**Networks and Systems**
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**Lecture-18**
**Evaluating Four Series Coefficients**

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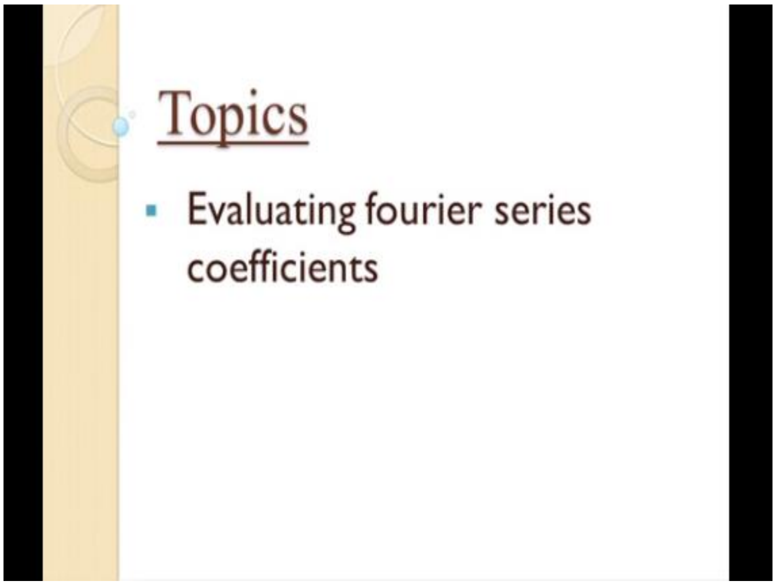


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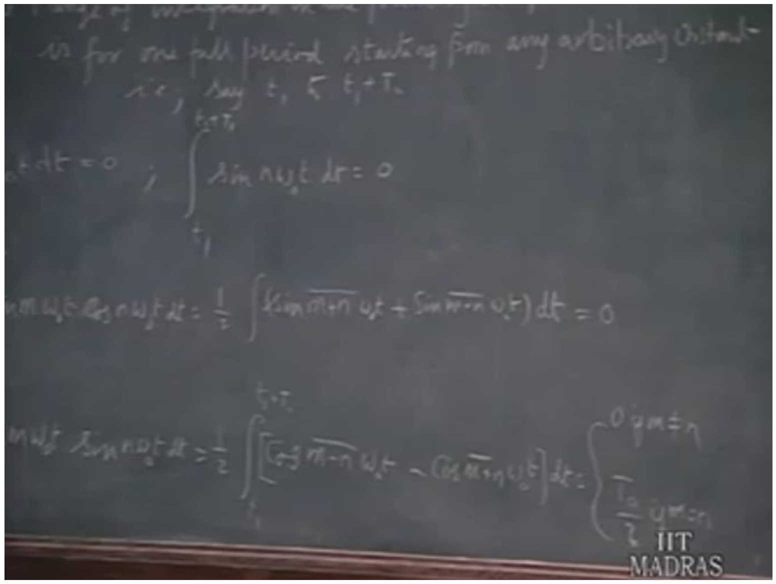


Before we set up expressions for the evaluation of the Fourier coefficients we need some background information. We have set up a series of integrals and show their values and we assume that, all these integrals the range of integration in the following integrals is for 1 full period starting from any arbitrary instant that is say t 1 to t 1 plus t 0.

Now over such an integral such a range we know that, cos n omega 0 t d t over 1 full period goes to 0 because in this period 1 period of fundamental you have n complete cycles of the cosine wave there will be as many positive loops or negative loops. So, the average will become 0.

Similarly, we also have for example, this could be t1 for t1 plus t 0 it could be from 0 to t 0 as well similarly, sine n omega t 0 t will also we likewise be 0. Now, if i have a product of a sine function and a cosine function sine m omega 0 t cos n omega 0 t d t this, can be expressed as half of over the same range sine m plus n omega 0 t plus sine m minus n omega 0 t d t again each of these terms will produce a 0 integral because integration is over a complete number of cycles integral number of cycles.

So, both these terms go to 0. So, this will be 0 for all m and n. On the other hand, if i take the product of 2 sine terms sine m omega 0 t sine n omega 0 t d t this will be half of the integral of cosine m minus n omega 0 t minus cosine m plus n omega 0 t d t. Now as far as m plus n omega 0 t is concerned that again is an integral over a complete number of cycles that goes to 0 no problem over that as for the first term is concerned.

If m minus n is an integer which is not 0 then once again it goes to 0. On the other hand, if m equals n this is the constant and therefore, over the complete period it gives the value t plus, it gives the value t 0. So, the upshot is we can say this will be 0, if m is not equal to n and this is equal to t 0 by 2 if m equals n.

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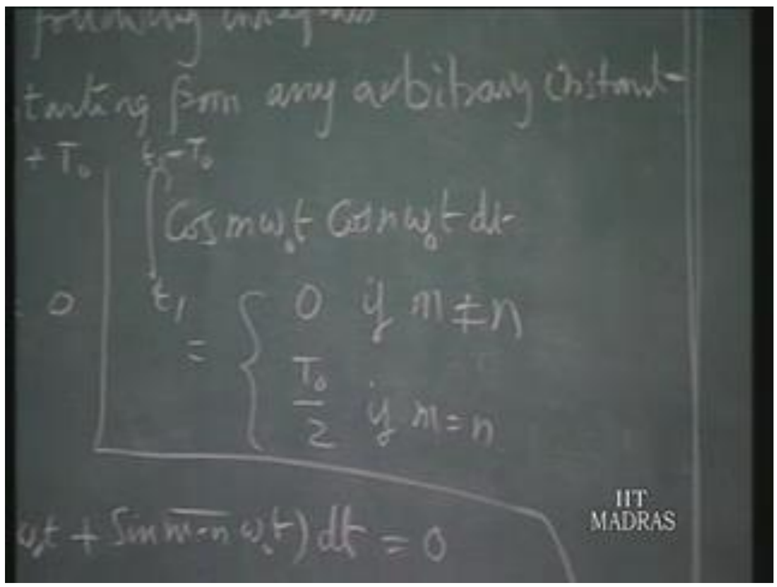


In the same manner, we can also write over a complete period cos m omega 0 t cos n omega 0 t d t is 0, if m is not equal to n equals t 0 by 2 say t1 to t1 minus t 0 t 0 by 2 if m is equal to n. So, these are the background results we need for setting up formulas for evaluating the Fourier coefficients.

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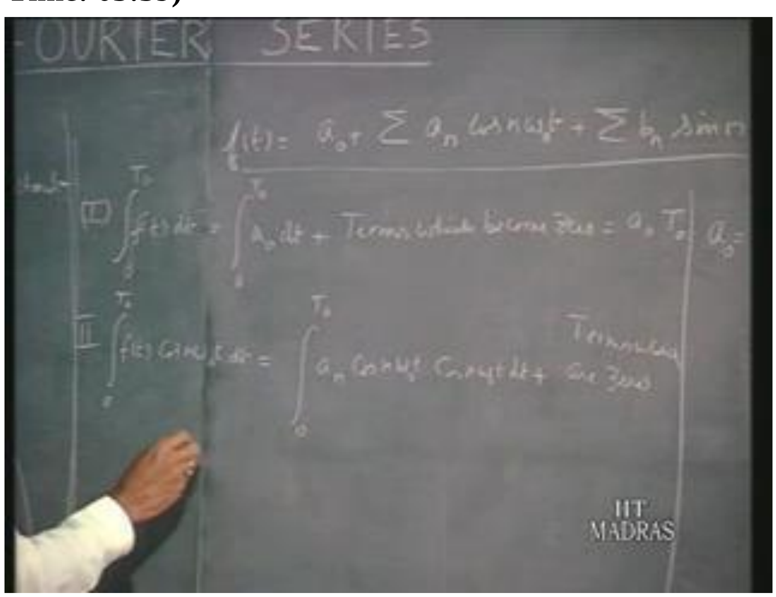

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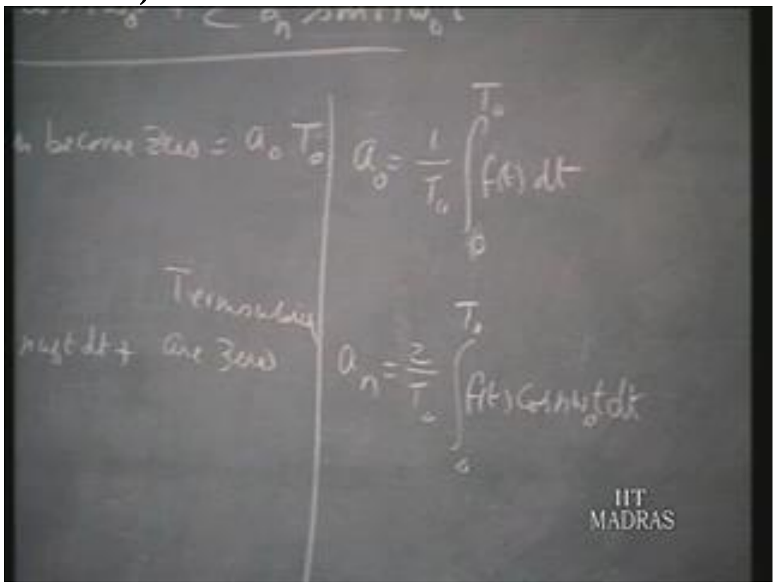


You recall that, the formula for we said that, f of t can be expanded as a 0 plus a n cos n omega 0 t plus b n sine n omega 0 t suppose, you multiply both sides no let us, first find out a 0.

Suppose, you integrate both sides and assume that as for the right-hand side is concerned you can carry out the integration term by term. Then we have f of t d t, once again the integration can be from t 0 t1 to t1 plus t 0 or for the sake of convenience let me, take it as 0 to t 0 it could be form any arbitrary small t 1 to small t 1 plus t 0.

Now, similarly on this side if you carry out the integration for each 1 of these terms by virtue of the results that, we already established each 1 of them will have f 0 average value over a period fundamental period. Each 1 of these terms will have a 0-average value over a fundamental period.

So, it is only this term which give rise to a contribution therefore, i can write this as 0 to t 0 a 0 d t plus terms which become 0 as a result we have this is a 0 into t 0 sorry a 0 times t 0. This immediately gives us a way of evaluating a 0 you have a 0 equal 1 over t 0 f t d t say 0 to t 0 or t 1 to t 1 plus t 0 likewise.

If i multiply the left-hand side by say cos n omega 0 t and then integrate the product f t cos n omega 0 t over, a complete period each 1 of these terms must be multiplied by cos n omega 0 t and integrated over a complete period. Obviously, a 0 multiplied by cos n omega 0 t over a complete period will vanish and every 1 of these terms will also vanish except for the term which involves index n because we have seen earlier that if you take the product cos m omega 0 t cos n omega 0 t and integrate this will yield non-0 only if m equals n.

Therefore, if you are multiplying this whole set of terms by cos n omega 0 t only when the index here happens to be n it will give rise to a non-0 product. Therefore, i can write this therefore, 0 to t 0 a n cos n omega 0 t multiplied by cos n omega 0 t d t plus other terms which go to 0 other terms which are 0.

So, this will now be a n times t 0 by 2 because when both these indices agree this is t 0 by 2 a n t 0 by 2. So, you can write this as a n equals 2 up on t 0 0 to t 0 f of t cos n omega 0 t d t.

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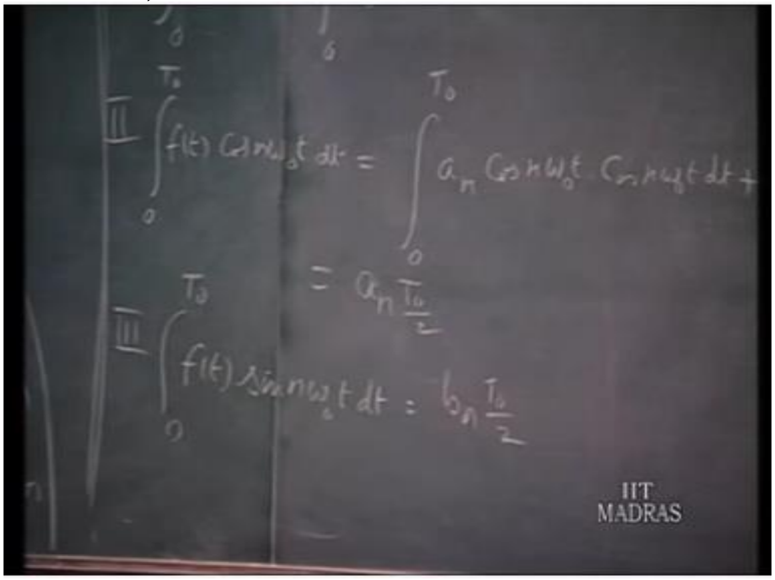


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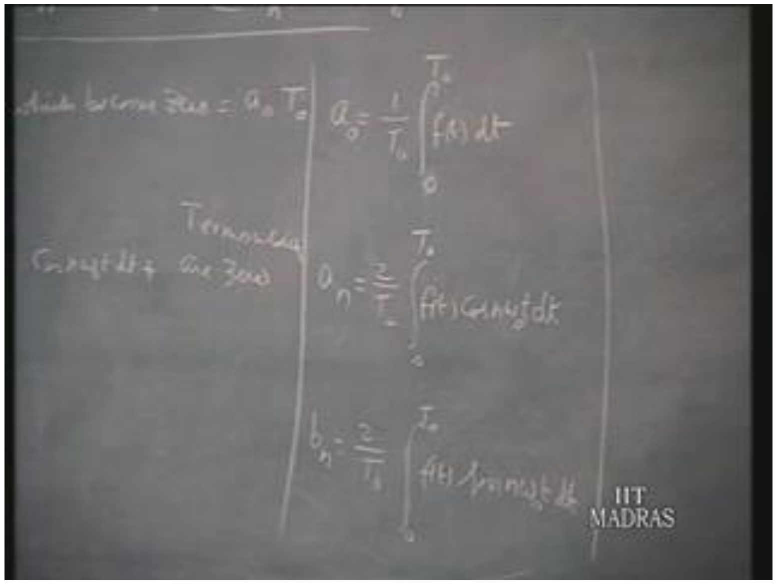


Similarly, it can be shown that by multiplying this by sine n omega 0 t d t you get b n times t 0 over 2. So, b n will be 2 up on t 0 0 to t 0 f of t sine n omega 0 t d t. So, these are 3 formulas which can be used to evaluate these various integrals.

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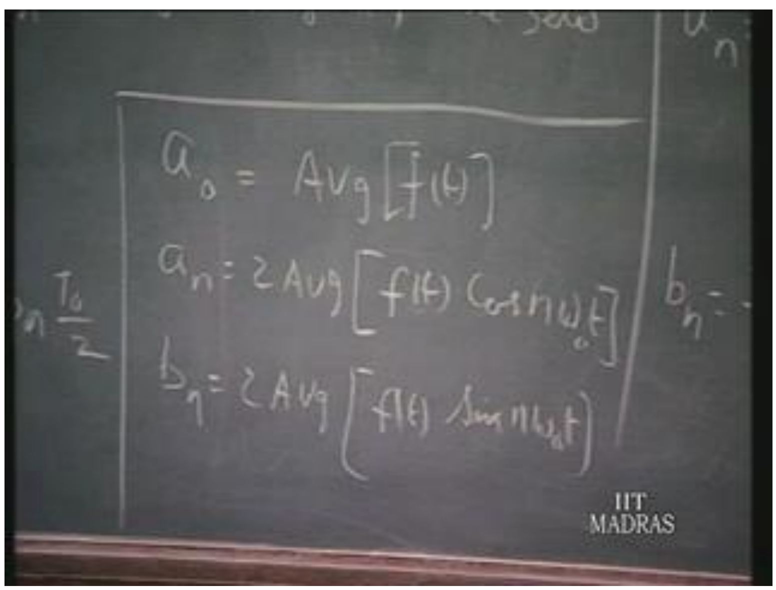


It is easier to remember these as saying a 0 equals the average value of f of t. Whenever, I am talking about average, it is average over 1 complete period and a n is 2 times the average value of the product f of t cos n omega 0 t and likewise b n is 2 times the average value of f of t sine n omega 0 t. So, these are the 3 integrals that 1 makes use of to evaluate the various Fourier coefficients.

It easy to remember this and this form or certainly these are equivalent to this. Now, as far as the integration is concerned, if you have got n little expressions for f of t you can carry them out analytically or you can also do them numerically. If f of t is given for example, in a graphical form a wave form is recorded and you would like to perform the Fourier series analysis of that then you can carry out these integrations in by numerical methods.

To sum up them in this lecture we started with the system function of a linear time invariant system h of s and how it relates the output and input and then we saw that. If the input is a sinusoidal function of time, how the output can be easily computed by multiplying the input phasor with h of j omega which is a system function under sinusoidal conditions and obtain the output phasor.

This advantage of phasor notation and the associated analysis of the sinusoidal inputs we would like to carry over to functions which are not sinusoidal as well. And as a first step we took up how a periodic function can be decomposed into various harmonic components of the basic frequency as well as a d c term and we started with this expression for periodic function in terms of a d c plus various harmonic components.

And once, we obtain the response to each 1 of these harmonic components we can using these phasor methods, we can obtain the total response by superposing the various responses. And for this purpose, we arrived at the formulas for the d c component coefficient for the cosine terms and the coefficient for the sine terms and these are the 3 expressions which we would like to remember in calculating this a and b coefficients.

In particular, the d c the average value of f of t over a complete period. A n is twice the average value of the product of f of t at cos n omega 0 t and b n is twice the average of f of t sine n omega 0 t.

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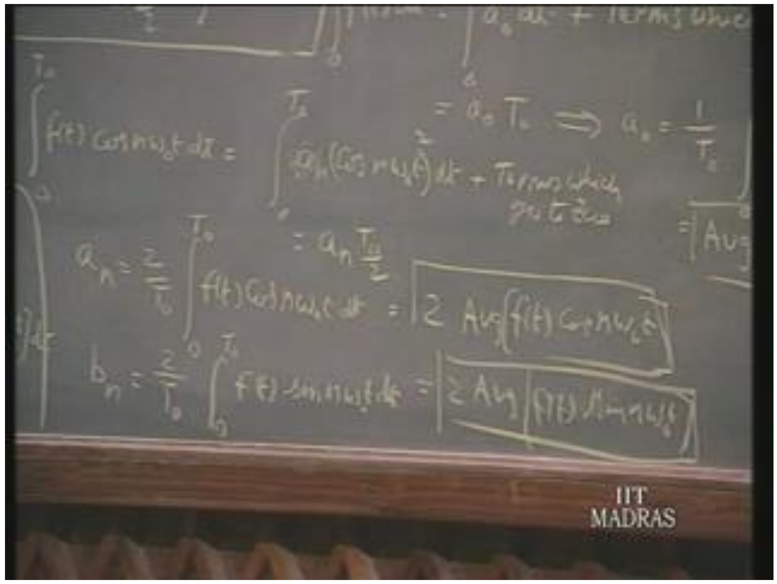


For every function that, we have we do not have to really calculate a 0 a n and b n it turns out that, there are symmetry conditions which enable us to guess even before hand, what parameters what coefficients are present and what are absent, these come under the symmetry conditions the topic, we will take up in the next lecture.