Digital Protection of Power System
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Lecture 06
Phasor Estimation Algorithm-II

Hello friends. So, in the previous lecture, we have discussed about the discrete Fourier transform algorithm and using full cycle DFT algorithm, how we are able to obtain the phasor value of the sampled values available for any given signal.

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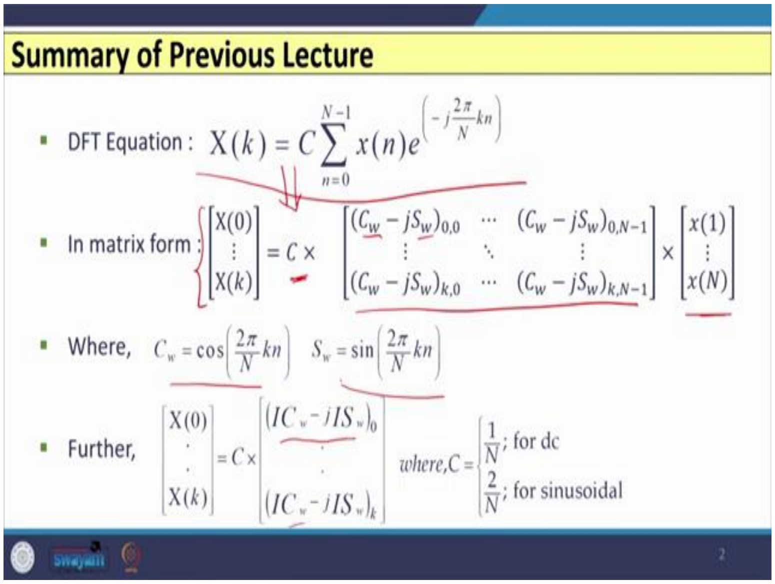

Now, in that class we have discussed regarding the equation of DFT:

$$X(k) = C\sum_{n=0}^{N-1} x(n)e^{\left(-j\frac{2\pi}{N}kn\right)}$$

And if we represent this equation in matrix form then,

$$\begin{bmatrix} X(0) \\ \vdots \\ X(k) \end{bmatrix} = C \times \begin{bmatrix} (C_w - jS_w)_{0,0} & \cdots & (C_w - jS_w)_{0,N-1} \\ \vdots & \ddots & \vdots \\ (C_w - jS_w)_{k,0} & \cdots & (C_w - jS_w)_{k,N-1} \end{bmatrix} \times \begin{bmatrix} x(1) \\ \vdots \\ x(N) \end{bmatrix}$$

Cw and S_w are given by these two equations, $C_w = \cos\left(\frac{2\pi}{N}kn\right), S_w = \sin\left(\frac{2\pi}{N}kn\right)$

And when we further multiply this finally, we obtained the value of the IC_w and IS_w.

$$\begin{bmatrix} X(0) \\ \vdots \\ X(k) \end{bmatrix} = C \times \begin{bmatrix} (IC_w - jIS_w)_0 \\ \vdots \\ (IC_w - jIS_w)_k \end{bmatrix}$$

$$where, C = \begin{cases} \frac{1}{N}; \text{ for dc} \\ \frac{2}{N}; \text{ for sinusoidal} \end{cases}$$

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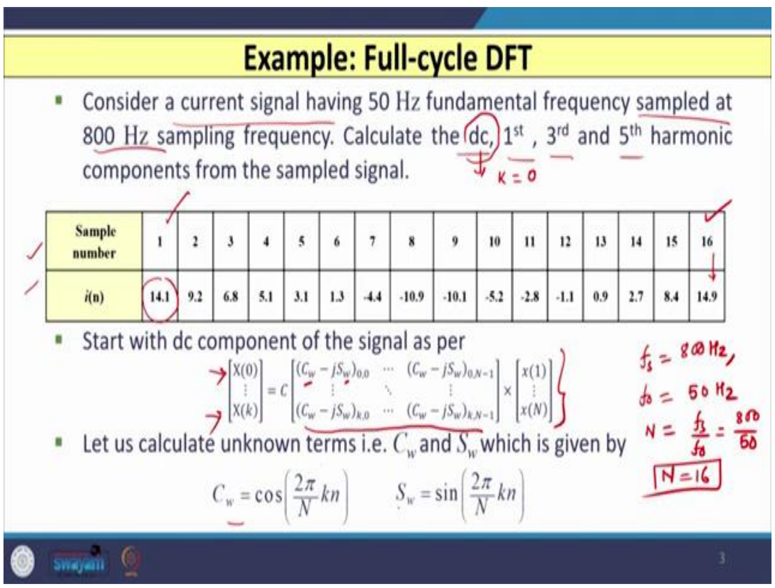

So, to understand this let us consider one example of a full cycle discrete Fourier transform algorithm. So, consider a signal, which has 50 Hz fundamental frequency, and this signal is acquired with a sampling frequency of 800 Hz. In this case, we wish to calculate dc component, first fundamental term, 3rd harmonic and 5th harmonic, from the available sampled values of the signal and we want to calculate phasor value of dc first, 3rd and 5th harmonics.

So, the available samples of the acquired signals are given here in this table. So, here you see the first row indicates the sample number, where 16 samples are shown. So, we have a sampling frequency, that is 800 Hz, and we have the fundamental frequency of the acquired signal, that is 50 Hz.

So, the number of samples in a cycle, that is capital N is given by f_s/f_0. So, that is 800 by 50. So, it comes out to be 16 samples in 1 cycle. So, that is why the sample number, that is given from 1 to 16. The value let us say when we acquired first sample, the value of that, let us say current quantity is 14.1. Similarly, when you acquired other samples up to 16, the values are also shown in the second row.

Now, let us start with the calculation of phasor value of dc term. So, this term is nothing, but we have to understand, or put the value of k that is 0. So, our original equation on left hand side that starts from X(0) to X(k) that is equal to C and we have weight matrix multiply by sampled value matrix. Now, let us first calculate the unknown terms of the weight matrix that is C_w and S_w. So,

we know that: $C_w = \cos\left(\frac{2\pi}{N}kn\right), S_w = sin\left(\frac{2\pi}{N}kn\right)$

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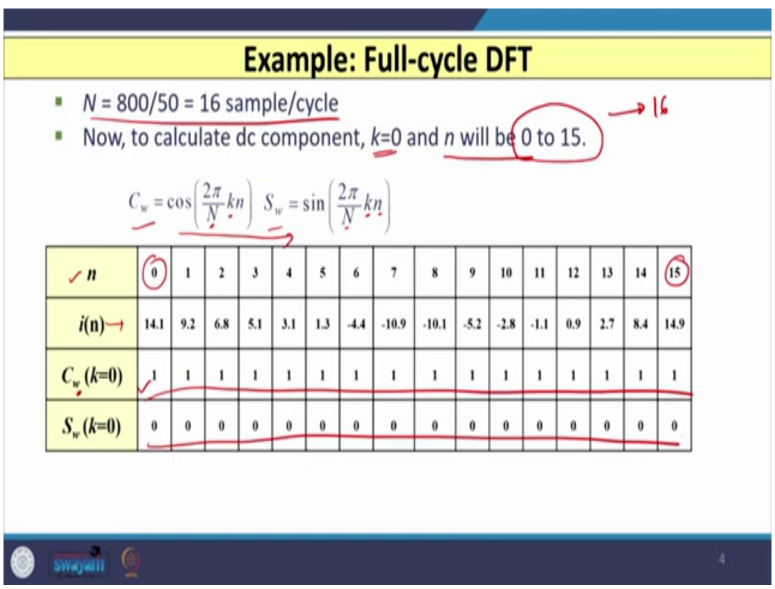

So, as we have 16 samples per cycle available, and to calculate the dc term as we have discussed that we have to put k is equal to 0, and small n, which is a sample number that varies from 0 to 15. So, a total of 16 values. So, let us see how we can calculate this C_w and S_w term?

So, we have the value of n, that is from 0 to 15, it varies, at each sample we have the values of in from 0 to 15. And C_w (k=0), N=16 and n that is again if you vary let us say 0 to 15, then you know that the value of k is 0. So, cos of 0, that is 1 for all the samples. And similarly, the value of S_w (k=0), N=16, and n varies from 0 to 15. So, S_w comes out to be 0. So, as C_w that comes out to be 1 and S_w that comes out to be 0.

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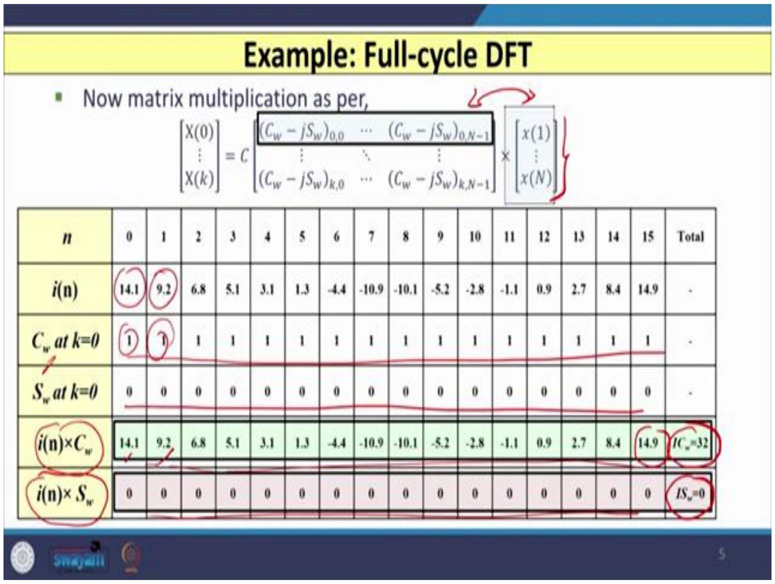

So, you can easily now calculate the values of this weight matrix after multiplying with this sample value matrix. So, here we have already calculated the value of C_w and the value of Sw. So, here now we have to multiply this first row with the sample value of the matrix. So, let us multiply these two, so when you multiply these two, so C_w has to be multiplied with i(n).

So, you have to multiply i(n)×C_w. So, it comes 14.1, similarly 9.2 with 1 and so on you can have the last value that is 14.9. And similarly, you can also have the imaginary term Sw, which is nothing but the multiplication of i(n)×S_w. So, as the value of S_w is 0, so your value of i(n)×S_w comes out to be 0.

And if you take addition of all these terms i(n)×C_w, that comes out to be IC_w equal to 32. And if you add i(n)×S_w all the terms ISw comes out to be 0. So, this comes out to be IC_w equal to 32 and IS_w that is equal to 0.

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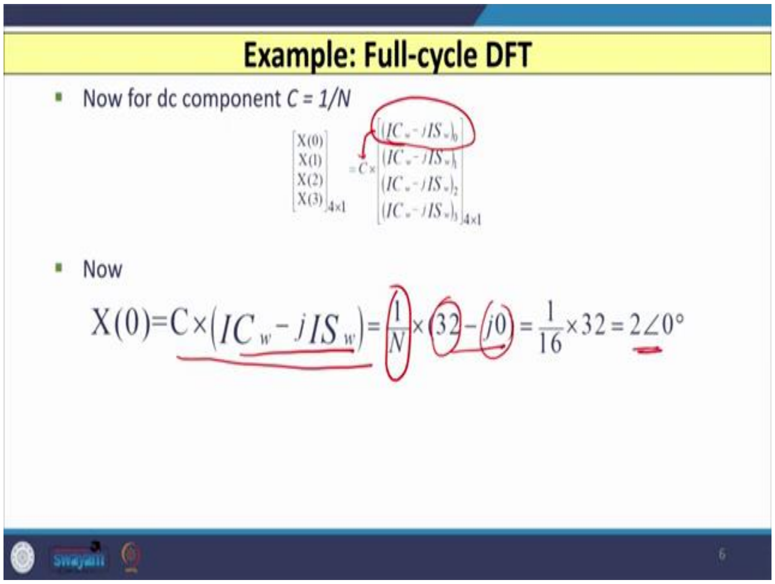

Now, we obtain this term. So, the only thing remaining that is we have to multiply it with a value of capital C. And we have discussed that when we have dc value, the value of C=1/N.

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix}_{4\times 1} = C \times \begin{bmatrix} (IC_w - jIS_w)_0 \\ (IC_w - jIS_w)_1 \\ (IC_w - jIS_w)_2 \\ (IC_w - jIS_w)_3 \end{bmatrix}_{4\times 1}$$

Now

$$X(0) = C \times (IC_w - jIS_w) = \frac{1}{N} \times (32 - j0) = \frac{1}{16} \times 32 = 2\angle 0^\circ$$

So, C is 1 by N into we have already obtained 32 minus j 0, this is your IC_w and this is your IS_w. So, you will have the value final $2 \angle 0^\circ$.

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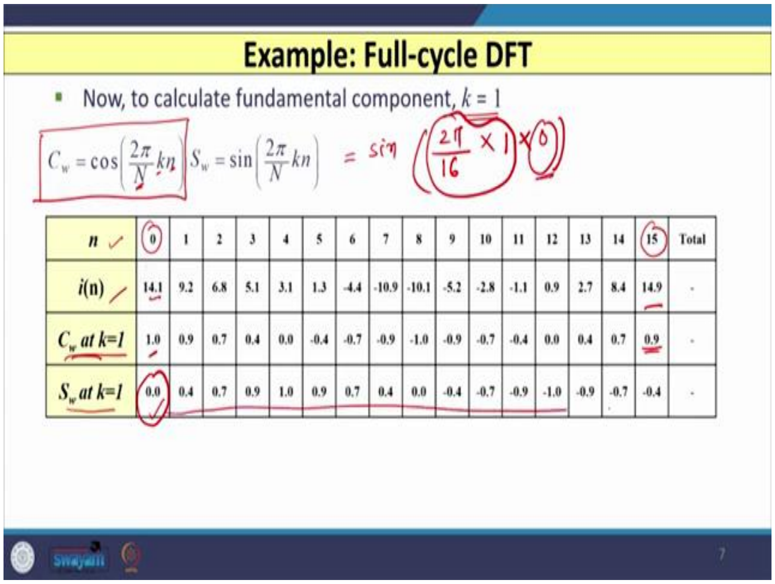

Now, let us calculate the value of the phasor of the fundamental component. So, here for that we have to use k that is equal to 1. So, when we have the value of n varies from 0 to 15 and we have i(n) already given for each sample, then C_w at (k=1), you can have the value like this. So, simply you have to put $C_w = cos\left(\frac{2\pi}{N}kn\right)$So, if you put this value here in this equation, then you will have the value of C_w, that is 1.9 and 0.9, last value. Similarly, you can have the value of S_w. $S_w = sin\left(\frac{2\pi}{N}kn\right)$. So, sin 0 that is 0. So, you will have this term. Similarly, this term that is constant term, which is going to change that is your sample number. So, if you put this value here for each sample, you will have the value of S_w also.

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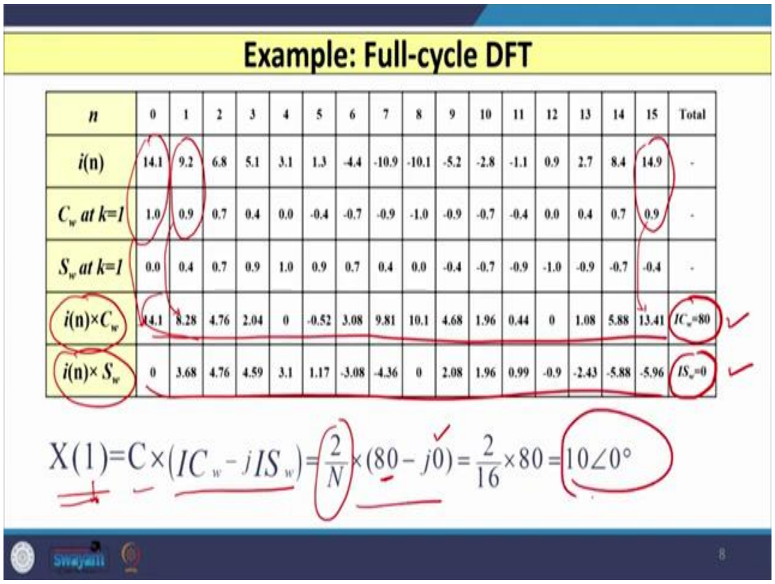

Once you have the values of C_w and S_w for k equal to 1, you can multiply this C_w with i(n). So, you will have the i(n)×C_w, that is this value, you will have again this i(n) into C_w, so you will have this term. Similarly, you can have the multiplication of this term and finally you will have the term that is this one. Same way you can also calculate i(n)×S_w. And similarly, you add all these values here. So, you will have IC_w is equal to 80 and you will have IS_w that is 0. So, here if you put X(1), which is the phasor value of fundamental term k equal to 1, that is

$$X(1)=C\times(IC_w - jIS_w)=\frac{2}{N}\times(80-j0)=\frac{2}{16}\times 80=10\angle 0°$$

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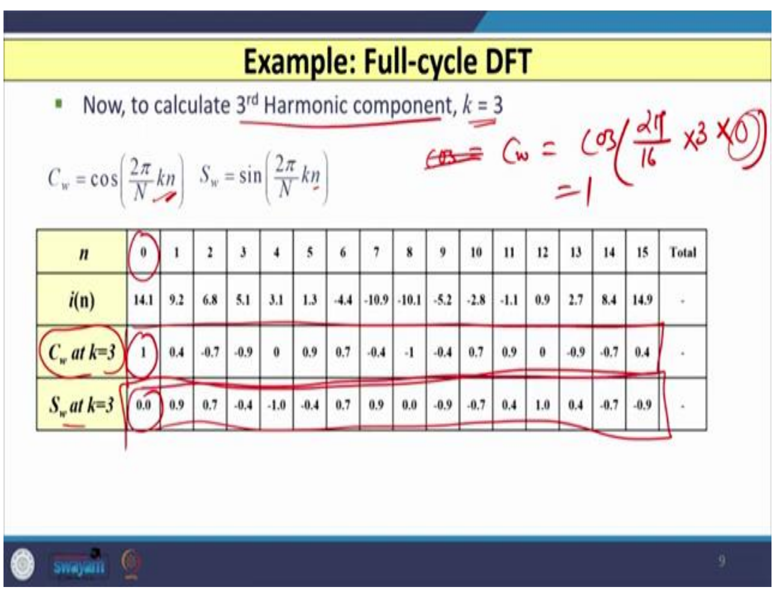

Now, similarly let us calculate the phasor value of third harmonic component. So, we have to put the value of k that is 3. So, if we put the value of k, that is 3 here, then you have C_w that is $C_w = cos\left(\frac{2\pi}{N}kn\right)$where N=16 k=3. So, n we are taking first sample that is 0, so cos of 0 that is 1. So, you will have this value, and this is also S_w that is 0.

Similarly, you can have the other values by varying the value of n, and you can put the value here in C_w Similarly you can put the value in the equation of S_w that is $S_w = sin\left(\frac{2\pi}{N}kn\right)$and you will have the value of this term, that is S_w and you will have all the values of C_w for different value of n.

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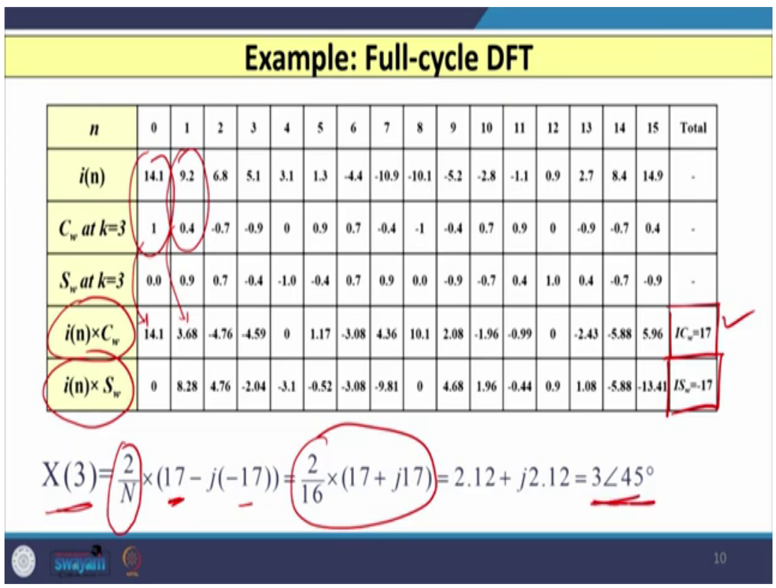

Example: Full-cycle DFT

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total
$i(n)$	14.1	9.2	6.8	5.1	3.1	1.3	-4.4	-10.9	-10.1	-5.2	-2.8	-1.1	0.9	2.7	8.4	14.9	-
C_w at $k=3$	1	0.4	-0.7	-0.9	0	0.9	0.7	-0.4	-1	-0.4	0.7	0.9	0	-0.9	-0.7	0.4	-
S_w at $k=3$	0.0	0.9	0.7	-0.4	-1.0	-0.4	0.7	0.9	0.0	-0.9	-0.7	0.4	1.0	0.4	-0.7	-0.9	-
$i(n)\times C_w$	14.1	3.68	-4.76	-4.59	0	1.17	-3.08	4.36	10.1	2.08	-1.96	-0.99	0	-2.43	-5.88	5.96	IC_w=17
$i(n)\times S_w$	0	8.28	4.76	-2.04	-3.1	-0.52	-3.08	-9.81	0	4.68	1.96	-0.44	0.9	1.08	-5.88	-13.41	IS_w=-17

$$X(3)=\frac{2}{N}\times(17-j(-17))=\frac{2}{16}\times(17+j17)=2.12+j2.12=3\angle45^\circ$$

Now, once we have the values of C_w and S_w for k equal to 3, you can multiply this C_w with i(n) and you can multiply S_w with i(n). So, when you multiply this C_w with i(n), you will have the value 14.1, similarly you will have the value 3.68. Same way you can have the value of i(n) into C_w and you have the value of i(n) into S_w. Then you add all these terms, that is 0, 8.28 and so on. So, you will have IC_w is equal to 17 and you will have IS_w that is equal to minus 17.

$$X(3)=\frac{2}{N}\times(17-j(-17))=\frac{2}{16}\times(17+j17)=2.12+j2.12=3\angle45^\circ$$

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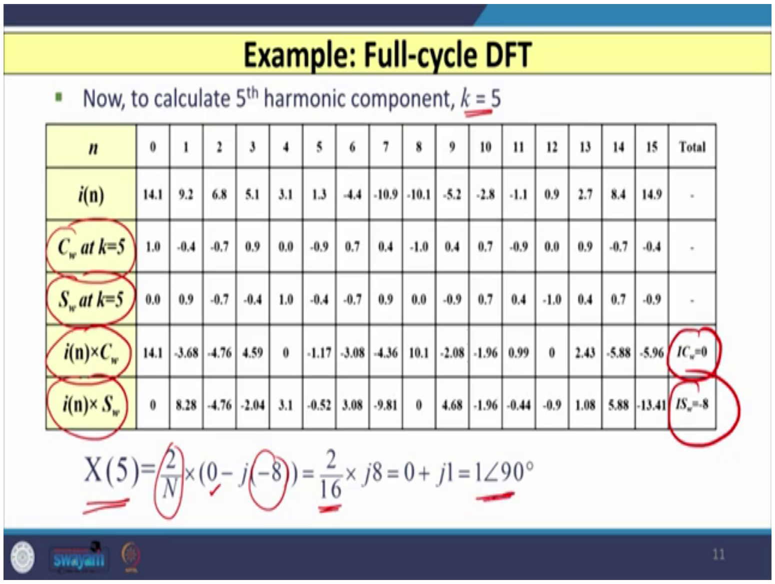

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total
$i(n)$	14.1	9.2	6.8	5.1	3.1	1.3	-4.4	-10.9	-10.1	-5.2	-2.8	-1.1	0.9	2.7	8.4	14.9	-
C_w at $k=5$	1.0	-0.4	-0.7	0.9	0.0	-0.9	0.7	0.4	-1.0	0.4	0.7	-0.9	0.0	0.9	-0.7	-0.4	-
S_w at $k=5$	0.0	0.9	-0.7	-0.4	1.0	-0.4	-0.7	0.9	0.0	-0.9	0.7	0.4	-1.0	0.4	0.7	-0.9	-
$i(n) \times C_w$	14.1	-3.68	-4.76	4.59	0	-1.17	-3.08	-4.36	10.1	-2.08	-1.96	0.99	0	2.43	-5.88	-5.96	$IC_w=0$
$i(n) \times S_w$	0	8.28	-4.76	-2.04	3.1	-0.52	3.08	-9.81	0	4.68	-1.96	-0.44	-0.9	1.08	5.88	-13.41	$IS_w=-8$

$$X(5) = \frac{2}{N} \times (0 - j(-8)) = \frac{2}{16} \times j8 = 0 + j1 = 1\angle 90^\circ$$

Similarly, if I want to calculate the fifth harmonic component phasor. Then we have to put the value of k that is 5. And similarly, you have to calculate the value of C_w and S_w using the equation $C_w = \cos\left(\frac{2\pi}{N}kn\right)$ $S_w = sin\left(\frac{2\pi}{N}kn\right)$. And then, you have to multiply that C_w with i(n) and S_w with i(n) and you have to add it. So, finally you will have IC_w and IS_w values that is 0 and minus 8 respectively.

So, if you put here X(5), that is phasor value of fifth harmonic component $X(5) = \frac{2}{N} \times (0 - j(-8)) = \frac{2}{16} \times j8 = 0 + j1 = 1\angle 90^\circ$

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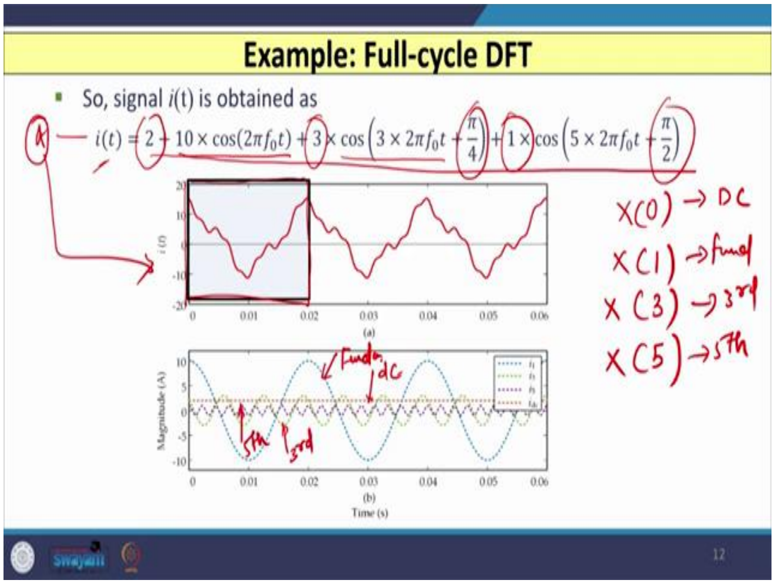

So, whatever samples we have taken and whatever the different phasors we have calculated, we have calculated the phasors, that is X (0), X(1), when X(3) and X (5), and this is your dc term, this is your fundamental term, and this is your third harmonic, and this is your fifth harmonic. So, whatever values we have obtained, phasor values of the signal that signal looks like this.

So, this is your dc term that is

$$i(t) = 2 + 10 \times \cos(2\pi f_0 t) + 3 \times \cos\left(3 \times 2\pi f_0 t + \frac{\pi}{4}\right) + 1 \times \cos\left(5 \times 2\pi f_0 t + \frac{\pi}{2}\right)$$

So, the signal, which is shown here in this equation A, that if we plot then it looks like this. And what we have done in the signal that is i(t) signal, we have considered 1 cycle window in, in 1 cycle, we have taken 16 samples, and then we have estimated the phasor value of this term that is dc term. Then we saw this is our fundamental term. Then we have 3rd harmonic term, and we have the this is your 3rd harmonic, and we have the 5th harmonic term, we have estimated.

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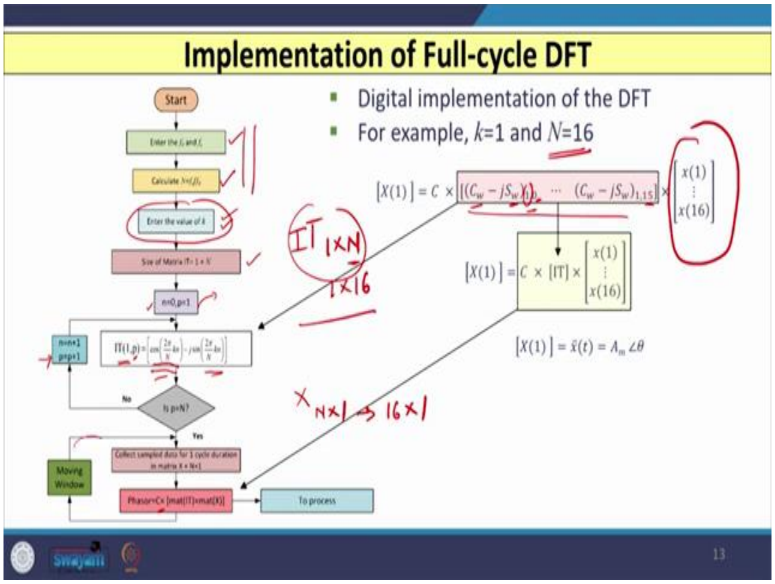

Now, if I want to implement this algorithm in terms of code, let us say in C, or MATLAB, then how we can do it? So, the flowchart for implementation of this full cycle DFT is shown here. So, here initially you have to enter two values, one is the sampling frequency f_s, and another is the fundamental frequency it can be 50 Hz, or it can be 60 Hz.

Once you have the value of f_0 and fs you can calculate the number of samples per cycle that is N by fs divided by f_0, once you have this, you have to enter the value of k that is harmonic number. So, let us say you enter the value of k that is 0. So, that means you want to estimate the phasor of dc term, or if you enter k, let us say 3, then you want to estimate the phasor of third harmonic. So, for that you need to enter the value of k.

Then we have to define the 1 matrix that is let us say i(t) matrix and that size of that i(t)matrix is nothing but the 1×N, where N we have already obtained, we have already defined. So, N is the number of samples in a cycle. So, let us say N is equal to 16, then the size of this i(t), that should be 1 by 16.

Now, what we want? We want to calculate this weight matrix. So, for that I have to initialize the sample number n that is 0, and I have taken another variable, let us say p, that starts from 1 and then I put the matrix IT, that is p=1. So, p that is going to vary initially it is 1 and then it increases from its original value to the next value that is p is equal to p plus 1.

So, here you can see the value of the C_w you can obtain by $C_w = cos\left(\frac{2\pi}{N}kn\right)$ and value of S_w you can obtain by $S_w = sin\left(\frac{2\pi}{N}kn\right)$. So, here similarly for in first row, if you have let us say 16 samples in a cycle, then you have to start with 1, 0. So, let us say 1 is your k value and you have to start this N with 0 and you have to go up to 15, then you have to end when your p is equal to N and when it is equal then the next task is you have to collect the sample data 1 cycle duration for another matrix, let us say the X matrix and the size of that matrix is N×1 and that is nothing but here 16 by 1.

So, this value of X, that is nothing but the matrix of sample values. So, once you have this you have to multiply this X matrix with IT matrix and then finally you have to multiply with capital C, which is fixed for dc it is 1/N for other it is 2/N. So, finally you will have the value of phasor for a particular value of k, which you have entered.

And then, in the moving window if you want to take next sample and if you want to discard the previous sample, then you have to move it here and you can calculate it and you can use this phasor value for any further application.

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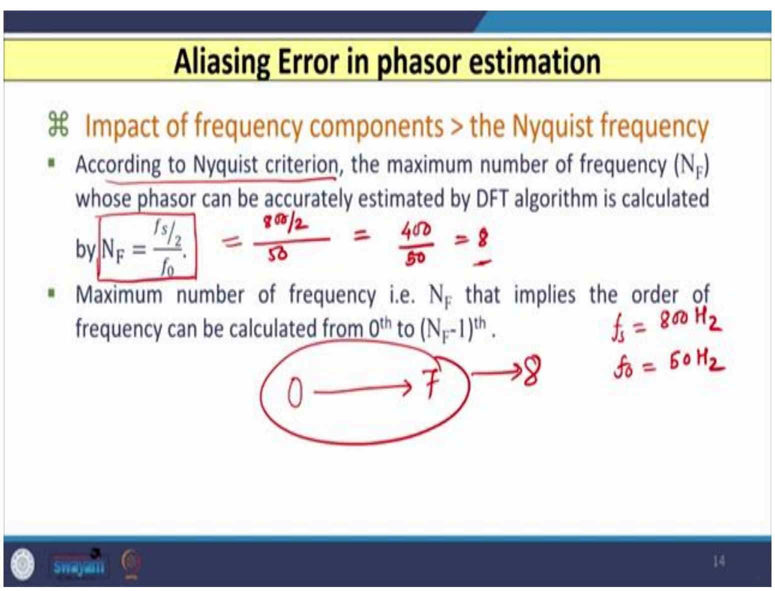

Now, after this discussion, let us see what is the aliasing error in Phasor estimation? So, when we have a frequency component available in the acquired signal, which is greater than Nyquist frequency, then this effect plays an important role. So, we know that we have discussed earlier also that according to Nyquist criteria, the maximum number of frequencies, let us say N_F whose phasor can be accurately estimated by full cycle DFT, that is calculated by the equation, $N_F = \frac{f_s/2}{f_0}$

So, let us consider the sampling frequency in earlier case, which we have considered, let us say 16 samples per cycle. So, let us consider my sampling frequency is 800 Hz, my f_0 will be let us say 50 Hz. So, if I use this value, then your N_F comes out to be 800/2 divided by 50. So, this is nothing but 400/50. So, this comes out to be 8.

So, that means when N_F is equal to 8, this indicates that the maximum number of frequencies, that is N_F that implies the order of frequency that is to be calculated. So, if your N_F value is 8, then you can calculate the maximum order of frequency only up to 8.

And normally it starts with 0, because we have to calculate dc value and it will go up to 7. So, if you add this you will have the value of N_F that is 8. So, that means when N_F is equal to 8, then the order of frequency that can be calculated is from 0 to N_F minus 1. So, 0 to 7 becomes 8.

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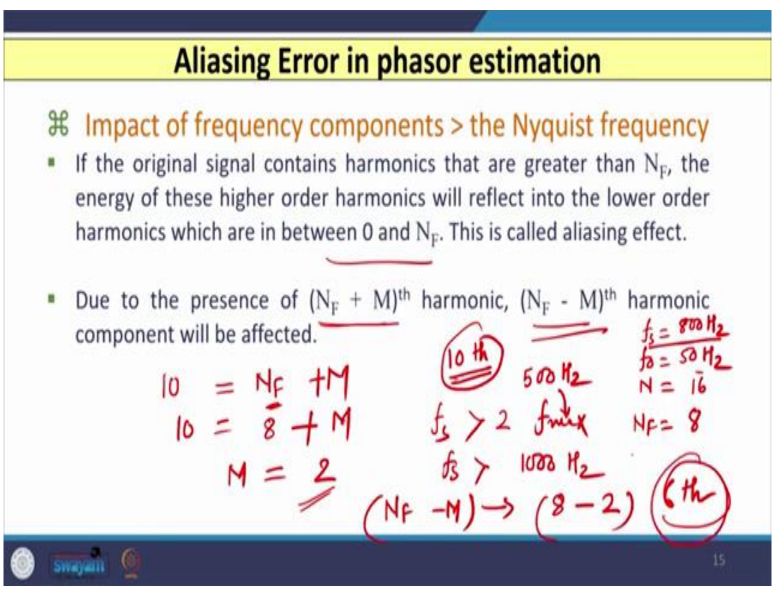

So, if the original signal contains harmonics, which is greater than your N_F, which we have calculated, then the energy of this higher order harmonics that is going to reflect it on the lower order harmonics, that is in between 0 to N_F and this effect is known as aliasing effect. So, because of the presence of this N_F+M^{th} harmonic term, N_F - M^{th} harmonic component will be affected.

Now, to understand this, let us consider our f_s is equal to 800 Hz. So, f_0 that is equal to let us say 50 Hz, your number of samples in a cycle that is 16 f_s by f_0. So, N_F comes out to be as we have already calculated earlier, it comes out to be 8. So, let us say N_F is equal to 8, which is f_s by 2 divided by f_0. So, the meaning is if we are acquiring, let us say some signal, so if we are taking samples of this signal and if that signal contains, let us say tenth harmonic.

So, tenth harmonic of the fundamental that is 50 Hz, that is nothing but 500 Hz, we have already discussed that the sampling frequency according to Nyquist criteria, that is greater than 2 times f_{max}. So, my f_{max} is 500 Hz. So, my sampling frequency should be greater than 1000 Hz, if I want to avoid aliasing error. But here you can see the sampling frequency is 800 Hz, the signal which we are acquiring it contains the tenth harmonic component.

So, in that case what will happen? Your N_F - M^{th} harmonic. So, let us say 10^{th} harmonic is present that is equal to your N_F + M. So, 10 is equal to your N_F that is 8 +M. So, here you can say your value of M, let us say that is 2. So, N_F- M. So, if I consider N_F- M^{th} harmonic, then that comes out to be N_F-M=8 -2=6, which we have calculated. So, 6^{th} harmonic is affected.

So, when we acquire any signal and for that signal if sampling frequency is not going to follow Nyquist rule or criteria and if that signal contains the frequency component, which is higher than N_F, then the let us say, if we are having our tenth harmonic component present in the acquired signal, then because of that tenth harmonic component, your estimation of 6^{th} harmonic that will be affected.

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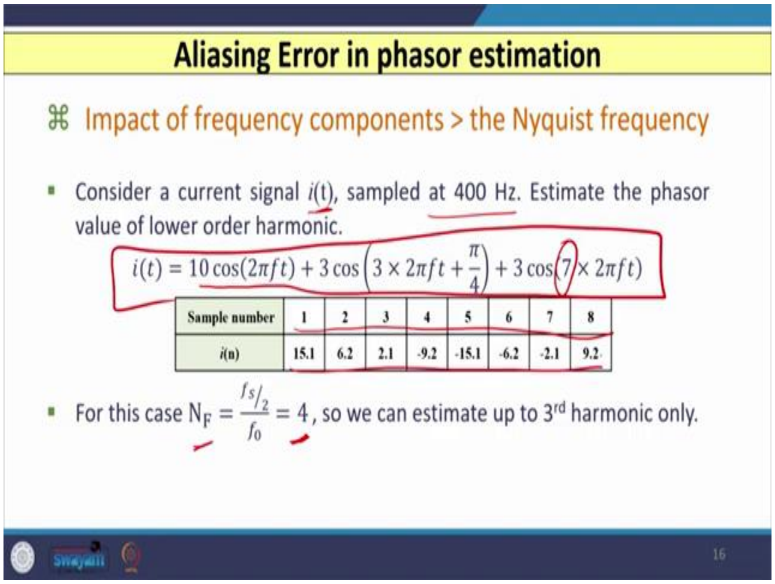

So, to understand this, let us consider a signal i(t), which is sampled at 400 Hz. So, when you sampled at 400 Hz with fundamental frequency that is 50 Hz, your N_F comes out to be 4, that is 400/2=200 divided by 50. So, that comes out to be 4.

Now, let us say this i(t), is a combination of this equation.

$$i(t) = 10\cos(2\pi f t) + 3\cos\left(3 \times 2\pi f t + \frac{\pi}{4}\right) + 3\cos(7 \times 2\pi f t)$$

Now, the number of samples in a cycle is 8. So, we have 8 samples, and we are going to acquire the value for each sample.

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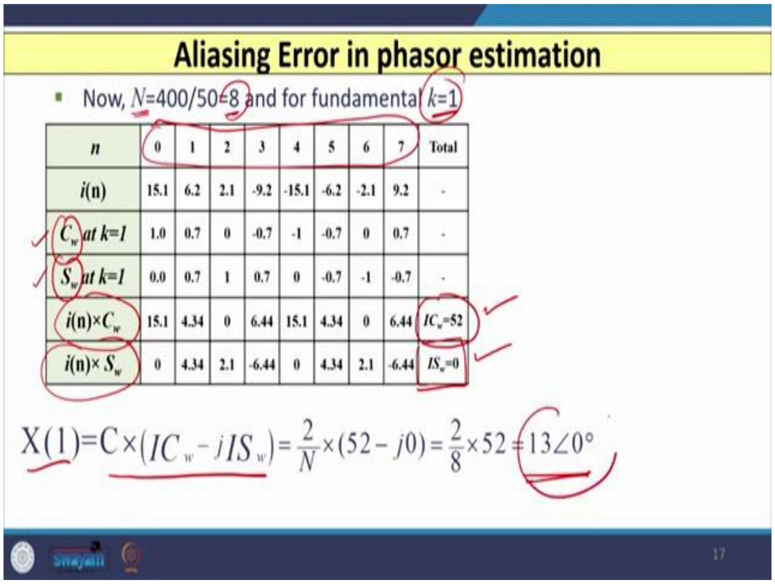

Now, here as the value of N, that is number of samples in a cycle a 400 Hz is the sampling frequency. So, N comes out to be 8. And let us say k=1 for fundamental. So, if you calculate the value of C_w and S_w, as we did earlier by putting the value of k =1, n varies from 0 to 7 and N that is 8 in the $C_w = cos\left(\frac{2\pi}{N}kn\right)$ and $C_w = cos\left(\frac{2\pi}{N}kn\right)$. You can have the values of C_w and S_w.

You multiply i(n) with C_w and i(n) with S_w. So, finally you will have the value of IC_w that is 52 and you will have the value of IS_w that is 0. So, your X (1) term that is k=1. So, fundamental phasor of fundamental term that is C into this equation. So, that comes out to be 13 ∠0.

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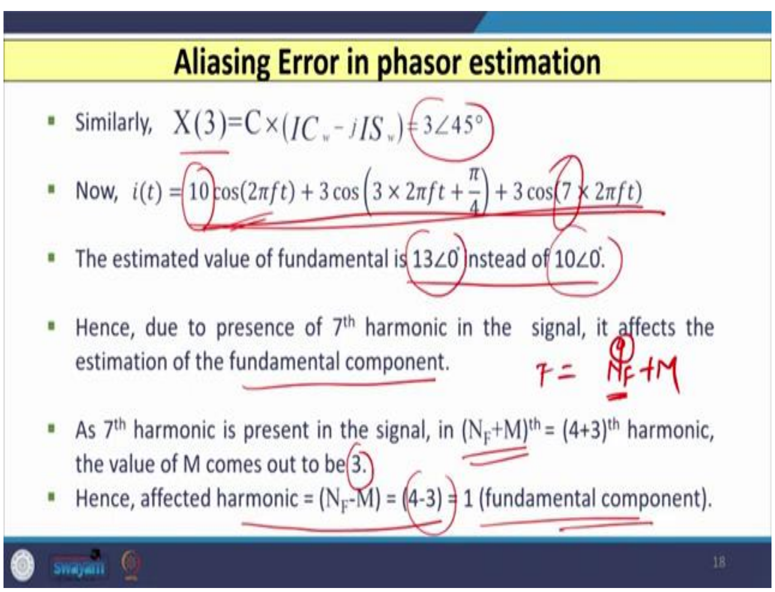

Similarly, you can calculate another component, let us say the phasor of third harmonic and you can have the value that is 3 ∠ 45. So, our original equation is:

$$i(t) = 10\cos(2\pi ft) + 3\cos\left(3 \times 2\pi ft + \frac{\pi}{4}\right) + 3\cos(7 \times 2\pi ft)$$

So, you can see the estimated value of fundamental term, that is 13∠0, whereas in the actual signal of i(t) the fundamental term present that is 10 ∠ 0.

So, that means because of the presence of seventh harmonic in the signal, the effect is going to occur on the fundamental component and fundamental component is wrongly estimated. So, as seventh harmonic is present in the signal. So, $N_F + M$, that is nothing but your 7th harmonic. So, 7 is equal to $N_F + M$, N_F is your 4, so M comes out to be 3. So, 7 is equal to $N_F + M$, N_F value is 4, so M value comes out to be 3.

So, hence the affected harmonic, that is $N_F - M$ that is 4 minus 3 that is fundamental and that we have observed that we obtain 13 ∠ 0 phasor value of fundamental instead of 10 ∠ 0.

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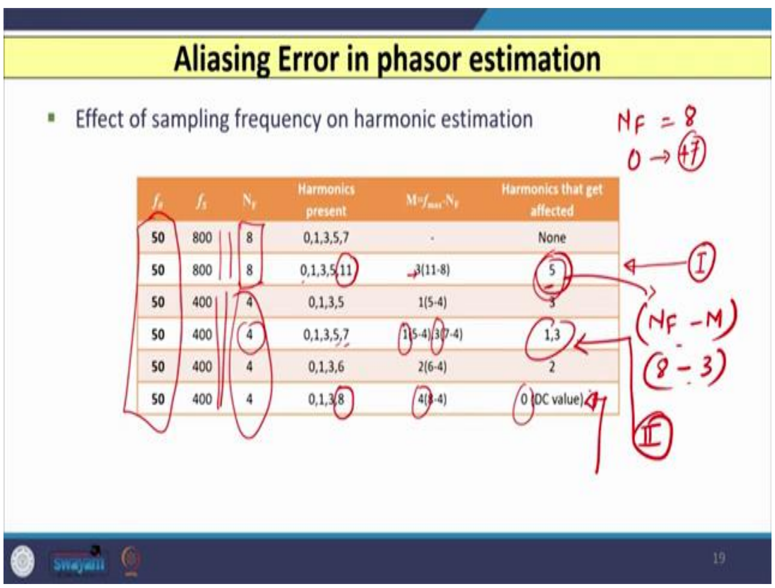

So, if I consider let us say different value of sampling frequency, here I have considered two sampling frequencies in one case 400 Hz, in another case 800 Hz. Fundamental frequency I have considered same that is 50 Hz. And accordingly, if I calculate N_F comes out to be 8 and 4. So, if I consider let us say this case second row, then in this case, you can see that the number of harmonics present that is 0, 1, 3, 5 and 11 and your N_F value is 8. So, as N_F is 8, so it can only estimate the frequency order from 0 to N_F -1 that is 7.

So, as eleventh harmonic is present, so your M comes out to be 3, that is 11 minus 8, that is 3. So, definitely the term, which is going to affect it, that is this fifth harmonic, it is going to affect. How this comes out to be? This comes out to be this is nothing but your N_F-M. So, M comes out to be 3, so N_F is 8, so 8 - 3. So, 5^{th} harmonic gets affected.

Similarly, let us consider this case that is second where the number of harmonics present that is fifth, seventh, third, first, and dc value, N_F is 4. So, here you can see the value of M, because as the value of N_F = 4, so it can estimate only up to 0 to 3^{rd} harmonic, but here 5^{th} and 7^{th} are also present. So, because of 5^{th} and 7^{th}, you can see that 5 - 4 that is first and again 7 - 4 that is third means, when you estimate first and third harmonic, then both phasor values are affected, because of the presence of 5^{th} and 7^{th} harmonic.

Similarly, if you consider this case, then here you can see the 8^{th} harmonic is present. So, your value of M comes out to be 8 - 4 that is 4 and again N_F -M that is 4 -4 that comes out to be 0. So, dc value gets affected. So, that means whenever we have any higher order harmonics that are present that is greater than N_F, then its effect will be reflected on the estimation of other harmonic components.

So, in this class, we have discussed initially with one example that how to calculate the weight matrix that is C_w and S_w. Then we also discussed how to calculate IC_w and IS_w. And then we finally discussed how we can calculate, or estimate the different phasors, let us say 0, first third, fourth, fifth harmonics.

And then we have discussed an important effect of the aliasing error and we have discussed that if the number of harmonics present in the acquired signal is greater than N_F, then the effect of those harmonics is reflected on the phasors, which we are going to calculate. So, thank you very much.