Networks and Systems
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Lecture-39
Fourier Transform of Signals That Are Absolutely Integrable

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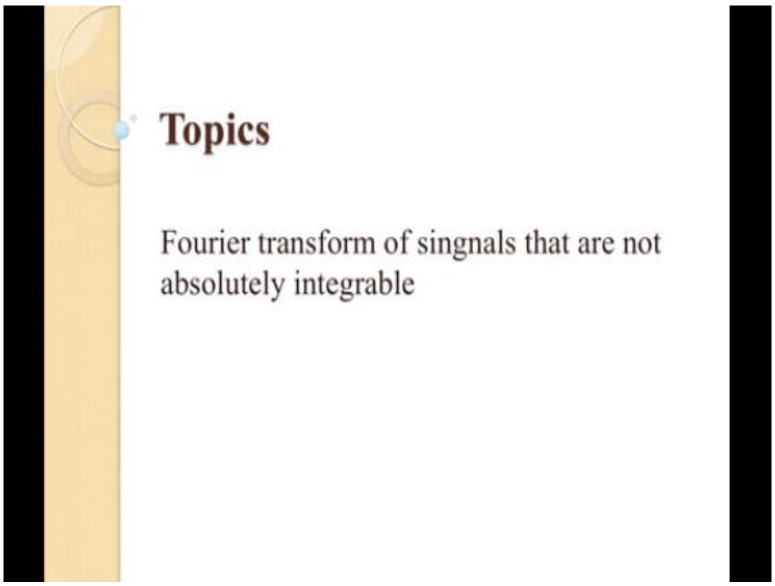

To discuss cases; where the Fourier transforms involve impulses. Let us see the origin of the trouble if we follow the classical rules.

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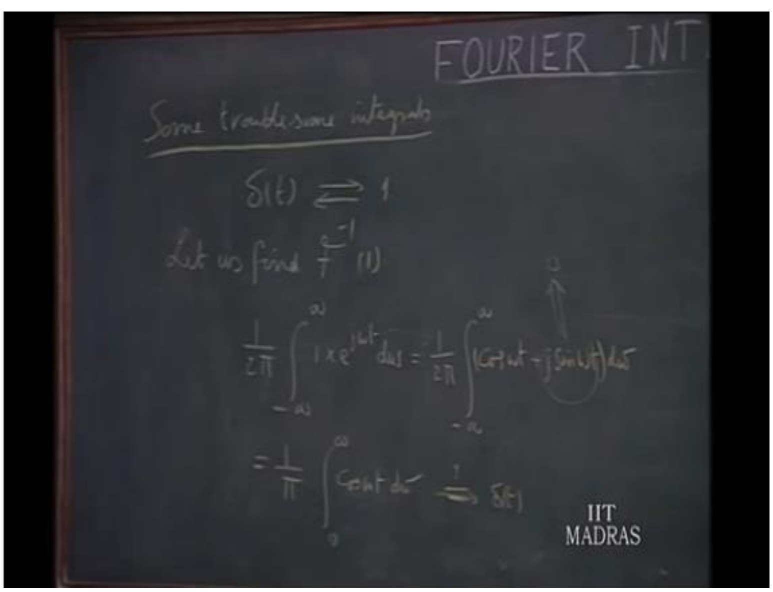

So, let me say some troubles of the integrals. We have just now seen a example, where in the forward direction we have got a pulse and then found out the Fourier transform. When you went in the reverse direction we have to work a little hard to get the final pulse function. Let us consider the another example, we know that delta t has the Fourier transform equal to 1.

Let us find the inverse Fourier transform of this 1 we should able to get delta t. How do we do it? then we write for the inverse Fourier transform of 1 one over 2 pi minus infinity to plus infinity of 1 e to the power of j omega t d omega. When you try to integrate we have some difficulty because you have got infinite limits.

So, in any case let us see, what it really means; so, minus infinity to plus infinity of cos omega t lus j sin omega t d omega. and once again sin omega t is an odd function of omega and you are integrating between the symmetrical limits we know the contribution from this is goes to 0 contribution from this goes to 0.

So, we can write this as 1 over 2 pi minus infinity to plus infinity cos omega t d omega and once again, this is cos omega t is an even function of omega. We can write this as 1 over pi. And take the limit from 0 to infinity of cos omega t d omega. now, 1 over pi cos omega t d omega so far so good.

But if you try to integrate, you get minus sin omega t by sin omega t by t and then when we try to substitute for omega infinity then you do not know, what that amount too. So this type of integration does not allow itself lend itself to analysis by the classical methods. However, since we started with inverse Fourier transform of 1 the inverse Fourier transform must lead to delta t0.

We know, delta t has this Fourier transform. So, apparently the whole thing must be equal to delta t, this must be equal to delta t. Now, that is where we have a problem in the classical mathematics. And after the impulses came to be used in analysis of systems then after their usefulness recognized mathematicians have developed a branch of mathematics is called theory of distributions.

With the help of which the analyses of the mathematical methods using impulses or put in more rigorous basis, but we will not go into that however make use of certain result of that and put them down here so that we will use them for our further work. Basically then we have some important results which i will list them out and these results which will make use of in our future work.

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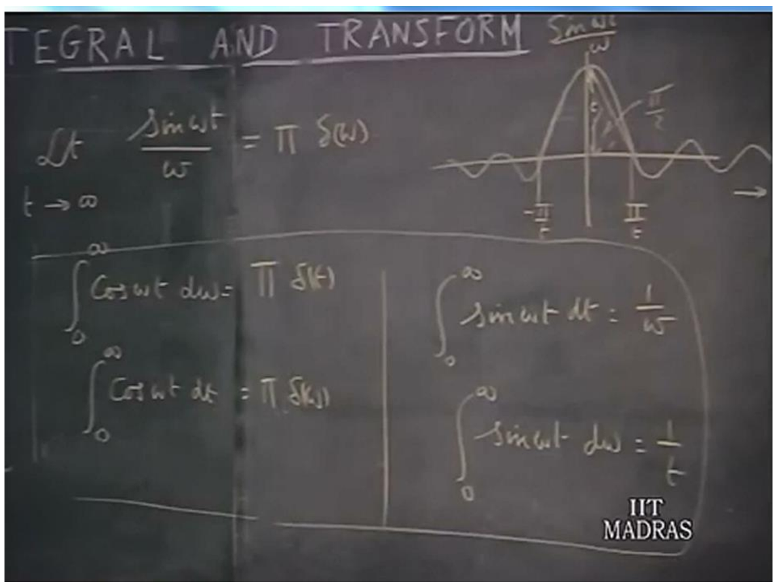

First of all limit as t tends to infinity of sin omega t by omega will be pi delta omega. It can be explained on the following basis; sin omega t by omega how it look like. So, this will be equal to t the height equal to t and the first 0 occurs when omega t equals to pi or t equals pi by or this is sin omega this is with respect to omega or omega equals to pi up on t.

Now, enquire as what happens when t becomes larger and larger. When t becomes larger and larger sin omega t by omega the function becomes this pi upon t so the first loop becomes smaller and smaller, but the height goes. And we already seen, that the area under this curve of this is as far as the positive side is concerned area equals to pi pi by t times t into half; therefore, this is pi by 2 and the other side is pi by 2.

That means, the area under this curve is pi area under this curve is pi no matter what t use. So, as the area from minus infinity to plus infinity of this is pi and d becomes larger and larger this loop strings and then you have very large amplitude. And the first loop becomes 0. That is essentially the character of the impulse and since the area under this curve is pi then it is impulse of value pi and the impulse is situated omega equal to 0 that is where we have a spike.

So, we can justify this in this fashion that sin omega t by omega t goes to infinity is tends the limit the pi delta omega. Similarly, we can have 0 to infinity of cos omega t d omega equals pi delta t. That we have seen, from this we have earlier come to this we got into difficulty 0 to t cos omega t d omega. What it means, it is pi times delta t. We did not know how to deriver this but we can assume this relation.

Similarly, we have 0 to infinity cos omega t d t after all we change the variables here this must be pi delta omega. And 2 more relations are useful 0 to infinity sin omega t dt equals 1 over omega and 0 to infinity sin omega t d omega is 1 over t. We assume these relations in our work when we derive the Fourier transforms of functions whose energy content is may be infinite may not be finite.

So, these 4 relations are important for us we will take this to be true in our further work. We will make use of this as I said in our further work but just as an example we will consider 1 particular case.
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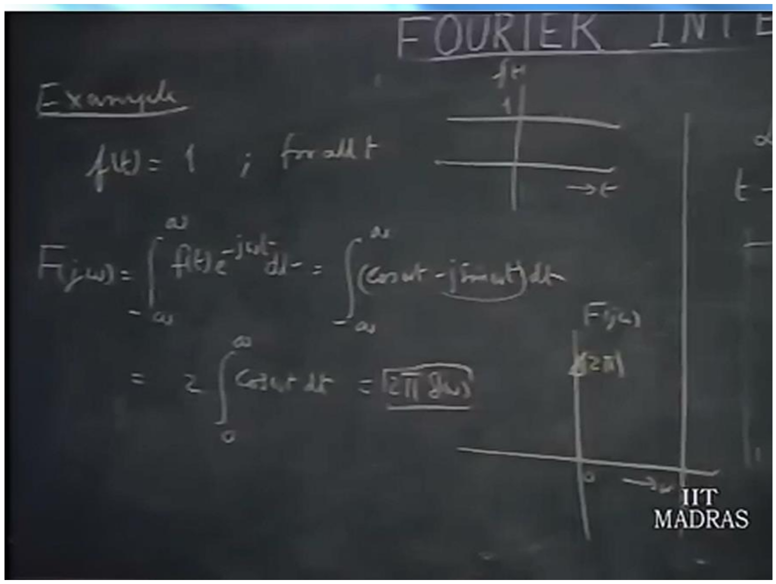

Let us take a simple example now, to illustrate these relations. We will take f of t to be a d c quantity equal to 1 for all t. So, that means f of t is a constant f of omega for this obviously from minus infinity to plus infinity of f of t e to the power of minus j omega t dt. Now, e to the power of j omega t can be written as cos omega t minus j sin omega t.

And once again we can make as the symmetry relation and sin omega t term will be this is minus infinity cos omega t minus j sin omega t dt. Because f of t happens to be equal to 1 and the contribution of this will be 0 because it is an odd function and we have symmetrical limits.

Therefore, I can write this as only integral involve cos omega t now i take from 0 to infinity of cos omega t d t and I introduce 2 sin because 2 here because instead of integrating minus infinity to plus infinity i am integrating 0 to infinity. And now, cos omega t dt integral from 0 to infinity is pi delta omega according to what we had already seen therefore this is 2 pi delta omega.

So, the Fourier spectrum for this f of j omega, you have an impulse of magnitude 2 pi and d c this is what we should expect this is a pure dc term; therefore, you will have only a dc component here and no other components. And at the dc we have finite amplitude for this voltage therefore, you have an impulse function that means you no longer even though we are talking about coefficient density.

Now, this coefficient density is infinite. Because there is a definite non 0 amplitude at d c; therefore, the coefficient density is going to be infinite. And it has got a magnitude equal to 2 pi we develop this further when we take about other periodic signals. So, in this lecture therefore, what we have done so far is, we started with the concept of energy in a periodic signals we said when we talk about a periodic signals with finite energy it no longer fruitful for us to talk in terms of power.

Because the average over a infinite interval goes to 0; therefore, we talk only in terms of energy available from the signal. And what we mean by energy available from the signal is. If this signal is a voltage signal if it is given to 1 ohm resister. What is the total energy dissipated in that 1 ohm resister from minus infinity to plus infinity is what we call the energy available from the signal.

And this energy available in the signal resides in the different frequency band by different amounts; therefore, we saw that if we integrate from minus infinity to plus infinity the energy density which is given by f j omega squared integrate it with reference to omega and divide by 2 pi because the energy in terms of frequency in hertz cycle per second.

That will you give the total energy and this we said answer refer to Parseval's theorem. Then we discussed the possibilities of extending in the Fourier analysis to signals whose energy content is not finite and in that contest we started with some interesting relations involving sin theta by theta functions and we familiarize with sin integrals.

And then we put down some results here, which are scientified by the distribution theory in mathematics which we cannot prove classical methods we assume this results. And making use of these results we try to find out the Fourier spectrum of a pure d c quantity. And came up with results which is a quite perhaps quite have been obvious this particular signals has only 1 frequency component and that is a d c. And therefore, you have only 1 this becomes line spectrum once again in the Fourier transform also but the coefficient there is coefficient which is non 0.

Therefore, the coefficient density has got to be infinity; therefore, we have a delta function. And the coefficient density is 2 pi omega and the density occurs at 0 and so we can expect that when we talk about periodic functions in the line spectrum the magnitudes are going to be infinite involving delta functions, more about this in the next lecture.