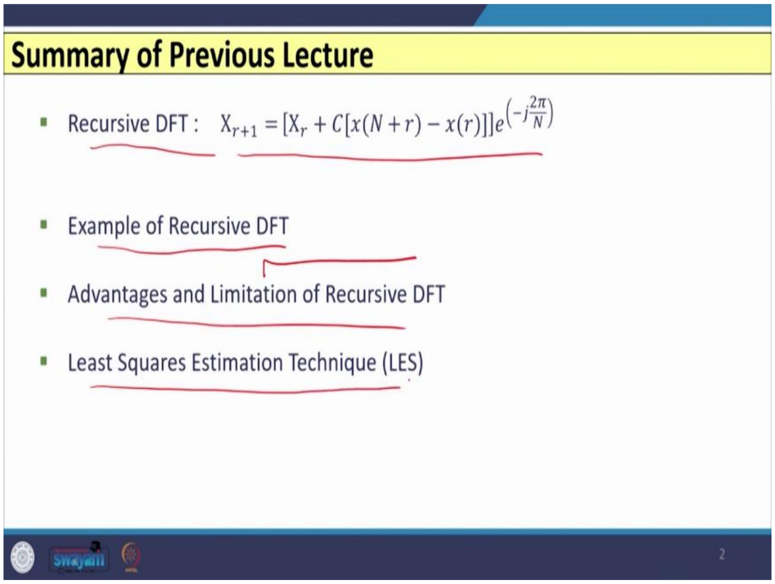
Digital Protection of Power System
Professor. Bhaveshkumar Bhalja
Department of Electrical Engineering
Indian Institute of Technology, Roorkee
Lecture No. 09
Phasor Estimation Algorithm-V

Hello friends. So, in the previous lecture, we have discussed regarding the recursive DFT.

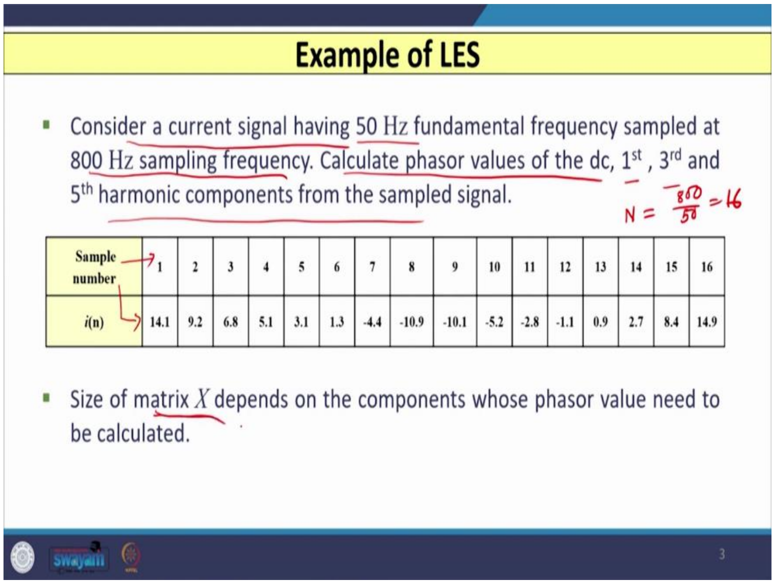
(Refer Time Slide: 00:29)



So, we have discussed that, if I use the formula of recursive DFT, then we need to calculate for the next window only those samples which are newly available in that particular window and we need to discard the sample which is not required in the new window. Moreover, we have also discussed the example of recursive DFT and after that we have discussed the advantages and limitation of recursive DFT.

And we have discussed that if I use the recursive DFT, then the important advantage we have that is the computational requirement reduces. However, at the same time the recursive DFT has the same limitation as the full cycle or upcycle DFT have that is when decaying DC component is present in the acquire signal say, in case of fault, then the accuracy of phasor estimation that is not proper. And finally, we have discussed a new phasor estimation algorithm that is LES technique.

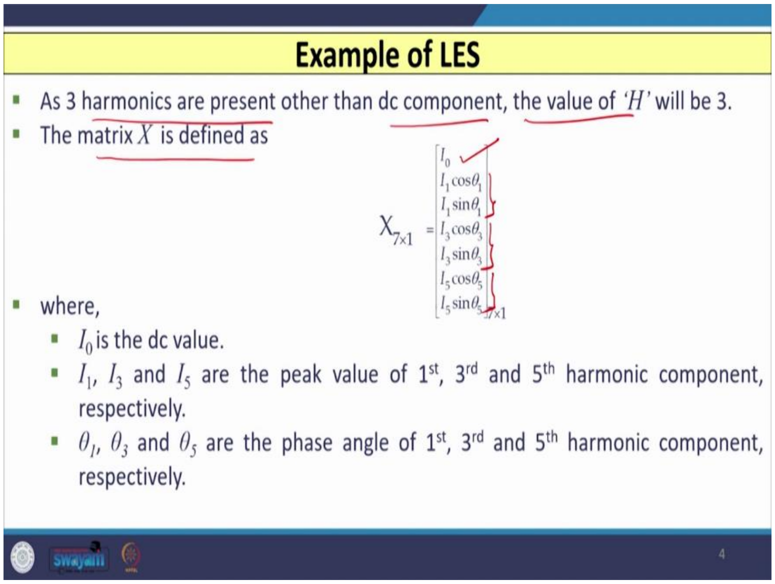
(Refer Time Slide: 01:39)



Now, let us consider one example of this LES technique. So, to understand this, let us consider a current signal having a fundamental frequency of 50 Hz and this signal is sampled with a sampling frequency F_s = 800 Hz. We wish to calculate the phasor values of DC value, first that is fundamental term, third harmonic and let us say fifth harmonic from the sampled values. And the given sampled values that is given in this table and the sample numbers are given here and for each sample the acquired value that is also given here and these values are for 16 samples.

So, as for 800-hertz sampling frequency the number of samples in one cycle that is 800 by 50. So, that is also 16. Now, we know that the size of matrix X which is unknown that depends on the components whose phasor value needs to be calculated. So, let us say if I wish to calculate phasor value or fundamental term or third harmonic or fifth harmonic, so, depending upon that the matrix size of X that is changed.

(Refer Time Slide: 02:58)

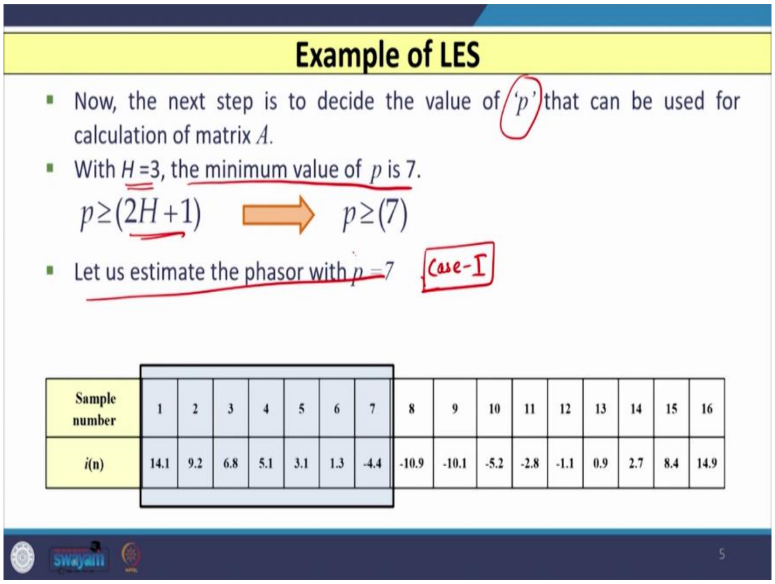


So, as 3 harmonics are present other than DC components. So, here in earlier case first, third and fifth these 3 harmonics are there other than DC component. So, we have the value of H that will be 3 in this case and the size of matrix X that is

$$X_{7\times1} = \begin{bmatrix} I_0 \\ I_1\, cos\, \theta_1 \\ I_1\, sin\, \theta_1 \\ I_3\, cos\, \theta_3 \\ I_3\, sin\, \theta_3 \\ I_5\, cos\, \theta_5 \\ I_5\, sin\, \theta_5 \end{bmatrix}_{7\times1}$$

So, as I told you I_0 is the dc value and I_1, I_3 and I_5 these are the peak value of first third and fifth harmonic component whereas, θ_1, θ_3 and θ_5 are the phase angle of first third and fifth harmonic component respectively.

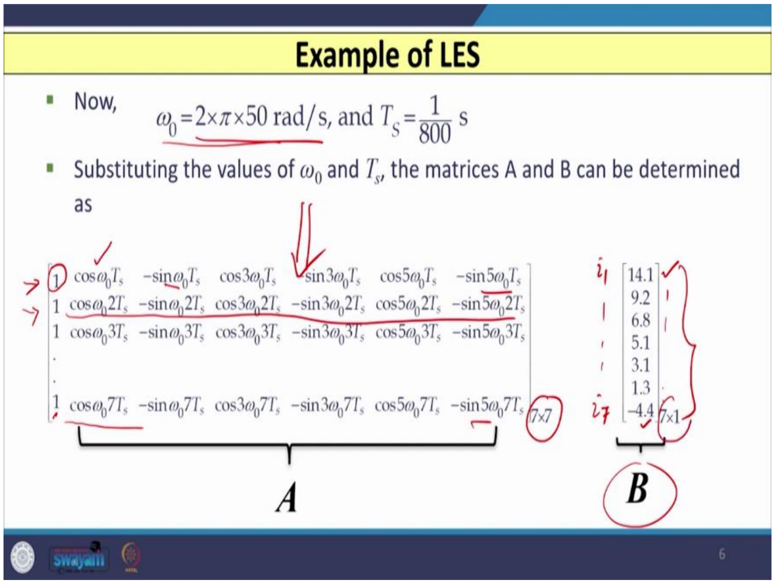
(Refer Time Slide: 03:58)



So, the next step is to decide the value of p. So, that can be used for the calculation of the other matrix. So, when we consider the value of H that is 3 because we need to calculate first third and fifth other than DC component. So, the minimum value of p that is 7 from this equation 2H+1, so, 2×H is 3+1. So, minimum we require 7 samples to estimate the unknown values of fundamental third harmonic, fifth harmonic and dc term.

So, let us consider the case number 1 in which we consider the value of p that is 7 exactly same as the number of unknowns. So, here these are the 16 samples and when we use these samples and when we consider p=7, so the window becomes like this, we need to consider sample number 1 to sample number 7.

(Refer Time Slide: 05:09)



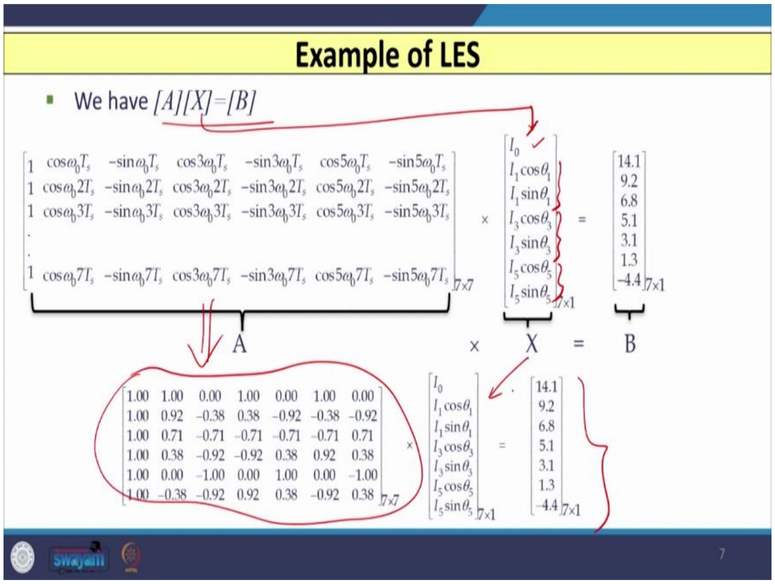
And when we use the value of $\omega_0 = 2 \times \pi \times 50$ rad/s, and $T_S = \frac{1}{800}$ s and if you substitute the values of ω_0 and T_s, the matrices of A and B that can be look like this.

$$\begin{bmatrix} 1 & \cos\omega_0 T_s & -\sin\omega_0 T_s & \cos 3\,\omega_0 T_s & -\sin 3\,\omega_0 T_s & \cos 5\,\omega_0 T_s & -\sin 5\,\omega_0 T_s \\ 1 & \cos\omega_0 2T_s & -\sin\omega_0 2T_s & \cos 3\,\omega_0 2T_s & -\sin 3\,\omega_0 2T_s & \cos 5\,\omega_0 2T_s & -\sin 5\,\omega_0 2T_s \\ 1 & \cos\omega_0 3T_s & -\sin\omega_0 3T_s & \cos 3\,\omega_0 3T_s & -\sin 3\,\omega_0 3T_s & \cos 5\,\omega_0 3T_s & -\sin 5\,\omega_0 3T_s \\ . & & & & & & \\ . & & & & & & \\ 1 & \cos\omega_0 7T_s & -\sin\omega_0 7T_s & \cos 3\,\omega_0 7T_s & -\sin 3\,\omega_0 7T_s & \cos 5\,\omega_0 7T_s & -\sin 5\,\omega_0 7T_s \end{bmatrix}_{7\times 7}$$

On the other hand, if I consider the B matrix, then B matrix is nothing but it contains your sampled values. So, it starts from i1 and it goes up to i7. $\begin{bmatrix} 14.1 \\ 9.2 \\ 6.8 \\ 5.1 \\ 3.1 \\ 1.3 \\ -4.4 \end{bmatrix}_{7\times 1}$

So, if I consider here, i1 is 14.1, and i7 is -4.4. So, you can see this is 14.1 and this is minus 4.4. So, accordingly, you have the matrix B, the size of which that is 7×1

(Refer Time Slide: 06:47)



Now, if I use this relation *[A][X] =[B]*then our unknowns X that is also 7, because it starts from the dc value, the fundamental term third harmonic and fifth harmonic. So, here, when we use this *[A][X]=[B]*and when we want the value of X, then finally, if I put the value, you will have the matrix A that looks like this $\begin{bmatrix} 1.00 & 1.00 & 0.00 & 1.00 & 0.00 & 1.00 & 0.00 \\ 1.00 & 0.92 & -0.38 & 0.38 & -0.92 & -0.38 & -0.92 \\ 1.00 & 0.71 & -0.71 & -0.71 & -0.71 & -0.71 & 0.71 \\ 1.00 & 0.38 & -0.92 & -0.92 & 0.38 & 0.92 & 0.38 \\ 1.00 & 0.00 & -1.00 & 0.00 & 1.00 & 0.00 & -1.00 \\ 1.00 & -0.38 & -0.92 & 0.92 & 0.38 & -0.92 & 0.38 \end{bmatrix}_{7\times7}$

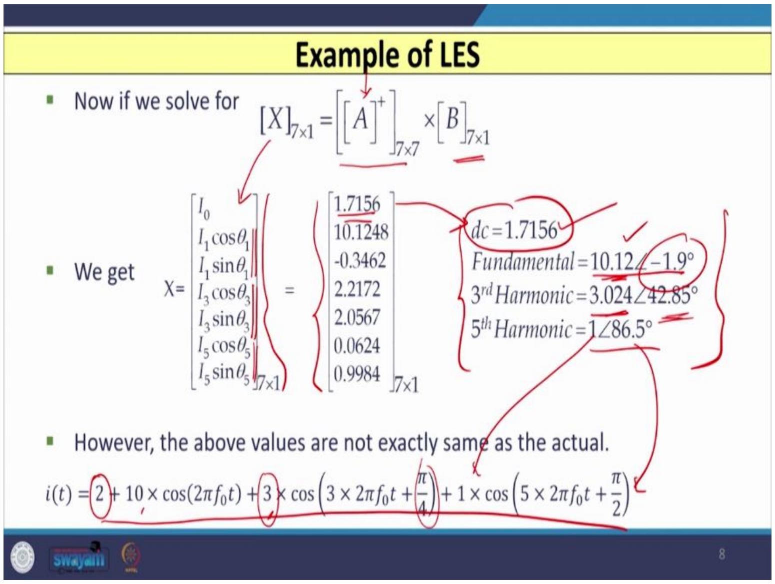
where I have calculated the value I put the value of T_s and ω_0 and hence you can have this matrix, you already have the matrix B which is given as

$$\begin{bmatrix} 14.1 \\ 9.2 \\ 6.8 \\ 5.1 \\ 3.1 \\ 1.3 \\ -4.4 \end{bmatrix}_{7\times1}$$

So, now, you can easily calculate the matrix X which is given as

$$\begin{bmatrix} I_0 \\ I_1 \cos\theta_1 \\ I_1 \sin\theta_1 \\ I_3 \cos\theta_3 \\ I_3 \sin\theta_3 \\ I_5 \cos\theta_5 \\ I_5 \sin\theta_5 \end{bmatrix}_{7\times1}$$

(Refer Time Slide: 07:32)



So, when you do that, as I told you, you have to take the pseudo inverse. So, that is the pseudo inverse is nothing but $[X]_{7\times1} = [[A]^+]_{7\times7} \times [B]_{7\times1}$

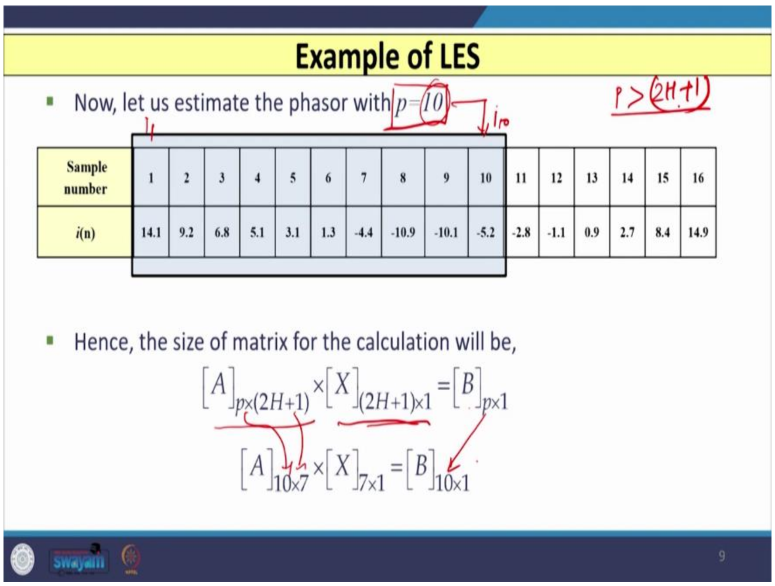
So, if you do that, then you will have the value of X here this 7 terms

$$X= \begin{bmatrix} I_0 \\ I_1 \cos\theta_1 \\ I_1 \sin\theta_1 \\ I_3 \cos\theta_3 \\ I_3 \sin\theta_3 \\ I_5 \cos\theta_5 \\ I_5 \sin\theta_5 \end{bmatrix}_{7\times1} = \begin{bmatrix} 1.7156 \\ 10.1248 \\ -0.3462 \\ 2.2172 \\ 2.0567 \\ 0.0624 \\ 0.9984 \end{bmatrix}_{7\times1}$$

So, here you can see that the dc value that comes out to be 1.7156. The fundamental term that is this first two term $I_1\cos\theta_1 + I_1\sin\theta_1$, if you obtain the magnitude and angle you will get $10.12\angle -1.9°$And similarly, if you see the third harmonic, that is $I_3\cos\theta_3 + I_3\sin\theta_3$, then you will have the magnitude $3.024\angle 42.85°$. And same way for fifth harmonic term, you will have the value $1\angle 86.5°$. However, if you see if you compare this value with the actual value of the signal, which is given by this, then you can see that the dc value actual value of dc that is 2 whereas you obtain 1.71.

Similarly, the fundamental term magnitude is 10 actual value you will have 10.12, angle is not there here you are getting some angle. Similarly, for third harmonic, the magnitude is 3.024, you have 3 and the ∠ is 42 whereas here the angle is $\frac{\pi}{4}$. And same way for fifth harmonic the magnitude is 1, but the ∠that is $\frac{\pi}{2}$ actual value, whereas whatever value you obtain, that is 86.5 so this values are not matching. So, further to get the accurate result of this phasor values of dc fundamental third and fifth harmonics, let us increase the number of samples that is p.

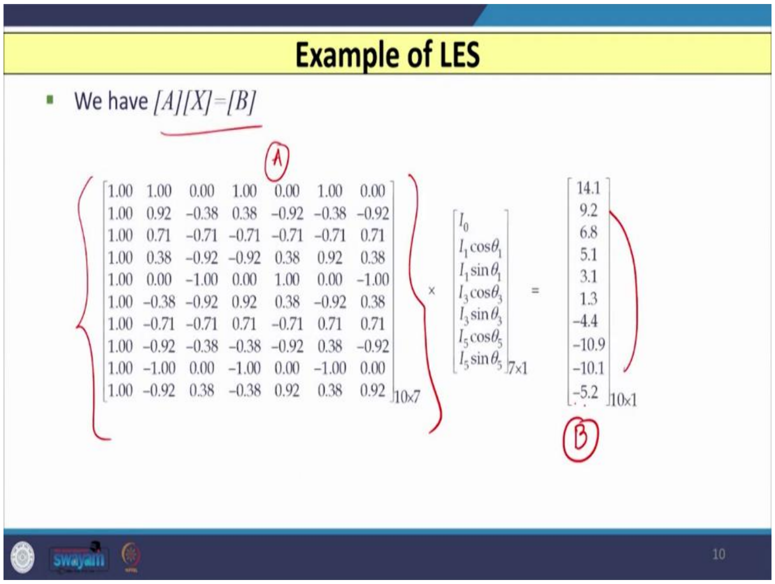
(Refer Time Slide: 09:47)



So as I told you, when we increase the value of p, which is greater than 2H +1, then your matrix A that becomes a non square and that is why you have to take the pseudo inverse. So, let us consider the p =10. So, here now, you are going to use your p that varies from the i1 to i10 and here in this case if you calculate the size of the matrix A, then that is $p \times (2H + 1)$.

So, here your p samples that is 10 so, it is $[A]_{10\times 7} \times [X]_{7\times 1} = [B]_{10\times 1}$

(Refer Time Slide: 10:48)

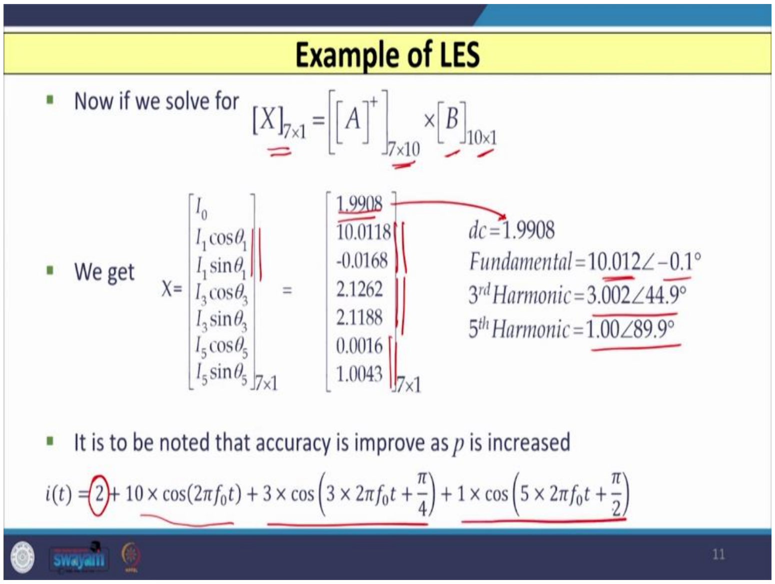


So, here if you calculate the value of matrix A, you will have the value of matrix A like this

$$\begin{bmatrix} 1.00 & 1.00 & 0.00 & 1.00 & 0.00 & 1.00 & 0.00 \\ 1.00 & 0.92 & -0.38 & 0.38 & -0.92 & -0.38 & -0.92 \\ 1.00 & 0.71 & -0.71 & -0.71 & -0.71 & -0.71 & 0.71 \\ 1.00 & 0.38 & -0.92 & -0.92 & 0.38 & 0.92 & 0.38 \\ 1.00 & 0.00 & -1.00 & 0.00 & 1.00 & 0.00 & -1.00 \\ 1.00 & -0.38 & -0.92 & 0.92 & 0.38 & -0.92 & 0.38 \\ 1.00 & -0.71 & -0.71 & 0.71 & -0.71 & 0.71 & 0.71 \\ 1.00 & -0.92 & -0.38 & -0.38 & -0.92 & 0.38 & -0.92 \\ 1.00 & -1.00 & 0.00 & -1.00 & 0.00 & -1.00 & 0.00 \\ 1.00 & -0.92 & 0.38 & -0.38 & 0.92 & 0.38 & 0.92 \end{bmatrix}_{10\times 7}$$

by putting the ω_0 and T_s the matrix B you have that is sampled values which starts from 14.1 to tenth value that is minus 5.2. So, you have this value $\begin{bmatrix} 14.1 \\ 9.2 \\ 6.8 \\ 5.1 \\ 3.1 \\ 1.3 \\ -4.4 \\ -10.9 \\ -10.1 \\ -5.2 \end{bmatrix}_{10\times 1}$

(Refer Time Slide: 11:20)



And now, if you calculate the unknown matrix X, $[X]_{7\times1} = [[A]^{+}]_{7\times10} \times [B]_{10\times1}$

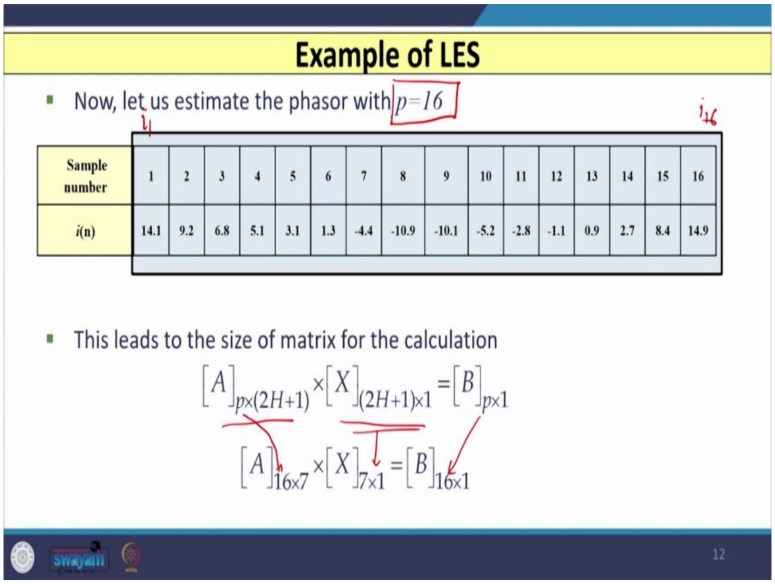
then you can have the value of X which is pseudo inverse of A that is a plus 7 cross 10 and B matrix that is 10 cross 1 or P×1. So, you have the value of X that is

$$X = \begin{bmatrix} I_0 \\ I_1\cos\theta_1 \\ I_1\sin\theta_1 \\ I_3\cos\theta_3 \\ I_3\sin\theta_3 \\ I_5\cos\theta_5 \\ I_5\sin\theta_5 \end{bmatrix}_{7\times1} = \begin{bmatrix} 1.9908 \\ 10.0118 \\ -0.0168 \\ 2.1262 \\ 2.1188 \\ 0.0016 \\ 1.0043 \end{bmatrix}_{7\times1}$$

So, whatever value you obtain, you can see that the dc value that is 1.99 which is again very close to the original actual value 2.

You have the fundamental component if you take magnitude of this and angle, you will have $10.012\angle -0.1°$. So, that is also very close to actual value and the third harmonic term, you can see it is $3.002\angle 44.9°$ which is also close to the actual value and again you can take the fifth harmonic value of which that is $1.00\angle 89.9°$that is also very close to actual value.

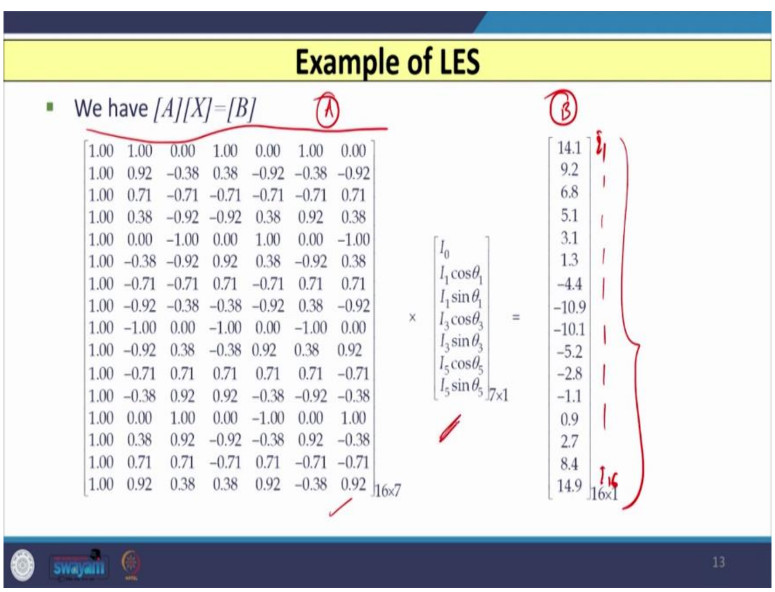
(Refer Time Slide: 12:24)



Now, let us further increase the value of p. So, now, let us consider the value of p =16. So, now, we are taking all 16 samples in one cycle from i1 toi16. So, if I consider 16 samples, then the size of this A matrix that is $[A]_{p\times(2H+1)} \times [X]_{(2H+1)\times 1} = [B]_{p\times 1}$.

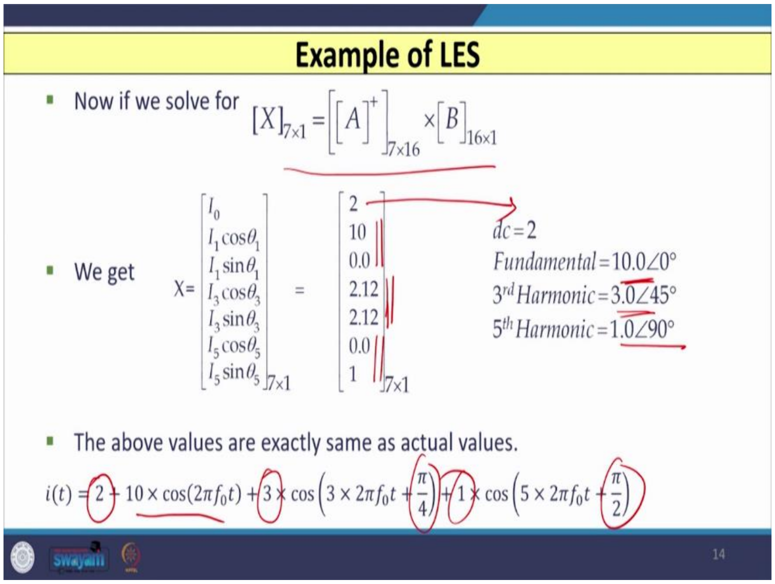
$[A]_{16\times 7} \times [X]_{7\times 1} = [B]_{16\times 1}$. So, now, let us see what the value of unknown matrix X is

(Refer Time Slide: 13:05)



So, here when you calculate the matrix A by putting the value of ω_0 and T_s, then you will have this value of A you will have the B matrix that is your sample values from i1 to i16. And then you have the unknown matrix X.

(Refer Time Slide: 13:29)

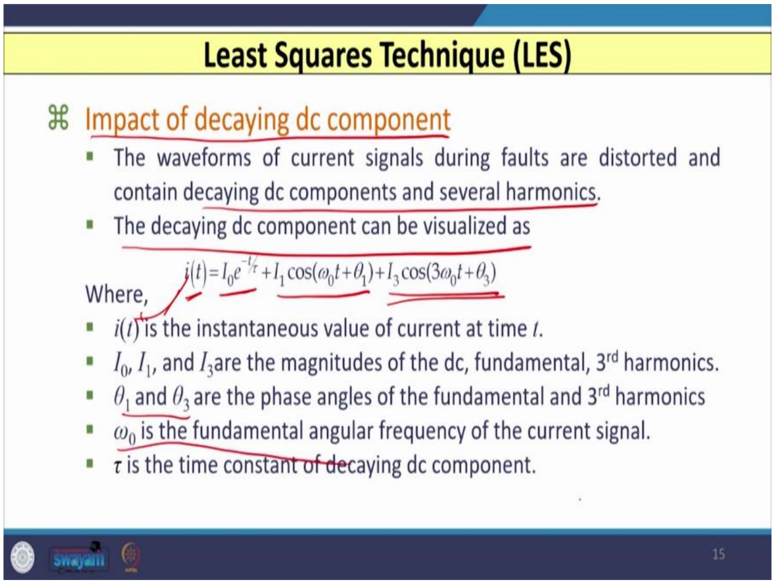


So, you have finally the unknown matrix X which is X= $\begin{bmatrix} I_0 \\ I_1\cos\theta_1 \\ I_1\sin\theta_1 \\ I_3\cos\theta_3 \\ I_3\sin\theta_3 \\ I_5\cos\theta_5 \\ I_5\sin\theta_5 \end{bmatrix}_{7\times1} = \begin{bmatrix} 2 \\ 10 \\ 0.0 \\ 2.12 \\ 2.12 \\ 0.0 \\ 1 \end{bmatrix}_{7\times1}$

So, now, you can see the dc value comes out to be 2 which is exactly same as the actual value the fundamental component that is this value, which is $10.0\angle0°$and this is also same as actual value. Similarly, if you consider third harmonic term, then the if you take magnitude and angle you will have $3.0\angle45°$ which is same as actual value and same way if you consider fifth harmonic magnitude and angle $1.0\angle90°$, you will have the same value.

So, as we increase the number of samples in a particular window, then the accuracy of the LES in terms of estimation of phasor values of unknowns that also increases, but as we increase the number of samples the computational requirement also increases. So, we have to optimize at one particular point that out to how much accuracy we want by increasing the number of samples.

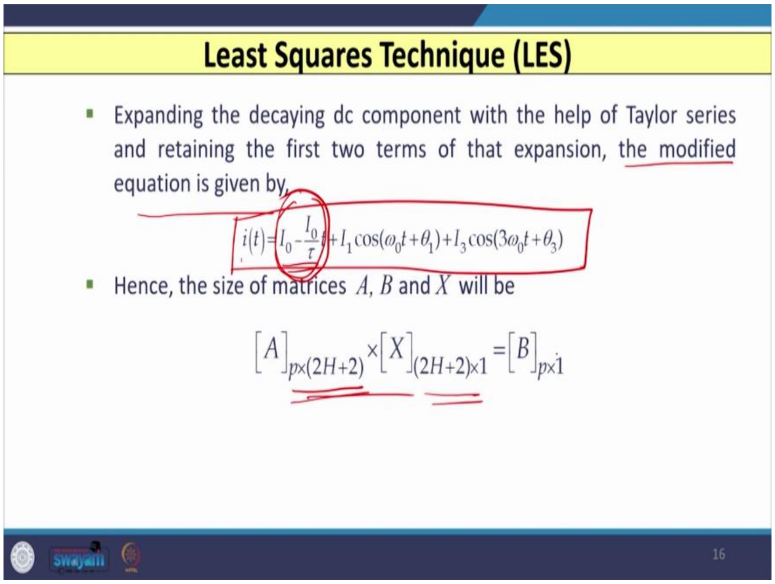
(Refer Time Slide: 14:39)



Now, to understand the impact of decaying dc component on LES algorithm. Let us consider the signal current signal. When we acquired the current signal in case of fault then we know that the waveform of this current signal is distorted and that signal contains decaying dc component along with several harmonics and fundamental term. So, decaying dc component that is given by let us say the equation $i(t) = I_0 e^{\frac{-t}{\tau}} + I_1 \cos(\omega_0 t + \theta_1) + I_3 \cos(3\omega_0 t + \theta_3)$

So, your i(t) that is nothing but the instantaneous value of current which changes with reference to time I_0, I_1, and I_3 are the magnitudes of dc fundamental and third harmonics and θ_1 and θ_3 are the phase angle or fundamental and third harmonics respectively. ω_0 is the fundamental angular frequency of the current signal.

(Refer Time Slide: 15:52)

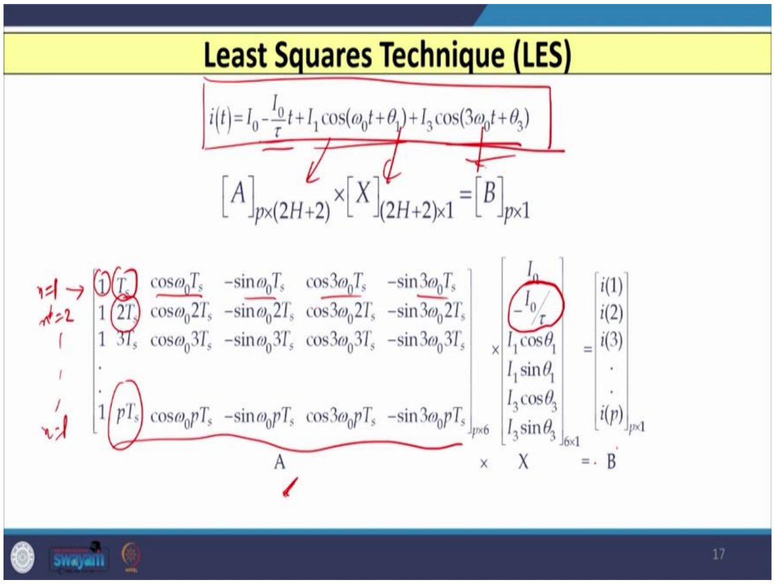


Now, if we expand this term using the Taylor series and if we retain the first two terms and neglecting the higher order terms, then we have the modified equation of this i(t) that is given by this equation $i(t) = I_0 - \frac{I_0}{\tau} t + I_1 \cos(\omega_0 t + \theta_1) + I_3 \cos(3\omega_0 t + \theta_3)$

So, here you can see we have the two terms of dc that is one is I_0 and another is minus I_0 by tau into t. So, now, the size of matrix A, B and X that becomes like this.

So, size of [A] that becomes p ×2H +2, because we have the two terms here for dc value. The size of [X] that also now 2H +2 × 1 this 2 are again unknowns instead of earlier 1 dc value unknown, now, 2 dc values are there and the size of [B] that is nothing but the p × 1 that is as it is.

(Refer Time Slide: 16:56)



So, if I consider the same signal of current which contains the dc value fundamental and third harmonic term, then the size of which is given by this $[A]_{p\times(2H+2)} \times [X]_{(2H+2)\times 1} = [B]_{p\times 1}$

and if you obtain the A matrix,

$$\begin{bmatrix} 1 & T_s & \cos\omega_0 T_s & -\sin\omega_0 T_s & \cos 3\,\omega_0 T_s & -\sin 3\,\omega_0 T_s \\ 1 & 2T_s & \cos\omega_0\, 2T_s & -\sin\omega_0\, 2T_s & \cos 3\,\omega_0 2T_s & -\sin 3\,\omega_0 2T_s \\ 1 & 3T_s & \cos\omega_0\, 3T_s & -\sin\omega_0\, 3T_s & \cos 3\,\omega_0 3T_s & -\sin 3\,\omega_0 3T_s \\ . & & & & & \\ . & & & & & \\ 1 & pT_s & \cos\omega_0\, pT_s & -\sin\omega_0\, pT_s & \cos 3\,\omega_0 pT_s & -\sin 3\,\omega_0 pT_s \end{bmatrix}_{p\times 6}$$

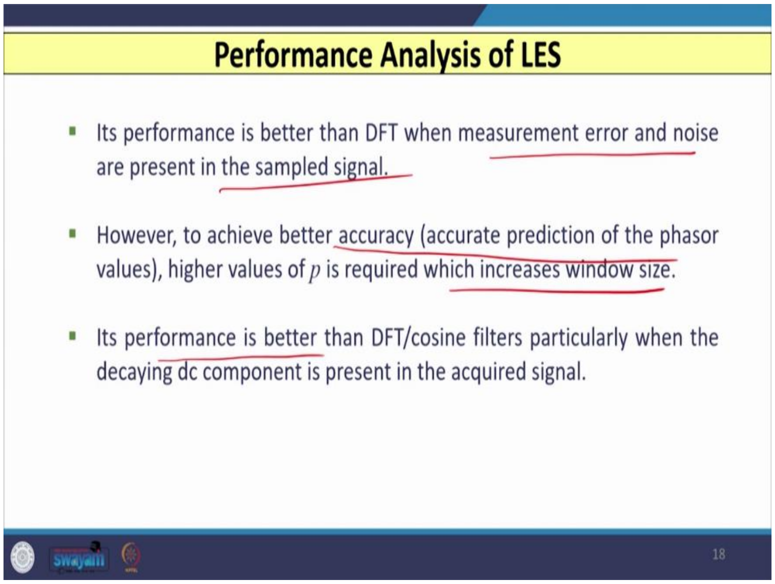
then you can see that here the first row when you start with sample number 1 n is equal to 1, but this term is again newly added that is T_s.

Same way, when you increase the number of samples n is equal to 2, then this term is added and as you go up to p, then this term that is added compared to the earlier one. So, here moreover in

matrix X unknown $\times \begin{bmatrix} I_0 \\ -\frac{I_0}{\tau} \\ I_1\cos\theta_1 \\ I_1\sin\theta_1 \\ I_3\cos\theta_3 \\ I_3\sin\theta_3 \end{bmatrix}_{6\times 1}$ this term is also now available.

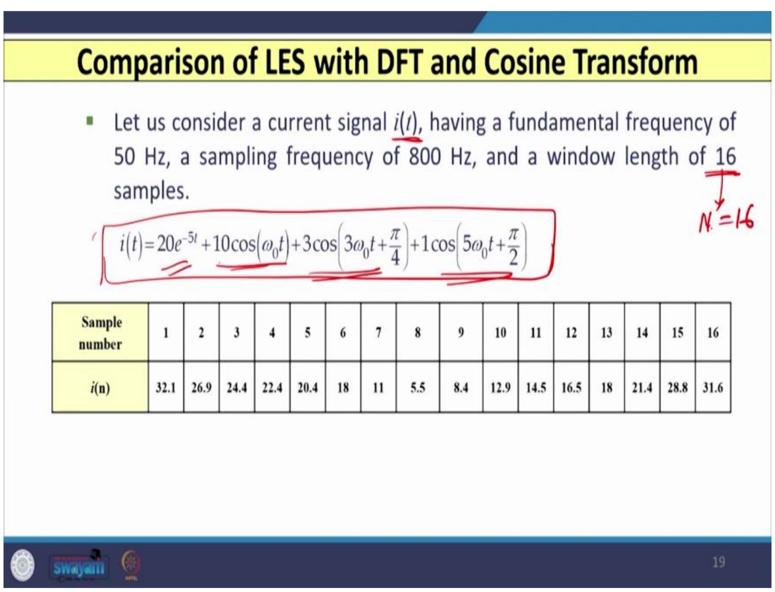
So, the number of unknowns that is more than the earlier one and the size of matrix A also changes whereas, the size of matrix B remains as it is.

(Refer Time Slide: 18:06)



So, if we carry out performance analysis of the LES, then the performance of LES is better than the DFT when we consider the errors and noise in the acquired signal. However, to achieve the better accuracy, obviously, as I told you, we have to increase the value of p and that increases the window size. However, the performance of LES is better than the DFT and cosine filters and recursive DFT particularly when the decaying dc component is present in the acquired signal.

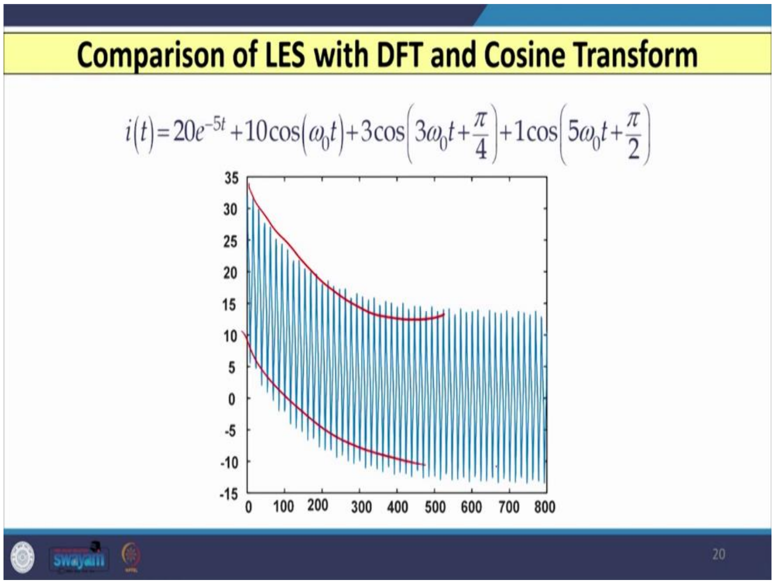
(Refer Time Slide: 18:41)



Now, to understand or to compare the performance of this LES technique with DFT and cosine transform, let us consider a current signal i(t) which is given by this equation $i(t) = 20e^{-5t} + 10\cos(\omega_0 t) + 3\cos\left(3\omega_0 t + \frac{\pi}{4}\right) + 1\cos\left(5\omega_0 t + \frac{\pi}{2}\right)$.

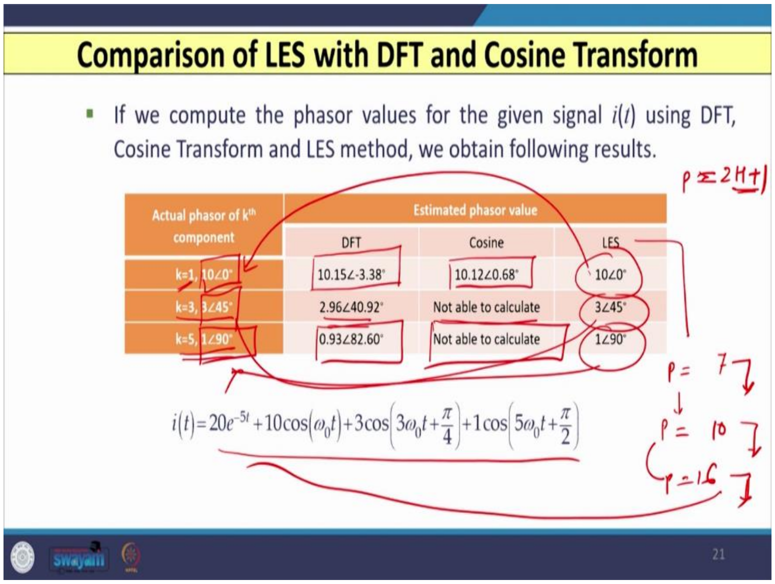
So, this equation contains dc term, it contains the fundamental component, it contains the third harmonic as well as it contains the fifth harmonic and this current signal is acquired with a sampling frequency of 800 Hz having fundamental frequency 50 Hz. So, number of samples in a cycle that is n that is equal to 16 and that 16 samples are shown here.

(Refer Time Slide: 19:23)



Now, if I plot this signal then this signal looks like this. So, you can see that there is a decaying dc term along with some fundamental term and some other harmonics.

(Refer Time Slide: 19:41)



So, if we obtain or calculate the phasor value of this equation using LES, DFT algorithm and cosine algorithm, then let us say we want to compare with the fundamental phasor. So, let us consider the value of k=1. So, in that case the actual value of phasor that is 10∠0°. And if I use the DFT algorithm the phaser value obtained that is 10.15 ∠ − 3.38° whereas, the cosine term we have the value that is 10.12∠0.68°.

But if I use the LES technique, we will have the exactly same value that is, we have in case of actual one. Same way, if I calculate the phasor value for third harmonic putting k=3, then we have the value of this phasor for third harmonic that is 3 ∠45, this is the actual value and if you compare if you obtain this value using DFT, then you have the value that is 2.96 ∠40.92°.

However, as we have discussed in case of cosine transform that cosine transform is applicable only for fundamental component, if we want to apply for third and higher order harmonics, then it is not able to calculate the phasor value. So, that is why it is unable to calculate this value. Whereas, if I use the LES algorithm, then the value obtained for third harmonic you can see that is exactly same as 3 ∠ 45° that is the actual values.

And similarly, if I use or if I calculate the phasor value of fifth harmonic term by putting k=5 then we have the value that is 1 ∠90° degree and if I use DFT then you will have the value that is 0.93 ∠82.6° degree whereas, as the usual cosine algorithm is not able to calculate and LES algorithm you will have the value 1∠90° again which is exactly same as the actual value.

So, you can definitely say that whenever we wish to calculate the phasor value for fundamental or third or any other harmonics including dc term, then LES algorithm performs better compared to the DFT algorithm and cosine transform algorithm. The only thing is that the number of samples required by this LES that is higher than the unknown values. So, as you increase more samples, you will have the better result which is actually very close to the actual values.

So, in this lecture, we have discussed initially one example of the LES algorithm and that example we have discussed by considering three different value of p. So, initially we have considered the value of p let us say that is seven. So, the number of unknowns that is same as the 2H+1. So, whatever samples you are acquiring, that is 7 and the number of unknowns that is also 7 and then we have seen that we do not get the required accuracy means whatever phasor value we obtain those values are not close to the actual values, then we have increased the value of p and let us say we have considered 10

And then we see that the accuracy of the phasor values that improved compared to the p equal to 7. And finally, we consider p equal to 16. And in that case, we have observed that phasor values of the all the terms which we obtain that is almost equal to the original actual value. After that we have discussed when we consider the impact of decaying dc component then how the LES is performed compared to the other algorithms like DFT and cosine transform. And we observed that LES performs better when decaying dc component is present in the acquired signal compared to the DFT and cosine transform. So, thank you.