Networks and Systems
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Lecture-23
Frequency Spectrum

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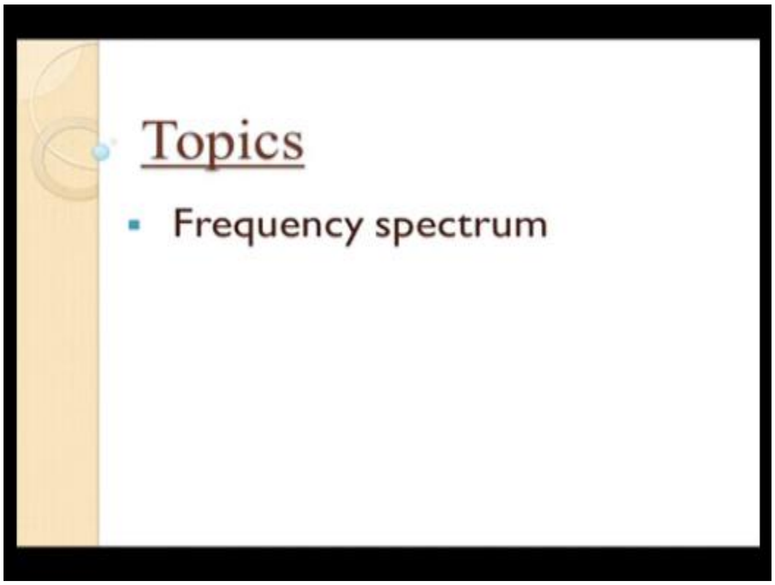

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What we have seen in the previous lectures, is first how to calculate the Fourier coefficients of a given periodic waveform and through an example. We saw, how this information could be utilized to access the steady state performance of an electrical network, excited by periodic non sinusoidal sources. Today, we will start with; we will take up the concept of frequency spectrum. To introduce this let us consider the statement of a periodic function of time.

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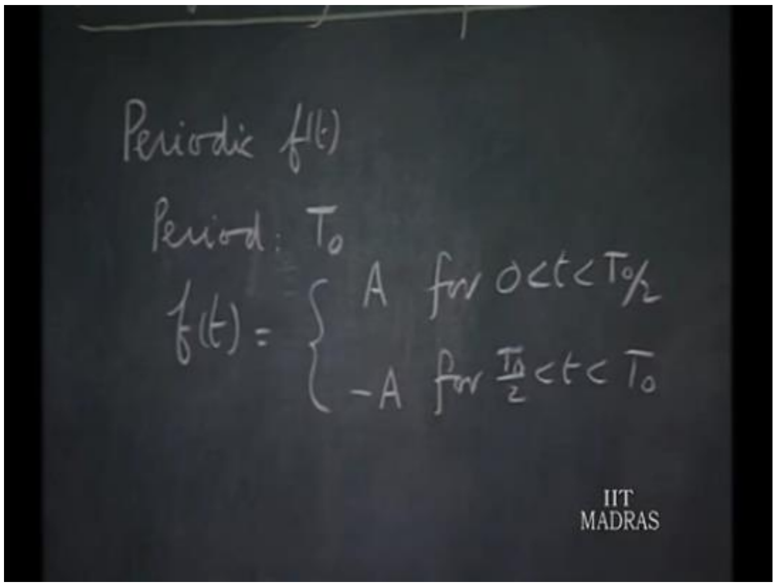

Suppose, I describe the periodic function of time as follows: period is T not and f t equals a for t ranging from 0 to T not upon 2 and minus a for t ranging from T not by 2 to T not. This is the complete description of the periodic function. But this is just a cold statement of facts. On the other hand, if I picture this information in the form of a waveform then, the whole situation comes alive.

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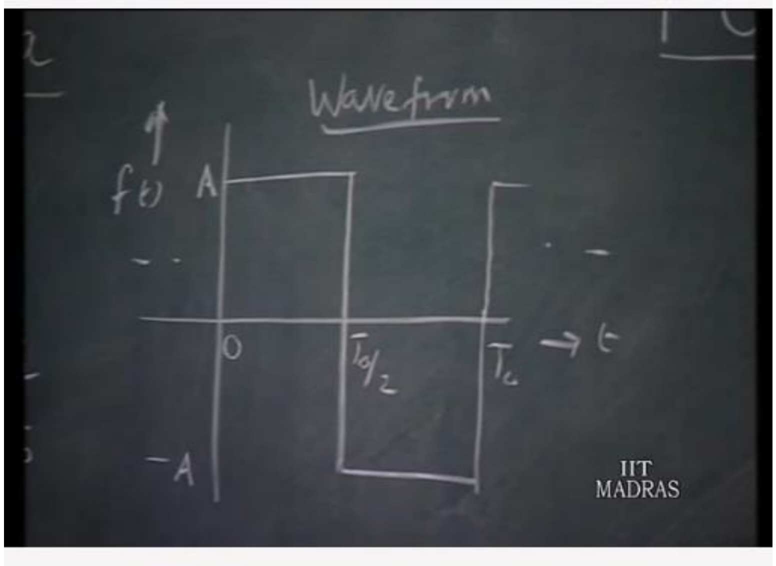

So, if you plot this for a whole period, you have this and repeats itself endlessly in both directions. This is A, this is minus A, 0, T not upon, 2 T not and this is f of t. So, information about the periodic waveform presented in this manner, gives you real feel for the variation and it is often useful therefore, for us, to plot these functions of time through waveforms like this. We know this is called a waveform and that is the reason why, an oscilloscope is such a useful instrument in the laboratory.

As we can, see the variations with respect to time which, are often masked in an electrical expression of this type.

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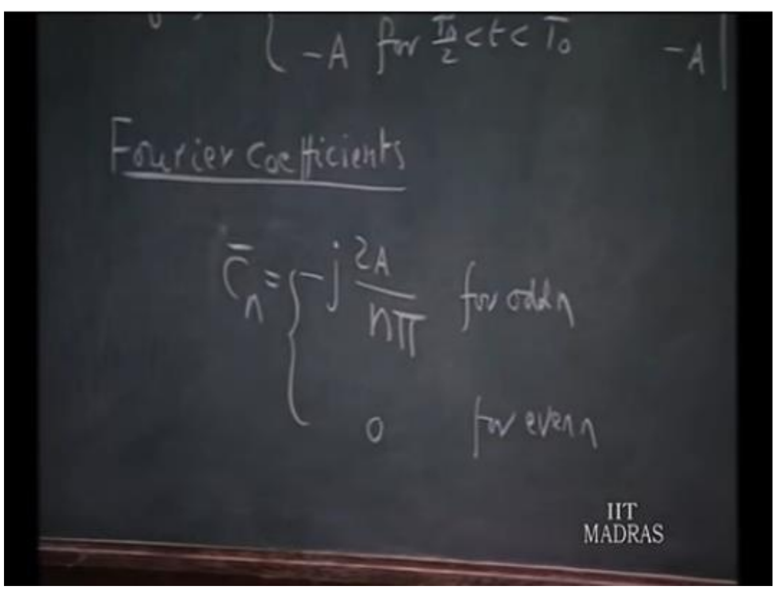

Now, when you perform the Fourier analysis of this, you can describe the Fourier coefficients of this waveform. We know that, C n the Fourier coefficient in the exponential representation is minus j 2 A by n pi. You recall, that in the trigonometric expansion of this function of time, the coefficients of sine n omega not t is 4 A by n pi and since, C n is a n minus j b n by 2, b n terms only are present 4 a by n pi for odd n. Therefore, C n will be minus j 2 A by n pi for odd n and 0 for even n.

Now, this statement again gives you the complete picture, but there is nothing like representing them in the form of a figure. So, suppose you want to do this a similar operation like this with respect to functions of time. So, we should be able to plot C n for different values of n in the form of a graph or a figure. But then there is a problem. C n is a complex number.

So, if you have to indicate a complex number, we have to indicate both its magnitude and phase. They are 2 parameters we associate with each value of C n. Therefore, a convenient way of doing this would be.

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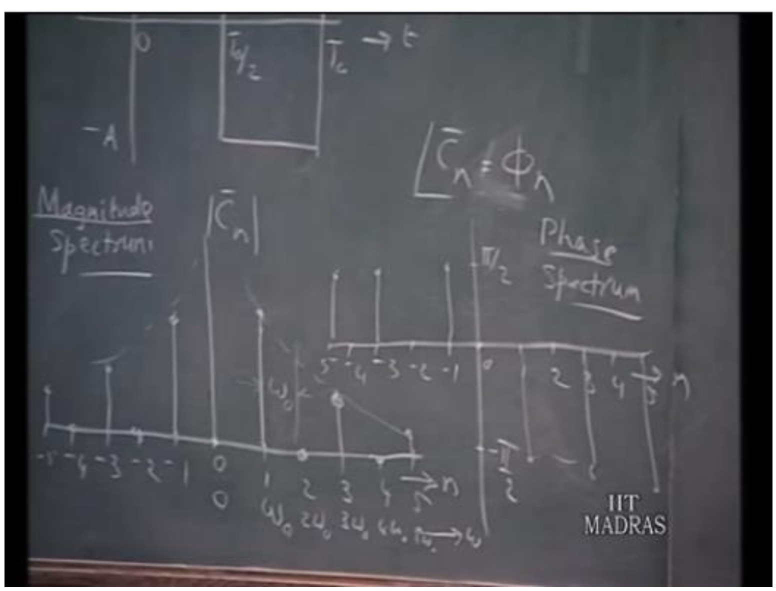

To plot the magnitudes for C n for different values of n and the angle associated with C n which, we call phi n for different values of n. So, in this case, we can graduate the x axis in terms of n or omega not; suppose you do it in terms of, n 0 1 2 3 4 5 etcetera and on the negative side minus 1 minus 2 minus 3 minus 4 minus 5 etcetera. So, when you plot the magnitude of C n then, there is no dc term. There is 1 component here, another component here, third component here. This is even harmonics are absent. Therefore, that will be how it looks like and phase every non vanishing C n will have minus 90 degrees as you can see here.

Therefore and since, for minus n it is the conjugate you have: plus phi pi by 2 minus pi by 2. You can graduate this x axis not necessarily in terms of n, we can as well do it in terms of frequency or angular frequency. Now, I can write this as 0 omega not 2 omega not 3 omega not and 4 omega not 5 omega not etcetera. The point to notice that, this is what is called a line spectrum. Such a representation is called a spectrum.

This is called the magnitude spectrum and this is called the phase spectrum and together they go by the name spectra: the magnitude and the phase spectrum. One point we should notice that, as long as the function is periodic that is the type of functions that, we are dealing with at this stage; the spectrum consists of components only at the d c and integrals multiples of the fundamental frequency. There cannot be any term in between.

So, these are called line spectrum. Both these magnitude and phase spectra are called, are now for a periodic situation are, line spectra and the spacing between 2 adjacent components this is also a component, but 0 magnitude, this spacing between 2 adjacent components is omega not, if you graduate this in terms of frequency. So, the entire information regarding Cn can be pictorially represented in this fashion.

It gives you in a graphic way: what are the frequency components present and what are the relative stages. All this information is given in a pictorial representation like this. Therefore, it is often convenient and useful for us, to plot this spectra of periodic waveforms in this fashion. For example, you draw an envelope like this. You can say this is how, the various Fourier coefficients decay for large values of n. So, generalizing this we can say now that.

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A periodic waveforms we have 2 kinds of spectra, the magnitude spectrum which is really a plot of the magnitude C n as a function of n or omega not or omega and. So, in general, you have values for integral values of n or integral multiples of omega not. You may be a little puzzled about this, meaning of minus omega not what is a negative frequency. It is just the coefficient of the term t, in the exponential representation we write minus j omega not t.

It has the dimensions of frequency as I mentioned earlier. These 2 together constitute a sinusoid of frequency omega not. So, this is only a mathematical terminology that we use and you should leave it at that. You should not try to find out a physical meaning for minus omega not as a frequency.

If you plot the phase spectrum. In general again you have some values. It is the angle of C n which is equal to phi n. The first spectrum turns out to be an odd function of n why because, we observed earlier that C n c minus n is C n conjugate which means: that the angle associated with C n and the angel associated with minus n are the negatives of each other. Therefore, for n equals 2 it has a phase angle, n equals minus 2 it is exactly the negative.

So, the phase spectrum is an odd function of n. On the other hand, the magnitude spectrum which is the magnitude of C n is a linear function, because the magnitude of c minus n is the same as the magnitude of C n. So, a magnitude spectrum is an even function of n. This is an odd function of n. So, these are 2 important properties of the spectra.

The magnitude spectrum is an even function of n the phase spectrum is an odd function of n. And it is conventional to plot the spectra for both the positive and negative indices n, even though as you can see if, you give this information pertaining to positive n for C n and positive n C n you can reproduce, what is happening on the negative side utilizing the property of evenness of the magnitude and recognizing that phi n is an odd function of n.
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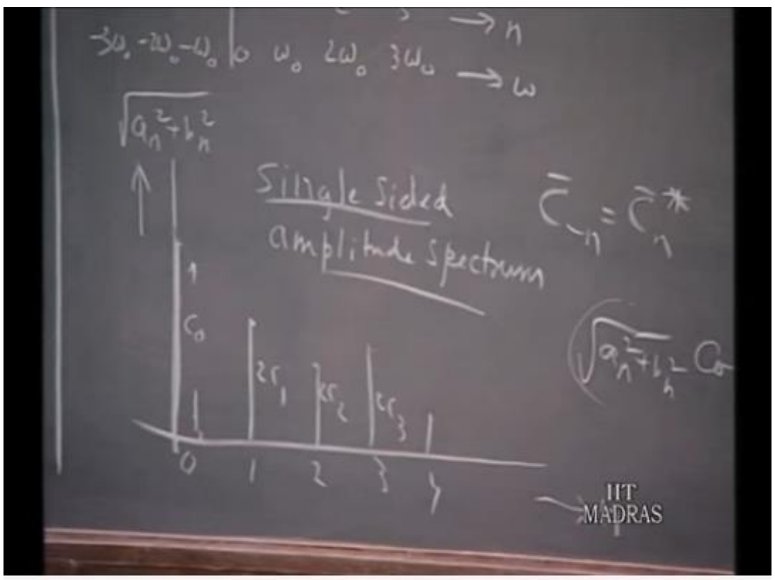

It is also sometimes. This is also done. Instead of plotting C n, 1 plots square root of a n square plus b n square, this being the amplitude of the n’th harmonic component. You recall, that in the trigonometric expansion of function of time, we can the amplitude of the n’th harmonic is square root of a n square plus b n square. And this is the coefficient of cos n omega not t plus phi n function of time.

So, this can be plotted also 0 1 2 3 4 etcetera. This is a single sided, sometimes called amplitude spectrum. Because what we are really plotting is; the coefficient you recall that the n’th harmonic is represented by cos n omega not t plus. So, we are plotting this as an alternative. So, instead of this magnitude spectrum, one can plot this, but in this case, we only have positive non negative values of n to deal with. Therefore, this turns out to be a single sided amplitude spectrum. This is an alternative way of plotting the magnitude spectrum, nothing more than this.

However, we normally confine our work to utilizing plotting the spectra in this manner rather than in this manner because, that will be convenient for us when, we go to the Fourier integral concept. You notice that if this is C not, this is C 1 this is C 2. When you go to the amplitude spectrum, square root of a n square plus b n square as you know is 2 times c of n. Therefore, this will be 2 C1 2 C2 2 C3 etcetera, but the d c term will be the same that is C not.

Okay, so the concept of spectra, is just a way of indicating the various harmonic coefficients in a pictorial way and since, each harmonic coefficient is associated with 2 quantities either a n or b n or magnitude of C n and phase of C n. For example, if you choose to plot in this fashion, the angle information is again once again this. So, as far as the phase spectrum is concerned you do not have any alternative.

On the other hand, if you like, you can plot a n and b n separately, but that is an unconventional way of doing it. So, since each Fourier coefficient is specified by 2 parameters, we need to have 2 spectra and usually we do this in the form of a magnitude spectrum and phase spectrum.