Networks and Systems
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Lecture-19
Symmetry conditions

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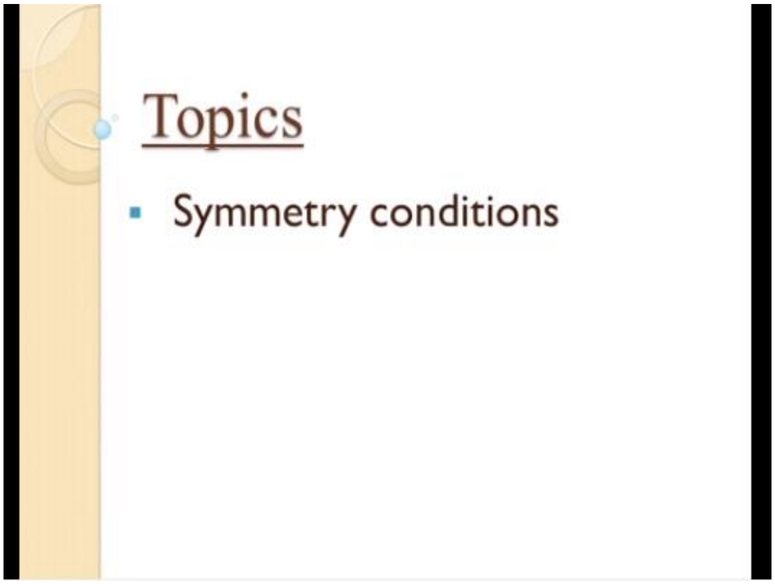

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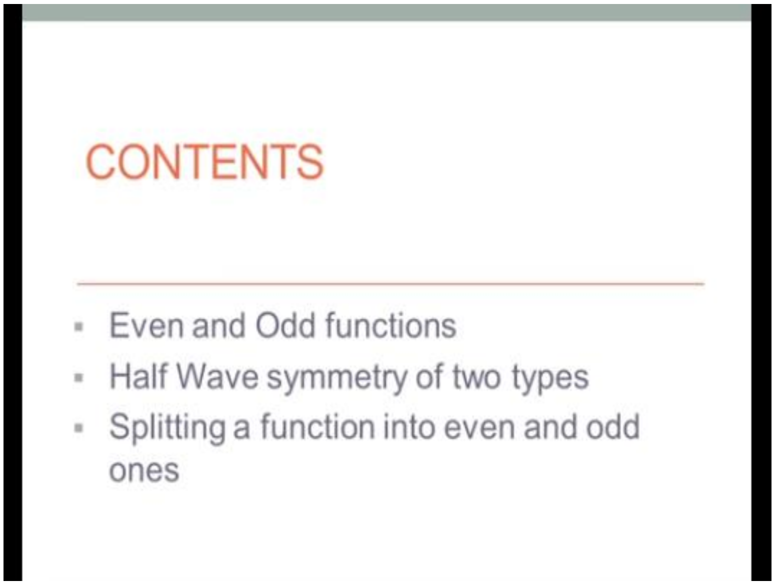

To the concept of Fourier series and to the evaluation of Fourier coefficients an and bn which are required the set up the series. In many situations, the waveforms that we have to deal with exhibit certain type of symmetries which we can take advantage of and simplify the work involved in the evaluation of these various Fourier coefficients.

This will now take up for study under the heading symmetry conditions. We will talk about 4 kinds of symmetry and then see how each individual set gives rise to some advantage in the matter of evaluation of the Fourier coefficients.

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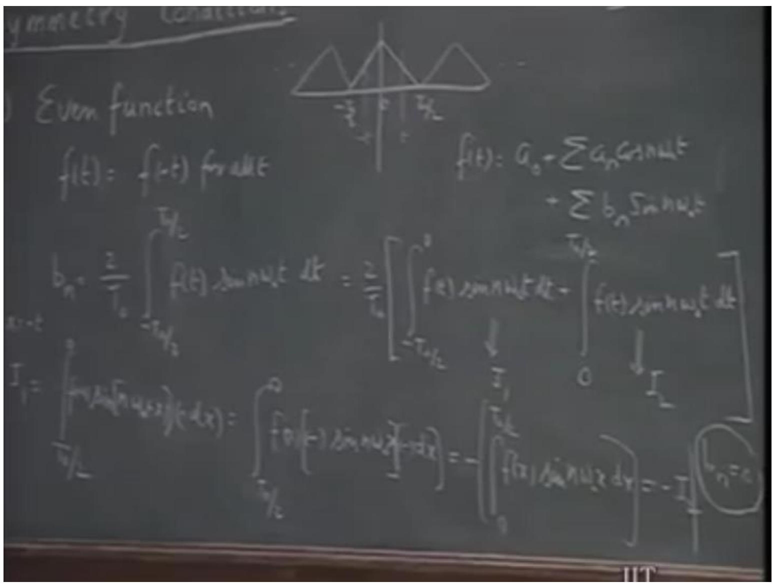

To start with, suppose we are given a periodic function which is an even function of time then the definition of even function is f of t equals f of minus t for all t. Example of that would be suppose I have the function which varies in this manner for positive t from 0 to t not upon 2. In the negative direction, it takes for any particular t at minus t the values must be the same.

So, if this is a periodic waveform if this is the basic period it repeats itself like this and this is the even function of time. You recall that the basic Fourier series expansion is a not an cos n omega not t plus bn sin n omega not t plus b n sin n omega not t the whole series of terms. Now these trigonometric functions here or this a not is of course is a constant. It maintains its value so we will consider that to be an even function, cos terms are even functions of time but sin terms are not even functions of time they are odd.

So, if this function even we can expect that we have no place for this sin terms in its expansion because, when you substitute minus t for t the values of these terms will remain the same but this set of terms will have their value reversed. Consequently, f of t can no longer f of minus t so we intuitively we can expect that all the bn terms go to 0. let us prove this more little more rigorously.

So, if we evaluate b of n you recall the expression for b of n is 2 upon t not 0 to t not or i can also for the sake of convenience in this regard. I will take this from minus t not upon 2 plus t upon 2 because this is more convenient in this contest f of t sin n omega not t dt.

I can break up this integral into two parts 2 upon t not minus t not upon 2 to 0 f of t sin n omega not t dt plus 0 to t upon t not upon 2 f of t sin n omega not t. Now we are going to show presently that this integral will be exactly the negative of this therefore bn will be 0. To do that let me call this integral I1 and this I2 our goal now is to show that I1 is the negative of I2.

So, to prove this let me evaluate I1 separately I1 will be this expression. But now in I1 I will be put x equals minus t so I will change the variable of integration instead t I use x therefore dt will become minus dx and since the variable of integration now is x I must put the appropriate limits x. So, when t equals minus t not upon 2 x will be t upon 2 and when t is 0 x of course will be 0 f of t will become f of minus x and then you have sin n omega not minus x minus dx.

By virtue of this even character of this function we can write this as sorry this is t not upon 2 again f of minus x equal f of x because of this relation and then sin n omega not minus x will be minus of sin n omega not x and then you have another dx is with negative sin out in front. Now, you see this negative sin and this negative sin can be cancelled and if I interchange limits of integration then I will get another negative sin.

Therefore, ultimately this will be minus of 0 to t not upon 2 I interchange that means you have got 3 negative sins one for virtue of interchange of integral limits one because of negative sin here and one here I have got f of x sin n omega not x. Now, this integral here is exactly the same as this because this is a definite integral and the variable of integration could be any value its dummy index really.

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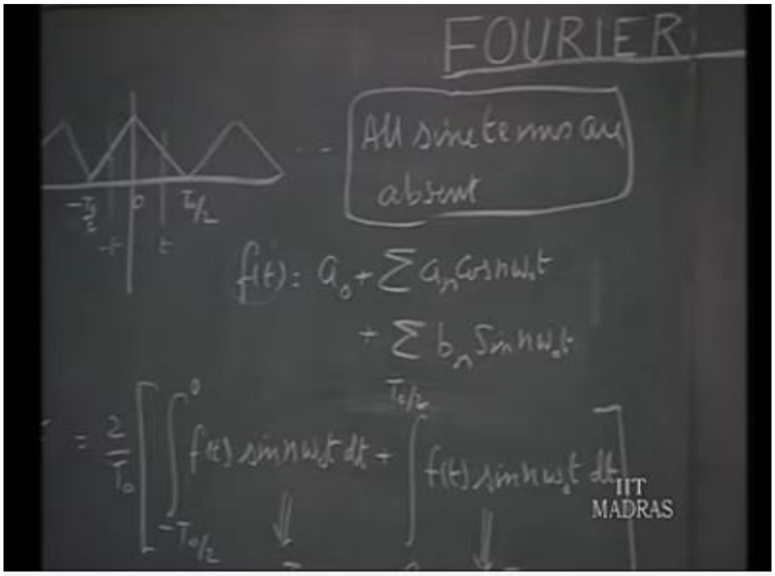

So, this integral f of x sin n omega not x dx could as well be written as f of t sin n omega not x dt which is indeed this and because this negative sin out in front this can be written as minus I2. So, the upshot the analysis is that bn 0. So, if the function is even we immediately can conclude that all sine terms are absent this is the result that we get if the function is even.

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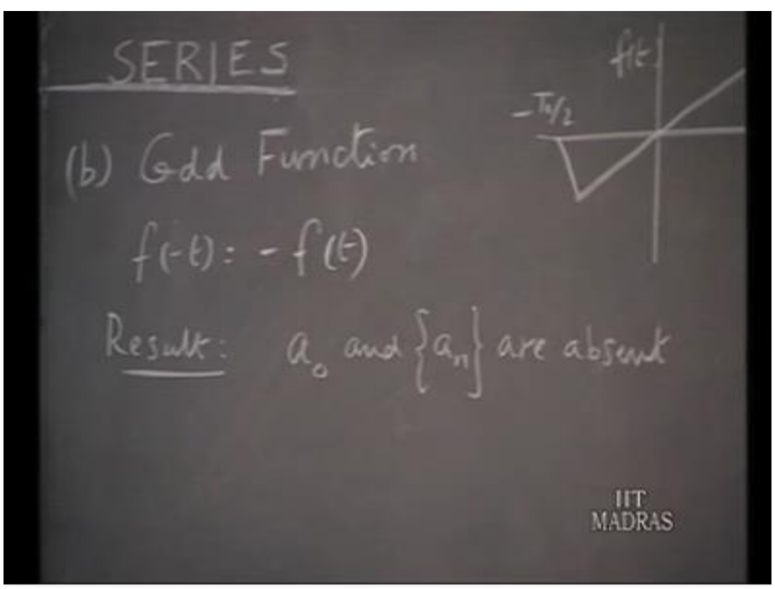

Naturally, we would like to ask what is the corresponding result for an odd function? How does an odd function look like? If I have this as the variation for positive t the negative t it would be like this. So, if I say this is t not upon 2 this is minus t not upon 2 so this is f of t over a one period whatever sequence of the values its takes in the positive direction of t the negative direction it will take the negative of those corresponding values. So, f of t we will say f of minus t equals minus of f of t minus of t.

Now, if you again look at the general expression of the Fourier series we observe that these are the terms which gives rise the odd character of the function and these are the terms which are even. Therefore, if you change t to minus t we like the entire function to value reverse it is sin. However, these terms a not and an terms will have the same value for minus t and plus t.

Consequently, they spoil the character of the odd function and therefore we expect that these terms to be absent and only bn terms will present and I will leave this is an exercise for you to show is the similar fashion as here that in this situation the result is. I will not work this out I can show it on similar lines result is a not and an terms are absent. So, when we see a periodic function which is an odd function of time you do not have to spend time in evaluating a not and an. We immediately know they are 0 so all we have to do is work out the values of the bn coefficients.

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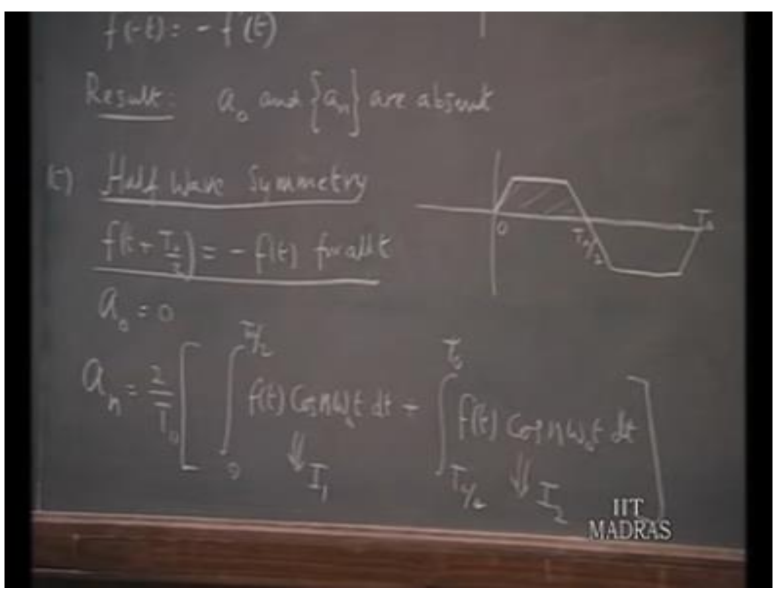

A third type of symmetry which is quite important because it comes up in several occasions is half wave symmetry. Basically, what it means is that if you have particular variation for half the wave the period this succeeding half period it will have exactly the negative of that variation f of t plus t not upon 2 is minus f of t for all t, such waveforms are quite common particularly in electrical machinery when you have a rotor rotating and inducing the waveforms in the stator.

Whatever, it is then alternatively the conductors comes under north pole and south pole and whatever sequence of values for emf are induced when the north pole is operation the values will be reversed in the south pole is an under operation therefore the waveform exhibits this kind of symmetry.

The same set of values are reproduced with a negative sin in the succeeding half cycle. So, what is the off shoot? What is the consequence of this first of all, we observe immediately that in the since the same set of the values of occurring the negative sin the succeeding half cycle the average of the period of the average the function over the complete cycle must be 0 which means the dc value is 0.

What are the conclusions? what are the conclusion you can drop from this may not be evident but let us work them out. Suppose, I take an 2 upon t and this time I break the integral for convenience 0 to t f t not upon 2 f of t cos n omega not t dt plus t not upon 2 to t not f of t cos n omega, by this time you must got an idea what we are going to do next you would like to see. Relate this integral with this therefore this purpose let us call this I1 call this I2 and you would like to compare the values of I1 and I2 making use of this result.

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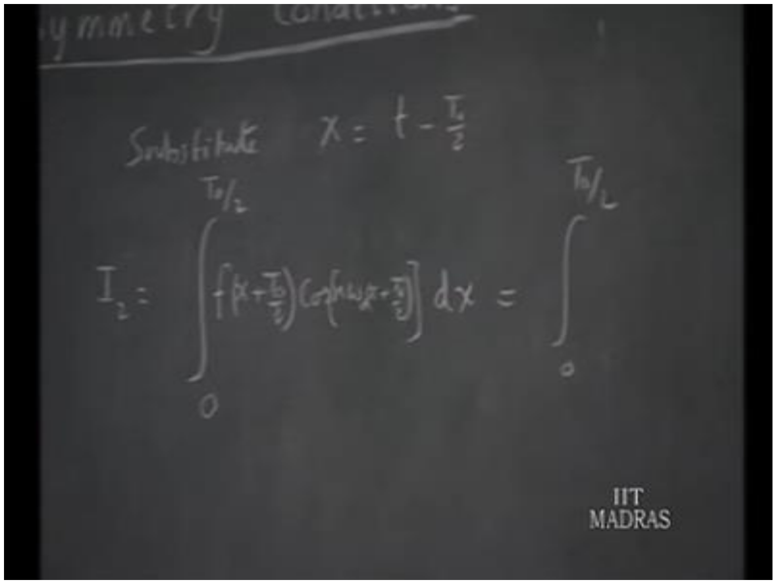

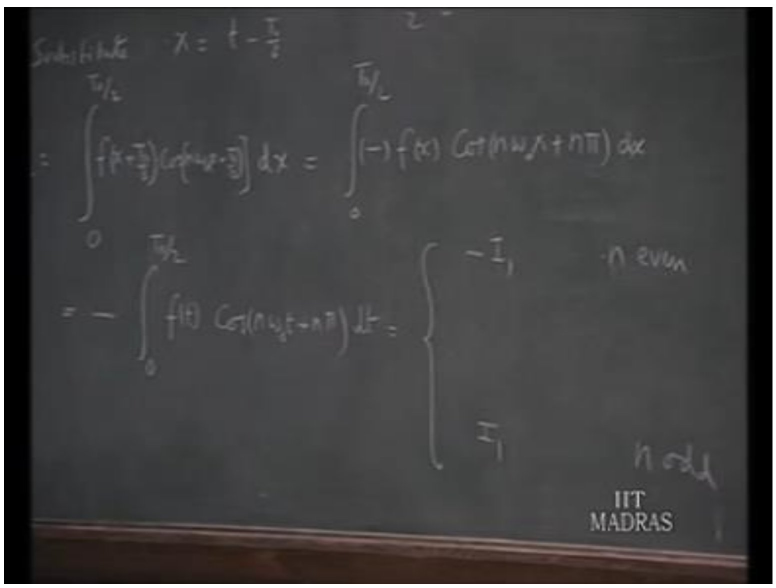

Let me evaluate I2. In this let me substitute x equals t plus t not upon 2 or rather x equals t minus t not upon 2, then I2 will be so I am going to substitute x for t dt equals dx therefore the variable integration is x. So, I have dx then the from small t equals t not upon 2 x will be 0 then the small t equals capital t not then x will be t not by 2.

So, you can see how the integral limits are corresponding to this straight away f of t will be a f of x plus t not upon 2 because t equal to x plus t not upon 2 and we have addition cos n omega x plus t not upon 2. Okay, we expects what it is okay now this will be 0 t not upon 2 and by the virtue of this relation that f of t plus t not upon 2 minus f of t f of x plus t not upon 2 is minus of f of x and then we have cos n omega x plus n omega t not upon 2 omega not t not is 2 pi therefore this is n pi therefore this is n pi.

So, this will be I take the minus sin out in front 0 to t not upon 2 now that I have outlived the residue for use of x. Let me go back to t because after all this is a dummy variable I can go back to t this will be f of t cos n omega not t plus n pi dt right. Now, we have the same pi that see what it becomes, so let us take two cases suppose n is even if n is even cos n omega not t plus 2 pi 4 pi and so on this simply n omega not t.

Therefore, this will be f of t cos n omega not t dt which is the same as I1 limits of integration is the same the integrand is the same therefore this will be the minus I1. If n is odd cos n omega not t plus pi will be minus cos n omega not t the 3 pi is also the same therefore that it will become I1 right.

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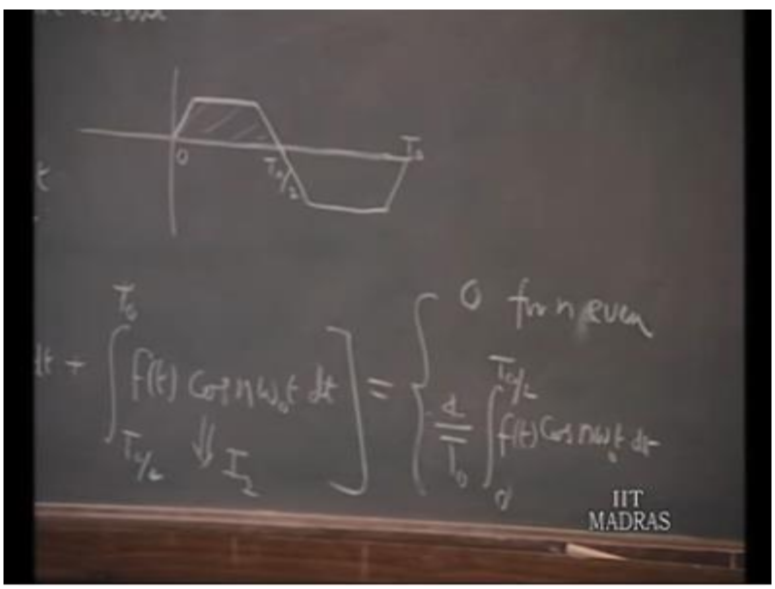

So, the upshot of this analysis that is an turns out to be 0 for n even and 2 times I1 for n odd right. Therefore, finally that conclusion we have is, all the a terms with even indices will become 0 same analysis can be extended for bn terms also.

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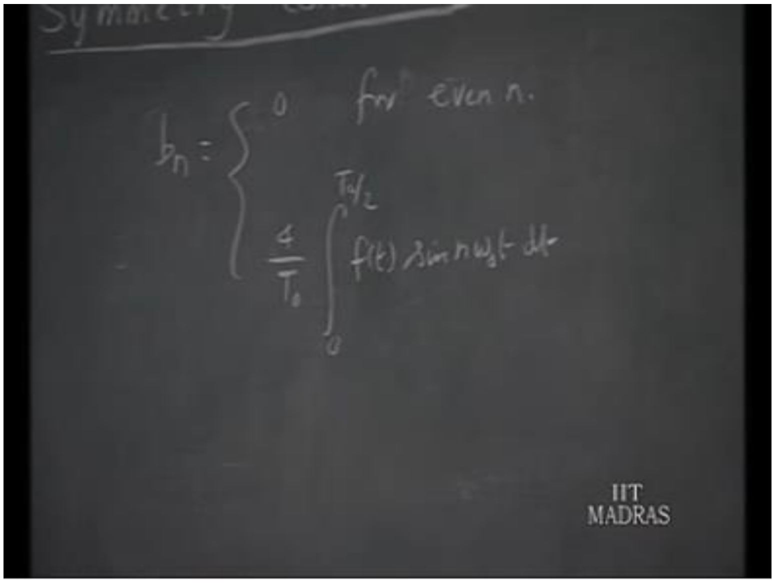

You can show that bn will be 0 for even n and for odd n you see an for odd n terms out to be. I am sorry 2 I1 I must also write here because I am equating this I must also multiply by 2 by t not. So, I will write complete expression here so this 2 by t not multiplied by 2 I1. Therefore, that means 4 by t not 0 to t not by 2 f of t cos n omega not t dt. So, here also you have 4 by t 4 by t not 0 to t not by 2 f of t sin n omega not t dt so that is the result.

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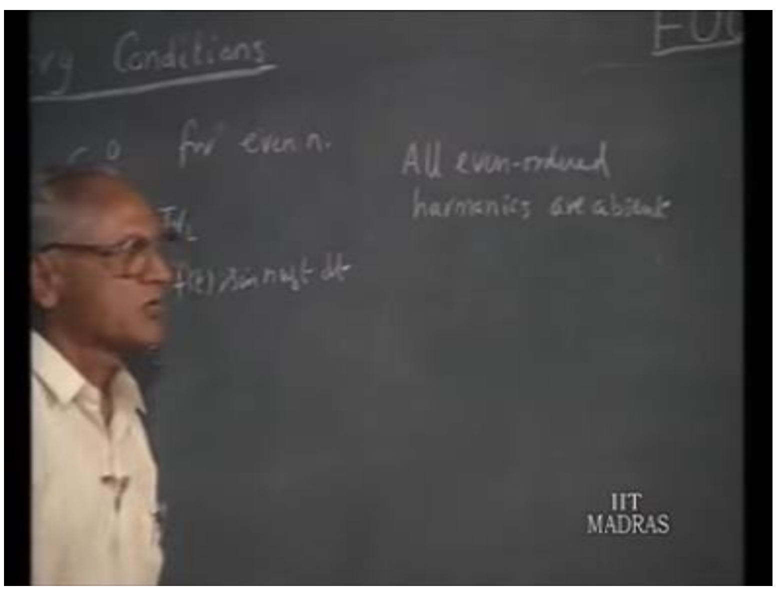

We can summarize this by saying that all even ordered harmonics are absent. So, when you have this kind of symmetry as you have in the waveforms generated by the electrical machines you will have only the fundamental third harmonic, fifth harmonic, seventh harmonic and so on. You cannot think of having in the second harmonic, fourth harmonic nor even the dc term because the average of it is going to 0 and for the evaluation of the odd harmonic components an and bn we have.

Instead of taking the integration over the complete period is the permissible for us to integrate over the half the period and consequently have 4 by t not instead of 2 by t not that means you are taking twice the average over for half the period rather than twice the average the complete period. So, you can also dispense with lot of integration and confine yourself to one half cycle whenever you have this kind of symmetry important to that all your harmonics are absent and dc is also considered to be even harmonics that is also absent.

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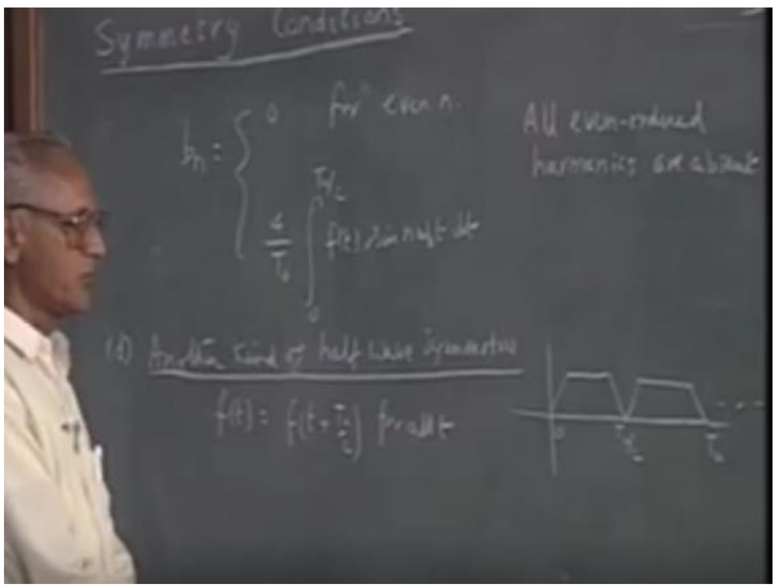

We have another kind of half wave symmetry where f of t equal f of t plus t not upon 2 for all t f of t previously f of t plus t not upon 2 minus f of t now f of t plus t not 2 plus f of t, which would been the same wave form. If you have for 0 to t not upon 2 it repeats itself like this now you should look at this closely if indeed this is situation that we have strictly speaking we should have t not by 2 has the period not t not right because we agreed the period is smallest value which satisfy this type of equation.

Therefore, basically t not 2 by is the period, which means the frequency of this is twice the frequency. If you consider this as a one period for consider this a one period is f not this is the period then it will be 2 f not.

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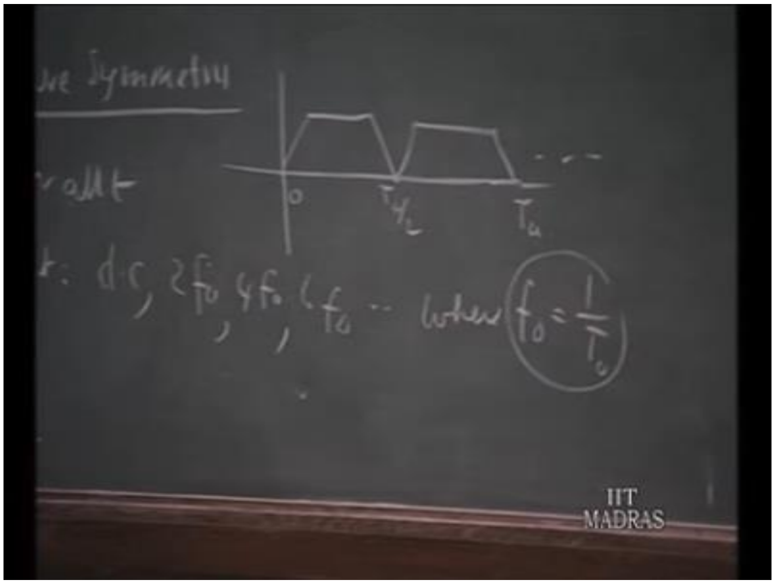

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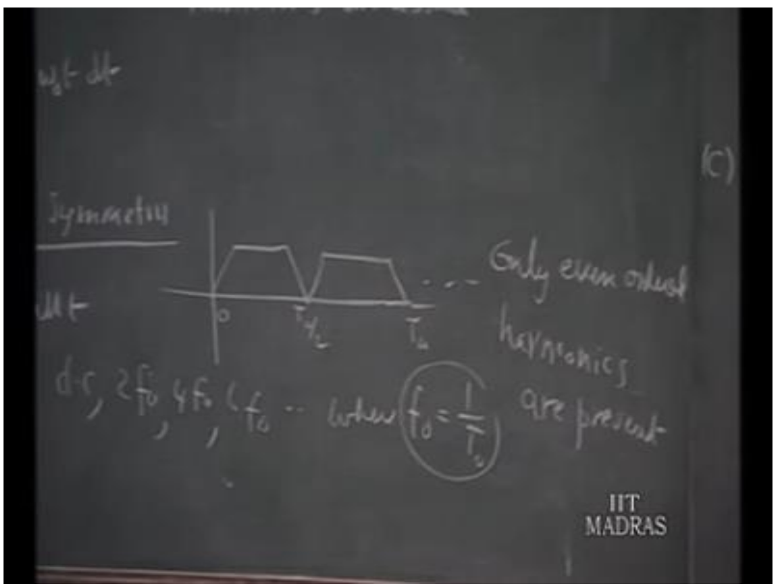

So, consequently the frequency present are frequencies that can be present are you can of course dc 2f not 4f not 6f not etc where f not is 1 over t not. So, here you have only even ordered harmonics are present all odd ordered harmonics are absent now naturally you question why do you make a big fuss about this if t not is the basic t not is the basic period why not called as the basic period call that as the t not and then deal will this.

The answer to this often we may have particular waveform which we split up into the number of component waveforms and in one of the component waveforms. You may have this kind of symmetry but you have to relate this frequencies to the original wave form which these are generated in particular.

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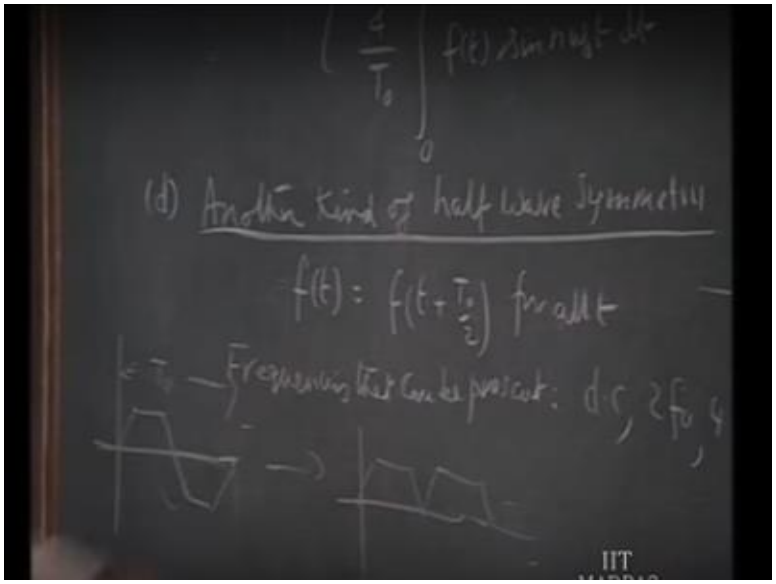

Suppose we have in example from a sin wave you generate are ok we will have not of sin wave really suppose you have a waveform something like this and then you rectify it you get this kind of waveform. So, we would like to whatever frequencies present in this waveform we would like to relate the fundamental frequency of this consequently. We still like to have this as the basic period even you are dealing with this right therefore in that context if you consider persist with calling t not here are the basic period.

Then in this signal you can have only the even ordered harmonics are present odd ordered harmonics are absent in the fundamental will not present naturally, because it is the first harmonic we will work out an example later which will illustrate this. So, this is really a matter of definition and if you call this as you want to deal with particular waveform on its own without reference any other waveform circulate this permissible for us to call this basic period and then analyze.

This in a conventional manner but if you call this as the period for this waveform then we can say that only the even ordered harmonics are present. These are the 4 important kinds of symmetry that one deals with also it is interesting to note that if you have any given function f of t you can always split this up into the its even part and the odd part. So, any arbitrary periodic function may not be even by itself may not be odd by itself but we can generate from it the even part and the odd part.

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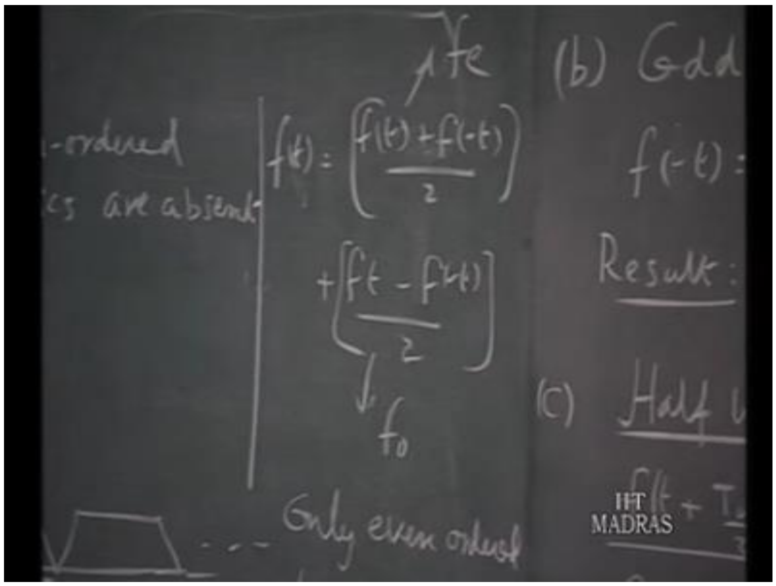

So, if you have f of t you can called this f of t plus f of t you can written as f of t plus f of minus t upon 2 plus f of t minus f of minus t upon 2. So, the first part is the even part the second part is the odd part so from any given function you can generate its even part and odd part. And naturally the Fourier series of this will contain only the dc and the cosine terms and this will contain only the sin terms.

So, if you have the component signal the group a not plus summation of an cos n omega t corresponds to this and summation bn sin n omega t corresponds to this and it might be possible for us. In some situations, to break this up an into its even part and odd part and find some additional symmetries in the respective parts we will work out a few examples which is illustrate this.