. Now, using this unit step function, w can describe a number of other functions of time which have discontinuities.

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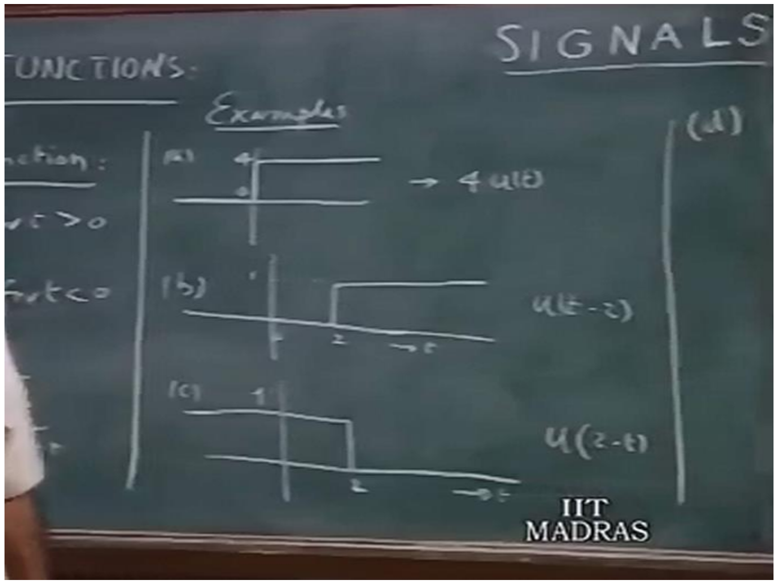

So, let us take a series of examples to illustrate, how the unit step function can be used to describe different types of functions.

Suppose, I have a function which has got unit 0 4 units are positive t and 0 for negative values of time then obviously, this will be described as 4 times u t because it is a same unit function enlarged 4 times. So, the amplitude is 4 4u t is this function. Suppose, I have a step function which starts at t equals 2 and 0 for negative values for values of t less than 2, then you would and the magnitude of this is suppose 1.

Then, you would call that u of t minus 2 because as long as your argument t minus 2 is positive t greater than 2 it has 1. If t is less than 2 u of t minus 2 is a unit function unit step function of the argument negative values then therefore, it becomes 0. So, this describes this function is described by u t minus 2 you can also see if this is u of t. It is translated in time delayed by 2 seconds therefore if this is f of t this must be f of t minus 2. Therefore, from that argument also you can show you can see that this is u of t minus 2.

Suppose I have a function like this one for t less than 2 and 0 for t greater than 2. So, what will you describe this function as? Now this function has, a value 1 as long as t is less than 2 therefore, you write u of 2 minus t that describes this function because as long as t is less than 2, 2 minus t is a positive number and therefore it must have a value 1 and as long as t exceeds 2, 2 minus t is a negative value therefore it becomes 0. That is how it goes.

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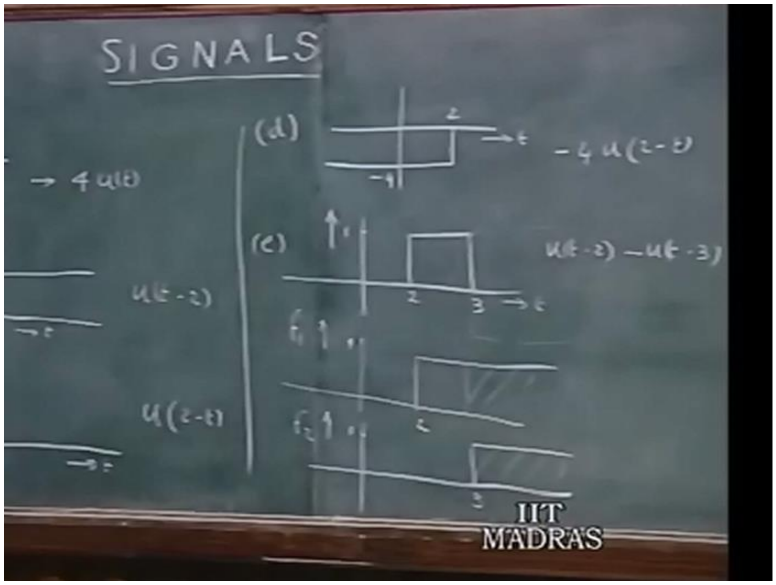

Let me now take a case, where you have a function whose value is minus 4 for t less than 2 therefore this is u of t minus t, but instead of having plus 1 it has a value minus 4. Therefore, this will be minus 4 u of 2 minus t. You can substitute different values of t and see that it is in accordance with the definition of unit step function. The unit step function is also useful in describing functions like these.

This is a kind of pulse sometimes called gate function because it is like a gate between this intervals this signal is there afterwards this signal is not there at all. Now, such functions can be easily shown to be the sum of 2 functions. For example, if i have a step function starting at 2 say f1 and another function f2 starting at 3. Notice, that if i subtract f2 from f1 from t equals 3 onwards this area is cancelled out by this area and what is left is only the value or the function between 2 and 3 and that is exactly what we are having.

So, a gate function or a pulse function like this can be written as u of t minus 2, u of t minus 2 would be a step starting at 2 seconds and going on forever. You want to pull down that step to 0. Therefore, at this point 3 you have to introduce a negative step of unit magnitude.

So, that the original step that was going like this from that you are subtracting a negative step. So, that from point t equals 3 onwards it becomes 0.

So, you need to subtract from this u of t minus 3. So, you can see that a square a pulse like this can be described in times of step functions and u of t minus 2 minus u of t minus 3. So, this is a very useful concept which we can use to describe discontinuous functions of this type.

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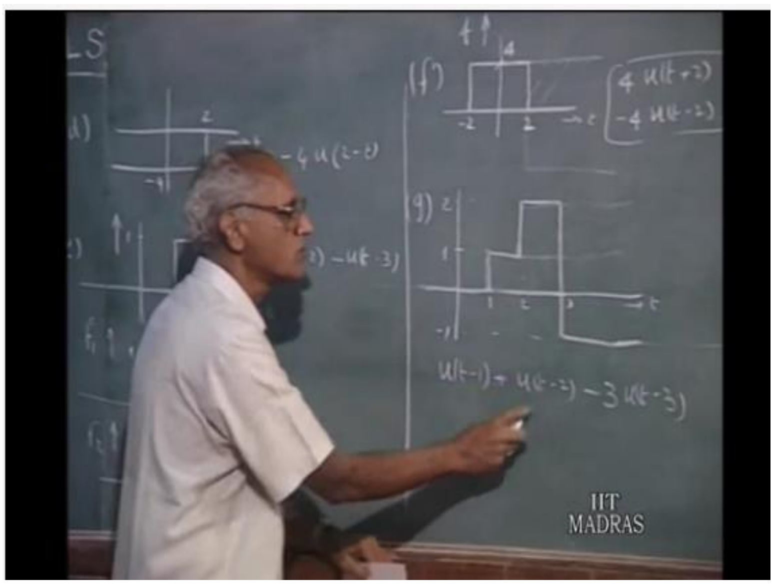

Another example, suppose you have this function which is symmetrically situated around the origin. Then you can think of this as a step function starting at this point and subtract from that a step function a negative step starting at this point. The step function starting at minus 2 has 4 units. Therefore, 4 u of t plus 2 you must write because for t less than minus 2 then t plus 2 is negative therefore, this is 0. As long as t is greater than minus 2 our t plus 2 is positive you have a value 4.

Therefore, 4 u of t plus 2 is a step which is going like this. But then you do not want this step to continue forever. At this point plus 2 you want to introduce a negative step of 4 units. So, that this portion is cancelled by this. So, from this you subtract minus 4 u of t minus 2. So, sum of these 2 is indeed this pulse that you are having a little more complicated situation. So, suppose this is 1 this is 2 this is minus 1 time 2 3.

So, this goes on like that forever beyond that t equals 3 it is continuous at minus 1. So, to describe this always what you should do is to start with the first step. And see what are the other steps that have to be added to that, in order to describe the actual situation. So, this function is 0 up to this point and at that point it jumps by value 1.

So, you can write this as u of t minus 1. That describes a unit step starting at t equals 1 and going on like that. And that explains satisfactorily the given function of time from t equals minus infinity on up to 2 this function describes the behavior at this point. At this point the function jumps up by another amount equal to 1. So, to this step you have to add another step of unit amplitude. So, that will be another unit step starting at t equals 2. So, u of t minus 2 and now, what is the effect of these 2 steps. You have 0 up to this point 1 and then it continues like this.

So, the effect of these 2 would lead to us would give us a curve which goes like this and then continues like this, but then what happens at t equals 3 you must bring this down to minus 1. So, you must introduce a negative step of 3 units in order to pull this value down to minus 1. So, minus of 3 u of t minus 3you do that then this value is pulled down to minus 1 and then from that point onwards it continues at a constant value. You can verify the result after all u of t equals 1 for t greater than 0. Therefore, suppose you take a large value of t say 6 or 7 then this is 1, this is 1 and this is minus 3.

So, 1 plus 1 minus 3 is minus 1. So, indeed that is minus 1. So, for any value of t greater than 3 this is going to be minus 1. So, these examples illustrate the usefulness of the step function in describing discontinuous functions and are piecewise constant functions and you must get the facility you must develop the facility of developing such functions.

By means of appropriate steps with appropriate weights and appropriate shifts in time as we have indicated by means of these examples.

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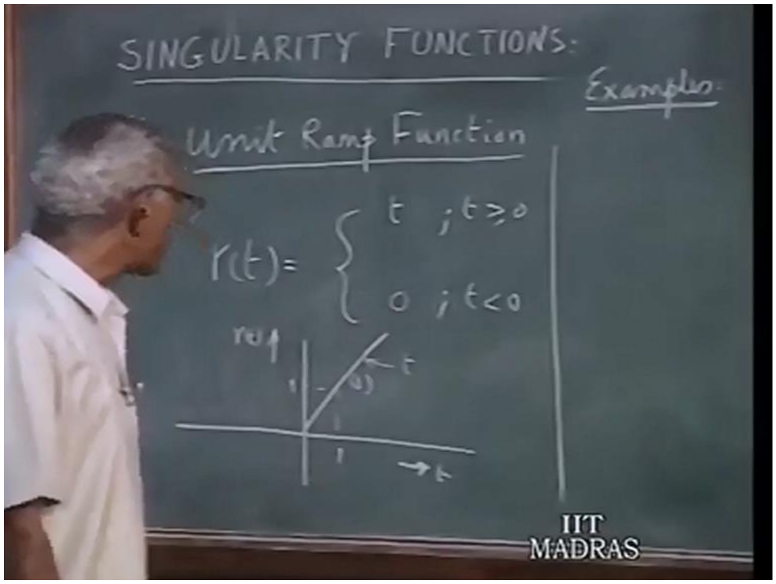

Similarly, the next singularity function that we would like to talk about is what is called a unit ramp function. This is also a causal signal like the u of t in this it is 0 for negative values of time. It is indicated as r t and it is defined as t for t greater than or equal to 0 and 0 for t less than 0. So, it is clear that its wave form would be like this goes on like this and it has a value of 1 at time t equals 1.

So, it is for positive t it is described as t and for negative values it is 0. So, this is called a ramp function because this is like a ramp a slope. So, it is called a ramp function and since this slope is equal to 1 it is called a unit ramp function and suppose i indicate this slope by brackets this slope equal to 1 it rises by unit amount the unit time this is called unit ramp function.

Now, unit ramp function once again let me illustrate its applications through examples. See how we can describe several interesting wave shapes through the unit ramp function.

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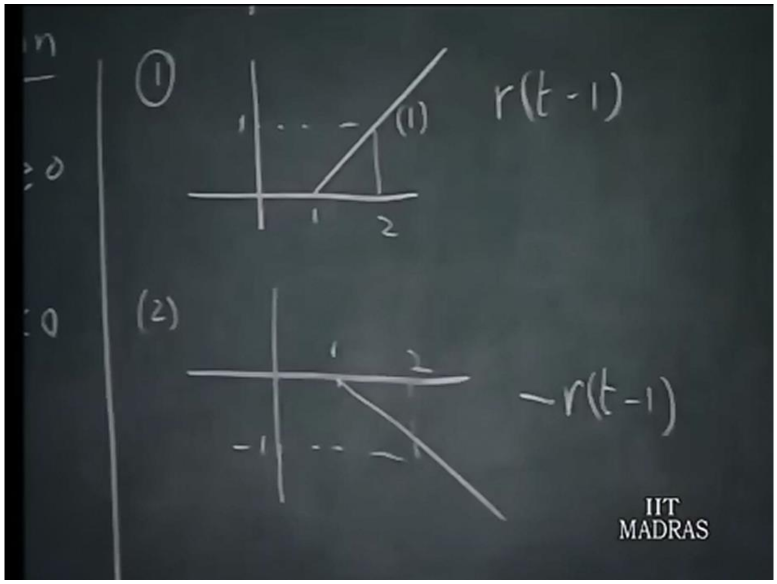

Suppose I have a function like this which is 0 for t less than 1 and increases at the rate of 1 unit per unit distance travelled along the x axis. Then this would be called r of t minus 1 because it is the same unit ramp function shifted in time delayed in time by 1 second and therefore, this will be r t minus 1.

We should be able to recognize whenever a function is delayed by an amount tou, then f of t becomes f of t minus tou. Suppose it starts at 1 and there is a negative going ramp. At 2 it is equal to minus 1 then it will be minus of r of t minus 1. It is a ramp function with a coefficient of minus 1 and it starts at t equals 1. That's how you describe it.

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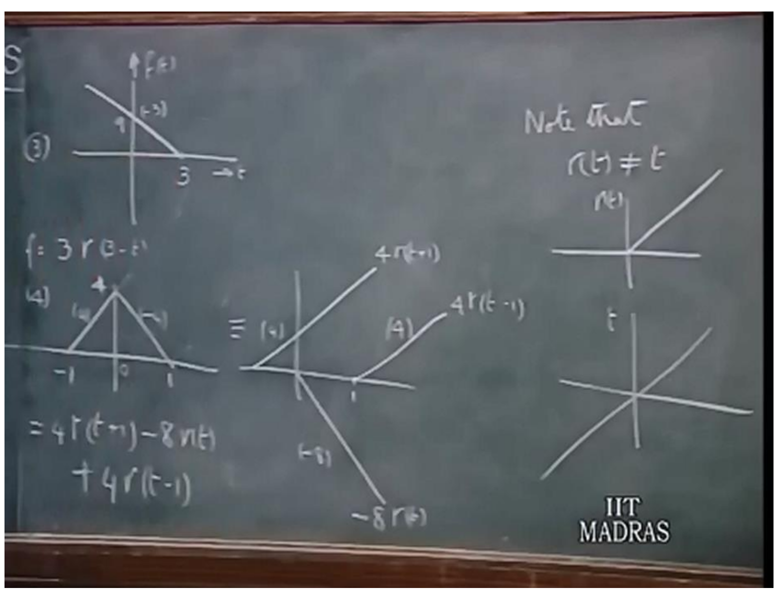

Suppose I have a function which is 0 for values of 3 beyond 3 in the positive direction and then it increases the negative direction of t with a slope equal to 3 or slope in the as t increases it is minus 3. Then first of all, it leads the ramp function has values for t less than 3. Therefore, you must have basically something like r 3 minus t because if t exceeds 3 this becomes 0. When t exceeds 3, 3 minus t is a negative number and therefore, this is 0.

So, basically you have r of 3 minus t. Now, you must fix the coefficient to suit this now when t equals 0 for example, then this is equal to 9. This slope is 3 therefore, if you put here 3 r t minus 3 this becomes answers this description. Because all you have to do is you must ensure that this is negative for t greater than 3 and therefore, the base point comes at t equal 3 and beyond that if you fix 1 point on the straight line the straight line gets fixed up.

Therefore when t equal 0 this is r of 3, r of t is equal to 3, r of 3 is equal to 3. Therefore, 3 times 3 is 9 and that answers this particular description. So, that is how 1 can fix up this you can do it in different ways, but at least this is 1 argument to show that f of t is equal 3 r 3 minus t. The fourth example suppose I have, a triangular wave shape 1 minus 1 4. This is a function which you can decompose as first of all at t equal minus 1 there is a ramp function going like this at t equals minus 1.

So, if a ramp starts at minus 1 and increases with a slope of 4, then you would describe this as 4 r t plus 1 because this is delayed this is advanced by 1 unit therefore, r of t plus 1. So, such a line that is a line starting from here and then going on like this could be called 4 r t plus 4 all right.

Now, if this is the line that is going then this describes a wave form which is going like this, but now at this point at t equals 0 you must introduce, you must pull this down by. So, that it will have a slope of minus 4 from that point onwards. Therefore, in order to pull this down to introduce a resultant slope of minus 4 this has a slop of plus 4 at this point you must introduce a negative going ramp to pull down this slope to minus 4.

So, at this point you must introduce here another ramp with a slope of minus 8 units. This has a slope of plus 4 units, this type of slope of minus 8 units and this ramp is introduced at t equals 0 therefore, this will be called minus 8 r of t. That will be that particular characteristic which starts at t equals 0 and has a negative slope.

Now, what is the result of these 2, if you have these 2 together then you load then no doubt this increases. At this point you pull this down and then it is going all like this. Now, at t equals 1 we must arrest this downward slope we must put this back to 0. That means this slope resultant slope of minus 4 must be stopped and then it must be restored to a D.C 0 slope.

Therefore, at t equals 1 you must introduce another ramp with a positive slope of 4 units. So that, that particular ramp and this what you had earlier picked up will lead to a 0 identically for values of t beyond 1 and therefore, this would be called 4 r. This ramp is introduced at t equals 1 therefore, 4 r t minus 1. So, ultimately this particular and if you do that then this resultant negative slope and then this positive slope will be cancelled out will cancel out and yield identically a 0 value from t equals 1 onwards.

Therefore, this particular triangle triangular wave form can be described as 4 r t plus 1 minus 8 r t plus 4 r t minus 1, the sum of these 3 that is how this ramp function can be employed. Notice that, r t is not equal to t they are not the same r t is a function like this whereas, t is function like this this is r t, this is t. T has a value for negative values of time as well whereas, r t is 0 for negative values of time.

You can however say r t equals t times u t. You can say that because if you take this t and multiply by unit step function the negative values will be multiplied by 0 it gets cancelled out and for positive t that t multiplied by 1 t u t will provide r t. So, r t can be written alternatively if you wish as t u t, but it cannot be equal to t by itself.