Digital Protection of Power System
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Lecture 05
Phasor Estimation Algorithm-I

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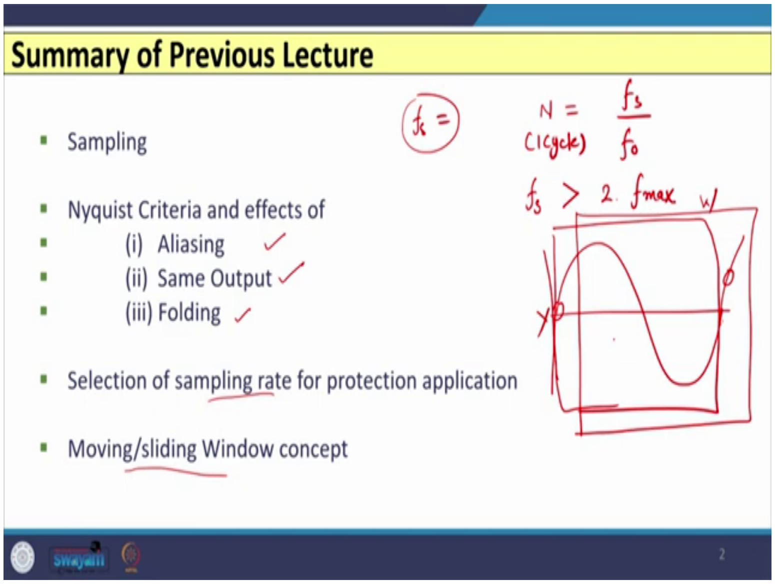

Hello friends, in the previous lecture we discussed the sampling. So, whenever sampling frequency is defined or given for a particular signal, then the number of samples in 1 cycle, that is given by the ratio of sampling frequency divided by fundamental frequency.

Then we discussed that, whenever we acquire any signal, we need to follow the Nyquist criteria, or Nyquist theorem, which indicates that the sampling frequency should be greater than 2 times the maximum frequency present in the acquired signal. If this criterion is not followed, then the effects known as aliasing, same output, and folding, these three are observed.

Further, we have also discussed regarding the selection of sampling rate for protection application, and we have also discussed that, if we use from for other application like power quality, then also as per IC 61850 sampling rate is defined. At last, we have discussed regarding the concept of sliding window, and we have discussed that whenever we have any sinusoidal signal and if we consider a window, let us say w, then whatever number of samples we acquired in a particular window, that remain constant.

So, when the next sample we take the previous sample should be discarded. So, whenever the next window is available, then in that window also the number of samples remains constant. So, this we have discussed.

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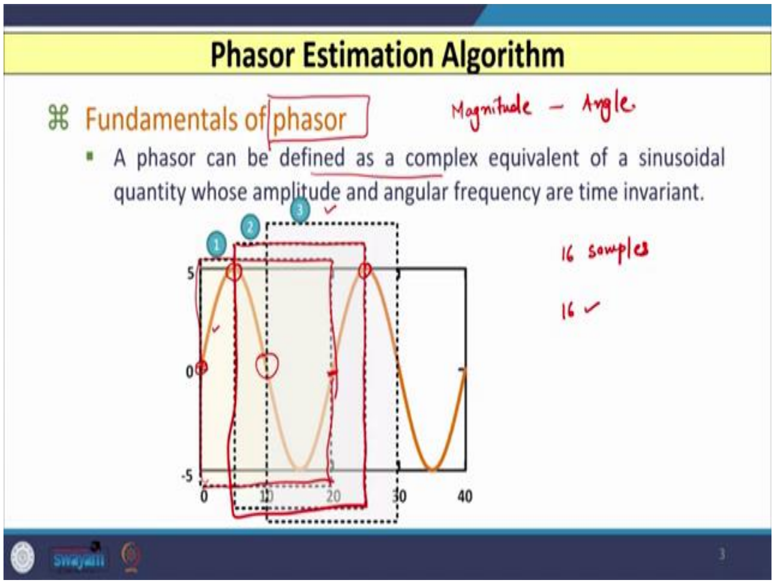

Now, in this class, we are going to discuss the Phasor estimation algorithm. The phasor estimation algorithm is very useful for our protection applications. So, let us see how we define phasors. So, a phasor is defined as a complex equivalent, so that contains magnitude as well as the angle. So, it is a complex equivalent of any sinusoidal quantity, in which amplitude and angular frequency remain constant.

So, to understand this, let us consider a sinusoidal wave as shown here. So, here you can see that we can take the samples, or we can start taking samples from this instant and we can stop at here, so this is going to become a first window, you can see this is the window in which we are going to acquire the samples.

So, in this window, if you calculate by taking whatever number of samples you consider in this first window, let us say you have taken 16 samples. So, using these 16 samples in 1 cycle, if you calculate the phasor, then it remains constant. Similarly, for the second window, let us say here you can see in second window, we have started with $\pi/2$. So, when you end with another $\pi/2$, let us say in this window and then in this second window also the number of samples, that remains 16 only.

And now if you calculate the phasor, that is amplitude and angle, then that also remain constant. And same is the case, when you acquire the sample in third window, which we start with π`. So, irrespective of whether we take the sample at 0, or 30, or 90, or π, and if we calculate the phasor value using the acquired samples, then that remains time invariant.

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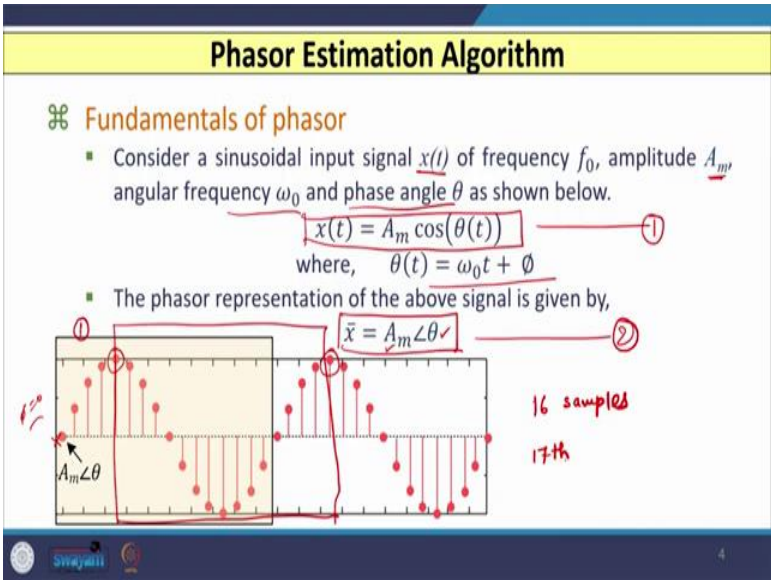

Now, to understand the fundamentals of phasor, let us consider a signal x(t), this signal has a fundamental frequency f_0, amplitude A_m and angular frequency that is ω_0, it also has a phase angle theta. So, if this is the case, then this input signal x(t) is defined by this equation, let us call it this equation as number 1.

So, this $x(t) = A_m \cos(\theta(t))$, where $\theta(t) = \omega_0 t + \Phi$. So, ω_0 is as I told you it is angular frequency and Φ is the angle. Now this angle Φ, if we let us consider this signal and if we start acquiring samples from here and if we take 1 window, then this phi in this case it is 0.

However, if I start the acquiring the sample, let us say at this point and if I go up to here, let us say this is the window, then in this case phi will be π/2and so on. So, if I calculate for this signal x(t), the phasor value, then it is given by this equation, let us say this is equation number 2. So, phasor value of this signal x(t), that is given by magnitude part A_m and angle that is θ, and that is what we are interested to calculate this phasor value, that is magnitude and angle θ.

Now, whenever the new sample is acquired. So, here let us say this window 1, we take few samples, let us say 16 samples we have taken, now whenever we acquire 17 sample obviously, we have to discard the first sample.

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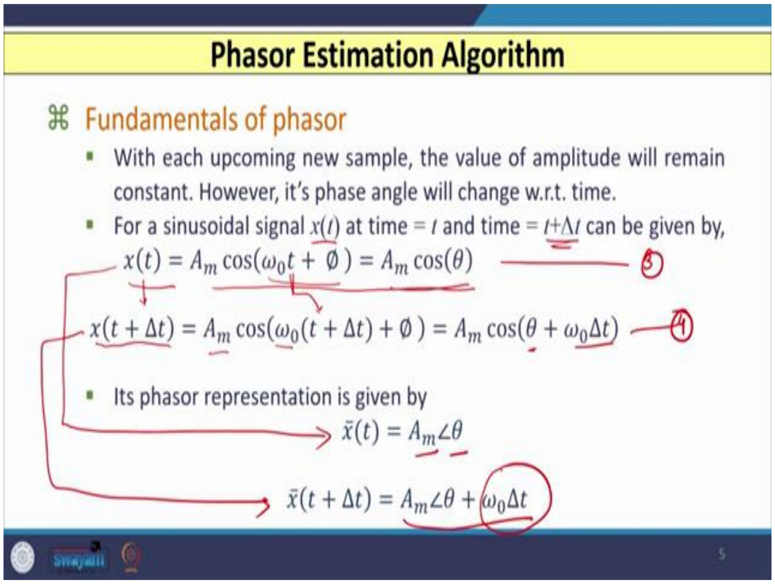

So, if we take the new sample, then the value of amplitude remains constant, however its phase angle changes with respect to time. How this is possible? To understand this, let us consider the same signal x(t), as we have considered earlier given by this $A_m \cos \theta(t)$ in equation 1. So, here if I take two different time instant, let us say for in case 1, I consider time is equal to t, then my x(t) will be $A_m \cos(\omega_0 t + \emptyset)$. So, x(t) will be equal to $A_m \cos(\theta)$.

At 2nd case, if I consider $t = t + \Delta t$, then this equation I replace t with $t + \Delta t$. So, x($t + \Delta t$) delta , that is equal to $A_m \cos(\omega_0 (t + \Delta t) + \emptyset)$, so that is nothing but theta. So, $A_m \cos(\theta + \omega_0 \Delta t)$.

So, if I have these two equations one is x (t) = to $A_m \cos(\theta)$, and another x($t + \Delta t$), then if I calculate the phasor value of equation 3, we will have the output in terms of phasor, that is $A_m \angle \theta$. However, if I calculate the phasor value of this x($t + \Delta t$), then its phasor value is given by$A_m \angle \theta + \omega_0 \Delta t$, here $A_m \angle \theta$ plus the additional term available that is $\omega_0 \Delta t$, that is why whenever you acquire a new sample, or whenever new sample is available the value of amplitude remain constant. However, there is only a change in the phase angle, and it changes with reference to time.

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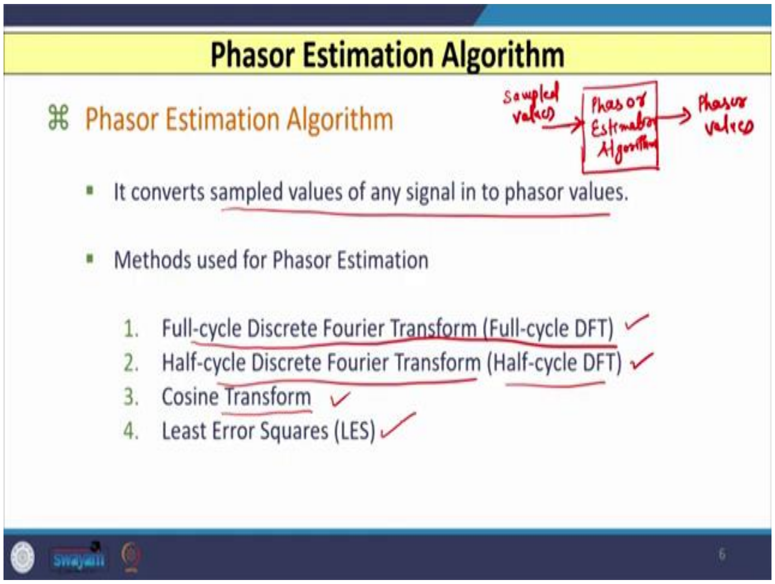

Now, let us see what are the different types of phasor estimation algorithms available? So, as I told you the function of phasor estimation algorithm is to convert sample values into any phasor values. So, if I have let us say one box, or black box, which is nothing but phasor estimation algorithm, then input of this blocks that is nothing but sampled values and output of this box that is nothing but phasor value, phasor values, that is magnitude and angle.

So, different methods are available. The first method available is known as full cycle discrete Fourier transform algorithm, it is also known as full cycle DFT. The second type of phasor estimation algorithm available is known as, half cycle discrete Fourier transform algorithm, or in short it is also known as half cycle DFT. The third algorithm is known as cosine transform algorithm, and fourth one that is least error square algorithm.

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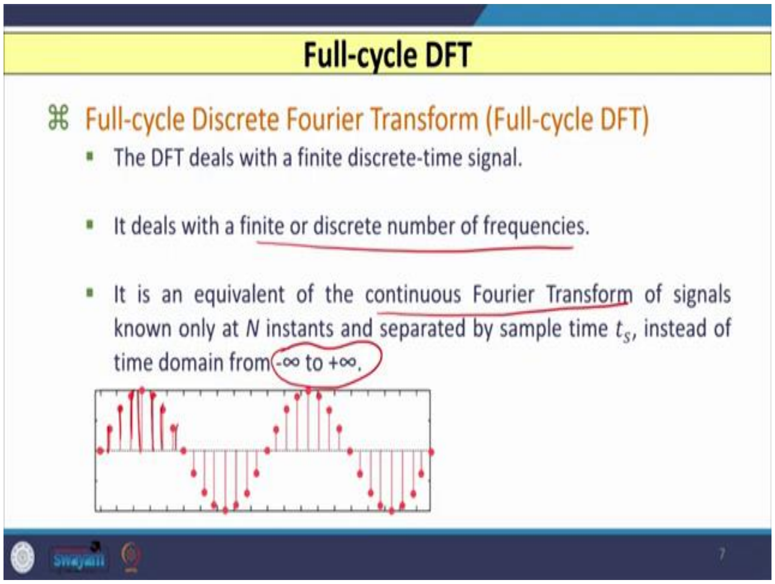

So, now let us see first, the full cycle discrete Fourier transform algorithm. So, let us see using full cycle DFT, how the phasor value can be estimated, if sample values are available for any given signal. So, discrete Fourier transform deals with finite discrete time signal. So, you can see in the waveform, we are taking finite discrete time signals. So, DFT basically deals with this, it also deals with finite, or discrete number of frequencies.

So, whenever you want to estimate phasor value of any given signal using full cycle DFT, you can estimate only phasor values of those signals, which are in multiple fundamental frequencies. If you want to estimate any in between frequency, let us say 75 Hz, or 55 Hz, then that is not possible.

So, if we compare discrete Fourier transform with continuous Fourier transform, then in continuous Fourier transform, the limit is given from minus infinite to infinite, whereas the limit of discrete Fourier transform is up to a particular window.

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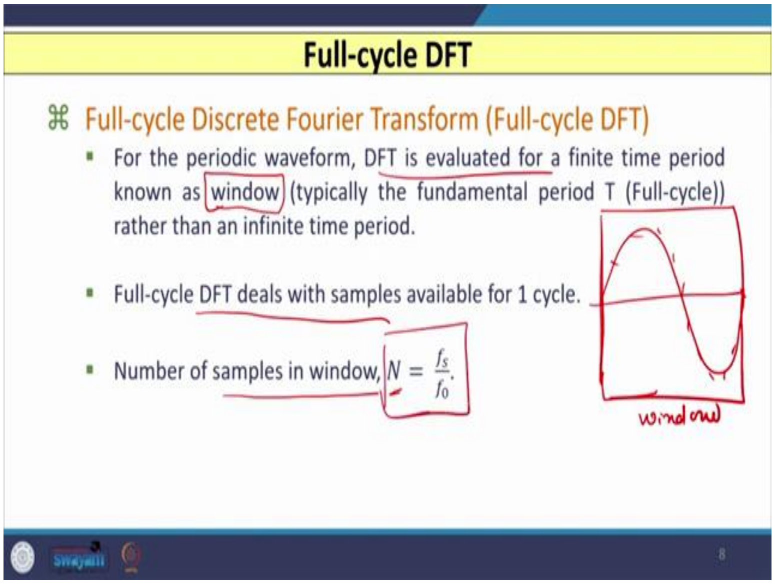

So, if I consider any periodic waveform, then DFT is evaluated for a finite time period and this finite time period is known as a window. So, in particular window, we are taking, let us say we have one signal and let us say we are taking this window, this is nothing but window. So, if you acquire the number of samples at different time instant, then using this number of samples for complete one cycle, this is evaluated using discrete Fourier transform.

So, full cycle DFT is dealing with the samples available for complete 1 cycle. So, if sampling frequency is given, let us say we are acquiring sample of particular signal, let us say current signal and if sampling frequency is given and if fundamental frequency is also known, then number of samples in a window, that is n that is given by it is a ratio of sampling frequency divided by fundamental frequency.

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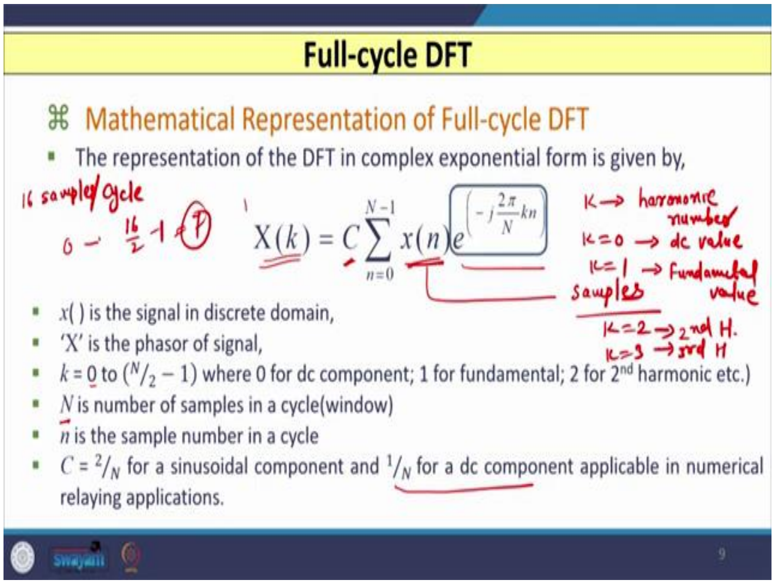

Now, let us see how mathematically full cycle DFT is represented. So, in complex exponential form the full cycle DFT is represented by the equation $X(k) = C\sum_{n=0}^{N1} x(n)e^{\left(-j\frac{2\pi}{N}kn\right)}$.

Here this x(n) is nothing but the signal in discrete domain. So, whatever number of samples are available this is nothing but the number of samples available for any signal, which we are going to acquire. X is the phasor value of the signal; k is the harmonic number. So, if I consider the k it indicates the harmonic number. So, if you consider let us say k is equal to 0, then that indicates the dc value, if you indicate let us say k equal to 1, then that indicates fundamental value. If you put let us say k is equal to 2, then that indicates second harmonic. If you put k equal to 3, it indicates 3rd harmonic and so on. So, the value of k varies from 0, where 0 indicates the dc value and it goes up to $\left(\frac{N}{2} - 1\right)$, where capital N is nothing but the number of samples in the cycle. So, as I told you if sampling frequency is given and fundamental frequency is given, then you can easily calculate the value of N, that is number of samples in a cycle.

So, if suppose 16 samples in a cycle are given, then the value of k varies from 0 to $\left(\frac{16}{2} - 1\right)$. So, that is nothing but 7. Small n is the sample number in a cycle, whereas the value of C, that is a constant and for any sinusoidal component, its value is given by $\frac{2}{N}$ and for any dc component, its value is given by C is equal to $\frac{1}{N}$, and this is fixed for all numerical relaying applications.

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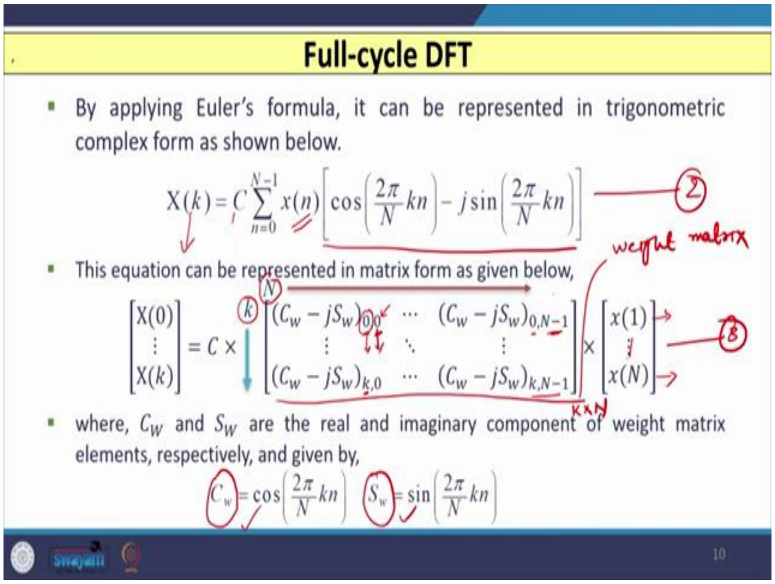

Now, in this equation, if I consider this exponential term and by applying Euler's formula, this exponential term can be represented in trigonometric term and this is given by:

$$X(k) = C\sum_{n=0}^{N-1} x(n)\left[\cos\left(\frac{2\pi}{N}kn\right) - j\sin\left(\frac{2\pi}{N}kn\right)\right]$$

So, if I put $cos\frac{2\pi}{N}$, kn as Cw and if I put $sin\frac{2\pi}{N}$, kn as Sw and if I write this equation, let us say this is equation 2 in matrix form, then this can be represented by equation 3.

So, here you can see that, this X(k,) where k represents harmonic number. So, this left-hand side, that is phasor value output, which we want to calculate, or estimate, that varies from X equal to 0 to X equal to k, where k limit is again restricted by $\frac{N}{2} - 1$.

On the right-hand side, C is constant, which is a scalar term and then some matrix, let us call it this is a weight matrix. And so, this matrix is nothing, but this is known as weight matrix and that is multiplied with the term $x(1)$ to $x\ (N)$. So, this term $x\ (N)$, which is nothing but the sampled values of the signal, which we are acquiring. So, that varies from 1 to N, which is nothing but the number of samples in a cycle.

Now, here there is a difference, you see the value in weight matrix that varies from 0 to $N - 1$, whereas the value of this $x(1)$ to $x\ (N)$, that varies from 1 to N. And the reason is whenever we acquire the samples from any signal, let us say current signal, then that signals we are having from first number to the number of samples in a cycle. Now, if we consider this in between weight matrix, then that matrix is nothing, but it is obtained and its size is nothing but $k \times N$, where k is the harmonic number and N is the number of samples in a cycle.

So, if we have the $C_w - j\,S_w$, where C_w and S_w is given by these two equations, then it varies from let us say if I consider the first row, then first element is 0, which indicates the value of k. And the second that indicates the column. So, that first row is indicated by k, that is harmonic number and second that is indicated by the N, that is number of samples in a cycle.

So, the first element is 0, and if you move further in the first row only, then you have 0, 0 to 0 to $N-1$. So, you have let us say if we are taking 16 samples, then you have in first row 16 elements. Similarly, you can increase the value of k. So, it becomes 0 to 1 and again you have to go up to $N-1$, again you increase the k you reach up to final value of k, which varies from k to 0 and it goes up to k to $N-1$. So, this is how the weight matrix, which contains C_w and S_w, the size depends on k and N.

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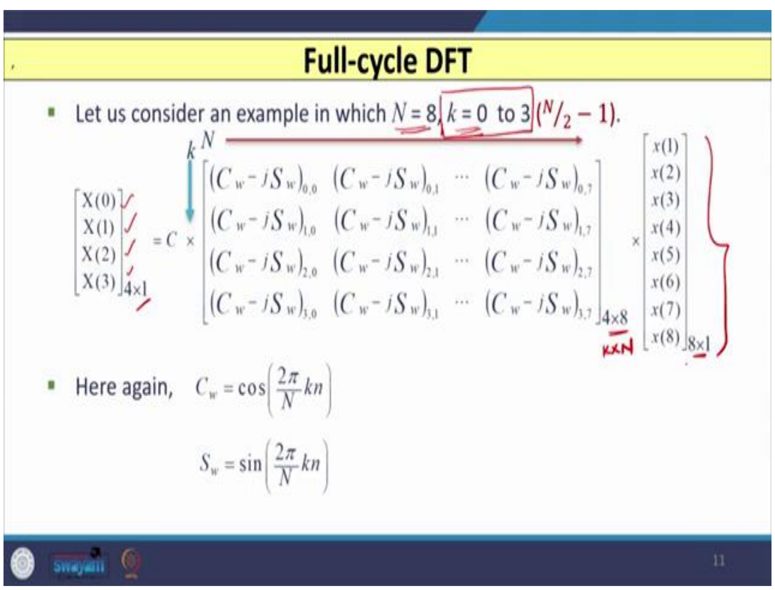

Now, let us consider one example in which the value of N number of samples in a cycle that is 8. Let us consider the value of k that is 0, so we want to estimate the dc value. So, here when I say k equal to 0, that means we want to estimate, or we want to calculate the phasor of dc term. And here the value of N is 8. So, k varies from 0 to $\frac{N}{2} - 1$. So, 8 by 2, that is 4-1, that is 3. So, k varies from 0 to maximum value, that is 3, in this case.

Now, if I consider the left-hand side, as I told you the phasor value which we want to estimate that is again varies up to k. So, X (0), X(1), X(2), and X(3) terms are available and the size of this that is 4 $\times$1. Now, as I told you the size of this weight matrix, that is from 4/8, or k b/N. So, here k value is 0 to 3, which includes 0, so total 4 and N that is 8. And this value is the samples which we are acquiring for any signal, let us say current signal, it has 8$\times$1 matrix.

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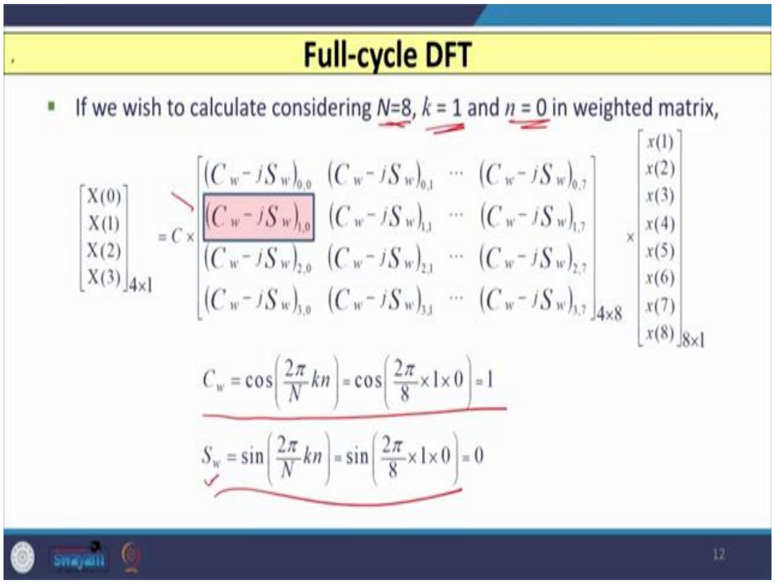

Now, let us say we want to calculate this term as highlighted, let us say 1, 0 term. So, considering N is equal to 8, as I told you the $C_w - jS_w$ 1, 0, so 1 indicates the value of k that indicates 0, and second 0 indicates the value of N sample number, which varies from 0 to N-1. So, if I put this value N=8, k=1 and n =0 in these two equations, then $cos\frac{2\pi}{N}$, that is 8$\times$ 1and n=0. Similarly, if I put in S_w also, then you will get the value of C_w and S_w, that is 1 and 0.

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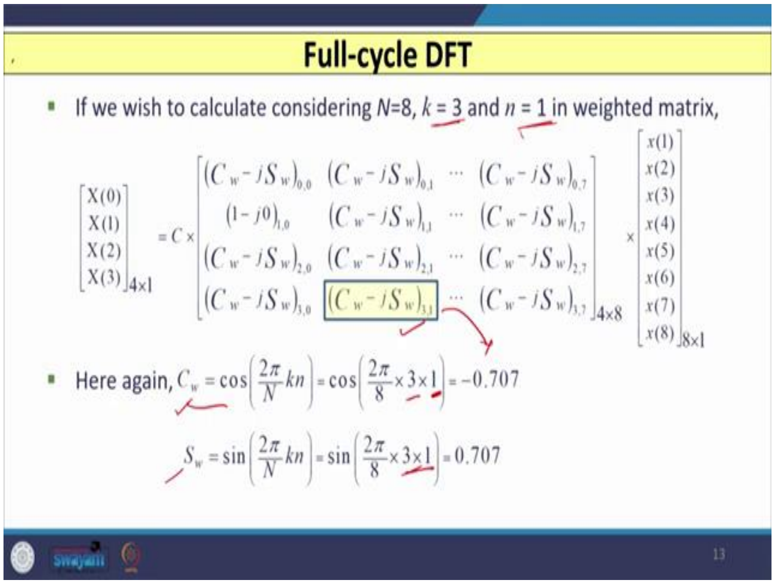

Similarly, if I wish to calculate this value, let us say this term. So, then that indicates 3, 1, so k is equal to 3 and n is equal to 1. So, again you put the value of k that is 3 and n that is 1. Here N remains constant, so you can obtain the value of C_w and S_w, that is minus 0.707 and plus 0.707.

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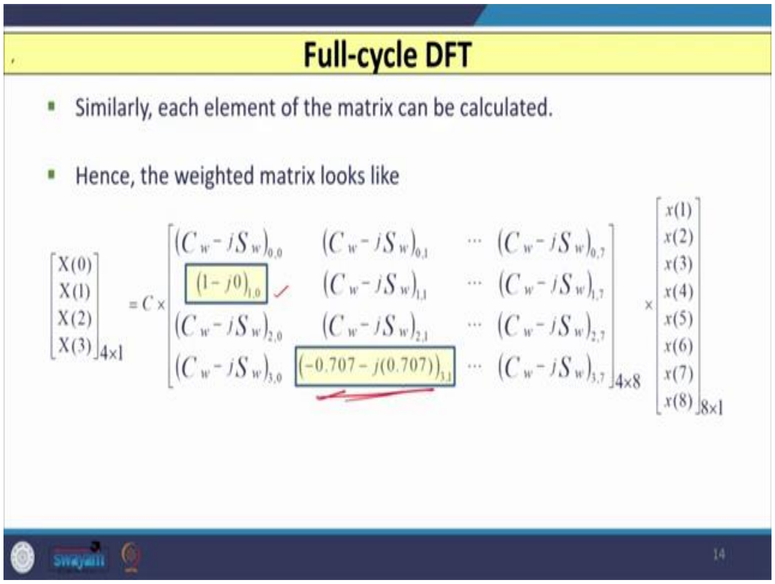

So, similarly you can calculate each element of the weight matrix and hence if you put all the values in weight matrix, then weight matrix looks like this. So, you can see that, last, we have obtained the value C_w that is minus 0.707 and S_w that is 0.707. So, we can put this value here. And this is how you can obtain the value of weight matrix.

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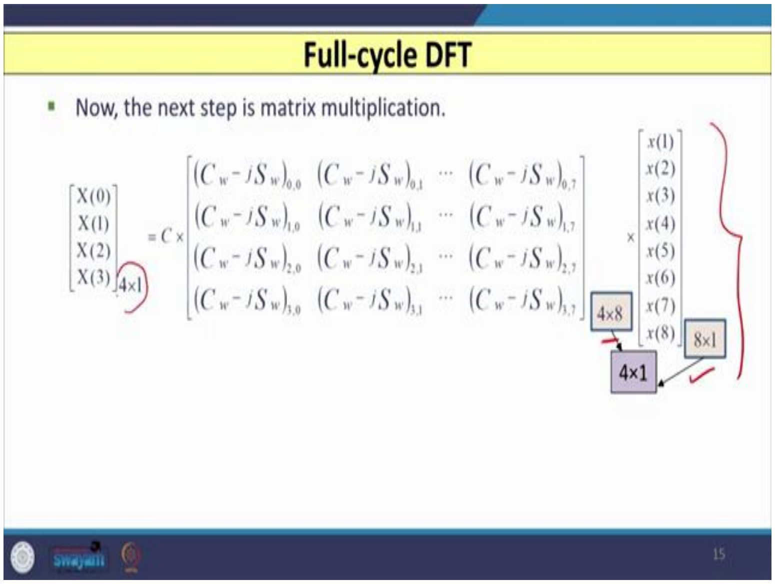

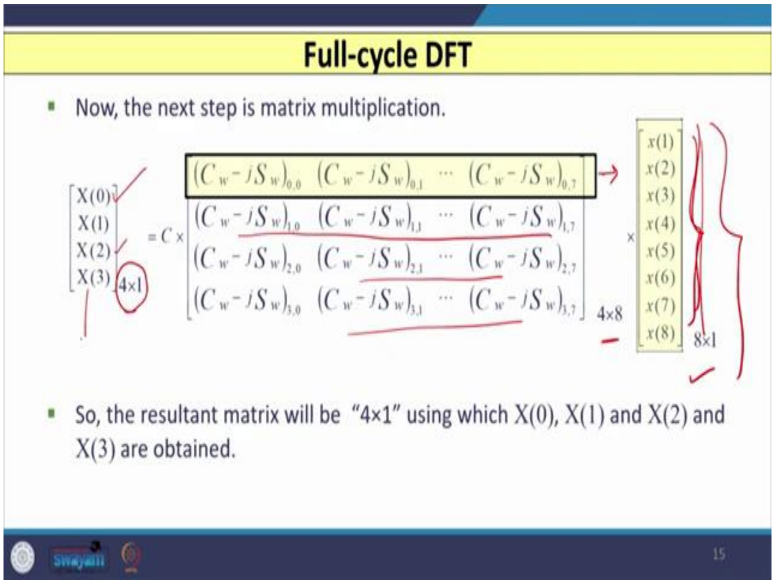

Now, the next step is matrix multiplication. So, here what you have to do is, your weight matrix is k by n, that is here in this case 4 by 8, and this value sampled value matrix, that is 8 ×1. So, finally the obtained value of the output, that is capital X you have that is 4×1. So, here in matrix multiplication, this row you need to multiply with these values. Similarly, in the second case you need to multiply this with this again. Then third row with all these values and fourth row with all these values of X (1) to X (8).

So finally, resultant will be 4×1 and that is X (0), X (1), X (2), and X (3). So, X(0) indicates the dc value phasor X (1) indicates the phasor of fundamental, X (2) indicates the phasor of second harmonic, and X(3)indicate phasor of third harmonics and so on.

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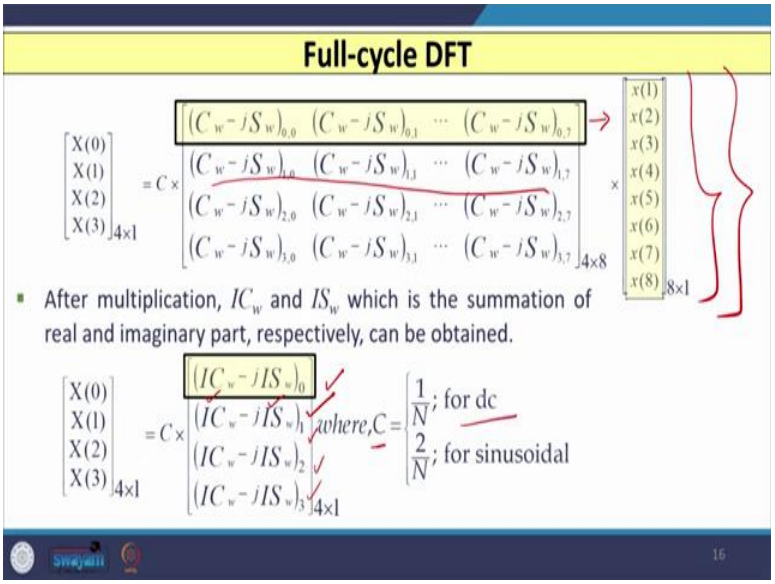

Now, if I multiply this first row with these sampled values, then if I call this multiplication output as ICw, that is real part and ISw that is imaginary part, then you will have the output that is 4 cross 1. So, we will have IC_w-j IS_w- that is 0. Then 2nd, 3rd and 4th, so it is 4×1.

So, when you multiply this second row with these sampled values you will get this thing, same way you can get third and fourth row. Now, the next task is to multiply this IC_w and IS_w with scalar value C and C is already we have defined for dc it is 1/N and for sinusoidal it is 2/N.

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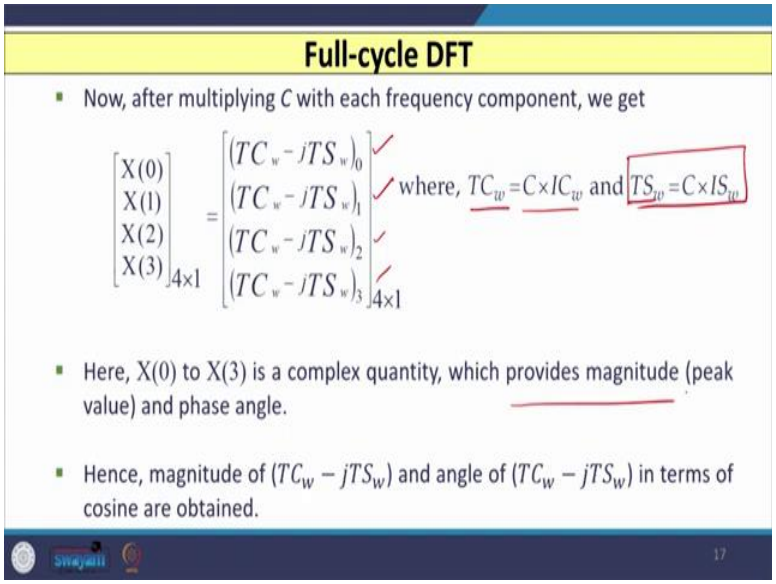

So, if you multiply this value of C with IC_w and IS_w, then you will have let us say TC_w and TS_w, in first, second, third, and fourth row. So, TC_w is nothing but C into TC_w and TS_w is nothing but the C into IS_w. So, here whatever value you obtain X(0), X(1), X(2), and X(3), these values are complex quantity and that provides magnitude that is peak value and it also provides phase angle.

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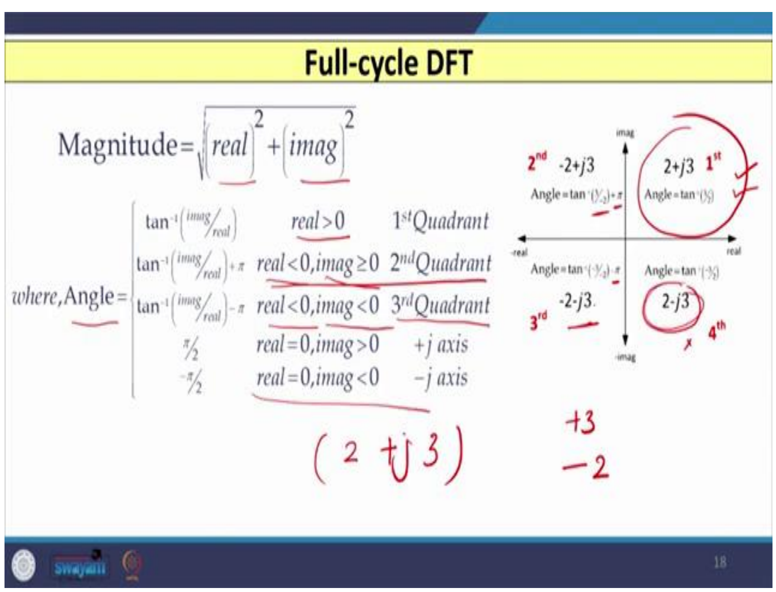

Now, one very important point is whenever we have magnitude, which is given by real square plus imaginary square and square root. And whenever we have angle, then angle is given by the equation $\tan^{-1}\frac{\text{imaginary}}{\text{real part}}$. Now, here when the real part is greater than 0, then your angle will be in the first quadrant, for example if I consider the real value is 2 and imaginary value is let us say 3, so that angle that is available, or falls in first quadrant.

But in the second case, when real value is < 0 and imaginary value $\geq$ to 0, then the angle let us say the imaginary value is +3 and real value is less than 0, let us say it I -2. So, if you take $\tan^{-1}(\frac{3}{-2})$, then the angle, which is obtained, or that is same as the angle which you obtain when your values is 2 -j 3. So, in this case $\tan^{-1}\frac{-3}{2}$, and $\tan^{-1}\frac{3}{2}$ both will give you same angle and it will fall in fourth quadrant.

So, to actually put into the second quadrant you have to add plus pi, so this is plus pi you have to add, this is very important. Similarly, when real part < 0 and imaginary part < 0, then the angle falls in third quadrant. But this is also same as the value which you obtain, let us say $\tan^{-1}\frac{-3}{-2}$, or $\tan^{-1}\frac{3}{2}$ both will give you same angle, but this angle will fall here in first quadrant.

So, to again put into the third quadrant you have to again put minus pi sine. So, you have to add minus pi in the angle, so that your actual angle will be in the third quadrant. And similarly in the fourth quadrant you have the final value, that is this one. So, here in this class, we started our discussion with the fundamentals of phasor. Then we considered one signal and then we obtained that whenever we want to calculate the phasor value of any sampled signal given, then amplitude remains constant, whereas phase angle changes with reference to time.

We have also discussed different methods available for estimation of phasors. If sampled values of signals are given. Then we discussed the full cycle discrete Fourier transform algorithm and how we are going to calculate each and every term of this discrete Fourier transform algorithm using a particular equation. Thank you.