Networks and Systems
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Lecture-67A
Full Circuit Example

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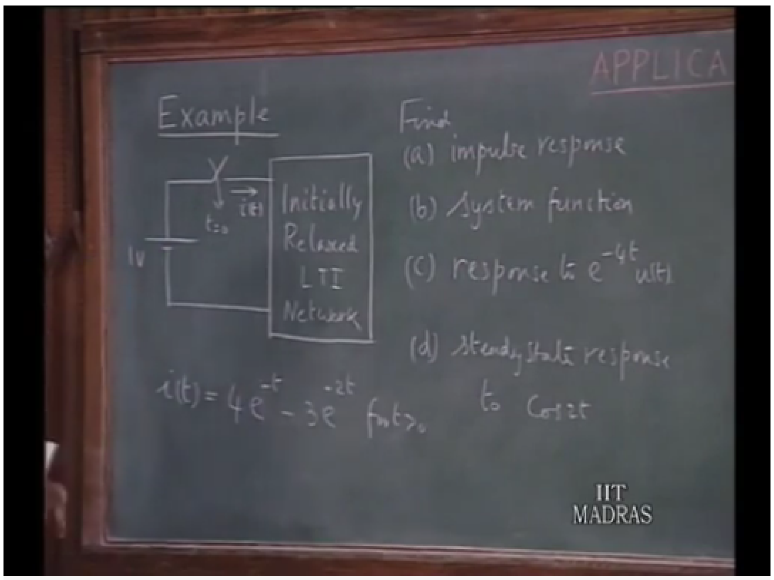

Let us now consider this example, where we have a network, which is linear timing variant and we treat this as a system the input is this whatever voltage you give and the response quantity identifies the current it. So, when the switch is closed t equals 0 in this particular network, it is observed that the current that flows in to the network is given by this 4 e to the power of minus t minus 3 e to the power of minus 3 t minus 2 t for t greater than 0 of course, it goes without saying it is 0 for t less than 0.

Given this data you are asked to find out. Find impulse response of the network that means if you the excitation is a unit impulse; What is the corresponding output? System function, taking the input voltage as the input and the resulting current as the output. c: Response to an input which is e to the power of minus 4 t ut,

d: steady state response to an input sinusoidal cos 2 t suppose, a sinusoidal voltage is given to this network what kind of steady state current you would get in this terminals that is the question that we asked. So, all this quantities are to be obtained. Now, the input is 1 volt and the output is 4 e to the power of minus t 3 e to the power of minus 2 t.

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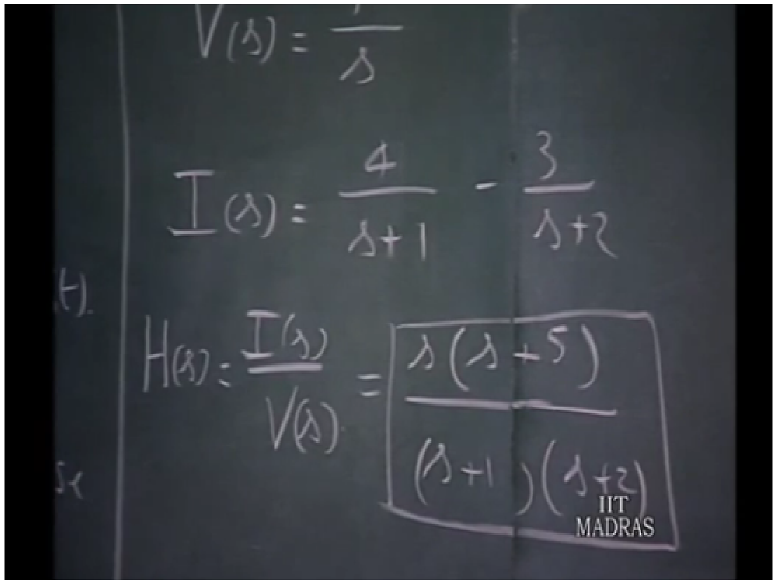

Therefore, the input Laplace transform is 1 volt. So 1 volt is given t equals 0 to this network therefore, it is a step function really v of s is 1 over s the current in the network as the Laplace transform of this 4 up on s plus 1 minus 3 up on s plus 2 4 up on s plus 1 minus 3 up on s plus 2 that is the expression for the current.

Therefore, I of s over v of s is the system function h of s that, will be obtained as you can work this out and show that this is equal to s times s plus 5 divided by s plus 1 times s plus 2 you can show that this is the system function h of s. So, that is the answer to b which we already obtained. If you look at the, this I of s can also be thought of as the step response because input is 1 unit step.

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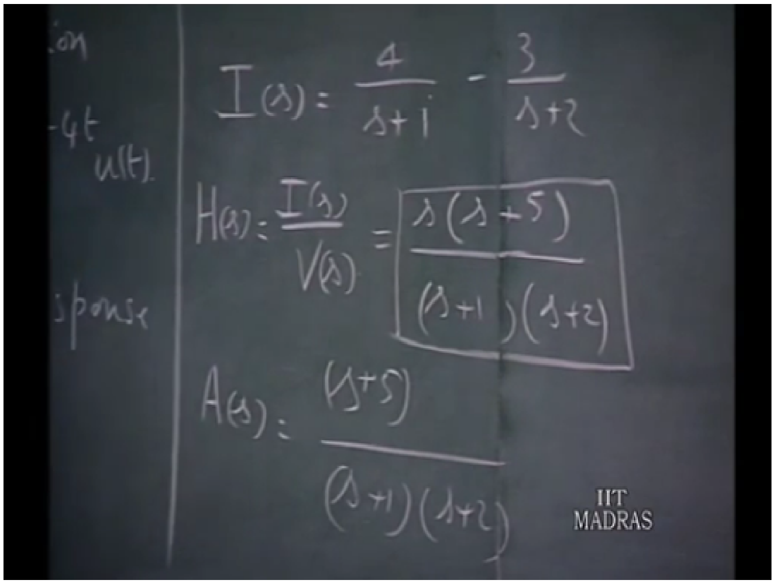

Therefore, a of is simply this 1. So, if this s plus 1 times s plus 2 so 4 s minus 3 is s 8 minus 3 5 so, this is a of s you can see that h of s is s times a of s that is the result which we already know.

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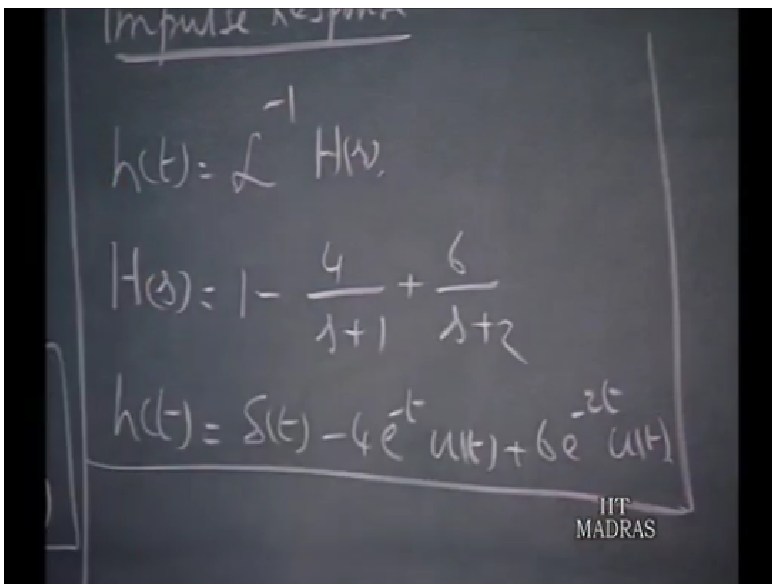

Impulse response, h of t is the Inverse Laplace transform of h of s this is h of s. So, h of can be expanded by means of partial fraction expansion so, it will turn out to be 1 minus 4 over s plus 1 plus 6 over s plus 2. So, if this rational function makes the partial fraction expansion of the rational fraction you get this because the numerator and denominator as the same degree polynomial s square divided by.

So, the ratio of the 2 leading coefficients is 1 that is the first term and then you have the residual the pole as at s equals minus 1 and the pole at s equals minus 2 will be like this. So, if you take the inverse Laplace transform of this delta t minus 4 e to the power of minus t plus 6 e to the power of minus 2 t of course ut you can write this 4 e to the power of minus t ut plus 6 e to the power of minus 2 t ut that is the impulse response.

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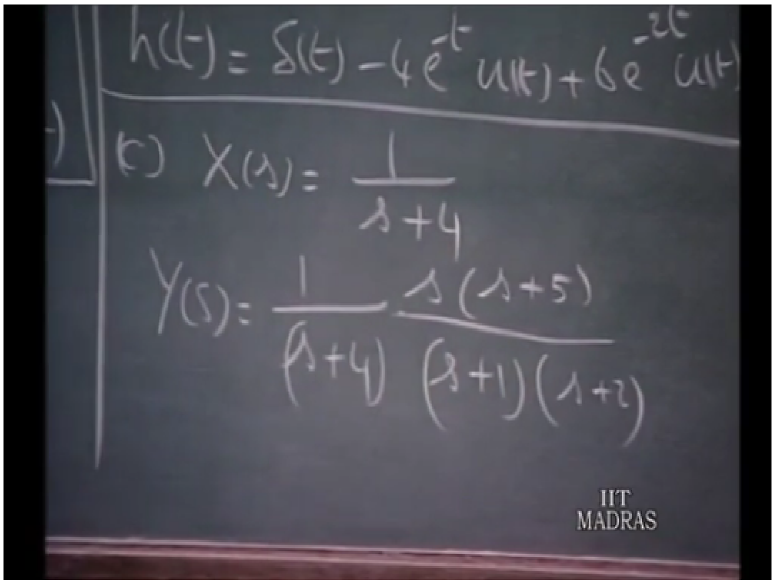

Now, response to e to the power of 4 t ut. So, x of s the input is e to the power of minus 4 t this is 1 over s plus 4. So, y of s corresponding to this is 1 over s plus 4; that is x of s times h of s h of s is s times s plus 5 divided by s plus 1 times s plus 2; that is the y of s. So, you make the partial fraction expansion of that.

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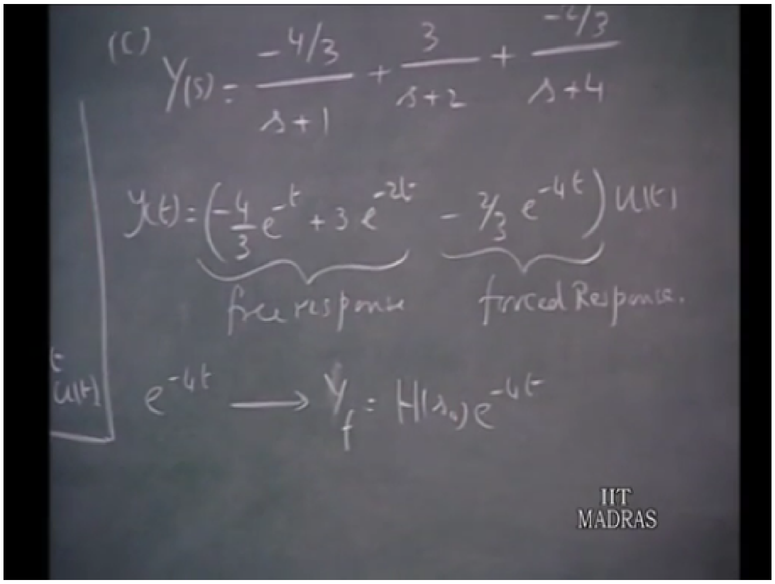

So, c continued y of s turns out to be minus 4 up on 3 over s plus 1 plus 3 up on s plus 2 plus minus 2 up on 3 over s plus 4 that is the partial fraction expansion of this we contain 3 terms those of the 3 terms. So yt will be minus 4 by 3 e to the power of minus t plus 3 e to the power of minus 2 t minus 2 by 3 e to the power of minus 4 t times ut the input is e to the power of minus 4 t and the output containing charm e to the power of minus 4 t.

This is the force response, the forced response contains exactly the same complex frequencies as the input. The system function has got 2 poles at minus 1 and minus 2 corresponding to this you have got this 2 terms e to the power of minus t e to the power of minus 2 t. So, this corresponds to this poles of x of s so, this represents the natural response of the free response.

So, as we mentioned earlier, the Laplace transform theory enables as at the total solution in 1 step the free response part and the forced response part you can apply get y of t using convolution integrals also all that you know ht and also you know this itself the it itself is at you can use the convolution and find out yt either from the impulse response or the step response wind up in the same result that you can work out exercise.

Now it will be interesting to see whether, you can get forced response independently suppose, you want only the forced response. So, the input is e to the power of minus 4 t.

So, the force response of that is h s 0 where s 0 is minus 4 times e to the power of minus 4 t.

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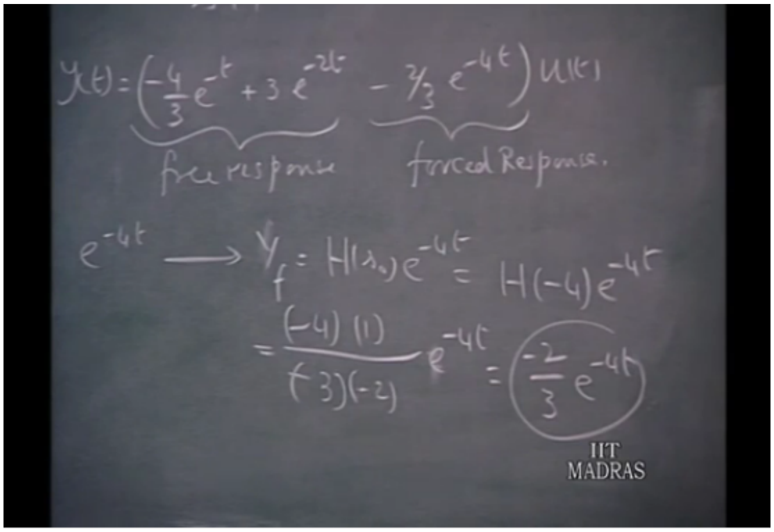

So, if you substitute this is really h minus 4 e to the power of minus 4 t we know h of s is s times s plus 5 divided by s plus 1 times s plus 4. So, when you substitute minus 4 for in this for s you get minus 4 s plus 1 divided by minus 3 times minus 2. So, that will become minus times e to the power of minus 4 t that becomes minus 2 up on 3 e to the power of minus 4 t.

So, this is exactly what you got here so, verification of the force response you can see that, the force response can be obtained by substituting s 0 for s, where s 0 e is the complex frequency in the input exponential signal. The last part is: what is the steady state response for cos 2 t.

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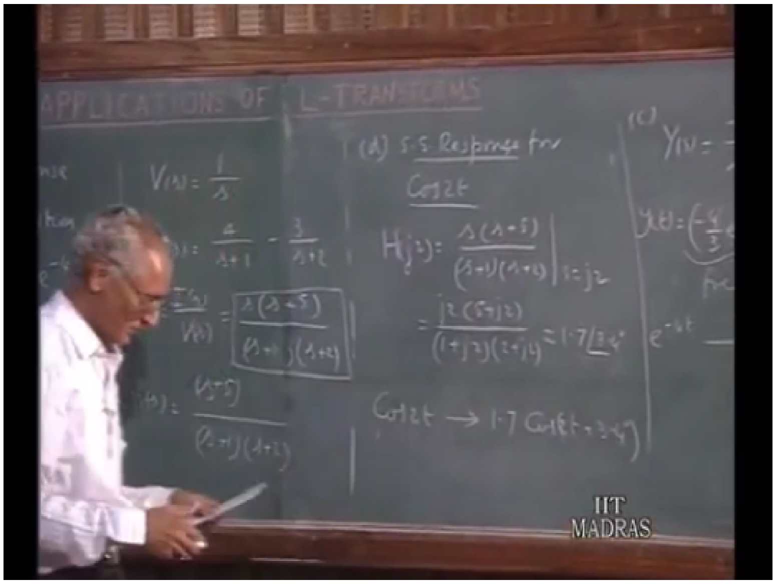

So the sinusoidal has the input sinusoidal frequency 2 so, hj 2 equals our h of s is s times s plus 5 divided by s plus 1 times s plus 2 this substitute s is equal to j 2. So, when we substitute s is equal to j 2 in this you get j 2 times 5 plus j 2 divided by 1 plus j 2 times 2 plus j 2.

If you simplify this rationalize in simplify this will turn out to be approximately 1.7 at an angle of 3.4 degrees. So, when you substitute this j 2 for s and evaluate this is the frequency response function h j 2 terms at 1.7 times 3.4. So, if cos 2 t is input the steady state response of that will be 1.7 this is the magnitude the frequency response expansion cos 2 t plus 3.4 degrees.

So, this frequency response function it cancels all the amplitude of the sinusoidal of the phases of the sinusoidal modified in the as for as the steady state response is concerned we will sin 2 t to still be 1.7 sin 2 t plus 3.4 degrees. So, this is how 1 can use the frequency response function evaluating the steady state response the sinusoidal trial in forces.

In the course of the last 4 lectures, we have seen how the Laplace transform whether can be portably employed to solve for the transients in networks and systems taking the case of network transients to be specified we saw that in the Laplace transform method, how the governing equations for the network in question or formed a algebraic form either from the transformation differential equation or to the transform diagram.

How the response quantities Laplace transform to the quantities can algebraically arrive that and then from through inverse transformation, how the corresponding time functions can be found out. There is no algebra involved for them are the initial there is no calculus involved is only the algebra that is involved furthermore the initial conditions are plugged into the formation of the algebraic equation.

Even at the very beginning itself and know separate evaluation of the arbitrary constants is called for as we had in the case of differential equation approach. Perhaps the greatest difficulty in the solution of the Laplace transform method lies in finding out inverse transformation particularly.

When the denominator polynomial is of a high order not only we have find the residues in the various poles. But, the determination of the poles itself, will become tedious if the denominator polynomial is of a high order. But, this is the problem which is attendant for large size problems by in any method even the differential equation approach and certainly the blame cannot be laid at the door of the Laplace transformation method.

We have also seen, how this system function can be performance of a general system, the system function embedded itself various properties pertaining to the system, we can use this system function the steady state response in exponential input by mere substitution of the corresponding complex frequency in the h of s.

We can see, how the system function can be used to find out the impulse response you can find, how the system function can be used to find out the step response and how the system function is almost equivalent to the frequency response function. So, the system function has number of versatile property which we are seen and in a general situation.

We can see how the system, how we can the once we know the frequency response the impulse response of the step response, we can find out the frequency response conversely. If you know the frequency response over a large entire range of frequency there also methods available by means of which we can find out the transient response.

In other words linear system if you know the transient response by that I mean the response to a step input or a impulse input we can find out the frequency response for any sinusoidal input and vice versa. There are algebra procedures and graphical procedures available where analytical expressions cannot be found out for either the frequency response or the transient response.

After cravings that all this, the Laplace transformation method must also say as some disadvantages certainly it cannot be used for systems which are either non-linear or time varying, even for the simple case of the product of 2 time functions the corresponding Laplace transform cannot be easily found out it involves complex convolution for other types of non-linear it may difficult to finds the Laplace transformation.

Similarly, when you time varying equation the convolution integral that you talked about cannot be used in the same form it becomes much more complex. So, for the general non-linear time varying systems, Laplace transformation is almost useless the differential equation is more fundamental.

And perhaps it may permits only feasible method of solution in such situations, even though this may involve complicated calculations and perhaps even numerical methods for the solution. But, then differential equation method is more fundamental and it can be used for a general situation where as the Laplace transform method cannot be used in such situations.

So, with that we close our discussion of the application of Laplace transformation method to networks and systems and now we look some problems which form an exercise on the topic which we are covered in the last 4 lectures.