Networks and Systems
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Lecture-31
Continuous Time Fourier Transform

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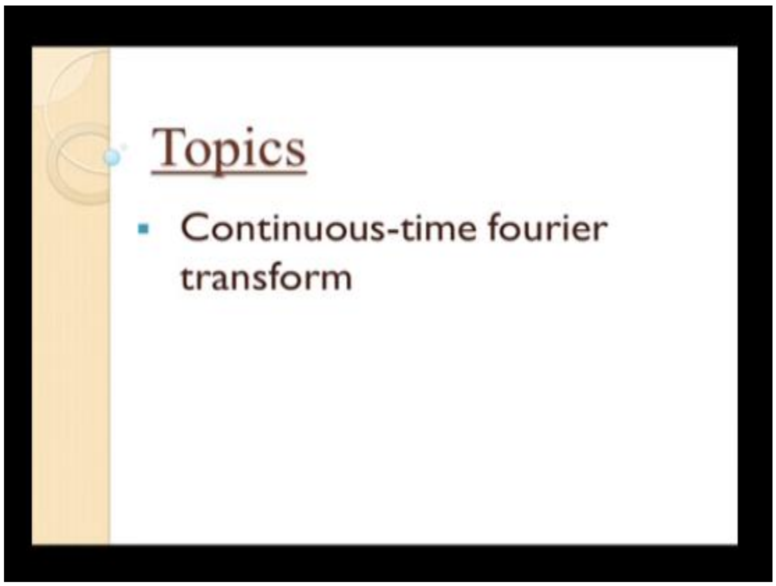

After introduced ourselves to the concept of Fourier integral and Fourier transform. Now, let us look at the points of comparison of the spectra of a periodic signal and a periodic signal.

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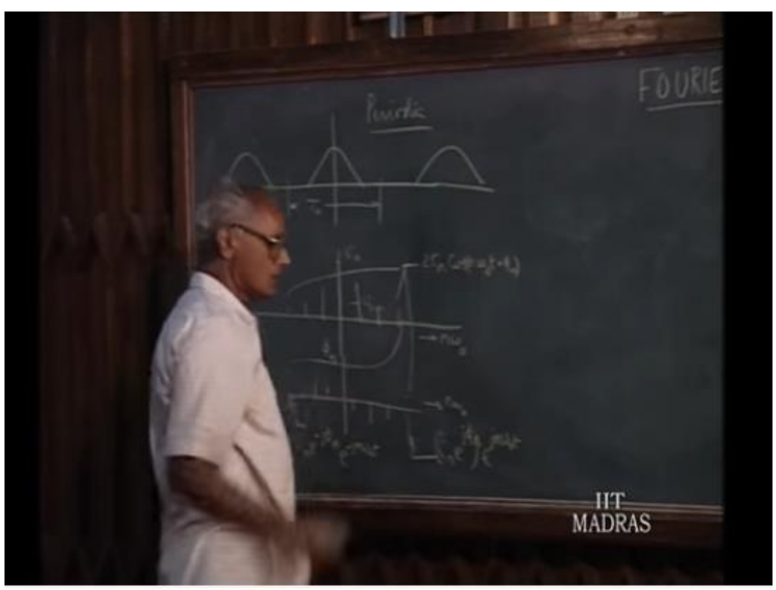

So, if you had a periodic function of time, say period T 0. Then, you have an amplitude spectrum magnitude spectrum C n. And a phase spectrum, as you recall the magnitude is the human function of n and the phase spectrum is not function of n. So, this x axis can be graduated in terms of frequency n omega 0 or in terms of n. Now, in particular C n here and the corresponding phase together these two components together will give me.

That means, this specifies the magnitude, this specifies the phase. So, it is C n, e to the power of j phi n, e to the power of j n omega 0 t. So, the significance of these two terms, one specifies the magnitude. The other is specifying the phase is C n, e to the power of j phi n is the coefficient of this exponential term, e to the power of j n omega 0 t.

Likewise, for the negative value of for this n, will have these two terms together will give me the conjugate of this. Because, when C n and c minus n are related by conjugate relationship. So, this will become C n same magnitude e to the power of j Phi n, e to the power of minus j omega 0 t. So, all these four put together. That means, the magnitude and phase together will give me 2 C n, cos n omega 0 t plus phi n.

So, the meaning of this negative frequency minus n omega 0 is only, it means that, this is the exponential. This is the coefficient of this j t terms in this exponential representation. Even though, the frequency is negative. These two terms together combine to provide you with a real frequency of n omega 0 t, n omega 0.

Now, this is the line spectrum and there will be frequency component present only at discrete value, along the frequency axis. And they are all integral multiples of fundamental frequency and as I said the phasing between two lines is omega 0. Whereas, when we have got the, this is the periodic case.

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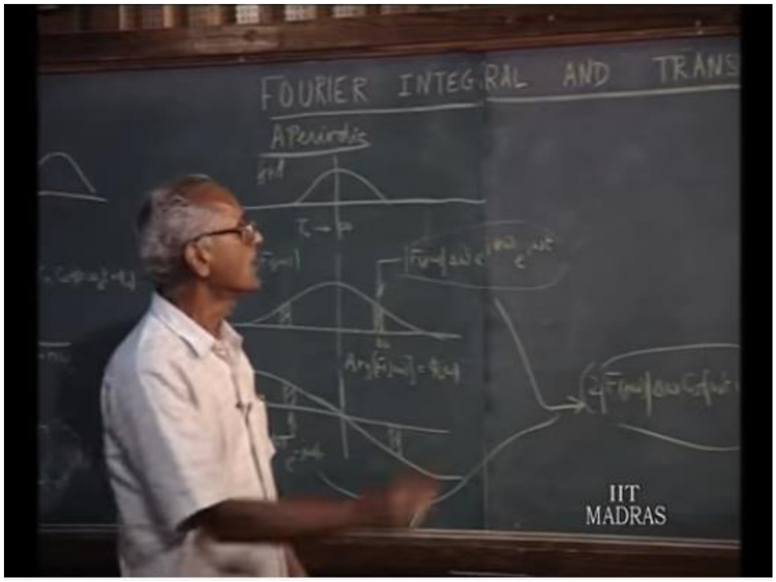

In the A periodic case, we have some phenomena, which lose, only one it does not repeat itself. The period T 0 tends to infinity and we said this will lead to a spectrum, which is continuous. And we call that coefficient density F of j omega 0, magnitude. And we also have associate with after all F of j omega 0 F of j omega in general is a complex number. It is complex function; therefore, we have magnitude and sign. So, also an angle associated with that it is a continuous function.

This is argument of f of t omega, which we may call phi j omega. Now, we said this coefficient density, what does it really mean. If you take any small interval delta omega and the corresponding here, this lies on the spectrum of the two spectra, the magnitude spectrum and the phase spectrum together. These two will imply a constituent of our f of t. What is that function, F of j omega magnitude, this is the coefficient density.

That means, over this interval of frequency. The density at the centre is F of j omega. Therefore, the total magnitude is F of j omega magnitude time delta omega. It has the phase e to the power of j t omega. And then, the exponential representation of function of time is e to the power of j omega t. So, this magnitude and this phase together will give rise function of time.

Naturally, on the other side, corresponding to, this is omega. This is minus omega and again, if you take two sections of the spectrum of width delta omega. These two together will give me, F of j omega magnitude, times delta omega times e to the power of minus j Phi omega, e to the power of minus j omega t. So, this portion of the spectrum combined with portion of the spectrum. These two together, if you combine these two, you will get 2 times F of j omega delta omega cos omega t.

So, even though you have components, corresponding to negative frequency, what it really means. For every negative frequency here, you have positive frequency and all together will lead a real function of a cosine term. This is magnitude of course. And you have frequency at every point. In other words, what we are having here is not and at any given, if you have a periodic case at any given point of time, at any given point of time at point of frequency, if you want to know, what is the amplitude, the amplitude is 0, vanishingly small.

But, if you take a small along the omega axis and if you consider this entire slice to be representative of one single frequency term at the centre. Then, the amplitude of that particular exponential term will be given by this. So, this is called coefficient density. So, in other words, f of t is in volts. If f of t is in volts the dimension of F of j omega is in volts per cycle per second or dimensions will be volt seconds.

Whereas, in the periodic case, if f of t is in volts, C n will be in volts. It gives the amplitude of each individual component, whereas, here it gives not the amplitude, but the magnitude density. It is something similar to, what we you know in your applied mechanics courses and so on.

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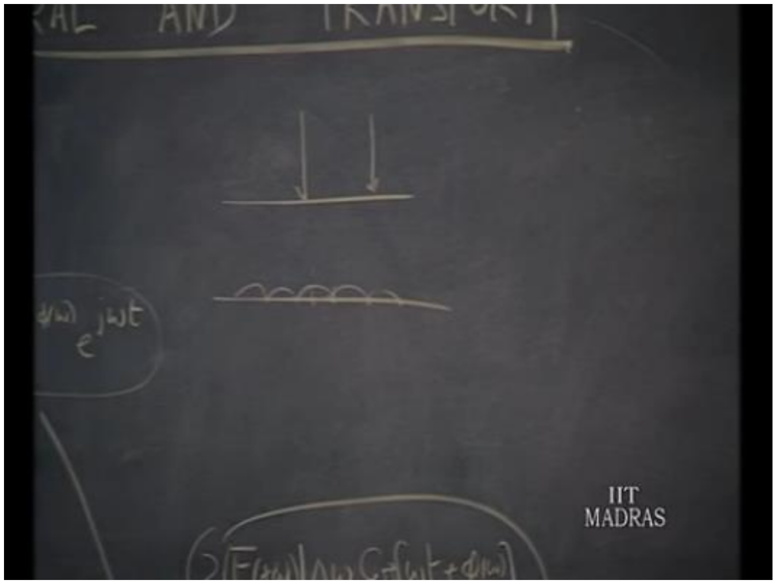

You have beam here, we have concentrated loads at point. This is one way of looking at it. And then, sometimes you have distributed loads. In case of distributed loads what you say is, so much kilogram per unit length. That means, at given point, there is no load. But, if you take a small interval and the total load in the interval will be the density times the small elemental distances.

So, it similarly here, you have situation where, each individual frequency component is almost negligible amplitude. But, you have over a small interval of along the omega axis. You can think of that the sections of the spectrum to represent a function of time, which has the amplitude, which is F j omega times delta omega. This is the density, this is the total amount.

So, what we have done is, to demonstrate by taking the limiting case of the Fourier series as the frequency becomes smaller and smaller and as the period becomes larger and larger. That even in A periodic case can be a thought of, as composed of number of different frequency components. In contrast to the Fourier series, which concerns itself, with the periodic case.

Here, we have a frequency component at every conceivable frequency all along the omega axis from minus infinity to plus infinity in the exponential representation. But, their amplitudes are vanishingly small. So, the amplitude at any given frequency is 0, but over a small width, you can think of an amplitude.

So, what we are really talking about is, the amplitude density of the magnitude density, otherwise called coefficient density. And that coefficient density is, we term it is F of j omega, because function of omega and you like associate j with the term omega. Therefore, it is F of j omega. So, we have basically these two relations, which are important, which will use to our further work.

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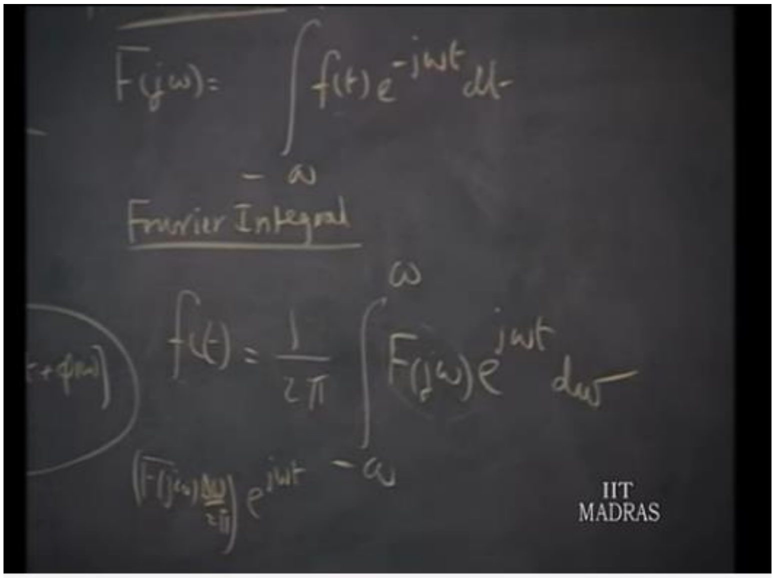

The coefficient density F of j omega is obtained from your given f of t as the integral from minus infinity to plus infinity of f of t, e to the power of minus j omega t d t. This is the coefficient density, which we call the Fourier transform. And once, we have the Fourier transform. We can recover the time function f of t by writing the integral relation from minus infinity to plus infinity of F of j omega, e to the power of j omega t d.

Both integrals from minus infinity to plus infinity, in one case, we get 1 over 2 pi here. The other case, we do not. And as I said, we get this 2 pi, because we are defining the coefficient density as the amplitude per cycle per second, f not instead of omega 0. And this is called the Fourier integral. That means, you always look at this as f of t is being composed of an exponential term the function of time like this.

And watch its amplitude, F of j omega this is the coefficient density delta omega over 2 pi. So, this is the amplitude, F of j omega is the coefficient density. So, at this frequency, if you want to look this, you take a small interval along the omega axis. So, delta omega over 2 pi this is the base, this is the amplitude density.

Therefore, this is the amplitude. And take the limits as delta omega goes to 0 overall possible frequencies and summation overall possible frequencies and that leads to this integral. So, what we have done in this lecture is to say that, the any periodic function can be thought of as an infinite summation of elementary frequency components. Each with vanishingly small amplitude and with all frequencies present along the omega axis. We continue next class, next lecture discussion of the properties of the Fourier transforms.