Networks and systems
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Lecture-10
Examples of Signals

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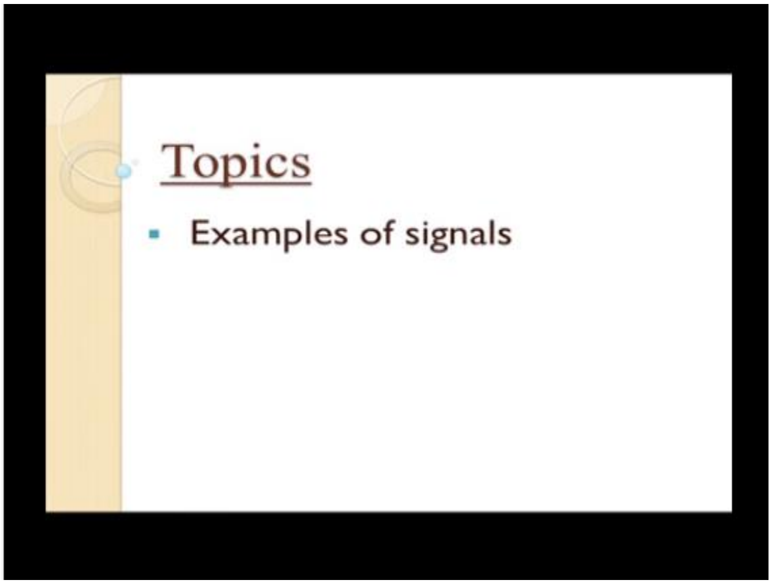

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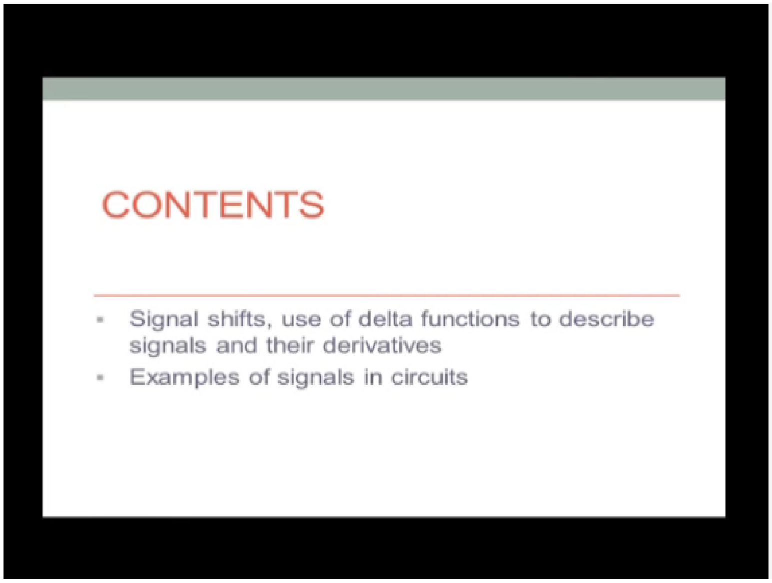

As a continuation of our discussion of the meaning and application of singularity functions.

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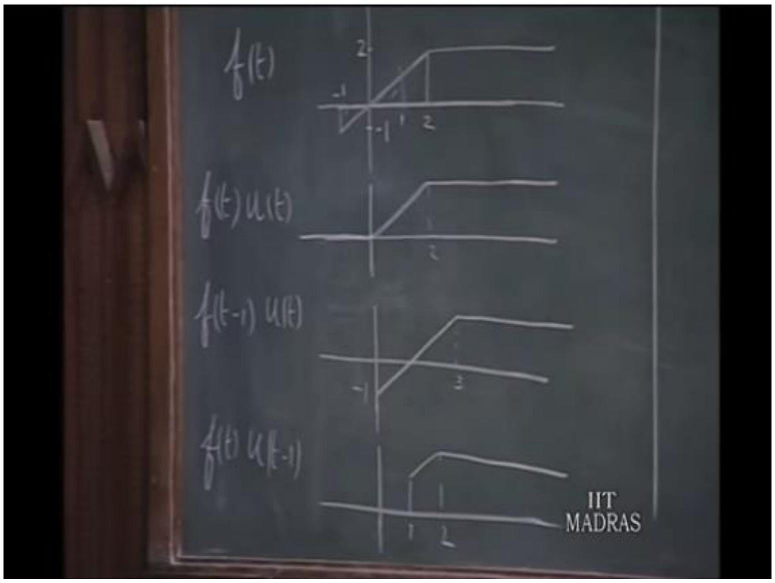

Let me, take an example suppose you are given this function f of t which has a discontinuity here. And also, discontinuous derivative and so on. F of t is described like this now, what is if f of t is like this. What is the meaning of f of t u t? So, this function is being multiplied by u of t.

That means the negative values of the function for negative values of t are cut off and for positive values of t it is multiplied by1. Therefore, f of t u t would be the reproduction of the same curve for positive values of time for negative values of time it is 0. So, this is exactly what will be f of t u t. If this is f of t this is f of t u t where the section for the negative values of time is cut off.

Now, suppose on the other hand if this is f of t I want to know, what is f of t minus 1u t. If this is f of t, f of t is a curve which is delayed by 1 second. Therefore, it will have this curve this is minus 1and this break point occurs at 3 units and this is f of t minus 1. And f of t minus 1multiplied by u of t f of t minus 1has values only for positive t multiplied by u of t will not disturb this.

Therefore, this will be f of t minus 1u of t. On the other hand, if I multiply f of t by u of t minus 1. What will be the result? If I multiply f t by u of t minus 1 I have u of t minus 1is a step starting at plus 1. So, at this point the step starts and then you are multiplying that this curve by u of t minus 1. Therefore, you have at 1 the curve will be reproduced from t equals 2 onwards only.

That means the this portion of the curve gets cut off truncated. So, that is you see the notation f of t f of t u t f of t minus 1u t f of t u t minus 1. So, this is how you can understand the meanings of the various functions of time whenever u of t minus 1 means it is starts from t equals 1 onwards and you should be able to get have the ability to recognize the waveforms which when, the f of t is multiplied by step functions with delayed step functions and. so on and so forth.

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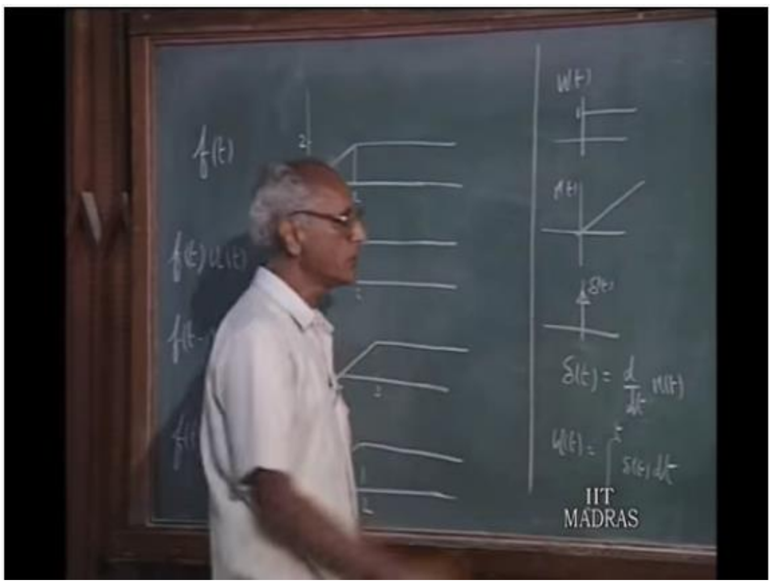

Let me take now we have seen 3 functions now U of t the ramp function r t and the delta function. How are they inter related? You see that if I differentiate u of t I get delta t; delta t is d by dt of u t. Because, when you differentiate this derivative is 0 here and here at this point there’s sudden jump of 1unit and the area under this delta curve is 1unit. That means when you integrate through the delta you get a rise of 1unit.

Therefore, u of t is the integral of delta t and delta t is the derivative of u of t. So, u of t can be written as from minus infinity to t delta t dt. So, that is how these 2 are related. In a similar fashion suppose you have r t you take the derivative you get this step function. If you integrate u t you get the ramp function.

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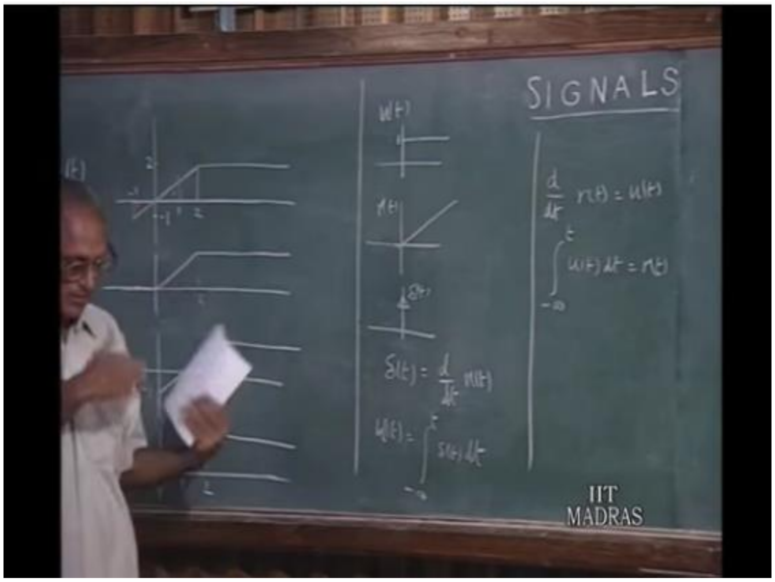

So, d by dt of r t gives an unit step function and integral minus infinity to t of u of t dt gives a ramp function. That means starting with the ramp you take the derivative you get this or starting from the impulse you take the integral you get u of t, you take the integral of that you take r. So, these are 3 are interrelated in this fashion.

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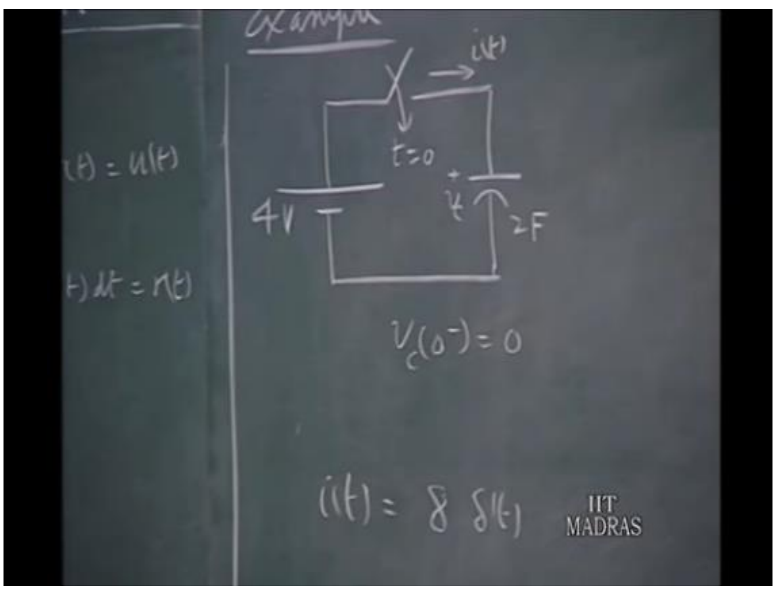

So, using this let me take an example suppose, I have a 4 volts d c source and connect this to a 2-ferret capacitor at t equals 0. I would like to find an expression for current in the circuit. You can see that initially; the capacitor voltage is 0. Let us, say vc 0 vc 0 minus before this vc is closed is equal to 0. If the capacitor is uncharged as soon as, you close the switch the capacitor by Kirchhoff's Voltage Law must acquire a voltage of 4 volts.

So, no way you can avoid it even though, the capacitor voltage is the same or continuous in the normal course of things, but here according to the rules of the game we you close the switch this 4 volts must appear across the capacitor.

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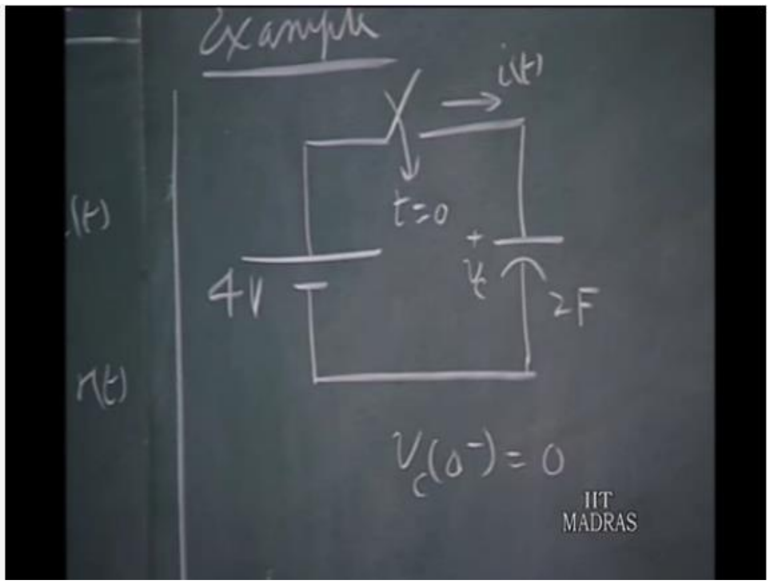

So, the capacitor voltage before t equals 0 is 0. Immediately after that it must jump to 4 volts. That means, 8 coulombs of charge must be dumped on the capacitor in 0 time. So, the charge in the capacitor t equals 0 minus is 0. At t equals 0 plus mean, it is 8 coulombs. So, 8 coulombs must be dumped on the capacitor in no time at all in the interval 0 minus to 0 plus.

That means, the current must be infinite, but the area under the curve of the current wave form which is really the charge must be 8 units. So, you can obviously, see that the current in the circuit is described by a delta function. So, the current in the circuit I of t can be written as 8 delta t. So, because the area under the curve is 8 units that is it represents the charge that has been put on the capacitor in infinitesimal interval between 0 minus to 0 plus. So, I of t is 8 delta t.

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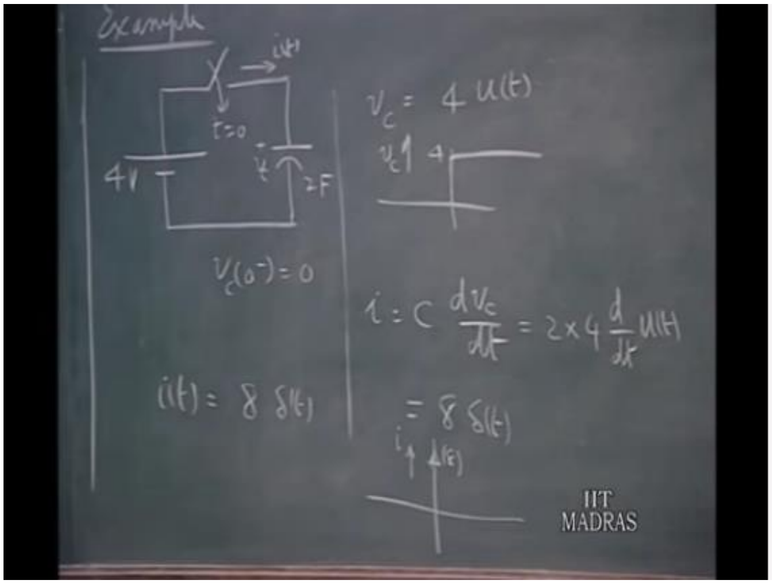

You can arrive at this result in a more formal way of course, we have physically argued that this should be the answer, but you can say that the capacitor voltage vc what is the expression for the capacitor voltage? For time t less than 0 it is 0 for time t greater than 0, it is 4 volts once you close the switch.

Therefore, vc can be written as 4 u of t. That is the voltage wave form across the capacitor.it will be like this. We know that, the current in the capacitor is c d vc dot I equals c d vc dt. That is the basic equation in terminal relation for a capacitor. Now, in this case the capacitor is 2 Ferrets. Therefore, 2 times 4 times d by dt into u t. That is the expression for the current and we have just now observed d by dt of u t is delta t. therefore this is 8 delta t.

So, if you look at the waveform of the current in the capacitor it is a delta of 8 units magnitude sitting at the origin that is the expression for the current in the capacitor.

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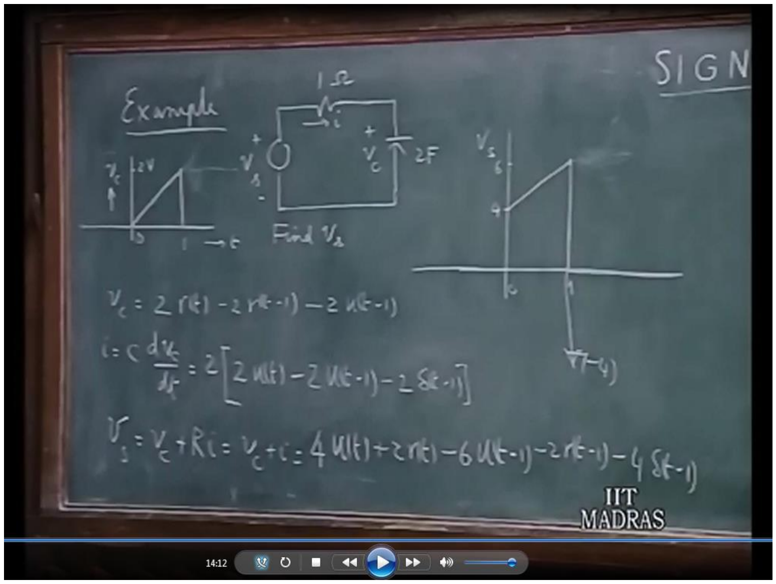

Let me, take a second example: we have a voltage source vs connected to a resistor capacitor combination. The capacitor is 2 Ferrets and the current I here. So, let the circuit we know the waveform of the Vc and we are asked to find out vs. So, let, Vc be in this circuit be having a waveform like this. So, this is a triangular pulse t this is 2 Volts. So, we are given this waveform the function for the expression or the waveform for v c and you are asked to find out Vs.

How do we go about it? Now, let us first find an expression for Vc. The plan of attack is once you find an expression for Vc c d Vc dt will give me the expression for the current. You have the voltage across the resistance plus, the voltage across the capacitance and that will give me the value Vs that’s how I proceed.

So, how do we find the expression for Vc? It is 0 up to t equals 0 and then, starts with a ramp which is has a slope of 2 units 2 Volts in 1second. Therefore, 2 r t would be an expression like this. At this point first of all the ramp the rate of growth must be arrested at this point which was earlier increasing. Therefore, you must introduce a negative ramp of 2 units of magnitude starting at t equals. Now, if only you had only these 2 ramps then you will have a growing ramp like this.

At this time this growth is arrested by a negative ramp and the curve would have continued like that. If you add these 2 you would observe that, the resultant of these 2 would be something like this. But then, we would like to bring this down to 0. So, you must introduce a negative step of unit magnitude starting at t equals 1 in order to pull this down to 0 which means you must introduce in further a 2 ut minus 1.

So, the sum of these 3 quantities will describe this triangular pulse, that is your v f c Vc. Therefore, the current in this is c dVc dt and c is 2 Ferrets. So, 2 times the derivative of these expressions 2r t the derivative of that would be 2ut minus 2r t minus 1. The derivative of this 2times u of t minus 1the derivative of the ramp function is the step function. And then, minus 2times the derivative of a step function is an impulse function.

So, delta t minus 1, that is your I and Vs equals Vc plus r times I which is equal to 1. Therefore, Vc plus I the sum of these 2 Vc plus I will give me the value of the Vs. Therefore, you have combining these 2 4 u t a 4 u t here plus 2r t this 1 minus 4 u t minus 1 form this minus 2r t minus 1 from this 1 1 minute minus 4 r t minus 4 u t minus 1 There's another 2u t minus 1 therefore, I must write here 6 u t minus 1 minus 2r t minus 1 and then, in addition you have minus 4 delta t minus 1.

That is the expression for this 4u t plus 2r t minus 6u t minus 1 minus 2r t minus 1 minus 4 delta t minus 1 that's your Vs. If you plot this Vs would have a waveform like this to start with 4u t. So, it starts with 4 till 0 or negative values of time and at that point of time it has a ramp with a slope of 2units. Now, it is going on increasing right up to t minus 1 right up to 1 so up to 1 this will build up to a value of 6 Volts. up to 1.

At this point of time you have a negative going ramp of 2 units. Therefore, it means that whatever was increasing earlier is now pulled down by the same slope that means, if you added this function it should have continued like this. But you also have a step of minus 6 so, instead of remaining constant here this would continue. This will come down here by 6 units. Therefore, the sum of these first total 4u t 2r t 6u t minus 1 minus 2r t minus 1 would have been just like this.

This would have been the result, but we also have a delta function of minus 4 units here. So, in addition to this we have a delta function minus 4. So, that would be the shape of the Vs curve. So, all these expressions describe this particular waveform. So, you observe now that whenever, you have waveforms of this type with discontinuities and discontinuous derivatives 1 can use the singularity functions with advantage to describe the corresponding waveforms.

In fact, we now in all these discussion we did not have to say this is the for t equals less than 0. This is the expression for t between 0 and 1, this is the expression for t greater than 1. We have 1 single expression valid for the entire range and this constitutes the central advantage of describing these functions with the help of singularity functions.