

**Digital Protection of Power System**  
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**Lecture 07**  
**Phasor Estimation Algorithm-III**

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**Summary of Previous Lecture**

- Full cycle DFT:  $X(k) = C \sum_{n=0}^{N-1} x(n) e^{-j \frac{2\pi}{N} kn}$
- Example of Full cycle DFT
- Digital Implementation of the Full-cycle DFT
- Aliasing error in phasor estimation
  - Impact of frequency components > the Nyquist frequency on estimation of the phasors.

Hello friends. So, in the previous lecture, we have discussed regarding the full cycle DFT and we have seen that the equation of DFT to calculate either dc value, or any other harmonics phasor value that is given by  $X(k) = C \sum_{n=0}^{N-1} x(n) e^{-j \frac{2\pi}{N} kn}$ . We have also considered one example of full cycle DFT and we have seen that how we are able to calculate the phasor value of either fundamental component, or dc value, or any other harmonic term.

Then we have discussed regarding the if we wish to implement the full cycle DFT algorithm in code, then how we are going to utilize the different matrices and how we have to use the either for loop or any other loop in the code. Then at last we have discussed the important aspect of aliasing error, which is going to occur in phasor estimation and this is specially occurs when the Nyquist criteria is not followed.

And we have discussed that the frequency components especially if it is greater than the Nyquist frequency, then the any higher order harmonics are present, then its impact that will be observed

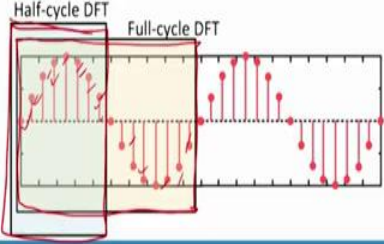
on the phasor estimation of several other components. So, this we have discussed in the previous lecture.

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### Half-cycle DFT

⌘ Half-cycle DFT

- Full-cycle DFT uses samples of one cycle duration to estimate the phasor.
- On the other hand, Half-cycle DFT utilizes only half cycle samples for computation of phasor.



The diagram illustrates the difference between Half-cycle DFT and Full-cycle DFT. It shows a sinusoidal waveform with red dots representing samples. A green box labeled 'Half-cycle DFT' covers the first half-cycle of the waveform. A yellow box labeled 'Full-cycle DFT' covers the entire first full cycle of the waveform. The waveform continues to the right, showing another full cycle.

Now, in this lecture, let us discuss the half cycle discrete Fourier transform algorithm. So, as the name suggests half cycle DFT algorithm utilizes only half cycle samples of the given signal. So, unlike full cycle DFT which utilizes one cycle duration, so whatever samples are available in one full cycle, let us say this is the window of full cycle.

So, all these samples available in this full cycle those are utilized by the full cycle DFT to estimate the phasor value. On the other hand, in case of half cycle DFT it utilizes this window. So, whatever number of samples are available in this window, that is utilized by half cycle DFT to estimate the phasor value.

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### Half-cycle DFT

⌘ **Half-cycle DFT**

 $0 \rightarrow N-1$   
 $0 \rightarrow \frac{N}{2}-1$

The representation of the half-cycle DFT in complex exponential form is given by,

$$X(k) = C \sum_{n=0}^{\frac{N}{2}-1} x(n) e^{-j \frac{2\pi}{N} kn}$$

where,  $C = \begin{cases} \frac{1}{N/2}; & \text{for dc} \\ \frac{2}{N/2}; & \text{for sinusoidal} \end{cases}$

Mathematically, the equation of half cycle DFT, that is almost similar to the full cycle DFT with a minor difference. So, if I just write down the equation of half cycle DFT in exponential form, then  $X(k) = C \sum_{n=0}^{\frac{N}{2}-1} x(n) e^{-j \frac{2\pi}{N} kn}$ . Here, the sigma limit is from  $n = 0$ , to  $(N/2) - 1$ . So, in case of full cycle DFT this limit varies from  $(0 \text{ to } N - 1)$ , whereas in this case half cycle DFT it varies from  $0$  to  $(N/2) - 1$ . Similarly, the value of  $C$  which is  $1/N$  for dc and  $2/N$  for sinusoidal in case

of full cycle DFT. So, in case of half cycle DFT, the value of  $C$ , where,  $C = \begin{cases} \frac{1}{N/2}; & \text{for dc} \\ \frac{2}{N/2}; & \text{for sinusoidal} \end{cases}$

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**Half-cycle DFT**

- By applying Euler's formula, it can be represented in trigonometric complex form as shown below.

$$X(k) = C \sum_{n=0}^{N/2-1} x(n) \left[ \cos\left(\frac{2\pi}{N}kn\right) - j \sin\left(\frac{2\pi}{N}kn\right) \right]$$

- This equation can be represented in matrix form as given below,

$$\begin{bmatrix} X(0) \\ \vdots \\ X(k) \end{bmatrix} = C \times \begin{bmatrix} (C_w - jS_w)_{0,0} & \cdots & (C_w - jS_w)_{0,N/2-1} \\ \vdots & \ddots & \vdots \\ (C_w - jS_w)_{k,0} & \cdots & (C_w - jS_w)_{k,N/2-1} \end{bmatrix} \times \begin{bmatrix} x(1) \\ \vdots \\ x(N/2) \end{bmatrix}$$

- where,  $C_w$  and  $S_w$  are the real and imaginary component of weight matrix elements, respectively, and given by,

$$C_w = \cos\left(\frac{2\pi}{N}kn\right) \quad S_w = \sin\left(\frac{2\pi}{N}kn\right)$$

Now, if we apply Euler's formula on these equations, let us say this is equation number A. Then in trigonometric complex form the same equation can be written as  $X(k) = C \sum_{n=0}^{N/2-1} x(n) \left[ \cos\left(\frac{2\pi}{N}kn\right) - j \sin\left(\frac{2\pi}{N}kn\right) \right]$ . And if we write this equation in matrix form. Then on left hand side, this X(k) that is written as X (0), to X (k), where k is the harmonic number that is equal to C constant and the weight matrix this is our weight matrix.

$$\begin{bmatrix} X(0) \\ \vdots \\ X(k) \end{bmatrix} = C \times \begin{bmatrix} (C_w - jS_w)_{0,0} & \cdots & (C_w - jS_w)_{0,N/2-1} \\ \vdots & \ddots & \vdots \\ (C_w - jS_w)_{k,0} & \cdots & (C_w - jS_w)_{k,N/2-1} \end{bmatrix} \times \begin{bmatrix} x(1) \\ \vdots \\ x(N/2) \end{bmatrix}$$

So, weight matrix is given by 2 components that is  $C_w$  and  $S_w$ , where  $C_w$  is calculated based on  $\cos\left(\frac{2\pi}{N}kn\right)$  and  $S_w$  is calculated using  $\sin\left(\frac{2\pi}{N}kn\right)$ . However, the only difference is you can see observe this first element of weight matrix. So, the first number varies from 0 to k, whereas the second number varies from 0 to  $(N/2) - 1$ .

So, in case of full cycle DFT this number in weight matrix varies from 0 to N-1, whereas in half cycle DFT this number varies from 0 to  $(N/2) - 1$ . And again, the X(N) value that also varies

from  $X(1)$  to  $X(N/2)$ , because we are taking only half of the samples compared to the full cycle DFT, in half cycle DFT algorithm.

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**Example of Half-cycle DFT**

- Consider a current signal having 50 Hz fundamental frequency sampled at 800 Hz sampling frequency. Calculate the value of fundamental component using Full-cycle and Half-cycle DFT.

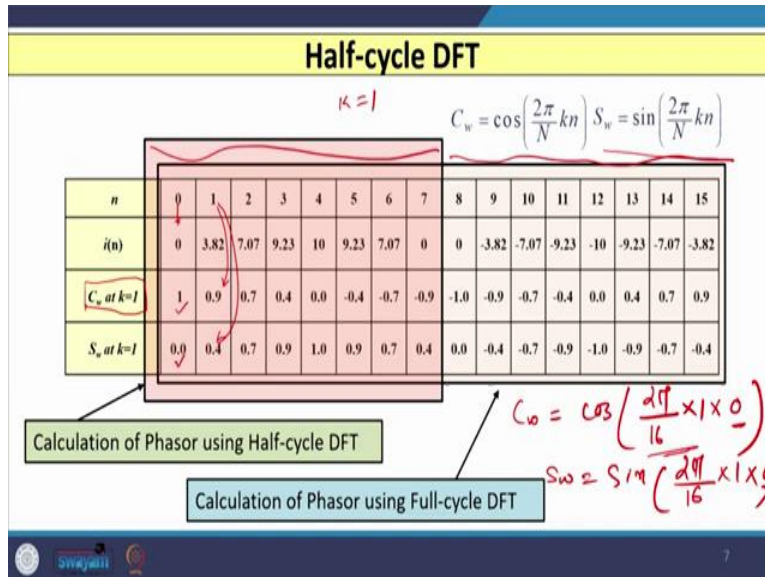
Sample number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$i(n)$	0	3.82	7.07	9.23	10	9.23	7.07	3.82	0	-3.82	-7.07	-9.23	-10	-9.23	-7.07	-3.82

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So, let us consider one current signal, which has 50 Hz fundamental frequency and let us assume that this signal is sampled with a sampling frequency of 800 Hz. And in this case, we want to calculate the fundamental component using full cycle DFT algorithm and also using half cycle DFT algorithm.

And after that we need to compare whatever phasor value we have obtained for a particular component, we want to see that what value we are getting, if we use full cycle DFT and what value we are getting if we use half cycle DFT algorithm. So, let us consider as we have considered 800 Hz sampling frequency. So, there are 16 samples available and with reference to each sample, we have considered, or we have acquired the values which are mentioned in this table.

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Now, with this table if I calculate first  $C_w$  component by putting  $k$  is equal to 1, which indicates that we are interested to calculate the phasor value of fundamental term. So, if we calculate the value of  $C_w$ , then we have to put in the equation of  $C_w = \cos\left(\frac{2\pi}{N}kn\right)$ ,  $N$  is 16 into  $k$  is 1. So, this is fixed into sample number  $N$ . So, we start from 0, so it is 0, so we will get the value that is 1.

Similarly, you can also use the term  $S_w$  and you can calculate the value. So,  $2\pi$  by 16 into 1 into  $N$ . So, only this parameter changes and accordingly you can calculate for each sample number, you can calculate the value of this  $C_w$  and  $S_w$ . So, once you have the values of both  $C_w$  and  $S_w$  for  $k$  equal to 1, that is fundamental component, then you can see if I use the entire 16 samples in one particular cycle, then that is nothing but the calculation of phasor value, in this case fundamental of phasor using full cycle DFT.

However, as we have discussed that in half cycle DFT algorithm, we are utilizing only half of the samples compared to the full cycle DFT algorithm. So, in half cycle DFT, we are going to use only half of the samples that are 0 to 7. And then, we are going to calculate the phasor of the fundamental term. So, here calculation of this term that is from 8 to 15, if I use half cycle DFT algorithm, then calculation of  $C_w$  and  $S_w$  and  $S_w$  for 8 to 15 are not required.

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Half-cycle DFT																	
$n$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total
$i(n)$	0	3.82	7.07	9.23	10	9.23	7.07	0	0	-3.82	-7.07	-9.23	-10	-9.23	-7.07	-3.82	-
$C_w$ at $k=1$	1	0.9	0.7	0.4	0.0	-0.4	-0.7	-0.9	-1.0	-0.9	-0.7	-0.4	0.0	0.4	0.7	0.9	-
$S_w$ at $k=1$	0.0	0.4	0.7	0.9	1.0	0.9	0.7	0.4	0.0	-0.4	-0.7	-0.9	-1.0	-0.9	-0.7	-0.4	-
$i(n) \times C_w$	0	3.44	4.95	3.69	0	-3.69	-4.95	0	0	3.44	4.95	3.69	0	-3.69	-4.95	-3.44	$IC_w=0$
$i(n) \times S_w$	0	1.53	4.95	8.31	10	8.31	4.95	0	0	1.53	4.95	8.31	10	8.31	4.95	1.53	$IS_w=80$

$$X(1) = C \times (IC_w - jIS_w) = \frac{2}{N} \times (0 - j80) = \frac{2}{16} \times -j80 = 10 \angle -90^\circ \quad \text{Full-cycle DFT}$$

Now, let us see how we are able to calculate the other terms. So, once we have calculated this  $C_w$  and  $S_w$ , then simply we have to multiply this  $C_w$  with  $i(n)$ . And then, we have to multiply this  $S_w$  with  $i(n)$ . So, that we can calculate  $i(n)$  into  $C_w$ , that is given here. And we can also calculate the  $i(n)$  into  $S_w$ , that is also given here. And then, after adding all this term  $i(n)$  into  $C_w$ , we will have the value of  $IC_w$  that is 0. And for adding  $i(n)$  into  $S_w$  all the terms we will have the value  $IS_w$ , that is 80.  $C_w$  and  $S_w$

And if I use this  $IC_w$  and  $IS_w$  that is 0 and 80 value here, then we will have  $X(1)$  that is a phasor of fundamental term, where  $C$  is given by  $2/N$ , this is for full cycle  $0 - j80$ . So, we will have the value  $10 \angle -90^\circ$ . If we use all the samples, that is full cycle DFT.

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Half-cycle DFT									
$n$	0	1	2	3	4	5	6	7	Total
$i(n)$	0	3.82	7.07	9.23	10	9.23	7.07	0	
$C_w$ at $k=1$	1	0.9	0.7	0.4	0.0	-0.4	-0.7	-0.9	
$S_w$ at $k=1$	0.0	0.4	0.7	0.9	1.0	0.9	0.7	0.4	
$i(n) \times C_w$	0	3.44	4.95	3.69	0	-3.69	-4.95	0	$IC_w=0$
$i(n) \times S_w$	0	1.53	4.95	8.31	10	8.31	4.95	0	$IS_w=40$

$$X(1) = C \times (IC_w - jIS_w) = \frac{2}{N/2} \times (0 - j40) = \frac{4}{16} \times -j40 = 10 \angle -90^\circ$$

Half-cycle DFT

$$X(1) = C \times (IC_w - jIS_w) = \frac{2}{N} \times (0 - j80) = \frac{2}{16} \times -j80 = 10 \angle -90^\circ$$

Full-cycle DFT

Now, compared to the full cycle DFT, if I use let us say half cycle DFT, then I told you that this samples, I need not required for calculation of phasor value using half cycle DFT. So, if I remove these samples, then we can say that we have to use only this samples, we have to calculate  $C_w$ ,  $S_w$  and then  $i(n)$  into  $C_w$ ,  $i(n)$  into  $S_w$ . And then finally, you can have the value of  $IC_w$  and you can have the value of  $IS_w$ .

So, if you put this in the equation that is  $X(1)$  that is  $C$ , where  $C$  is nothing but  $\frac{2}{N/2}$  and  $IC_w$  that is 0 and  $IS_w$  that is 40. So, if you solve this, you will again get the value that is  $10 \angle -90^\circ$ . And using full cycle DFT also we obtain the same value that is  $10 \angle -90^\circ$ .



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Comparison between Full-cycle and Half-cycle DFT	
Half-cycle DFT	Full-cycle DFT
→ Accuracy is better	Accuracy is best
→ Faster response	Fast response
→ Can not reject even harmonics	Can reject odd as well as even harmonics

So, that means if I use the half cycle DFT compared to the full cycle DFT, then there is a significant difference between the half cycle DFT and the full cycle DFT as far as the performance is concerned.

So, if I consider the first parameter of the performance that is accuracy, then accuracy is better in case of half cycle DFT. Whereas, in case of full cycle DFT accuracy is best, obviously because full cycle DFT use more number of samples. So, it is able to estimate the phasor value more accurately compared to half cycle DFT.

If I use the response time, that is the second parameter for performance parameter, then the response of half cycle DFT is faster, whereas the response of full cycle DFT is comparatively lower, because half cycle DFT use less number of samples. So, lower calculations are required, whereas full cycle DFT utilizes more number of samples than double the half cycle DFT. So, response time is a little bit lower than the half cycle DFT.

And the third important point is if I use half cycle DFT, then it is not able to reject even harmonics, whereas full cycle DFT is capable to reject both odd as well as even harmonics. So, this is the important difference between the full cycle DFT and half cycle DFT.

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The slide is titled "Limitations of DFT" in a yellow header. Below the title, there is a handwritten red circle containing the mathematical expression  $e^{-Rt/L}$ . The main text of the slide is a numbered list item: "1. In protection systems, the fault current contains a decaying dc component. The estimated phasor value of fundamental component from such a fault current signal using DFT will give erroneous value. Therefore, the DFT cannot eliminate the effect of decaying DC." The slide also features a footer with logos and the number 11.

Now, we have considered two important phasor estimation algorithms, one is half cycle DFT and the other one is the full cycle DFT. If I use either full cycle DFT algorithm, or half cycle DFT algorithm for phasor estimation, then the fundamental limitation of both this algorithm is whenever the decaying dc component is present in the acquire signal. Let us say this is possibly present when we acquire the signal during fault condition.

So, whenever we acquire the voltage signal, let us say current signal then that signal contains decaying dc component, and which is nothing but given by  $e^{-Rt/L}$  this term. So, in this case, if phasor value is estimated of the fundamental component using either full cycle DFT, or half cycle DFT, then you may not get accurate result. So, we can say that either full cycle DFT, or half cycle DFT algorithm is not capable of eliminating the effect of decaying dc component, if it is present in the acquired signal.

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### Phasor Estimation Algorithm

⌘ **Cosine Transform**

- Cosine filter is an extension of one cycle DFT.
- If observed in filter aspects, one cycle DFT is constituted with cosine and sine filter.
- The output of cosine filter and sine filter is the real part and the imaginary part of one cycle DFT, respectively.

$$X(k) = C \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn} \Rightarrow X(k) = C \sum_{n=0}^{N-1} x(n) \left[ \cos\left(\frac{2\pi}{N}kn\right) - j \sin\left(\frac{2\pi}{N}kn\right) \right]$$

$$\text{Cosine} = C \sum_{n=0}^{N-1} x(n) \left[ \cos\left(\frac{2\pi}{N}kn\right) \right] \quad \text{Sine} = C \sum_{n=0}^{N-1} x(n) \left[ \sin\left(\frac{2\pi}{N}kn\right) \right]$$

Now, let us see the third type of algorithm phasor estimation algorithm, that is known as the cosine transform. So, cosine filter it is basically an extension of the one cycle discrete Fourier transform algorithm. And if we absorbed in filter aspects, then one cycle DFT is basically constituted with cosine and sine filter.

$$X(k) = C \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$$

$$X(k) = C \sum_{n=0}^{N-1} x(n) \left[ \cos\left(\frac{2\pi}{N}kn\right) - j \sin\left(\frac{2\pi}{N}kn\right) \right]$$

$$\text{Cosine} = C \sum_{n=0}^{N-1} x(n) \left[ \cos\left(\frac{2\pi}{N}kn\right) \right], \text{Sine} = C \sum_{n=0}^{N-1} x(n) \left[ \sin\left(\frac{2\pi}{N}kn\right) \right]$$

So, output of cosine filter and sine filter, if we consider in real part and imaginary part of one cycle DFT, then it is given by this is the equation of full cycle DFT and if we write this term in cosine and sine term, then it is given by this equation, where we have replaced  $-j\frac{2\pi}{N}kn$  term with  $\cos\left(\frac{2\pi}{N}kn\right)$  and  $\sin\left(\frac{2\pi}{N}kn\right)$ . So, if I write down the equation of cosine filters you can write down  $C \sum_{n=0}^{N-1} x(n) \left[ \cos\left(\frac{2\pi}{N}kn\right) \right]$  and sine filter that is given by  $C \sum_{n=0}^{N-1} x(n) \left[ \sin\left(\frac{2\pi}{N}kn\right) \right]$ .

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### Cosine Transform

⌘ Cosine Transform

- The relation between sine component and cosine component is of the order of  $\frac{\pi}{2}$  angle.

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Now, if I consider the wave shape of either cosine wave, or sine wave, then the relationship between cosine and sine component exist in terms of angle and this angle is either  $\frac{\pi}{2}$  in radian or  $90^\circ$ . So, this you can visualize from these two windows of different color, you can see here that for example, if I have the signal available that is this signal is available and if I just delay the signal  $\frac{\pi}{2}$ , then the other signal available is like this. And the same is also applicable in other cases.

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### Cosine Transform

⌘ Cosine Transform

- The equation of one cycle DFT computation in the form of real and imaginary component is represented as,

$$X(k) = C \sum_{n=0}^{N-1} x(n) \left[ \cos\left(\frac{2\pi}{N}kn\right) - j \sin\left(\frac{2\pi}{N}kn\right) \right]$$

- This equation can also be defined in terms of cosine ( $X_c$ ) and sine ( $X_s$ ) filters.

$$X(k) = X_c - jX_s \quad \text{--- (B)}$$

- Where,

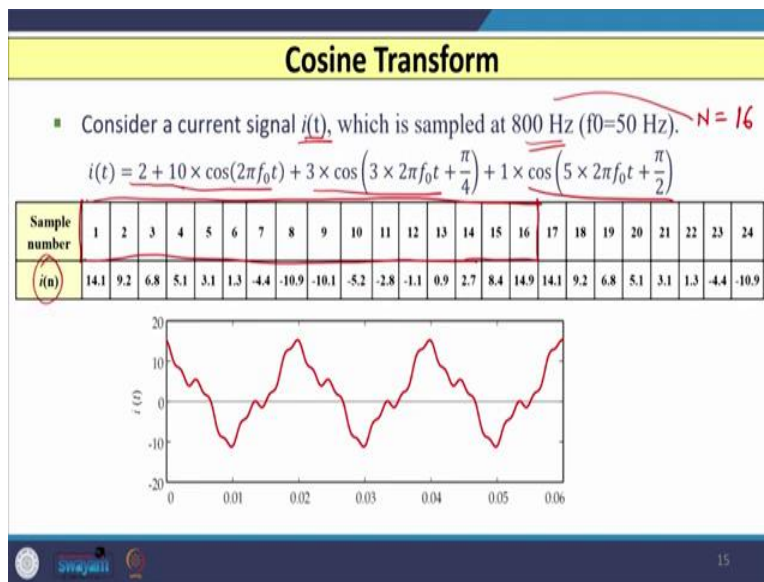
$$X_c(k) = C \sum_{n=0}^{N-1} x(n) \cos\left(\frac{2\pi}{N}kn\right) \quad ; \quad X_s(k) = C \sum_{n=0}^{N-1} x(n) \sin\left(\frac{2\pi}{N}kn\right)$$

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So, we can say that, if I use the equation of one cycle DFT for the computation of real and imaginary component. Then it can be represented by this equation, which we have discussed earlier. Now, in this equation, if we define the cosine filter as  $X_c$  and sine filter as  $X_s$ , then cosine filter  $X_c$  is given by this equation that is  $X_c$  of  $k$  that is  $X_c(k) = C \sum_{n=0}^{N-1} x(n) \cos\left(\frac{2\pi}{N} kn\right)$  and  $X_s$  that is sine filter is given by  $X_s$  of  $X_s(k) = C \sum_{n=0}^{N-1} x(n) \sin\left(\frac{2\pi}{N} kn\right)$ .

And this overall term, if I subtract this  $X_c$  with  $X_s$ , then we will have the value of  $X(k)$ , where  $k$  indicates the harmonic number. So, this equation is important as far as cosine transform is concerned, that is  $X(k) = X_c - jX_s$ , where  $X_c$  is the cosine filter and  $X_s$  is the sine filter given by these two equations.

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Now, to understand this cosine transform in a better way, let us consider a current signal  $i(t)$ , which is given by some dc value, some fundamental component, some harmonic components are also there. And let us say this signal is sampled at 800 Hz sampling frequency with a fundamental frequency of 50 Hz.

So, here I have shown you the number of samples, so as 800 Hz is the sampling frequency, so the number of samples in a cycle that is 16. So, here if I consider this window that is up to this 16, and along with that, I have also given, or represented few additional 8 samples also in this sample

number. And accordingly, I have also represented the values corresponding to each sample number. Now, the same signal  $i(t)$ , it looks like this.

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### Cosine Transform

Sample number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$i(n)$	14.1	9.2	6.8	5.1	3.1	1.3	-4.4	-10.9	-10.1	-5.2	-2.8	-1.1	0.9	2.7	8.4	14.9	4.1	9.2	6.8	5.1

- The phasor output of fundamental component for the first window (16 samples), computed using one cycle DFT, is found to be  $X_1(1) = 10 \angle 0^\circ$ .
- The cosine and sine filter parts of  $X_1(1)$  are given by

$X_{1c}(1) = 10 \cos 0^\circ = 10$

$X_{1s}(1) = 0 \sin 0^\circ = 0$

Handwritten notes:

 $2\sqrt{2} \angle 45^\circ$   
 $\swarrow \searrow$   
 $2\sqrt{2} \cos 45^\circ \rightarrow 2$   
 $2\sqrt{2} \sin 45^\circ \rightarrow 2$   
 $2 \rightarrow 2$

### Cosine Transform

**✂ Cosine Transform**

- The equation of one cycle DFT computation in the form of real and imaginary component is represented as,

$$X(k) = C \sum_{n=0}^{N-1} x(n) \left[ \cos\left(\frac{2\pi}{N}kn\right) - j \sin\left(\frac{2\pi}{N}kn\right) \right]$$

- This equation can also be defined in terms of cosine ( $X_c$ ) and sine ( $X_s$ ) filters.

$X(k) = X_c - jX_s$

Where,

$X_c(k) = C \sum_{n=0}^{N-1} x(n) \cos\left(\frac{2\pi}{N}kn\right)$

$X_s(k) = C \sum_{n=0}^{N-1} x(n) \sin\left(\frac{2\pi}{N}kn\right)$

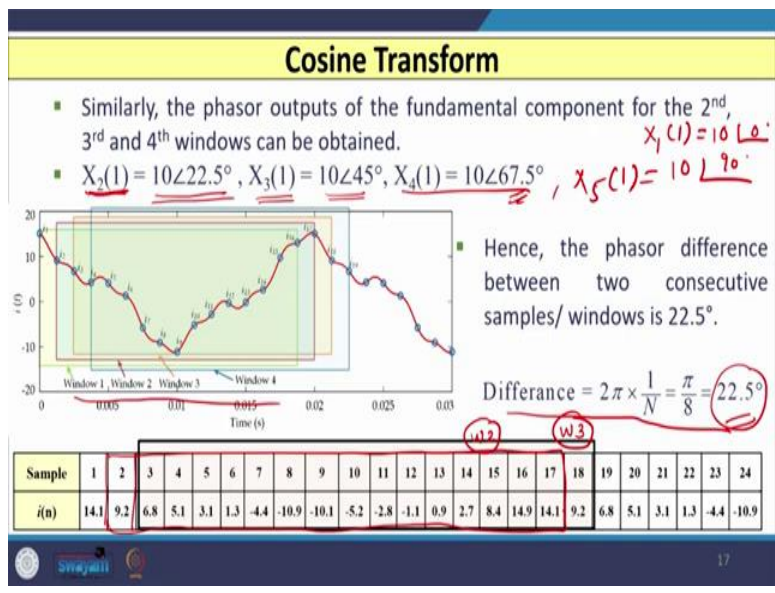
And now if I consider the first window, which contains the sample number from 1 to 16 as shown here, you can see, there are 16 samples in this window, let us call it this as window 1. So, when I consider this window 1 containing 16 samples, and if you calculate using this the value of the phasor, that is of fundamental term. So, by putting  $k=1$ , if you calculate the value of  $X$ , that is the phasor value of fundamental, then you obtain the value that is  $10 \angle 0^\circ$ .

Now, here if I use this equation, and if I find out the cosine filter and sine filter using this  $X(1)$ , that is  $10 \angle 0^\circ$ . Then cosine filter  $X_{1C}$  of fundamental term that is given by  $10 \cos 0$ , that is 10.

And the sine filter term and  $X_{1X}$  of fundamental that is given by  $10 \sin 0$ . Now, here you see I have used the negative sign here and the reason for this negative sign is whenever we have the value  $2\sqrt{2} \angle 45$ , some value in polar form.

Now, if you take the  $2\sqrt{2} \cos 45$ , then you will get the value 2. And if you take  $2\sqrt{2} \sin 45$ , then you will have the value again 2. And this is represented by  $2 + j 2$  in this form. However, you see the earlier case in our case, when we use this equation, equation B, we have the term that is cosine filter is  $X_c$ , whereas sine filter we have the negative sign. So, when we have the negative sign here, we have to put the negative sign in case of sine filter term.

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Now, with this background, you can also take the other windows, let us say the other windows I am considering is let us say 2 to 17. So, first window we have considered that is 1 to 16. Now, I am considering second window, let us say window 2, that varies from sample number 2 to sample number 17. So, number of samples again remain constant that is 16.

And if you calculate the value of again the fundamental term phasor of fundamental that is  $X_2(1)$ , then you will have the value that is  $10 \angle 22.5^\circ$ . If you take third window, let us say from 3 to 18,

this is your window 3. And if you calculate the fundamental phasor, then you will have the value  $X_3(1)$  that is given by  $10\angle 45^\circ$ .

So, your  $X_1(1)$  value will be  $10\angle 0^\circ$ , your  $X_2(1)$  will be  $10\angle 22.5^\circ$ ,  $X_3(1)$  will be  $10\angle 45^\circ$ . And if you further move if you take fourth window for let us say from 3, 4, to 19, then you will have the value  $X_4(1)$ , that is  $10\angle 67.5^\circ$  and so on. If you further calculate  $X_5(1)$ , then you will have the value 10 angle if you have to add again  $67.5$  into again  $22.5$ . So, you will have again  $90^\circ$ . So, as you move further, you can see that the magnitude remains constant, but there is the only change that is going to occur in the angle part.

So, the phasor difference between two consecutive samples, or between two consecutive windows. So, see I have shown here four different windows and you can see the difference is  $22.5^\circ$ . So, this difference  $22.5^\circ$ , if I wish to calculate, or put in equation term, then it is  $2\pi \times \frac{1}{N} = \frac{\pi}{8} = 22.5^\circ$ . So, this difference that is going to obtain as you move from one window to another window.

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### Cosine Transform

- The phasor output of the fundamental component for the window occurring after every quarter cycle ( $N/4$ ) is represented by,
 

$$X_{\frac{N}{4}+1}(1) = 10\angle 0^\circ + \left(\frac{2\pi \times N}{N} \times \frac{N}{4}\right) = 10\angle 90^\circ$$
- Now, the imaginary part of the said phasor value is given by
 

$$X_{\frac{N}{4}+1\text{imag}}(1) = 10 \sin 90^\circ = 10$$
- Sine filter  $X_{\frac{N}{4}+1s}(1) = -10$

$N = 16$   
 $\frac{N}{4} = 4$

But, due to  $X(k) = X_s(-)X_s$

So, with this background, the phasor output of the fundamental component of the window occurring at every  $N$  by 4 quarter cycle that is given by this. So, here you can see  $N=16$ . So, when I say  $N/4$ , so that is nothing but 4. So, when I say  $X_{\frac{N}{4}+1}$ , I am talking about basically this is nothing



but your  $X(5)$  value. So, that is nothing but  $10\angle 0$ . So, this is your  $X_1(1)$  value plus I have added this term that is  $\frac{2\pi}{N} \times \frac{N}{4}$ .

So, at every quarter cycle, the difference will be in  $N/4$ . So, you will have the final value of  $X_5(1)$ , that is  $10\angle 90^\circ$ . So, here you can see that imaginary part of this this said phasor, if you say this phasor then that is given by  $X_{\frac{N}{4}+1}$  imaginary part that is  $10\angle 90^\circ$ , that is 10. But as I told you earlier that our original equation, equation B, that is  $X_c$  minus  $X_s$ . So, cosine filter is. But sine filter you have the negative sign, that is why this value, we have to consider as minus 10.

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### Cosine Transform

$X_{\frac{N}{4}+1s}(1) = -10$

$X_{1c}(1) = 10 \cos 0^\circ = 10$

- Here, it can be observed that the sine filter part of  $(\frac{N}{4} + 1)$  window is equal and negative of the cosine filter part of the first window.

- This  $N/4$  (one-quarter cycle) adds  $\pi/2$  angle in the phasor, which is valid for the fundamental component only.

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So, we have the  $X_{\frac{N}{4}+1s}(1) = -10$  and we have  $X_{1c}(1) = 10 \cos 0^\circ = 10$ . So, here we can say that if the sine filter part that is of  $\frac{N}{4} + 1$  window and that is equal to a negative of the cosine filter part.

So, if I have let us say this window and if I wish to calculate the sine part of this window according to our equation, then I have to calculate here just the  $N/4$  just quarter cycle ago, I can calculate the cosine term and I have to take negative of that, that is same as the sine term of this window.

Also we have to understand this that this  $N/4$  quarter cycle, which is going to add  $\pi/2$  angle in the phasor, this is valid only for fundamental component, if you consider harmonics or any other third harmonics, fifth harmonic, then this is not valid.

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### Cosine Transform

- In general, for  $r^{\text{th}}$  window, it can be written in terms of cosine and sine filter as

$$X_{rs} = -X_{\left(\frac{r-N}{4}\right)c}$$

- where, 'r' represents window number and 'c' and 's' represents the output of cosine and sine filter, respectively.

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### Cosine Transform

- Therefore, sine filter can be replaced by cosine filter in DFT and the cosine transform for  $r^{\text{th}}$  window can be represented by,

$$X_r = X_{rc} - jX_{rs} = X_{rc} - j(-X_{\left(\frac{r-N}{4}\right)c}) = X_{rc} + jX_{\left(\frac{r-N}{4}\right)c}$$

- The term  $X_r$  provides the phasor of fundamental component using cosine transform for any  $r^{\text{th}}$  window.
- Hence, phasor estimation of cosine transform is given by,

$$X_r = X_{rc} + jX_{\left(\frac{r-N}{4}\right)c}$$

- Where 'r' is window number and 'N' is sample/cycle and 'c' stands for cosine filter.

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So, in general form if I consider  $r^{\text{th}}$  window, then the same can be returned in terms of cosine and sine filter as  $X_{rs} = -X_{\left(\frac{r-N}{4}\right)c}$ . So, r is nothing but the window number and c and s both represents the cosine and sine filter respectively.

So, here if I replace the sine filter by cosine filter in DFT, then the cosine transforming  $r^{\text{th}}$  window that can be represented by the equation  $X_r = X_{rc} - j(-X_{\left(\frac{r-N}{4}\right)c}) = X_{rc} + jX_{\left(\frac{r-N}{4}\right)c}$

So, we are replacing this term  $X_{rs}$  with this term. So, finally you will have the equation of  $X_r = X_{rc} + jX_{(r-\frac{N}{4})c}$ . So, both this term you can obtain from the cosine filter only and there is no need of sine filter. So, finally our phasor estimation of cosine transform is given by  $X_r = X_{rc} + jX_{(r-\frac{N}{4})c}$ . So, as I told you both this terms are obtained by cosine filters only and there is no need of sine filters.

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**Cosine Transform**

- For signal  $i(t)$ , the output of cosine and sine filter for the fundamental component for windows 1 to 7 are given in Table.

Window number ( $r$ )	cosine filter ( $X_{rc}$ )	sine filter ( $X_{rs}$ )	Phasor ( $X_r$ )
1	10	0	$10\angle 0^\circ$
2	9.239	-3.827	$10\angle 22.5^\circ$
3	7.071	-7.071	$10\angle 45^\circ$
4	3.827	-9.239	$10\angle 67.5^\circ$
5	0	-10	$10\angle 90^\circ$
6	-3.827	-9.239	$10\angle 112.5^\circ$
7	-7.071	-7.071	$10\angle 135^\circ$

Hence, the output of the sine filter for a particular ( $r^{\text{th}}$ ) window is same as the negative value of the cosine for  $(r-N/4)$  window.

So, what we have discussed? We can also consider, or we can also understand by considering this table. So, in this table what I have done? We have considered a signal  $i(t)$ , and that signal we have considered with several windows. So, here I have considered the 7 windows total and each window contains 16 samples, but it is a sliding window. So, one value of the sample is discarded and new value of sample is available in the next window.

So, here in this window number you can see the  $r^{\text{th}}$  window, that is this window fifth, and you can see that the value of sine filter  $X_{rs}$ , that is coming 10, which is same as the this value except the sine difference. Here you have to take this value and negative of that, that is the value of sine filter.

And this difference between these two that is  $N/4$ . So, if you take let us say we are considering fifth window, then  $5-4$ , you have to consider the first window and whatever value is available of cosine filter, you take the same value with negative sign. Similarly, you can see that if you consider the sixth window, then  $6-4$  that comes out to be 2. So, whatever value is available of cosine filter, you can have the sine filter value only with negative sign.

And same way when you consider the seventh window 7-4, that comes out to be 3. So, same value of cosine filter you have to consider with sine filter with only negative difference, or negative value. Hence, we can say that the output of the sine filter for any  $r^{\text{th}}$  window that is same as the negative value of the cosine filter for  $(r-N/4)$  window.

With this background, let us see how we can apply the cosine transform and when we apply what are the important things which we need to consider.

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### Application perspective of Cosine Transform

- The computational complexity of the phasor estimation reduces as there is no requirement of sine filter (component).
- However, there is a need of cosine filter output of the window occurring every quarter cycle ( $N/4$ ) prior to the current ( $r^{\text{th}}$ ) window. This causes delay of one-quarter cycle in phasor estimation compared to the DFT algorithm.
 

$$N + \frac{N}{4} = \frac{5N}{4}$$
- The phasor calculated by cosine filter is less affected particularly when the decaying dc component is present (compared to DFT) as cosine filter suppresses the decaying dc component more than the sine filters.

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The first important point is the computational complexity of the phasor estimation, that is carried out by cosine transform. So, as there is no requirement of sine filter, though computational requirement reduces, if I use the cosine transform for phasor estimation. The second thing is the delay, as we need the samples of the cosine filter that is actual sample, prior than the  $N/4$  samples, or the  $r^{\text{th}}$  window.

So, this is going to cause a delay of one quarter cycle in phasor estimation compared to the DFT algorithm. So, if DFT algorithm, let us say needs  $N$  samples, in one cycle, this much time is required, then this cosine transform, it needs this  $N+N/4$  samples for estimation of phasor value. So, this is nothing but  $5N/4$ .

And the third important point is that whatever phasor that is calculated by this cosine filter, this is capable to remove, or reject dc component, if it is present in the acquired signal compared to our

conventional DFT algorithm. So, these are the three important points which we need to consider regarding the cosine transform when we apply in the actual field.

So, in this lecture, we have considered the half cycle discrete Fourier transform algorithm. We have also considered the example related to the half cycle discrete Fourier transform algorithm. And after that we discussed the cosine transform, how we can use the cosine transform for phasor estimation of any acquired signal. And we have seen that this can be applied only for fundamental components, and we have discussed with one example also. Thank you.