Networks and Systems
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Lecture-49
Properties: Linearity, Differentiation in the time domain

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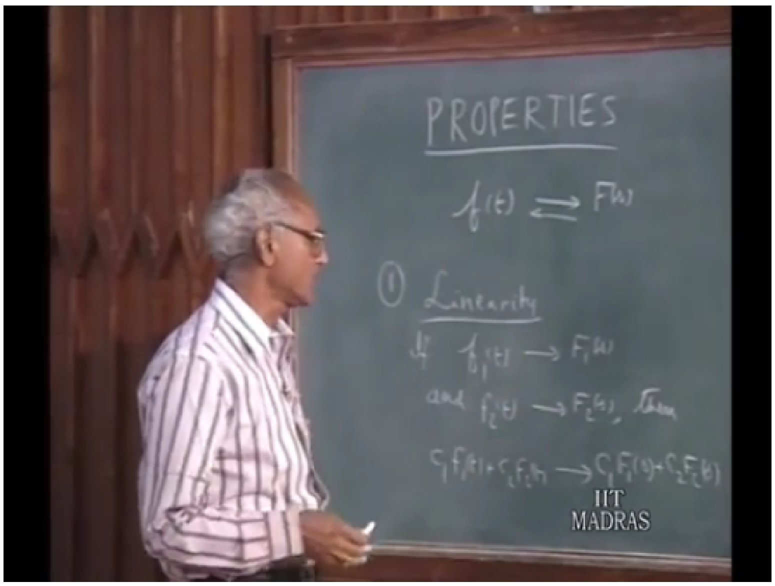

Let us now take a look at some of the important properties of the Laplace transforms. We use, the notation that f of t has the Laplace transform f of s and f of t inverse the Laplace transform of f of s capital F of s. Most of the properties that we are going to discuss parallel goes which we have already discussed in the context Fourier a transform.

So, as we go along, you compare the properties that we are having discussing here with those that we discussed earlier in the context of Fourier transform. The more or less are similar except for minor variation which arise as a result to the fact that the Laplace transform concerns itself essentially with causal time functions. Whereas, the Fourier transform deals with functions from which can exists from t equals minus infinite t equals plus infinite.

So, basically, that is the difference and that create some differences otherwise the properties are quit similar. First property is something which we already assumed is the linearity property. What we mean is; if f1 of t has the Laplace transform f 1 of s if f1 of t is the Laplace transform f1 s. And f 2t has the Laplace transform, F2 of s then a combination of c1 f 1 of t and c 2 f2 of t where c1 and c2 are arbitrary constants the linear combination of this 2 is having Laplace transform c1 of f1 of s plus c2, f2 of s is nothing much poor about this.

You plug this find out the Laplace transform of this plus this in to the expression for the Laplace transform defining integral break it up the 2 parts and you can show it. There is no difficulty doing this. As a matter of fact where assume this relationship, if you recall when we are trying to find out the Laplace transform cos omega not t. We said cos omega not t is 1 half of the e to the power of j omega t and plus e to the power of minus j omega t and therefore, there we are already made use of this area to property.

So, we read that discuss it any for them this is quit obvious and it can be really demonstrated.

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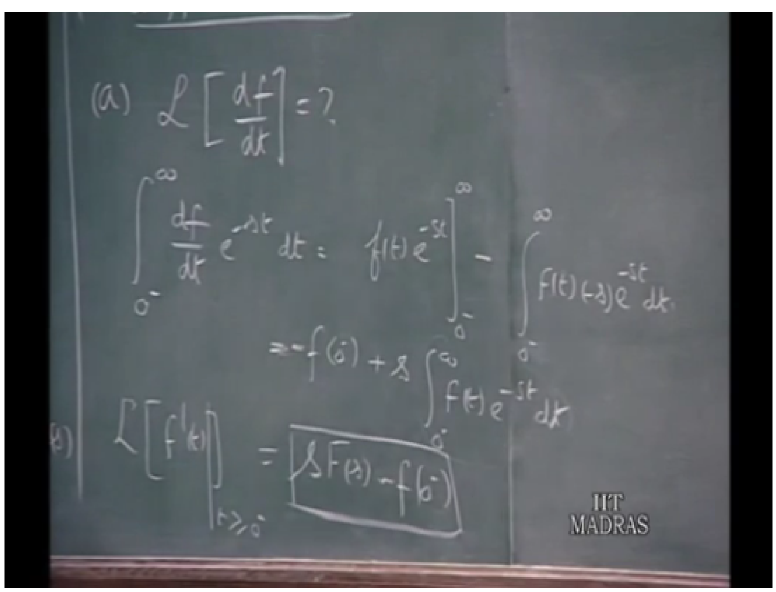

The second property is going to talk about is the differentiation in time domain we ask the question what is the Laplace transform of df by dt? This is the question which we like to also. Given that f of t as the Laplace transform capital f of s you will like to find out what is the differential what is the Laplace transform of df by dt. Assuming that the Laplace transform of the differentiated function exists.

Then we set up the differential equation defining the integral 0 minus to infinite of df by dt e to the power of minus st dt could be the Laplace transform of this derivative of this function of time. Now, this can be integrated by parts can be integrated by parts, you take the integral of df by dt which is f of t e to the power of minus st evaluated with in the limits 0 minus to infinite minus integral from 0 to infinite of f of t and take the derivative of e to the power of st with reference to time.

So, minus s e to the power of minus st dt. So, when we substitute this 2 limits in the first expression in the regional convergence as t goes to infinite limit as s t goes to infinite of f of t e to the power of minus st that goes to 0. That is, the definition of the value of s that we are going to take in the regional converters. So, at the upper limit this is 0 at the lower limit e to the power of minus st becomes 1 and therefore, you have got f of 0 minus. Because it is the lower limit minus of f of 0 minus.

And here the integration reference to s; therefore, s times 0 minus to infinite of f of t e to the power of minus st dt. This integral is capital f of s is Laplace transform of f of t itself therefore, this is s times f of s. So, this will have s times f of s minus f 0 minus. That is, the Laplace transform of f of t which is the derivative of f of t. This is s 2 link that f of t talking about we want to take the behavior of the f of t for t greater than are equal to 0 minus; that means, you are taking the derivative of the function of time.

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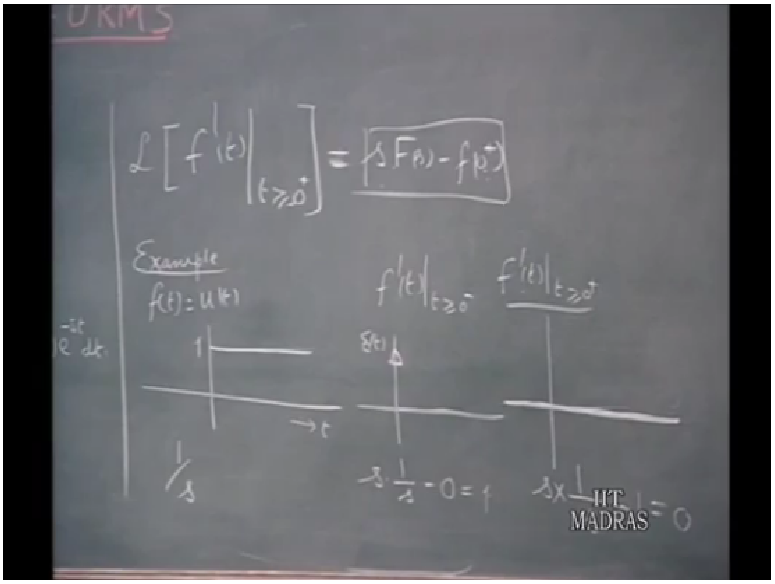

We take into account the variation of f of t from 0 minus onwards often we would like to find out the Laplace transform of f prime t for t greater than or equal to 0 plus; that means, any jump that this function has got in transiting from the 0 minus to 0 plus would like to exclude. That the derivative you want to take only from 0 plus onwards we would like to this will got whatever is happening before that. In such case this will be the Laplace transform of that will be s f s minus f 0 plus.

That means, you are now taking about the Laplace transformation of the derivative. Starting from 0 plus instead of 0 minus dealings of this will become clear as we take an example. So, in the literature find the Laplace transform of f prime t is write in this manner or in this manner both are equal both are correct depends up on whether you want to consider the derivative starting from 0 minus or 0 plus. Let be illustrate what we meant by this by example, let us take f of t to be a unit step function of t.

So, you have this is f of t. Now let me say, that I want to find out to derivative f prime t for t greater than are equal to 0 minus. So, i would like to take the derivative of this functions starting from 0 minus. So, as take into account the variation of f of t starting from 0 minus at from 0 minus to 0 plus it goes through jump a 1 0 minus to 0 plus.

So, the derivative of this function a t equal to 0 is an impulse. Because we jumping from 0 to 1. So, derivative is infinite and we integrate with impulse we get arise a step 1 that the derivative desired the behavior of the function 0 minus to 0 plus is an impulse. And from t equals 0 plus onwards there is no derivative the 0 because it is a flat function of time. So, f prime of t, t greater than taking into account our variation where t equal 0 minus is in delta function delta t.

On the other hand, if you want to consider the derivative only starting from e equals 0 plus onwards. From 0 plus onwards the variation of this function there is no variation; therefore, it is 0 and from all negative values of time we assume the function to a 0 value; therefore, f prime t is identically 0. So, for this that is the function. Now, let us see the Laplace transform of this we know this 1 over s.

The Laplace transform the unit f function is 1 over s. What is the Laplace transform of this if you ask this question f prime of t is greater than equal to 0 minus? You must use this formula s f of s use this formula s f of s minus f 0 minus. This is the formula that we have to use we want to take the derivative that starting from t equal 0 minus; therefore, you write s times 1 over s minus. What is the 0 minus value of this f of t is 0 this is equal to 1 and we know that, the Laplace transform of the unit impulse function which is derivative of this is equal to 1.

On the other hand, if you want to consider the derivative to be 1 which the value from t equal 0 plus onwards. Than we use this form which is s f of s minus f 0 plus. So, that will be s times, the Laplace transform of f of t 1 over s minus f0 plus f0 plus are the unit time function is equal to 1. Therefore, we substitute 1 here and that will be 0 and that is should be because our f prime t in this case is identically 0 if Laplace transform as well will be identically 0.

So, that is the distinction between this 2 formulas; depending up on how you want to take the derivative whether you want to consider the derivative from t equal 0 minus or 0 plus.

So, you can use either of this the difference between this 2 will be the Laplace transform of impulse if there is any existing the jobs. Now, let us now proceed further and consider the second derivative of this.

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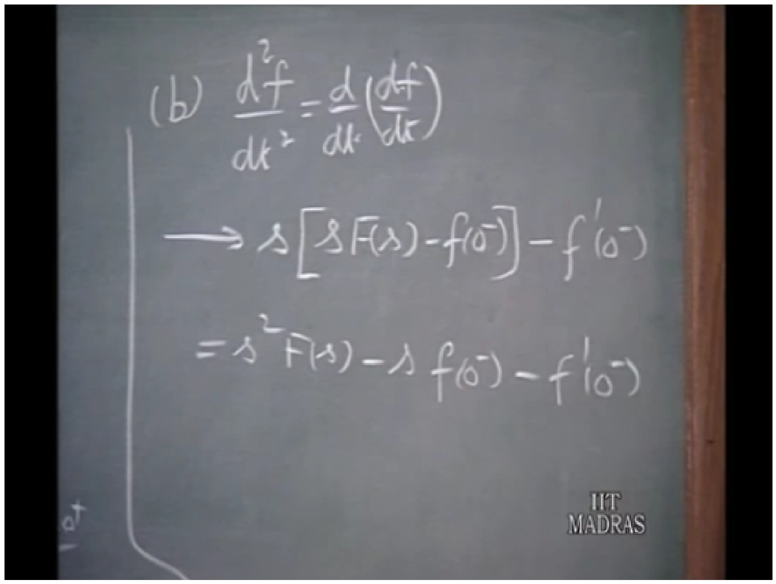

That means; what is the Laplace transform of this d square f dt square. Now, the second derivative d square f dt square required is after all, the derivative of d by dt of df by dt. Since we know, the Laplace transform of this we have to apply the formula which we already derived. So, this will be s time the Laplace transform of df by dt is s f of s minus f 0 minus i will continue writing 0 minus because that more general, minus the value of the derivative df by dt t equal 0 minus I will write this f prime 0 minus.

So, that will be s square f of s minus s f 0 minus f prime 0 minus. So, the Laplace transform the second derivative of the time function in terms of, the Laplace transform the original time function is given in this and this can be continued.

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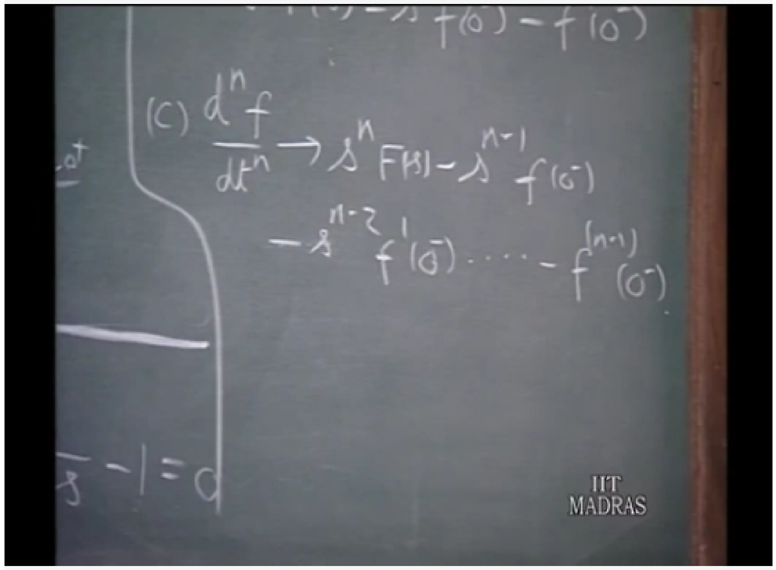

So, a general formula of the Laplace transform of the n'th derivative dn f dt n can be written as, s to the power of n f of s we taking about the n'th derivative of time function. minus s n minus 1 f0 minus like that you take next of the sn minus 2f prime 0 minus down the line to f n minus 1 the derivative evaluated 0 minus that all would be, you don't have to remember this formula.

All we know is, all we should know is the formula for the derivative and then 1 can extrapolate this in this fashion that will be. In fact, you can easily get this by repeating the application of the derivative formula.