Networks and Systems
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Lecture-14
Impulse Response

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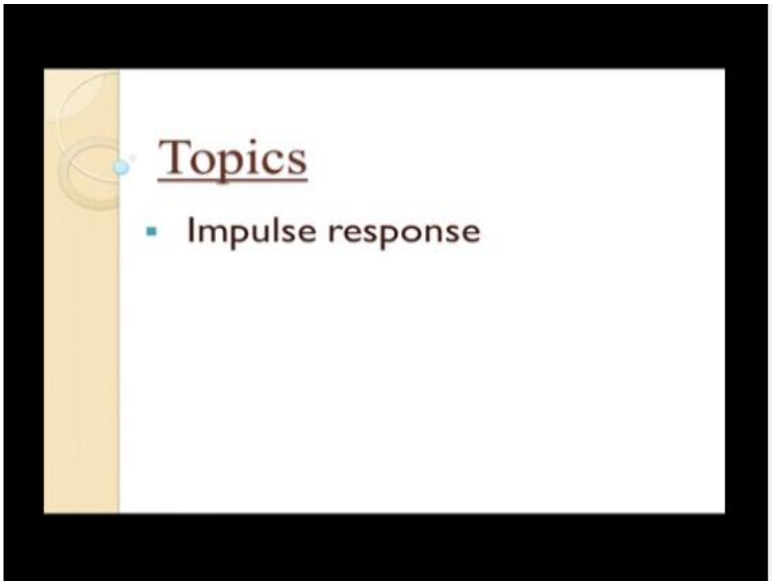

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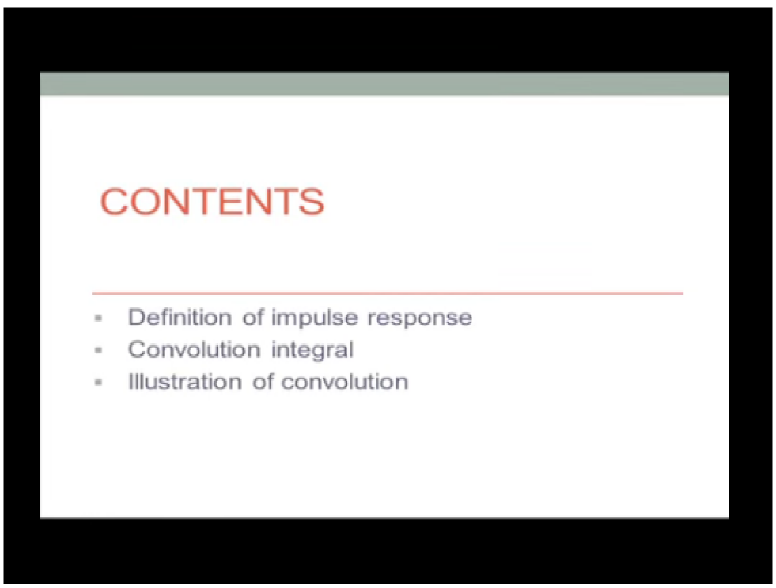

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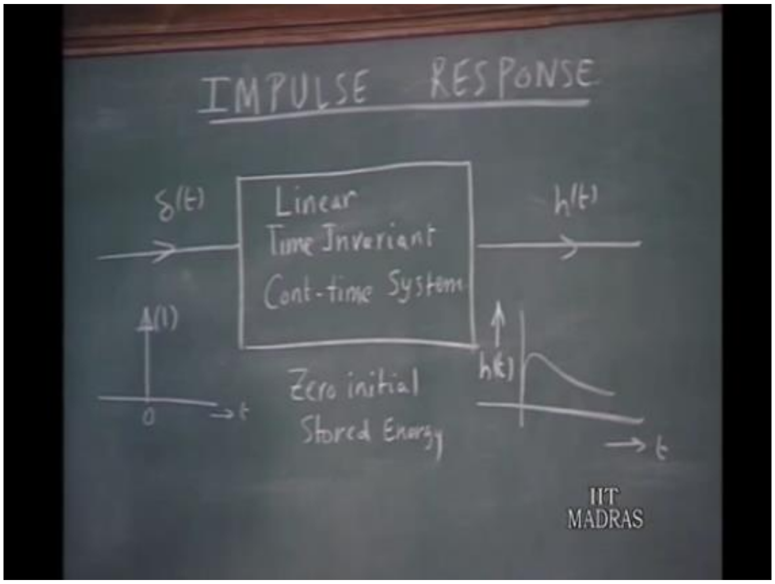

To a unit impulse will give all the information that is needed for us, to find out the response to any arbitrary input. Let us see how we go about it. This is a linear time invariant continuous-time system. It is represented as a box like this. Suppose the input is delta t; a unit impulse at the origin and let the corresponding response h of t is given like this.

We also assume in this discussion that, if this is a network, it has no initial conditions on the capacitors and inductors or in other words in the system 0 initial stored energy. The reason is, we are going to apply the principle of superposition and whenever number of excitations are present simultaneously; the responses could be superpose only if, the system does not have any 0 any initial stored energy. Otherwise, the superposition principle is violated.

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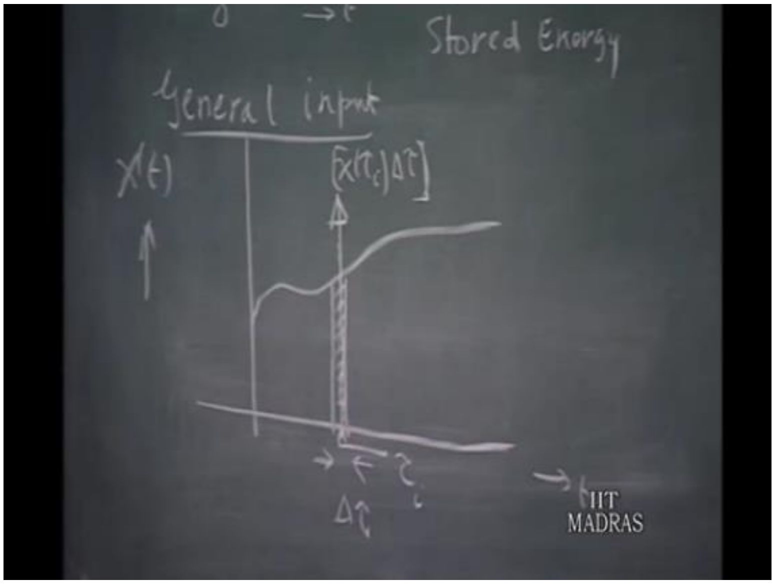

Now, any arbitrary except t given as an input something like this; this is the general input, can always be thought of; suppose, you take a small narrow section of width delta tou and then let this, at the center we will call that tou y. This point tou y and this section of the input; a narrow section of time, this area can be replaced by an equivalent impulse.

So, if you regard this, the impulse will have essentially the same area as this narrow strip and the area of the narrow strip will be x of tou y times delta tou. So, this will be x tou y delta tou that is, the magnitude of the impulse and that impulse is situated at t equals tou y. So, provided you make these slices thinner and thinner, then the approximation will be so, much more accurate.

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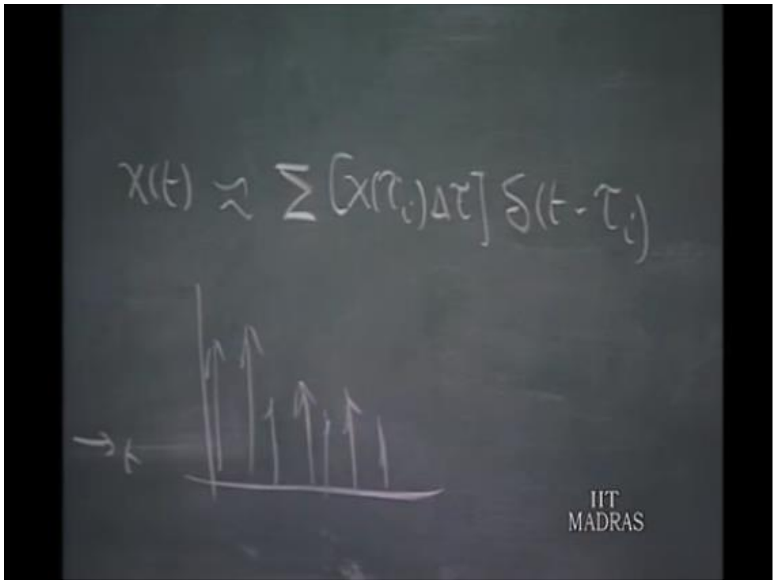

So, x t can be regarded as: approximated as number of impulses. A particular impulse which we have sketched in the figure is x tou y delta tou that is, the area under the curve, that is the magnitude of the impulse and that impulse is situated at tou y. So, this is the impulse that we are talking about.

This section of the input is replaced, by an impulse and several such impulses starting from for the whole duration of the time, will be approximately will be equal to x of t. So, x of t can be regarded as summation of several impulses.

So, that means, your x of t is now being regarded as several impulses. So, if we want to find out the response to this arbitrary input, we can find out the response due to the individual impulses and add them up, that will be your total response. And in the limit as delta tou goes to 0 then, the response will be faithfully the 1 that will be obtained when, the actual x of t is present.

That is the philosophy; that is the strategy that we adopt. Because we know the impulse response; an impulse at the origin gives rise to h of t. So, any impulse displaced by some amount will also give rise to an impulse response which, is displaced by the same amount, translated by the same amount and by additivity property; if unit impulse gives h of t, an impulse of this magnitude will be this times h of t. So, that is the principle that we are going to adopt.

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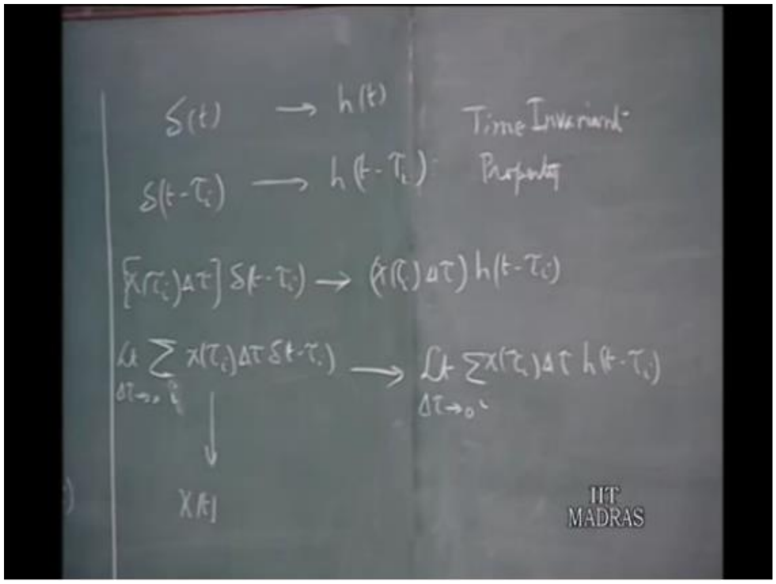

So now, we now say delta t gives rise to a response h of t. Therefore, any impulse which is displaced by a certain time interval will, give rise to h of t minus tou i. What is the principle which sanctions this? This is a time invariant property. Because, the network is time invariant, any shift in the input will correspond to the corresponding shift in the response.

That is what we discussed earlier. And instead of an impulse of unit magnitude; suppose you have x tou y delta tou that is, the magnitude of the impulse delta t minus tou i. If, that is the input you get correspondingly x tou i delta tou h of t minus tou i. So, a shifted impulse will give rise to shifted impulse response and an impulse magnitude increase by this amount will also give rise to a response which is increased by the same amount.

And now here we have summation of all such things; x tou i delta tou delta t minus tou i summation, limit delta tou goes to 0 that is your x of t and that correspondingly give me limit as delta tou goes to 0 of x tou i delta tou h of t minus tou i not 1 such response number of such responses, some done in various tou i's or i will put some done in various i's. So, in the limit, this entire thing goes to x of t.

So, as you make delta go to 0, all these impulses is equivalent to the input x t as we have seen. From x t we will generate this impulse. You make them closer and closer, approximately that is the same as x of t. So, x of t is regarded as the series of impulses and correspondingly this will now be; you have a number of these things delta t is an integration of x tou h t minus tou.

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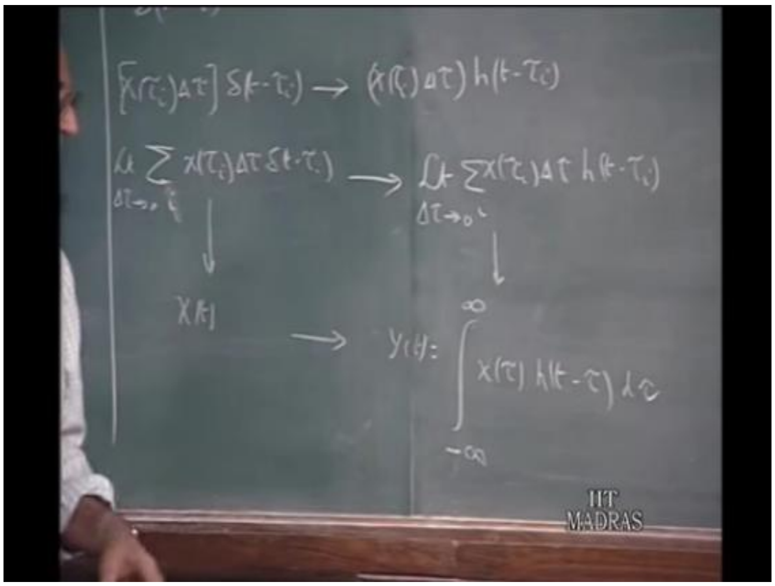

So, tou i is not discrete points. Continuously it exists delta t d tou, that is your y of t. So, y of t is obtained as x tou h t minus tou d tou. That is how we can obtain in the limit, the output y t corresponding to this x of t and in general, the most general case; this integration must last for minus infinity to plus infinity, but in the more usual cases the limits can be further restricted as we will see at a later point of time.

So, in other words given any input x t; the output can be obtained as x tou h t minus tou integrated between minus infinity and plus infinity. This is the function of time. Tou is the variable of integration that gets cancelled out, final result will be a function of time. So, this t stays and that is your y of t. And this particular type of formation is usually represented as h t x t star h k. This is referred to as convolution integral.

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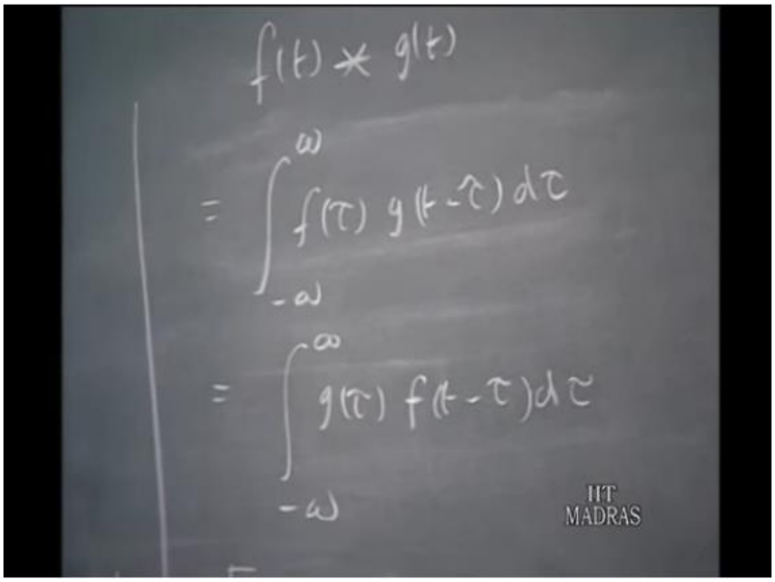

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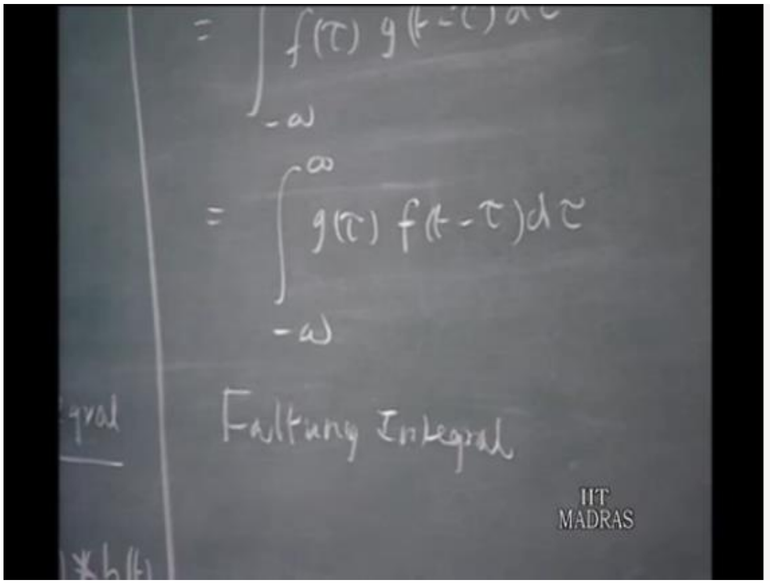

So, if you convolve 2 functions of time; x t and h of t the meaning is; this particular integration: x tou h t minus tou d tou. So, let me rewrite this. We will come back to the impulse response and the calculation of y of t, but let us spend a few minutes on the meaning of convolution integral. So, convolution integral of 2 quantities; f t convolution of f t and g t represented in this manner is f of tou g of t minus tou d tou. This is a symmetrical arrangement.

We can also write this as: g tou f of t minus tou. Normally the integration is between minus infinity and plus infinity. This is called the convolution integral sometimes called Faltung Integral. That is the German name is Faltung Integral.

So, what is involved in this integration? It is instructive to see, physically how you interpret this integration; this convolution principle. So, let me illustrate this by means of this picture here.

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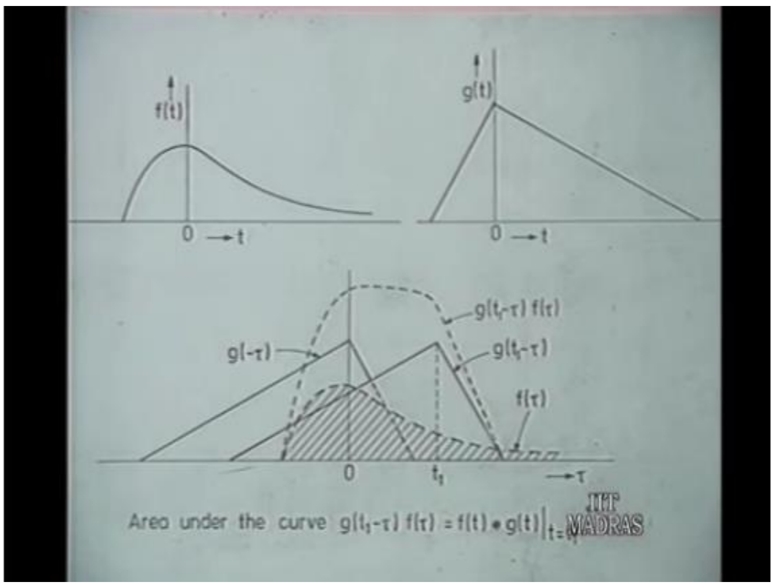

You have 2 time functions f of t and g of t. You would like to find the convolution of these 2. What you do is; you keep 1 of these f t fixed. So, let us say this is f t. This is fixed. And this g of t suppose, you reverse this time axis that means: g of t is like this. This, what we are having here is g of minus t. This is g of minus t that means, you fold it, g of t is folded along the y axis.

So, the sequence of values which it takes for positive t will now, take for negative t and vice versa. So, you fold it along the y axis. This is g of minus t. And then, suppose you want to find out the convolution integral for particular value say t 1. Then, this g of minus t you advance it in the forward direction by an amount equal to t 1. That mean, this is g of minus t, you advance it by an amount t 1.

This becomes g of minus of t minus t 1 that is, t of t 1 minus t. That means you shift that, you are having this. Now, we have the product of in this convolution here, f tou g t minus tou. Therefore, what you are doing now is, this original f of x that you are having or f of tou that you are having and this shifted g of t, you multiply them out and this will be your result. The product of these 2 functions will be this and you take the area under that curve that is the integration that, will give the convolution value at t equals t 1.

In other words, what we are doing is; you take 1 time function, keep 1 time function as it is. The other time function you fold it then shift it, by the required amount then, multiply then take the area under the curve that you integrate. So, the keywords are: fold, shift, multiply and integrate. These are the 4 words keywords that are involved here. This will be explained a little clearly more clearly perhaps, by taking this particular thing.

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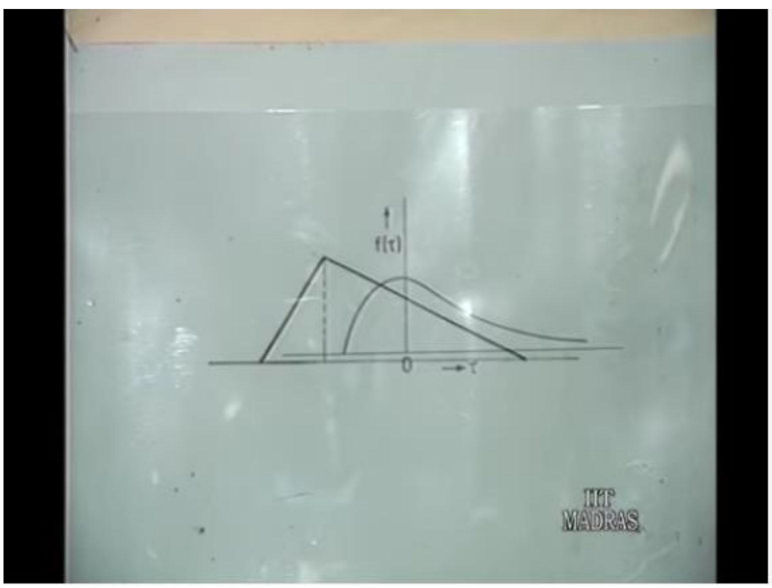

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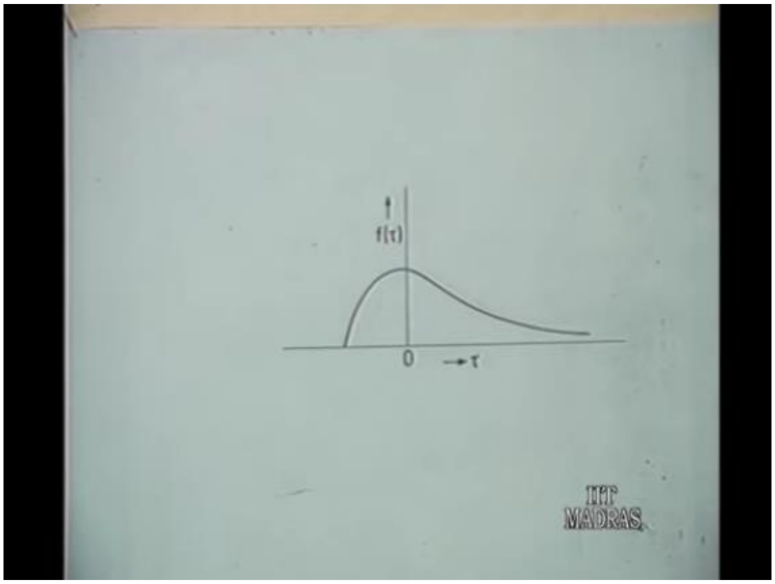

Suppose, this is f of t and then you are having a g of t here. This is g of t; the triangular curve. So, what we are doing now is; instead of having like this g of t you shift, you fold it like this.

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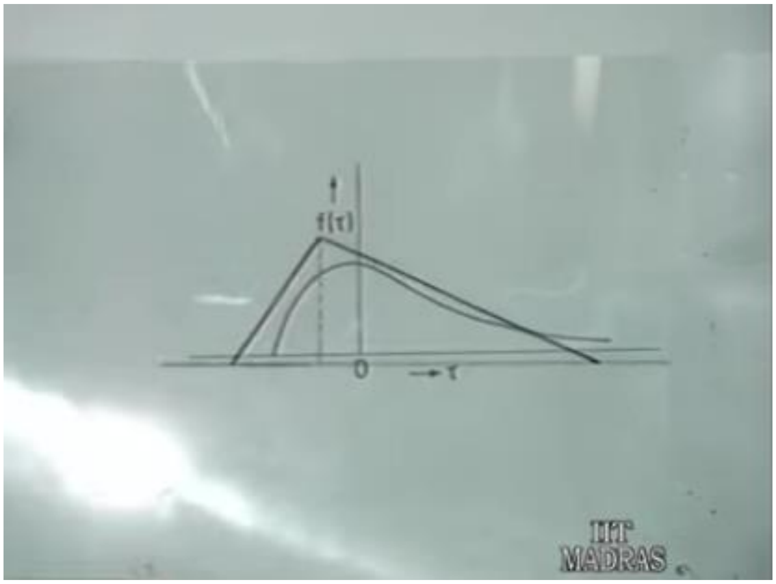

So, g of t originally was like this. Now, you folded it

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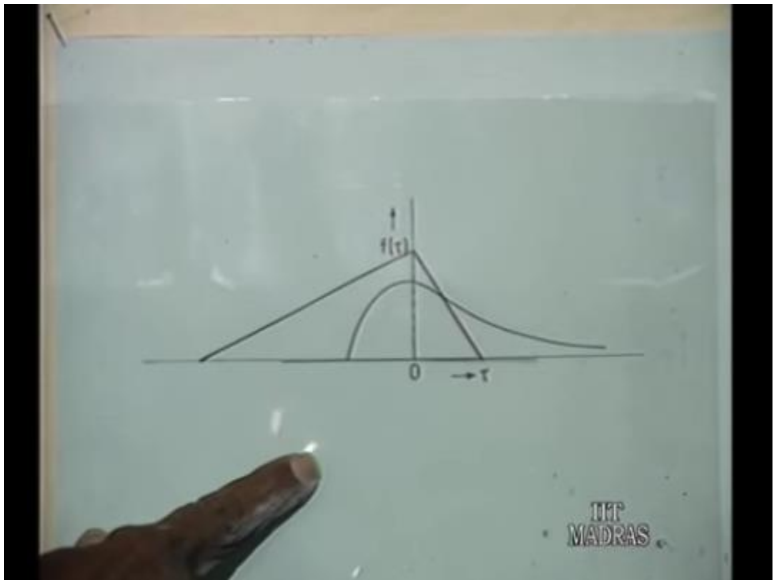

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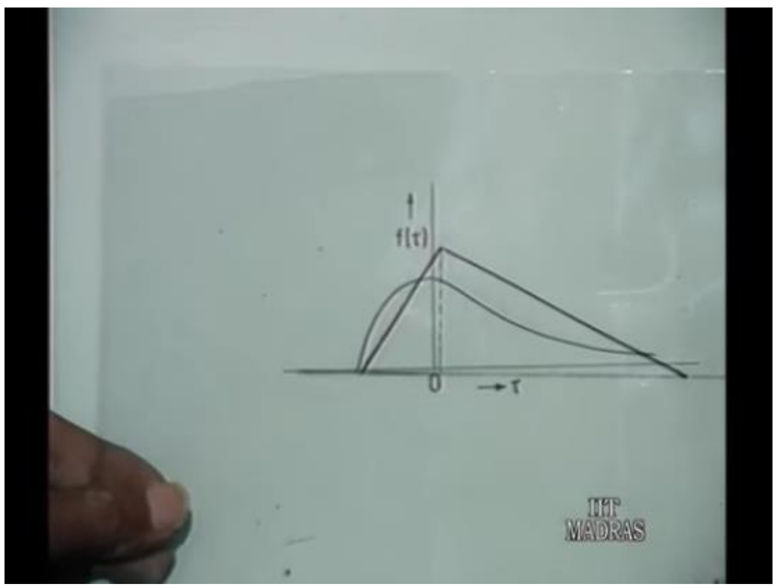

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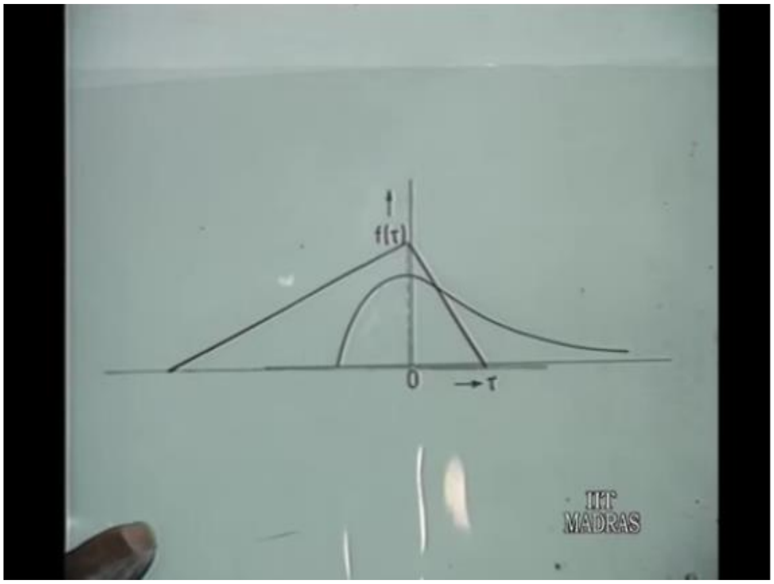

That means you made this run backwards. This is now g of minus t. Now, depending up on the point where you want to find the convolution, you fold, you shift it like this.

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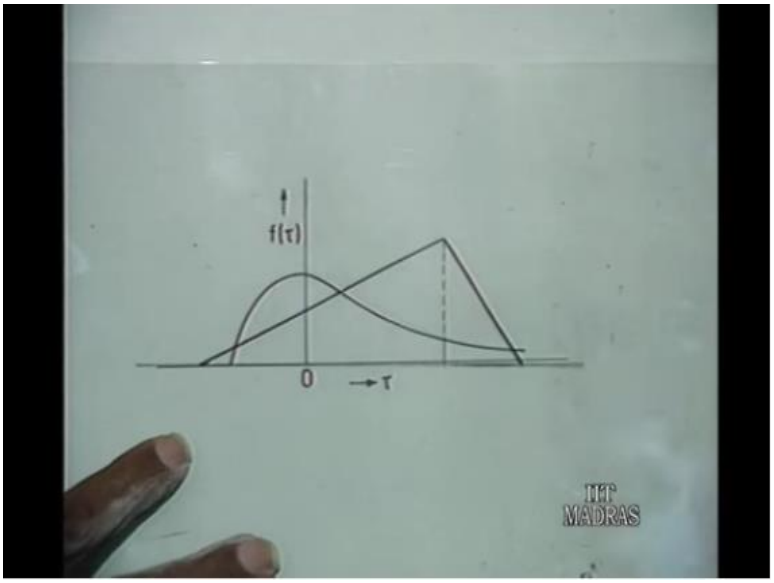

So, for various shifts you find out the product of these 2 curves and find out the area under that curve. And this is how, 1 can calculate the convolution values, integral values for elementary time products. So, the key words here are: fold, shift, multiply and integrate.

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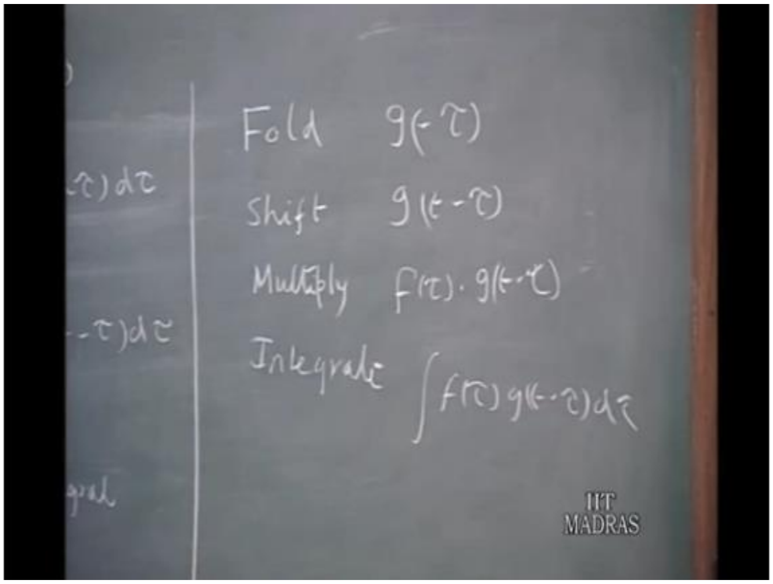

So, you are having here f g of t, is a time function, g of minus t is what is obtained by folding. Shift it; so g of t minus tou. You shift it by the amount at which you want to calculate the integral. At t equals, t 1 you want to find out the product of this and t 1 you shift it by the amount t 1. You have general t g of t minus t 1. Multiply; so f tou is multiplied by g of t minus tou and then integrate. That means you find the area under this curve.

So, these are the steps that are involved. If you keep this at the back of your mind, we can find out the convolution of simple functions quite easily, without going for the mathematics, from physical graphical approach, you can find out the convolution of different functions very easily.