Networks and Systems
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Lecture-59C
Relating Fourier and Laplace Transform

So, far we have discussed the definition of the Laplace transformation, its properties and also method of finding out the inverse Laplace transformation. We have notice that many of the properties of the Laplace transform parallel to those that we have already studied under the Fourier transform method.

And we also notice that for certain functions the Laplace transformation and Fourier transform are quit related to each other, in sense that we substitute j omega for s you get the Fourier transform in the Laplace transform.
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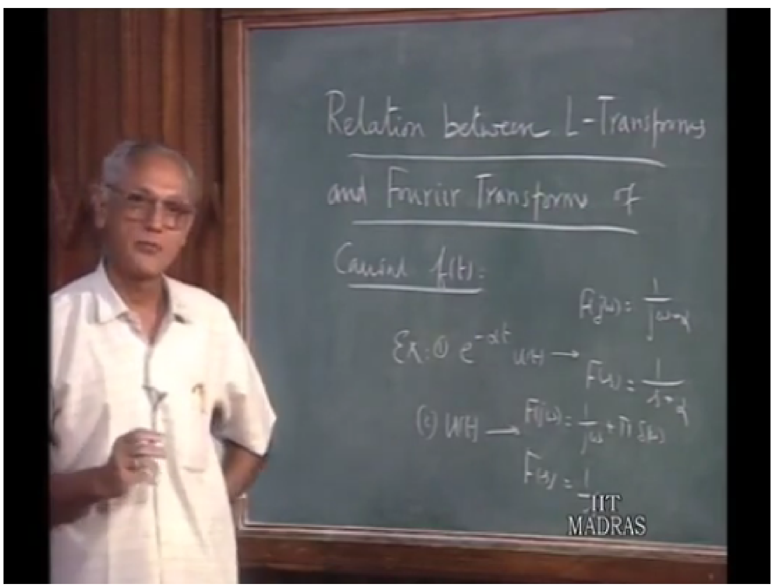

For example, you have e to the power of minus alpha t u t; the Fourier transform is 1 over j omega plus alpha. The Laplace transformation is 1 over s plus alpha. So, for substituting j omega for s you get the Fourier transform, from the Laplace transformation. On the other hand, suppose you have u of t itself, the Fourier transform of this is 1 over j omega plus pie delta omega.

On the other hand, the Laplace transformation is 1 over s. So, here there is a discrepancy. So, what would like to know, under what conditions will be the Fourier transform obtained the mere substitution of j omega for s from the Laplace transformation, and under what cases will be differ. This substitution will not yield as the appropriate Fourier transform. This is what, this is the question with to like to address.

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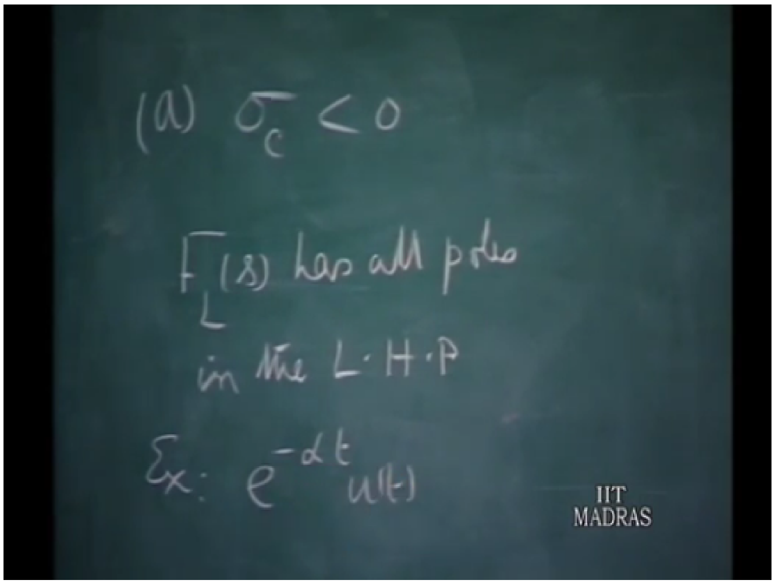

Let us consider three cases; a, in the Laplace transformation the abscissa of convergence is less than 0. Now, before this it must be clearly mentioned, that when we compare the Laplace transform in the Fourier transform, we can do it only for the causal time functions, because the Fourier transform integrates for minus infinitive can plus infinitive the time axis.

So, f of t values for t less than 0, there is no reasons for the Laplace transform in Fourier transform to be related to each there. So, whatever comparison we can make, can be done only for casual time functions. So, the first case corresponds to situations, where the abscissa of convergence is less than 0, which means that f l of s has all poles in the left of plane.

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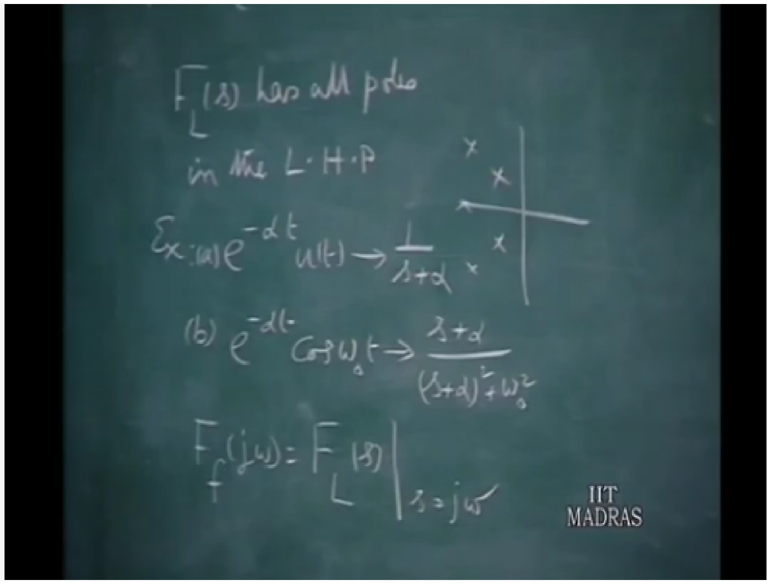

Examples e power minus alpha t u t has the Laplace transform 1 over s plus alpha example a b e power minus alpha t cos omega 0 t will have the Laplace transform of s plus alpha over s plus alpha whole square plus omega 0 square; that means, here poles will be the left of plane. In all this cases, the Fourier transform f of j omega is obtained from the Laplace transform of the corresponding function with substitution s is equal to j omega.

So, we have the Fourier transform can be obtained to be the Laplace transforms by the mere substitution of s by j omega. Conversely, the Laplace transform is obtained to be the Fourier transform by substituting s for j omega. So, when the abscissa of convergence is less than 0, Fourier transform and Laplace transform are very closely related, 1 is obtained from the other by substitution of s for j omega r j omega for s.

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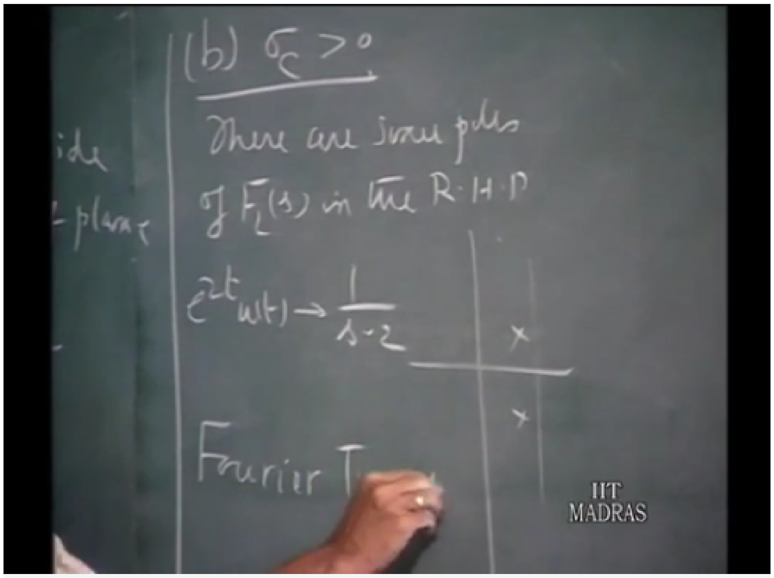

Suppose, I have the abscissa of convergence to be greater than 0; that means, there are some poles in the Laplace transformation of f l of s in the right half plane. So, may be you are having some poles here. So; that means, that is why the abscissa of convergence is a positive number. Example of such situation will be e to the power of 2 t u t will have the Laplace transform 1 over s minus 2.

Now, all we can say is, for such functions Fourier transform does not exists, because the integral does not converge. You have the f of t e to the power of minus j omega t d t that integral cannot converge, because you have exponentially increasing time function. In such cases Fourier transform does not exist.

So, there is no way in which we can relate the Laplace transform and the Fourier transforms. Simply a Fourier transforms does not exist for such functions.

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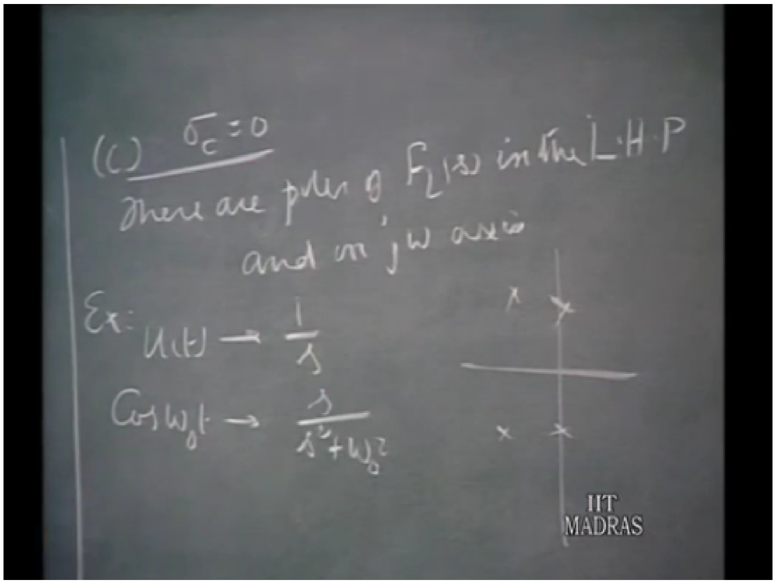

The third case will be the abscissa of convergence is equal to 0; that means, there are poles of f l of s in the left half plane, and on the j omega axis. The abscissa of convergence is 0, because you have poles on the imaginary axis as well example u t 1 over s cos omega 0 t s over s square plus omega 0 square. So, all these are cases, where you have poles on the imaginary axis.

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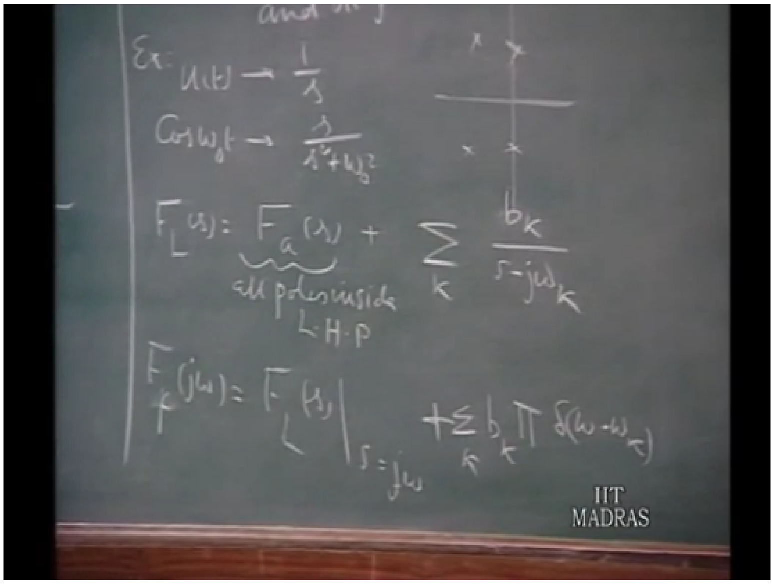

What we do in such cases. The general rule is, f l of s in general will have f a of s which has got all poles inside left half plane plus some poles here on the imaginary axis. Let us say b k over s minus j omega k. So, it will have b k over s minus j omega k where k; the summation is on k.

That means, the Laplace transform f l of s can be thought of, as the combination of the terms which correspond to poles in the imaginary axis plus terms which do not all poles on the imaginary axis, which all poles inside the left half plane corresponding to case a.

In such equation, it can be easily visualize, that before a transform of this, will turn out to be the substitute in j omega for this f l s s is equal to j omega plus corresponding to each one of this poles of the imaginary axis you have got b k pie delta omega minus omega k b k pie b k delta omega minus omega k sum of k.

So, it tells out that in such equation for every pole and imaginary axis and corresponding residue, you have an extra delta function, and that is why when you go to u of t 1 over j omega pie delta omega you are getting, the corresponding residues is 1. So, pie delta omega you are getting. So, this is what it would be.

So, to summarize them wherever you have the abscissa of convergence is less than 0, then the Laplace transform and Fourier transform are very closely related substitution s is equal j omega will get one from the other. But if you have poles on the imaginary axis, you have delta functions in the Fourier transform. You do not have delta functions in the Laplace transformation.

We will close this discussion of properties of Laplace transformation at this stage and in the next lecture will continue with the application of Laplace transformation technique to system analysis and circuit analysis. But now I thing is appropriate time to break at this point, and look at an exercise on the topic that I already discussed, related to Laplace transformation.