Digital Protection of Power System
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Lecture No. 08
Phasor Estimation Algorithm –IV

So hello friends, in the previous lecture we have discussed regarding the half cycle DFT.

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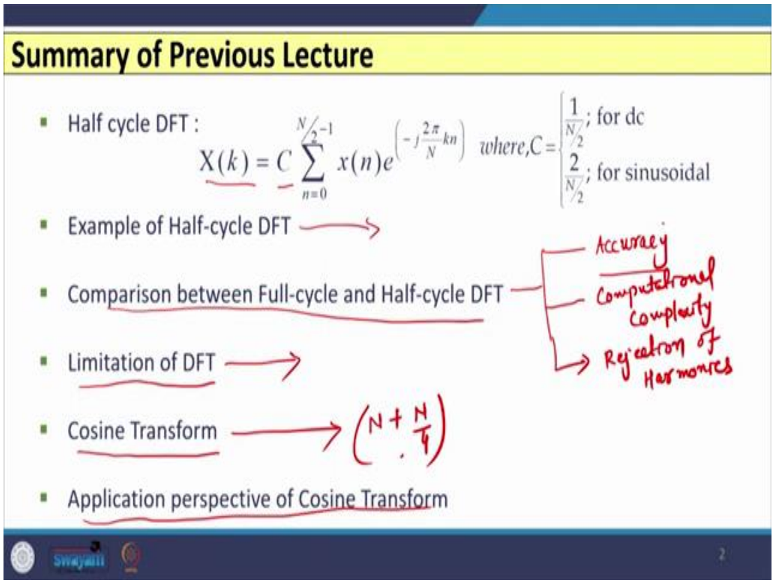

And in that we have seen that the phasor value that is to be estimated for any harmonic component or fundamental or dc term that is given by the equation:

$$X(k) = C\sum_{n=0}^{\frac{N}{2}-1} x(n)e^{\left(-j\frac{2\pi}{N}kn\right)} \; where, C = \begin{cases} \frac{1}{\frac{N}{2}}; \text{ for dc} \\ \frac{2}{\frac{N}{2}}; \text{ for sinusoidal} \end{cases}$$

After that we have also discussed one example of half cycle DFT and in that example we have seen that whatever phasor number of samples we considered is practically half the value of full cycle DFT, then also the phasor value that is to be estimated are almost equal to the same value as estimated by full cycle DFT.

After that we have discussed the comparison of full cycle DFT and half cycle DFT. So, in this comparison we have done based on three important parameters. So, the first parameter is the accuracy. And we have discussed that as the number of samples required per window in case of full cycle DFT that is higher than the number of samples required in half cycle DFT. So, accuracy of full cycle DFT is better compared to the half cycle DFT.

Then we have considered another parameter for the comparison of performance of full cycle DFT and half cycle DFT that is computational complexity. So, as the number of samples required in full sample is DFT is higher compared to half cycle DFT, so computational requirement reduces if we use or if we go for half cycle DFT for phasor estimation. So, this is also another important parameter. So, that is computational complexity.

And the third thing is the rejection of harmonics. So, this is also an important parameter for comparison. So, we have discussed that if we consider the full cycle DFT, then this algorithm is capable to reject both even as well as odd harmonics, whereas half cycle DFT is not capable to reject even harmonics. So, this we have discussed.

After that we have discussed the limitation of DFT whether full cycle or half cycle. And we have discussed that if decaying dc component is involved in the acquired signal and this is the case when fault occurs. So, when we acquire the samples from fault current signal, then decaying dc component is present. So, in that case the DFT is not capable to accurately estimate the phasor values.

After that we have discussed another algorithm that is cosine transform. And in this transform we have discussed that the sine component we can replace with the cosine term. So, the sample calculations required for cosine transform that can be reduced. However, the unavoidable time delay is included when we use cosine transform. So, it requires N+N/4 samples compared to the DFT which required N samples.

And after that we have discussed the application perspective of cosine transform. And that we have discussed that cosine transform compared to the DFT whether full cycle or half cycle how effective we can utilize the cosine transform algorithm for phasor estimation of the signals.

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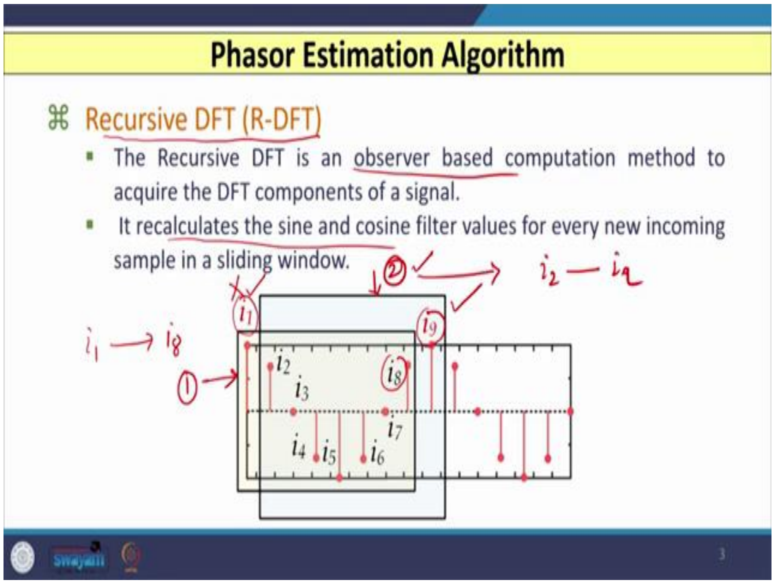

Now in today's class let us extend our discussion and let us discuss about the recursive DFT. So, recursive DFT is an extension of our conventional full cycle DFT and in this DFT an observer based computational algorithm is used to acquire the components or the samples from the signals. So, in this algorithm the re-calculation of sine and cosine filter values for every new incoming sample in a particular window that is to be carried out.

So, if we consider here you can see there are two windows. This is the first window. And this is the second window. So, when in the first window you know that the samples from i_1 to i_8 are available and when you use these 8 samples in window 1, then you can easily estimate the phasor value. So, phasor value of window 1 is already available.

And now when you move from first window to the second window the only change is that the first sample that is i_1 that is discarded and the new sample i_9 that is to be included. So, for window 1 sample number varies from i_1 to i_8 whereas, for window 2 the sample number varies from i_2 to i_9. So, the only change that is going to occur in window 2 that is the sample number i_9 and sample number i_1. So, this clearly indicates that there is no need of calculation of samples i_2 to i_8 because that we have already done in the first window or the previous window.

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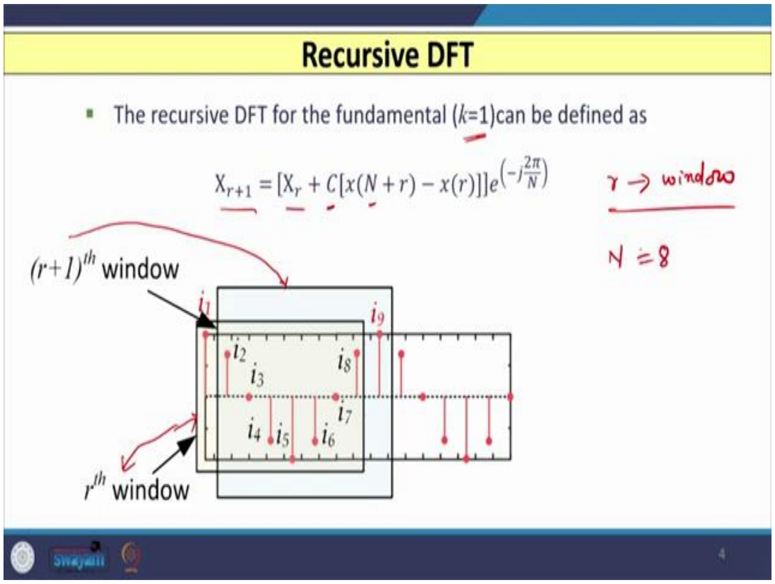

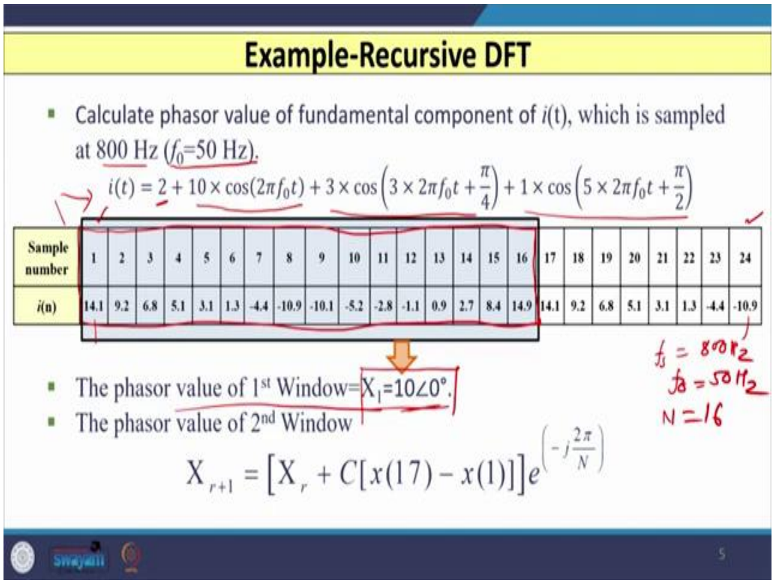

Sample number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
i(n)	14.1	9.2	6.8	5.1	3.1	1.3	-4.4	-10.9	-10.1	-5.2	-2.8	-1.1	0.9	2.7	8.4	14.9	14.1	9.2	6.8	5.1	3.1	1.3	-4.4	-10.9

So, to understand the recursive DFT for any fundamental phasor, so assuming k=1 mathematically we can represent the recursive DFT as $X_{r+1} = [X_r + C[x(N+r) - x(r)]]e^{\left(-j\frac{2\pi}{N}\right)}$. Now here r is nothing but the window, a particular window.

So, here you can see the two windows I have shown. This window is your r^{th} window and the next window that is your $(r+1)^{th}$ window. And as I told you as I explained earlier that when you move from first window or r^{th} window to the $(r+1)^{th}$ window then only the two samples that is i_1 and i_9 you need to consider. The remaining samples are same with reference to the previous r^{th} window.

Here, you can see that N is the number of samples in a cycle. So, that is also here in this case these samples are 8 in this case. So, if I consider in this case for example let us calculate the phasor value of fundamental component of this signal i(t) which is given by some dc value let us 2 with some fundamental component $10\angle 0$, some third harmonic term $3\angle\frac{\pi}{4}$ and the fifth harmonic term that is $1\angle\frac{\pi}{2}$ or $1\angle 90°$.

Here this signal when we acquire the samples the fundamental frequency is 50 Hz and the sampling frequency is 800 Hz. Whatever value we acquired for respective sample are given in this table. So, you can see that sample Number 1 to sample number 24 are given. And for each sample the acquired value that is also listed in the table.

Now using this if we calculate the phasor value of the first window, so here as the sampling frequency f_s is 800 Hz and fundamental frequency f_0 is 50 Hz so your number of samples in a cycle that is 16 f_s by f_0. So, if you consider the first window from sample number 1 to sample number 16 and if you calculate using this 16 samples in a window or in a cycle, the phasor value that value is given by X_1 and that value you can obtain that is $10\angle 0$. And this is for fundamental component. So, here you can see that for this first 16 samples the phasor value of fundamental component comes out to be $10\angle 0$.

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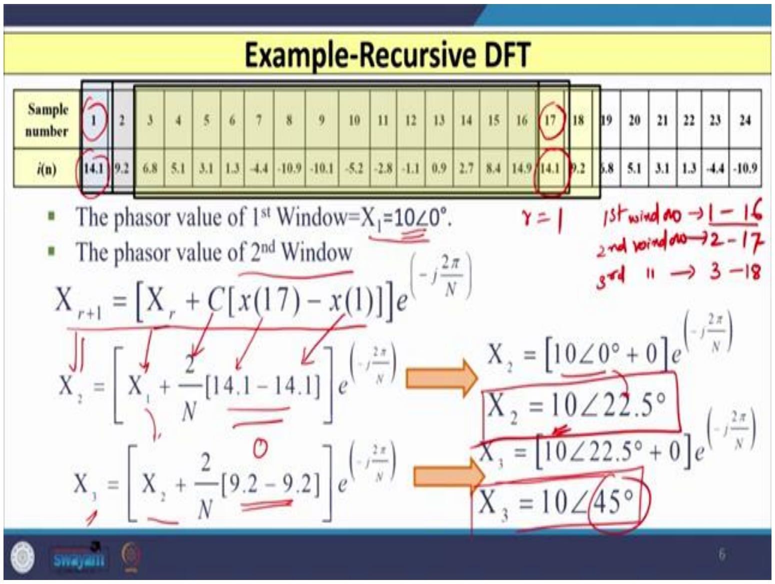

Now when you move to the next window that is the second window, the second window is from 2 to 17. So, your first window that is from 1 to 16 samples and your second window is from 2 to 17 samples. So, the only difference that is for sample number 1 that is, that needs to be discarded and sample number 17 that is to be included.

So we have already calculated the phasor value X_1 for fundamental component that is $10\angle 0$ for first window when we considered 1 to 16 samples. Now when you want to calculate the phasor value for second window which contains sample number 2 to sample numbers 17 then using this equation:

$$X_{r+1} = [X_r + C[x(17) - x(1)]]e^{\left(-j\frac{2\pi}{N}\right)}$$

So, here let us consider r =1. So, when I consider r = 1 this value that is X_{r+1} So, X_2 for which second window we want to calculate the phasor value that is X_r and r is 1. So, this is X_1. So, X_1 we have already calculated the value that is 10 ∠0°. The C that is 2 by N and X(N)+r, so N is 16 and r is 1 so this is X 17 that is 14.1 value and minus X_r so this is X_1 so that is 14.1 again.

So, when you calculate the phasor value for second window:

$$X_2 = \left[X_1 + \frac{2}{N}[14.1 - 14.1]\right]e^{\left(-j\frac{2\pi}{N}\right)} \quad X_2 = 10\angle 22.5°$$

Now if we further proceed let us say third window that is from sample number 3 to sample number 18. So, when we consider the third window and if we calculate the value of X_3 then the equation is $X_3 = \left[X_2 + \frac{2}{N}[9.2 - 9.2]\right]e^{\left(-j\frac{2\pi}{N}\right)}$. So, you can see that when you take new window then the only difference that comes out to be here that is in the phase angle.

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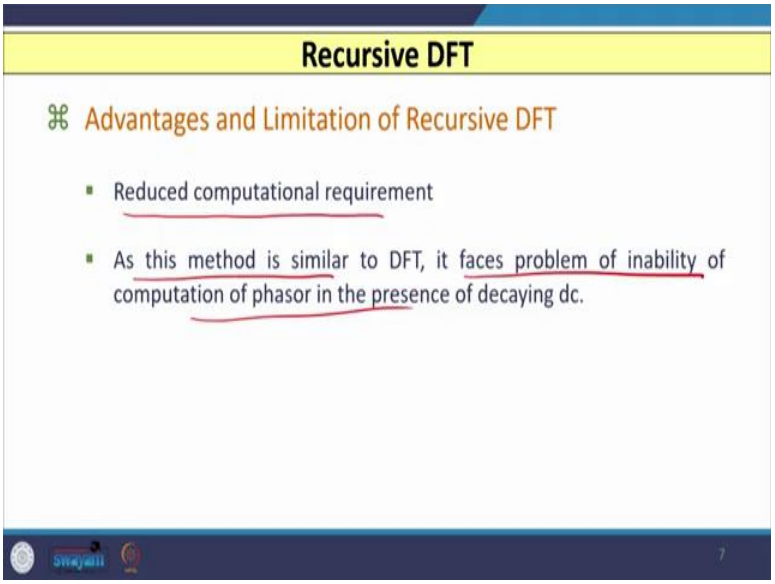

So, we can say that when we use the recursive DFT computational requirement reduces because when you move from one window to other window you have to consider only those samples which are available. So, new sample as well as the sample which needs to be discarded.

Moreover, as this method is similar to DFT so it also faces a problem of the inability of computation of phasor when decaying dc component is present in the signal. So, when fault occurs, fault signals normally contain decaying dc component. So, when such component is there and when you acquire the sample for estimation of phasor value then recursive DFT is also not able to accurately estimate the phasor value.

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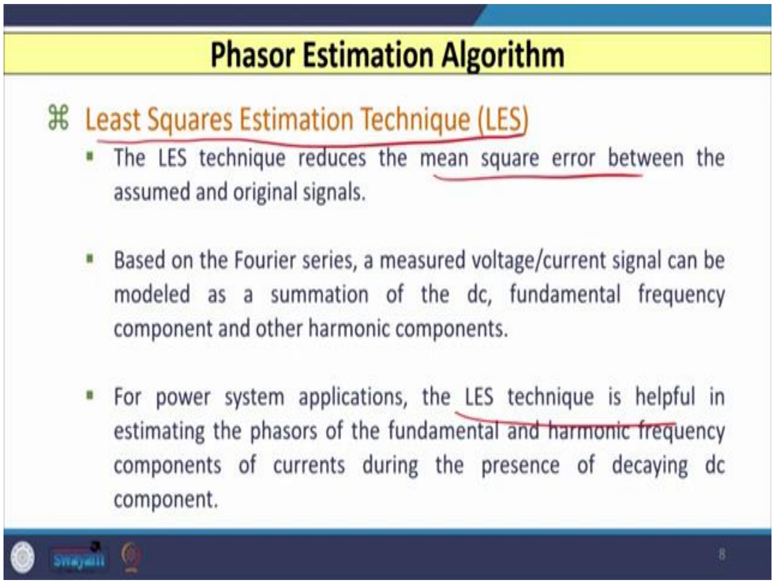

Now let us move to the next phasor estimation algorithm which is known as Least Square Estimation or LES algorithm. So, this LES technique reduces the mean square error value between the assumed value and the original values. So, when we consider DFT that is Discrete Fourier Transform algorithm and when we acquired samples of either voltage or current signals for the estimation of dc term or fundamental term or any other higher order harmonics, then in that case when decaying dc component is present in the acquired signals specially in case of fault LES technique performs better. We will see how it performs better.

So, when decaying dc component is present, then instead of utilizing DFT or cosine transform or any other recursive DFT or any other extended version of DFT algorithm, we must go for the LES technique.

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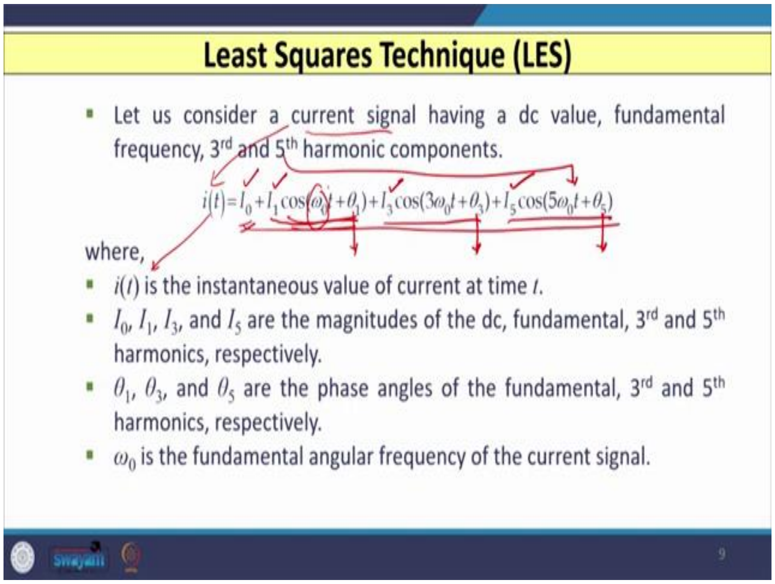

Now let us see how the LES technique works. So, let us consider a current signal i(t) which is given by $i(t) = I_0 + I_1\, cos(\,\omega_0 t + \theta_1) + I_3\, cos(\,3\omega_0 t + \theta_3) + I_5\, cos(\,5\omega_0 t + \theta_5)$.

So, this equation of current it has a term I_0 that is a dc component. It has a fundamental term I_1. It has a third harmonic term that is this one, $I_3\, cos(\,3\omega_0 t + \theta_3)$. And it also has as a fifth harmonic component that is $I_5\, cos(\,5\omega_0 t + \theta_5)$.

So here, you can see that this i(t) is nothing but the instantaneous value of current which changes with reference to time. The I_0, I_1, I_3 and I_5 are the magnitudes of dc component fundamental third and fifth 6harmonics respectively. If I consider the theta 1, theta 3 and theta 5 then these are the phase angles of fundamental third and fifth harmonics respectively. And ω_0 is nothing but the fundamental angular frequency of the current signal that is i(t) for which we are acquired the samples.

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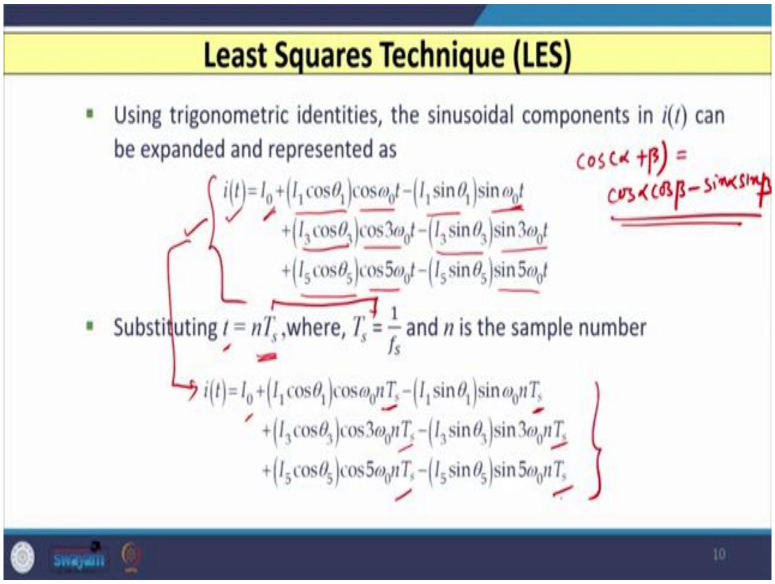

Now if I use the trigonometric identities then we know that the equation of $\cos(\alpha+\beta)$ that is given by $(\cos\alpha\cos\beta)$ – (sine α sine β). So, if I use this then we can say that the earlier equation in this equation the term $I_1 \cos\omega_0 t + \theta_1$. So, we can extend this $\cos\omega_0 t + \theta_1$ in terms of $\cos(\alpha+\beta)$. So, where α is $\omega_0 t$ and β is θ_1.

So, if I extend using this then we have $i(t) = I_0 + (I_1 \cos\theta_1)\cos\omega_0 t - (I_1 \sin\theta_1)\sin\omega_0 t$.

Similarly, if I again extend this term with reference to cos of alpha plus beta this equation, then we can have $(I_3 \cos\theta_3)\cos 3\,\omega_0 t - (I_3 \sin\theta_3)\sin 3\,\omega_0 t$.

Same way you have $(I_5 \cos\theta_5)\cos 5\,\omega_0 t - (I_5 \sin\theta_5)\sin 5\,\omega_0 t$.

Now here in this equation if I substitute $t = nT_s$ where n is this sample number which varies for a particular window. If I have 16 samples in a window then n varies from 1 to 16, like that. And if this T_s is nothing but your sampling time which is nothing but the reciprocal of sampling frequency that is $1/f_s$.

So, if I replaced $t = nT_s$ in this equation, then we have the value of i(t) that is given by

$$i(t) = I_0 + (I_1 \cos\theta_1)\cos\omega_0\, nT_s - (I_1 \sin\theta_1)\sin\omega_0\, nT_s$$

$$+(I_3 \cos\theta_3)\cos 3\,\omega_0 nT_s - (I_3 \sin\theta_3)\sin 3\,\omega_0 nT_s$$

$$+(I_5 \cos\theta_5)\cos 5\,\omega_0 nT_s - (I_5 \sin\theta_5)\sin 5\,\omega_0 nT_s$$

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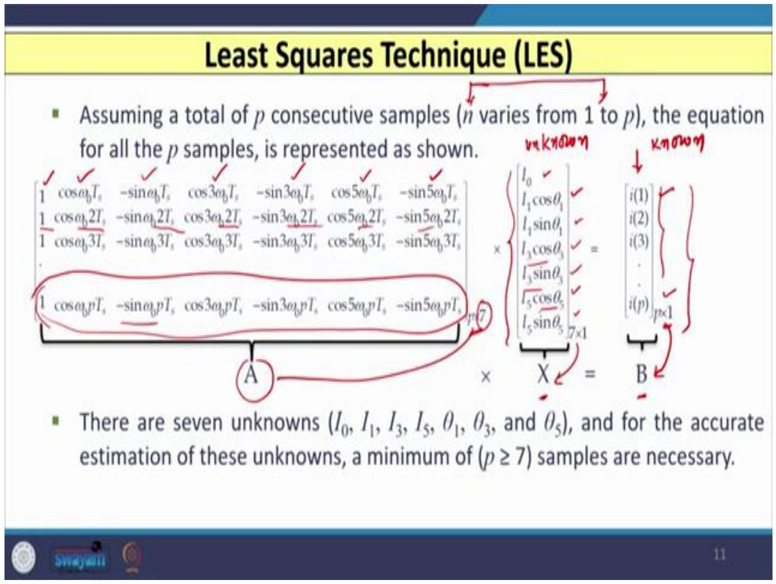

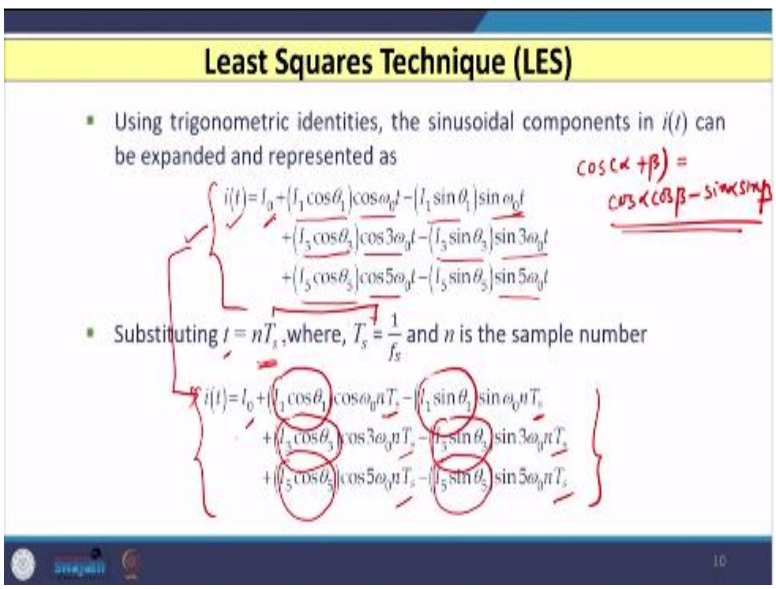

Now this equation if I write down in the matrix form then we have the term that is this $I_1 \cos\theta_1$. We have the term $I_1 \sin\theta_1$. We have the term $I_3 \cos\theta_3$ and we have the term $I_3 \sin\theta_3$. Similarly, we have $I_5 \cos\theta_5$ and we have $I_5 \sin\theta_5$ and we have the I_0 term. So, these are the unknown values. On right hand side the known values that is nothing but the samples which we are acquiring for this current signal. So, that sample varies from i(1) to let say i(p) where p is nothing but the consecutive samples. So, that value of p that is variable in that case n varies from 1 to p.

So, if I have 16 samples then this n varies from 1 to 16. So, in this equation if I replace this or rewrite in terms of matrix form then we have the term you can see here I_0 is there. So, if I take I_0 on unknown side and this becomes 1 so we have the 1. Then we have the term that is this term that you have here $cos\ \omega_0$, n we are putting as sample number 1, so here we have $cos\ \omega_0$, n is 1, into T_s.

Similarly, we have here sinω_0 , n is 1, into T_s. So, we have this term $cos\ \omega_0 T_s$ here. Similarly we have $sin\ \omega_0 T_s$ with negative sign. So, we have this term that is $sin\ \omega_0 T_s$. Same way if you write down $cos\ 3\ \omega_0 nT_s$ and $sin\ 3\ \omega_0 nT_s$ so you have the term, this one and this one, and similarly you have this term. Now this is for n is equal to 1.

So, if you again put n is equal to 2 then you have 1 $cos\ \omega_0 2T_s$, again sine term again this term. So, similarly if you extend you will have the last row in which you have these elements that is $cos\ \omega_0\ pT_s \quad -sin\ \omega_0\ pT_s \quad cos\ 3\ \omega_0 pT_s \quad -sin\ 3\ \omega_0 pT_s \quad cos\ 5\ \omega_0 pT_s \quad -sin\ 5\ \omega_0 pT_s$ and so on. So, this matrix which is nothing but $p \times 7$. The 7 number we will explain because we have the 7 unknowns that is why this 7 number comes here.

And on right hand side we have this samples which are known that varies from i(1) to i(p). So, size of this matrix is $p \times 1$. So, if I called this matrix as A if I called unknown as matrix X and if I call known value of samples that is matrix B then the size of B is p cross 1. The size of X that is 7 cross 1 because we have 7 unknowns. And the size of A that is again p cross 7. So, here you can see that the unknown values are I_0 and $I_1\ cos\ \theta_1 sin\ \theta_1$. Similarly, $I_3\ cos\ \theta_3$and $sin\ \theta_3$. And we have $I_5\ cos\ \theta_5$and $I_5\ sin\ \theta_5$. So, the total unknowns we have that is 7.

So, for any equation if we want to estimate the unknowns then the Thumb rule is that we need minimum that many samples for the minimum unknown values. So, here unknown values are 7. So, we need at least 7 samples. So, here these samples vary from I_1 to, up to 7 values. So, 7 samples I_1 to I_7 we required. Or maybe we have higher samples. That is also okay. But a minimum of 7 samples are needed.

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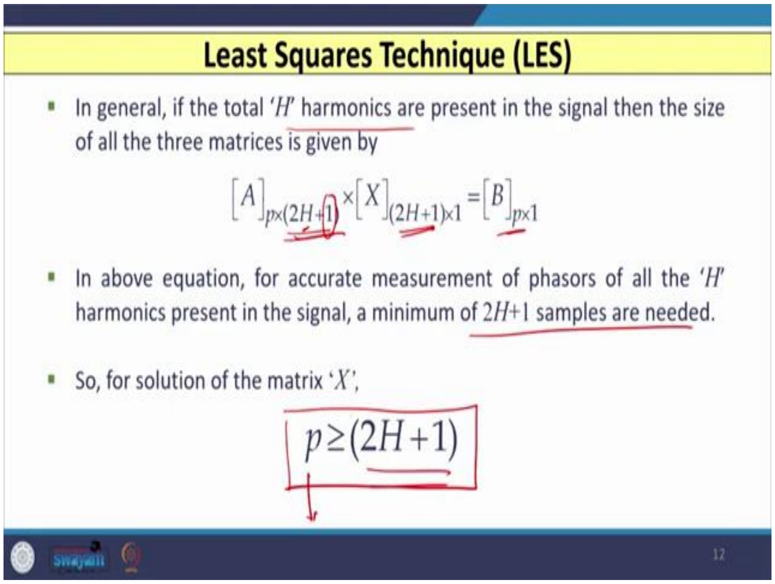

So, in general form if I write this equation that is [A] into [X] that is equal to [B] , then I can write A which size is you know p x 7. So, 7 I am replacing with the term 2H +1 where H is nothing, but the total harmonics present in the signal. So, here you can see in the earlier signal harmonics dc value and the other harmonics are fundamental, third and fifth, two values for each. So, 2 into 3 6 plus I0 that is 7. So, that is why this 2H+1. So, 1 indicates the dc value, H indicates the harmonics. So, this is how the size of A matrix is available.

Similarly, the size of [X] you can see here that is unknowns that is 7 cross 1. So, 7 I am replacing again with 2H +1. And same way we have the B matrix that is p cross 1. Now in this above equation you can see that for accurate measurement of phasors of all the harmonics present in the acquired signal so minimum we need 2H + 1 samples.

So, as I told you we need at least 7 samples if 7 unknowns are there. That is why the solution for [X] we need p that is the number of samples which is always greater than or equal to 2H + 1. If it is less than, then we are not able to calculate the or find out the unknown values.

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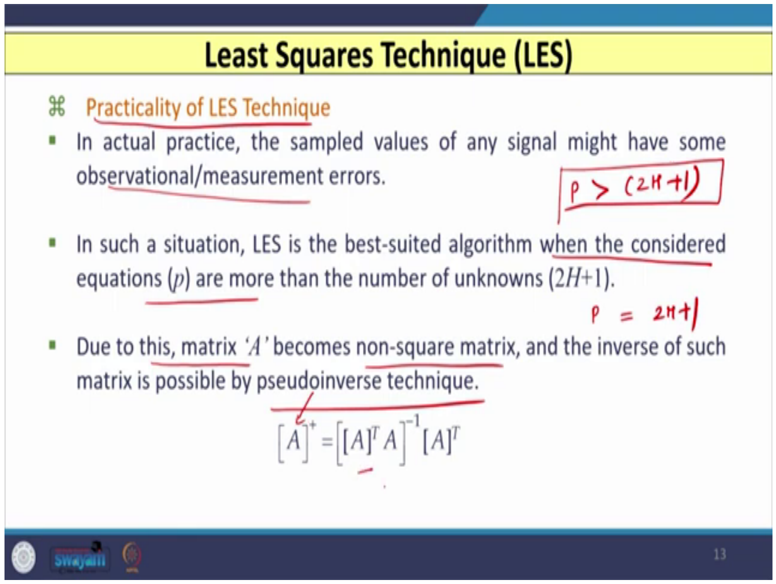

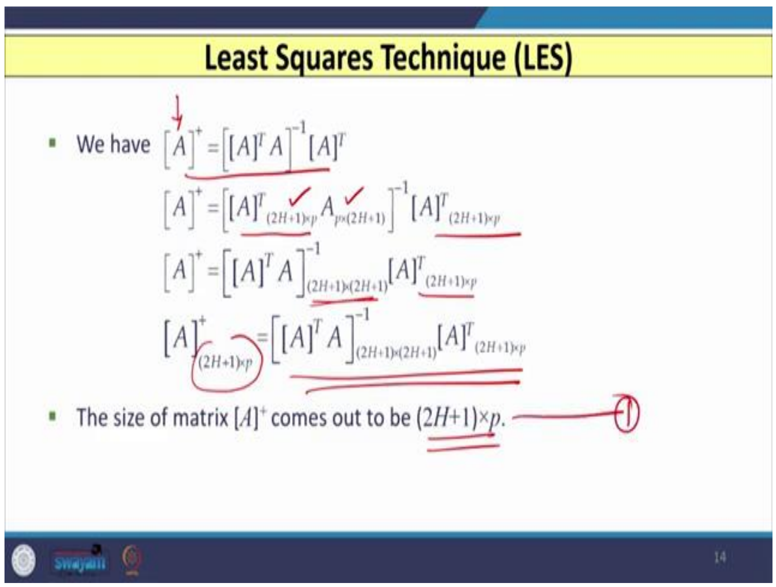

Now let us discuss practicality of LES technique. So, in actual practice we know that wherever we acquire the samples then your signals may contain some errors like measurement errors or observational errors. So, in such a situation if I use LES algorithm for estimation of phasors then this is the best technique. How? So, when the considered equations, samples p available are more than the unknowns so when the value of p is greater than 2H+1, minimum is p is equal to 2H+1 but if the samples are more than the unknowns, then in that case LES algorithm performs better compared to the earlier algorithms.

Now let us discuss one important point. When we consider the matrix A and when the number of samples available that is same as 2H+1 so if we have 7 unknowns and if 7 samples are available, then the size of this matrix A that is a square matrix. And we can easily find out the inverse. But when we have the value of number of samples available that is greater than the 2H+1, that is more than the unknown values, then the size of the matrix A that becomes non-square matrix.

And inverse of this matrix is possible by pseudo inverse technique. So, here if I consider pseudo inverse of $[A]^+$, that is nothing but $[A]^+ = [[A]^T A]^{-1}[A]^T$

So, if I write down this value that is pseudo inverse A^+ here same equation, then we can say that $[A]^T$ size is $(2H + 1) \times p$. this we have already discussed earlier. So, the size of [A] that is $p\times(2H+ 1)$, so size of $[A]^T$ that is reverse $(2H + 1) \times p$. So, that is here available.

Again, [A] that is $p\times (2H +1)$ and whole inverse into $[A]^T$ that is this value. So, if you combine these two then you can have, you see the size that is $(2H + 1)\times (2H +1)$. So, your matrix becomes square. And then you have the $[A]^T$. So, finally the pseudo inverse of [A] that is A plus it has the size that is $(2H + 1)\times p$ which is given by this thing. As I told you the size of the [A], pseudo inverse of A] that is $(2H + 1)\times p$. So, this is really important equation which we need to remember.

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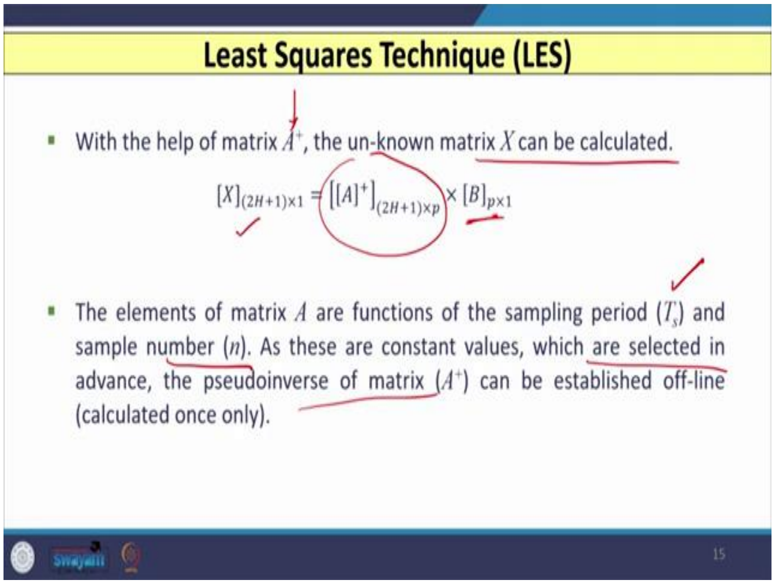

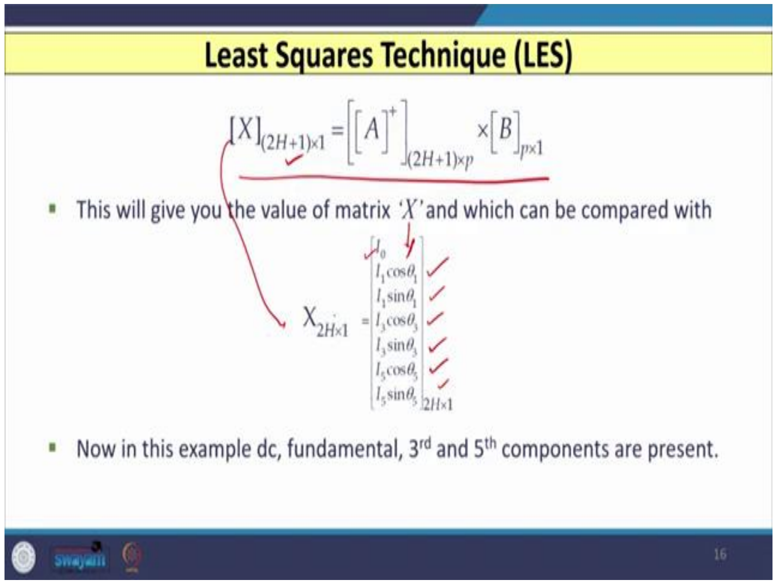

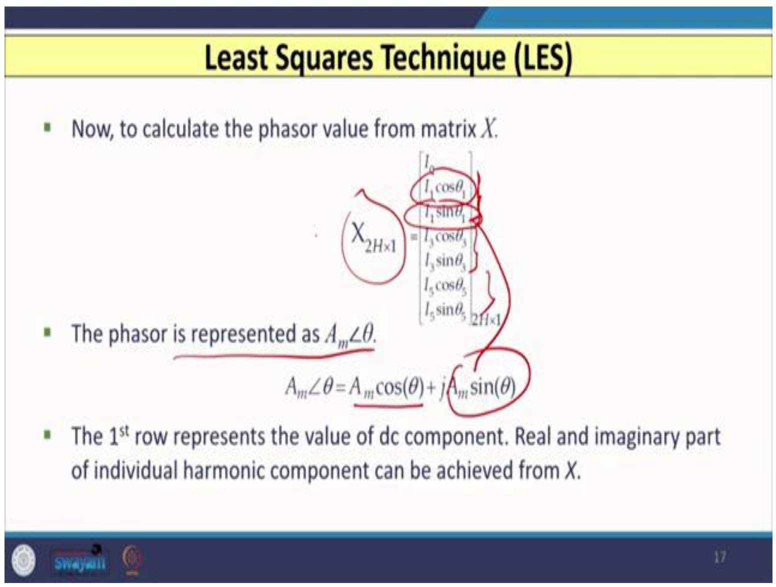

Now if I use this pseudo inverse of A, then the unknown value of X that can be easily calculated and size of this pseudo inverse of A that is $(2H+1)\times p$and size of B is $[B]_{p\times 1}$.

So, we have the unknown matrix X and its size is $(2H+1)\times 1$.

Now the important point is the elements of this matrix A depends on two parameters. One is the T_s sampling period and another is the sample numbers. And these values are constant. So, whenever we consider or when we calculate the pseudo inverse of matrix A we have to calculate this matrix once only. There is no need to calculate this matrix again and again for each and every iteration.

So, our final equation of unknown value of X that is given by this equation:

$$X_{2H\times 1} = \begin{bmatrix} I_0 \\ I_1\cos\theta_1 \\ I_1\sin\theta_1 \\ I_3\cos\theta_3 \\ I_3\sin\theta_3 \\ I_5\cos\theta_5 \\ I_5\sin\theta_5 \end{bmatrix}_{2H\times 1}$$

So, here you can see that when you consider the value of this X unknown size of which is $(2H+1)\times 1$, so in that case if we wish to calculate the phasor value then phasor is normally represented by $A_m\angle\theta$. So, here you can see that if I want to calculate, phasor of this then A_m cos Ɵ is nothing but your $I_1\,cos\,\theta_1$ 1 and your $I_1\sin\theta_1$ is nothing but $A_m\sin\theta$. So, $I_1\,cos\,\theta_1$ +j $I_1\,sin\,\theta_1$, that is nothing but the phasor value of this term.

Similarly, you can calculate the phasor value of third and 5th harmonic. So, that you can easily calculate. So, here you can see that in this first row represents the dc value whereas the real and imaginary part of other harmonic component that can be achieved when you obtain the value of X. So, in this class we have discussed initially what is the equation of recursive DFT algorithm.

And we have discussed that when we want to use recursive DFT then there is no need of calculation of several samples except the new sample and the discarded sample. So, we can tremendously reduce the computational requirement when we use recursive DFT. We have also discussed one example of recursive DFT and thereafter we have discussed the new phasor estimation algorithm that is LES technique.

We have discussed how mathematically LES can be represented and how we can obtain the phasor value of LES technique when the number of unknown or samples are greater than or equal to the unknown values that is 2H+1. Thank you.