Introduction to Atmospheric Science
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Lecture-11
The hydrostatic equation

In the last class we are looking at the concept of virtual temperature which is very important because it lets us use the gas constant of dry air and we can work out problems with ease okay. So, it is an important construct or a concept in atmospheric thermodynamics.

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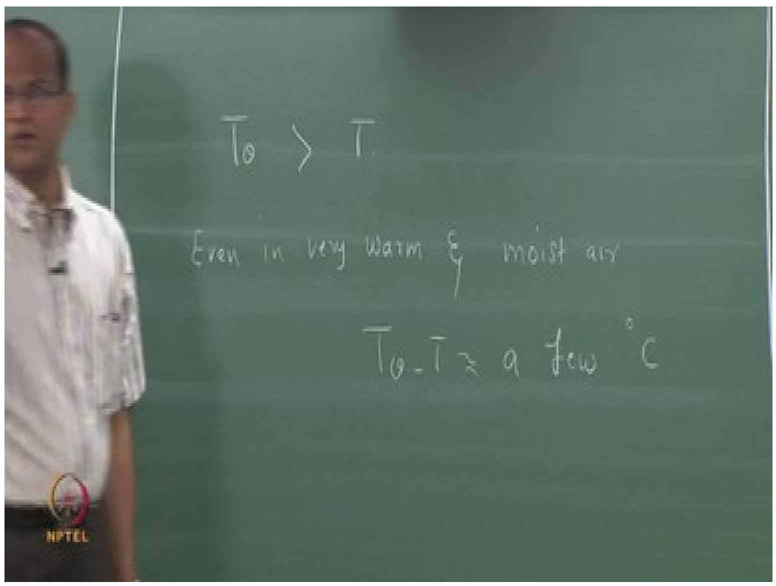

We will try to work out some problems involving this definition so for the sake of completeness let us quickly go through this is virtual temperature given by Tv is nothing but the ratio of the actual temperature divided by 1 - Pv dash by P where epsilon is Rd okay, so it goes without saying that since the denominator is less than 1, Tv is always greater than T even in very warm and moist air is only a few degrees centigrade all right.

Towards the end of the last class we just saw a very formal definition of virtual temperature. We define the virtual temperature the temperature which the gas must be or atmospheric air must be raised in order that it has the same density as that of moist air correct but at the same say at the same pressure P all right. So, because it is via the gas constant is more for the moist air it is less dense and so on. Therefore it has to be the dry air has to be heated to a high temperature so that it matches with this all right.

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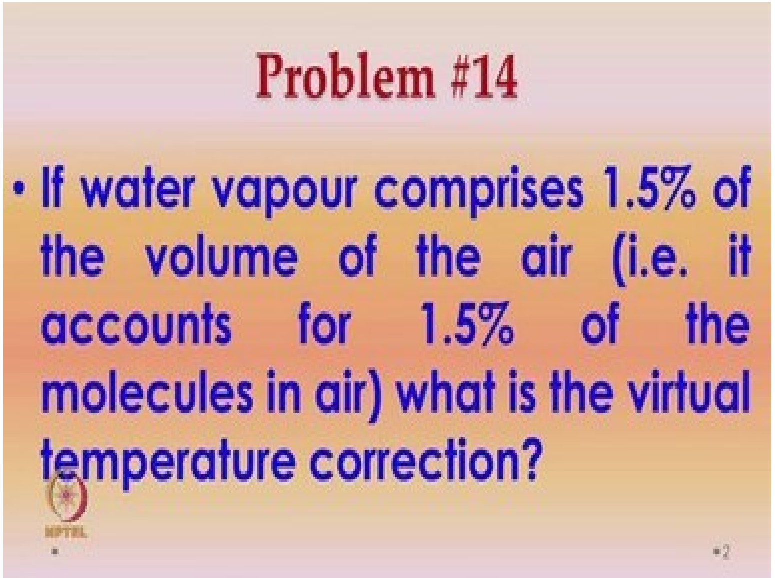

Let us solve some problems in today's class problem number 14 some students will be entering it is okay so problem number 14 if water vapor comprises please take down the problem if water vapor comprises 1.5 % of the volume of the air that is it accounts for 1.5 % of the molecules in air, if water comprises 1.5 % of the volume of the earth that is it accounts for 1.5 % of the molecules in air what is the virtual temperature correction.

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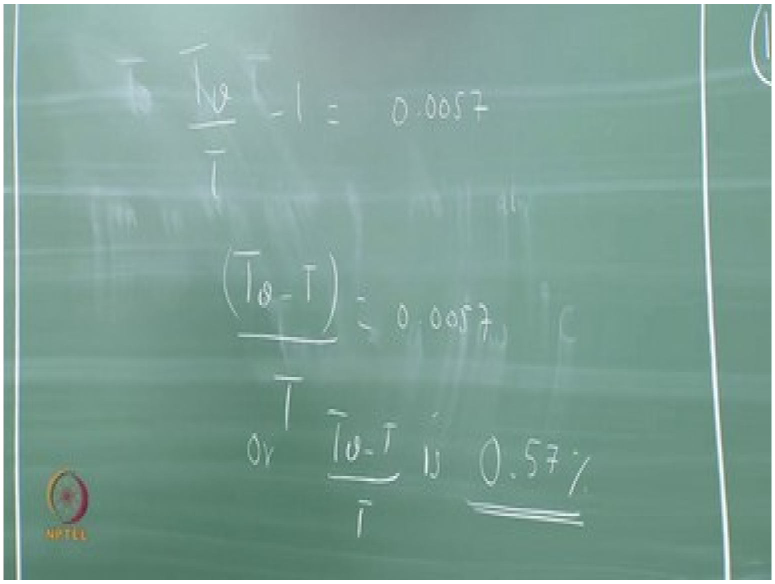

We start solving it, so when you say the water vapor compression is 1.5 % of the volume in mind as well say that Pv dash by P okay the ratio of the partial pressure of water bay whatever I put it the total is 1.5 % you make it into remove the % it is .15 okay. Now Tv, what is this value 1 point, all right is half a % rate it is not much it is alright okay. But if you make this correction you will get better estimates when you work out problems put it through with this, Marius is it okay. I think you should bring calculators to all the classes, shall I proceed to the next one, done okay.

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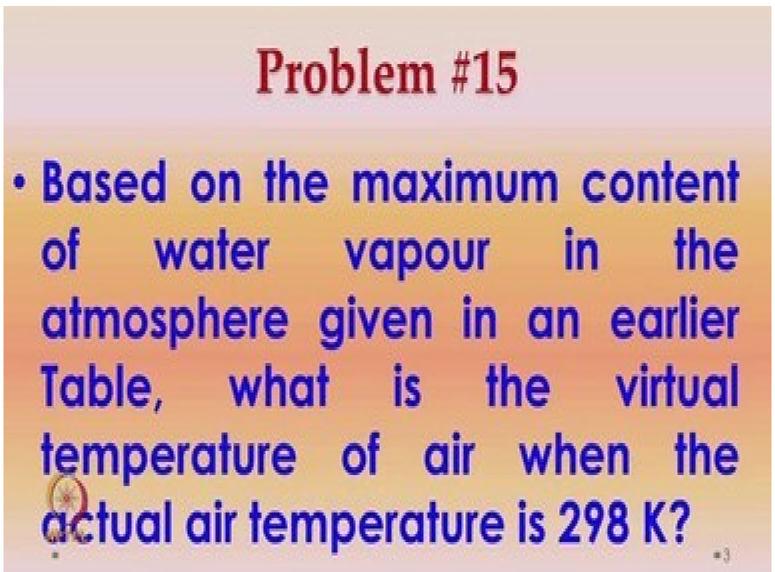

Problem 15 based on the maximum content the water vapor in the atmosphere given in an earlier table what is the virtual temperature of air based on the maximum content of whatever content on the atmosphere given in an earlier table what is the virtual temperature of air when the actual air temperature is 298 Kelvin okay.

Based on the maximum content of water vapor in the atmosphere given in an earlier table what is the virtual temperature of air when the actual temperature is 298 Kelvin okay? It is actually the aptly the application the same formula which we used for problem number 14 except that in this case I am asking you to take the data from a previous table and in this case instead of giving it as a %, I actually gave you the air temperature.

So, you can actually give the virtual temperature in Kelvin so that you know what is the difference between the actual temperature and the Tv the virtual temperature please start solving this. First you should have the data on the previous day;

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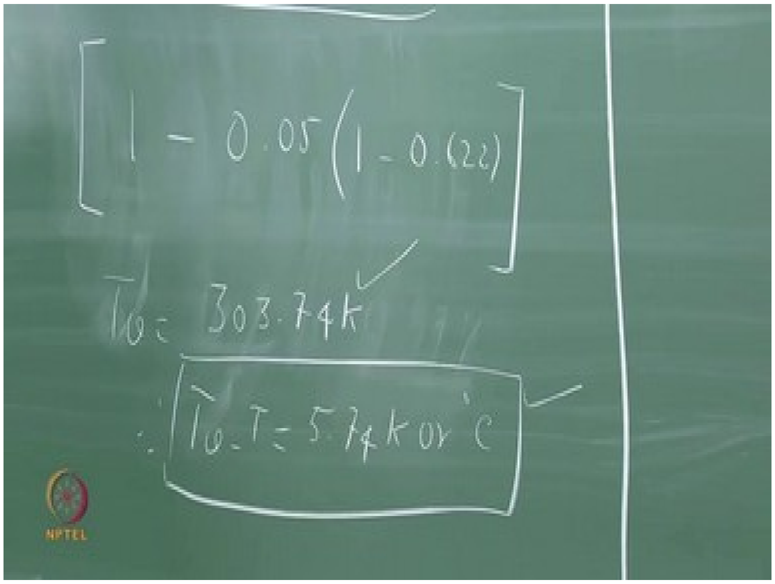

I hope you are not in the wrong class, so by the way the 4:45 class will be in studio 2 something is going on here today we are meeting again at 4:45 done there. A virtual temperature okay, what is the maximum content of water vapor okay very good so maximum, so this is from an earlier table 5% okay. Therefore fine very good, how much is this .07, 5., so the worst case scenario it does not exceed 6 Kelvin that is not okay. So, I made a statement in an earlier class in fact at the beginning of today's class.

I made the same statement that the Tv - T is of order of a few degree centigrade only at the max at the worst it is not at the order of 10’s of degree centigrade or of the order of 100’s of degree centigrade. It is a few degree centigrade for 1.5% it was something like half a %, so if you keep the same 298 that may translate 1 and a half or 2 Kelvin, so the maximum is 5 Kelvin. But 5 Kelvin is not small enough to be ignored you have to factor in the calculation.

But if it is half or 1 Kelvin up to 1 and half to 2 unit 6 Kelvin we make some changes in your calculation is that okay all right. Now we were to get deeper into atmospheric thermodynamics is everybody through with this okay. So, the answer is this so the virtual temperature is three note 3.74 the temperature correction is 5.7 degree centigrade.

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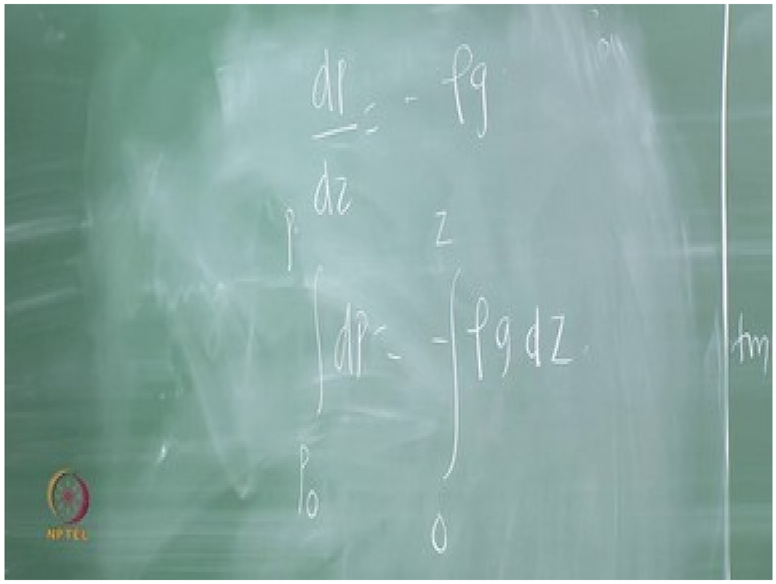

So, now we have to start in the hydrostatic equation, consider reasonably stationary atmosphere okay. Consider this a slice consider the horizontal slice the atmosphere okay. The slice is having a height of dz thickness sorry thickness dz okay. This is the ground okay unit horizontal cross sectional area. Let us do a force balance on this okay. What is the pressure will the pressure is always acting towards on the surface by convention so the pressure is P.

And the pressure acting from the top is P + dp, so if you do a force balance what do you get? Yes that one is a cross sectional area fine. So, because I am g Rho z that you are taking it as positive in the downward direction so please mark this right, the direction of the gravity vector is important it is always acting downwards right okay. So, now if you cancel this out what do you get equation 1, this is a hydrostatic equation dp by dz = if you consider as a function of several variables we can also change it to dou P by dou z if you want okay.

So, dp by dz this is a hydrostatic equation what is the relationship between Rho and specific volume V okay. So, if you know the variation of g and if you know the variation of Rho with respect to z, the right hand side can be integrated and the left hand side is pretty straightforward

P - P naught okay or you can start from; you can go all the way up to infinity when you go all the way up to the up to infinity what is P of infinity 0.

So, you can change the limits and do something you want to try that so this is one way of that is up to some intermediate level okay.

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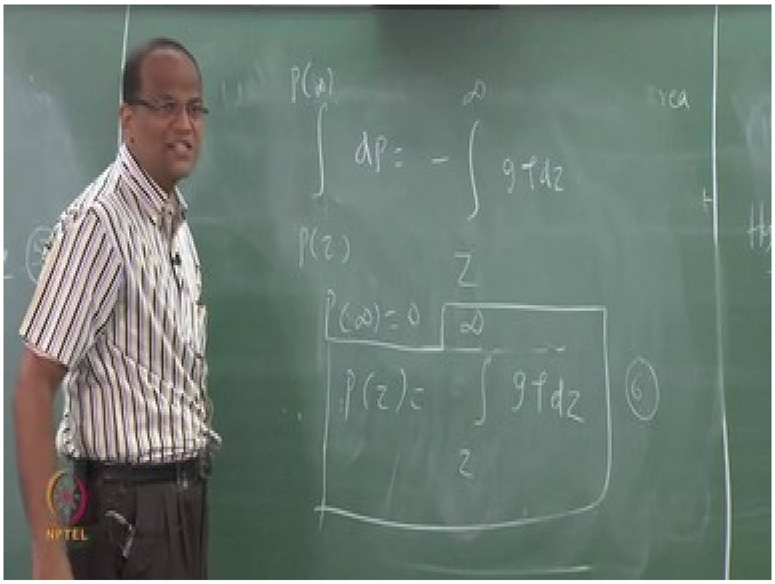

We want to integrate it all the way up to infinity, it can be integrated between any two limits right, so P infinity there is a – right, you can find the pressure at any height z it is basically the integral of 0 dz it from z to infinity. So, the pressure at a particular height is actually the weight of the water column sorry weight of the air column above it all right, no wait hang on so it is rate we cannot take g constant and sometimes you can take.

If the limits of the integration you do not otherwise g you have to find out g varies okay. We will work out problems for g varies and all that ok. What is the mean sea level pressure? What is that from equation 6, look at equation 6, so P of 0 will be 0 to infinity 0 dz when that is done and when that is averaged over the globe that P of 0 has a value = 1.013 into 10 to the power of 5 Newton per meter square which is frequently referred as 1 atmosphere, shall you do that ok.

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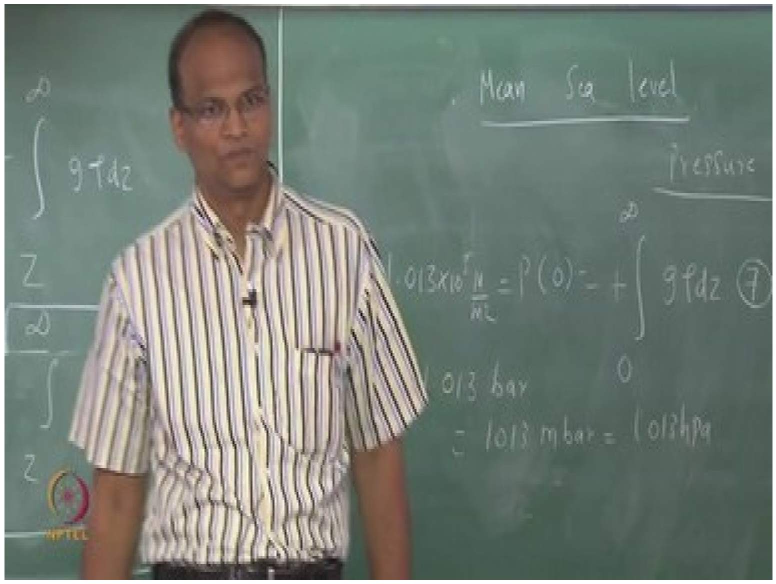

So, this is nothing but 1.01310 to the power of 5 Newton per meter square it is also 1.01 in 10 to the power of 5 Pascal it is called 1 atmosphere it is also 1.023 bar it is 130 millibar it is also = 113 hectopascal - is gone, why? Very good, so you have some torque some in childhood you learnt all that matter is we pack all the torque and all we do not use already there are enough units for pressure we struggle it some more okay.

Please take down this the P naught of the mean sea level pressure is based on the following okay. The mass of P naught or mean sea level pressure is based on the following. The mass of the Earth is uniformly distributed over the globe; sorry ah the mass of the Earth is the mass of the atmosphere right with the mass of the atmosphere is uniformly distributed over the globe.

And the Earth's present topography is retained okay. Based on this you are getting all right, this is the mean sea level pressure. So, the hydrostatic equation is very important in fact the hydrostatic equation you also learned in fluid mechanics right okay where instead of air you would have looked at water mechanical and civil engineering people would have studied in aero engineering. I think right now let us go to some further definition some of the definitions so you look at what is called geo potential.

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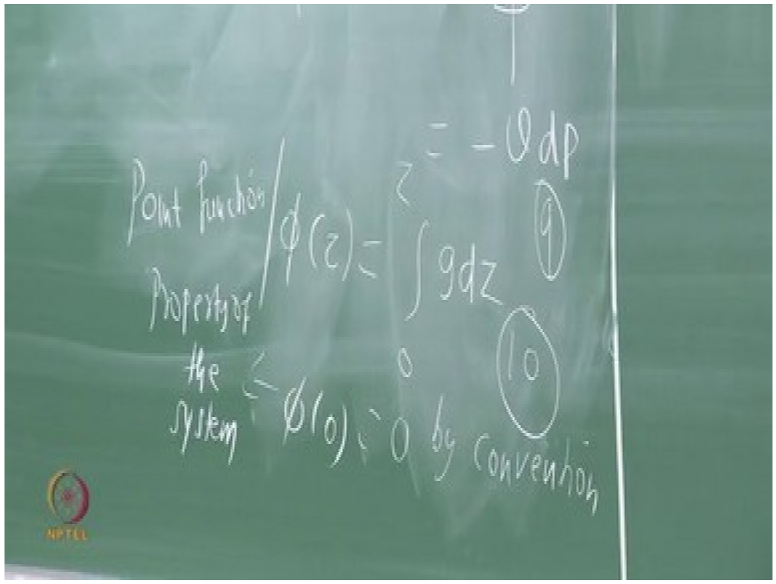

The geo potential is given by Phi, d Phi = g dz but what is the definition of geo potential itself, please take down. The geo potential Phi at any point in the Earth's atmosphere is defined as the work that must be done the work that must be done against the gravitational field the work that okay. The geo potential Phi or fee is defined in the Earth's atmosphere is defined as a amount of work that must be done against the Earth's gravitational field to raise a body of mass 1 kg.

To raise a body of mass 1 kg from sea level to that point okay. What is the potential energy if you raise it by some dz at what is the potential energy change a body of mass m mg dz. What is the potential energy change for a body of mass 1 kg? What did I write? Fine that is correct right we cannot leave it there so we will torture it by doing some integrations and limits and all that right.

So, we let it do that so d Phi is some number some numbers please 8, so this is 1, that 1 is there, so d Phi is = what is g Rho dz, dp therefore gdz is dp by so - is there or it is okay what is 1 by Rho, so you feel the checker but that is a unavoidable we keep wondering one way around but it is all same we did not sort, very difficult concepts but just to reinforce I am just putting it in various forms okay, 9 therefore Phi of z.

What is Phi of z? There is a slight of hand here, what did I do? Phi of 0 there is actually Phi of z - Phi of 0 but Phi of 0 = 0 that is simply by convention that is the datum okay. Anyway you to raise it by 0 to 0 there is no work done that answer is somebody's question is g, if you know the variation of g just go ahead and you can calculate the Phi of z okay.

Now it can be verified that Phi of z is a point function and it is a property it is independent of the path. You can raise something from 0 to 45 meters reduce it to 25 meters bring it to 84 meters and bring you to finally leave it at 65 meters and its final Phi will be the same as well if you take it from 0 to 65 meters the path does not matter at all. It is not like work or heat okay it depends on where it is and the z at which you are looking at all right. So, this is a point function in thermodynamics we call it a it is a property of the system.

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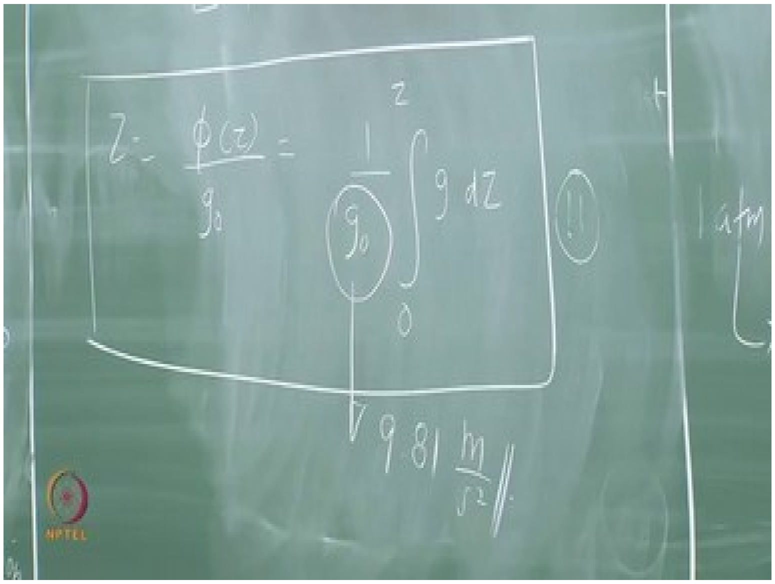

Next is a concept called geo potential height which is very important Phi of z by g naught this is nothing but what is your potential I said please take down the geo potential height z is the height to which 1kg of mass must be raised if g = 9.81 meters per second square instead of a decreasing g correct okay. So, what is this g naught, correct but it is not complete, g naught is a globally average acceleration due to gravity at the Earth's surface which has been assigned a value of 9.81 meters per second square okay.

Please take down g naught is the globally averaged acceleration due to gravity at the Earth's surface and the value of 9.81 meters per second squared is the agreed upon value for g naught. The geo potential height z is a frequently used vertical coordinate in atmospheric science wherever energy interactions are taking place okay. So, the geo potential height z will be frequently seen in plots in the area but not in the field of atmospheric science.

Now I will give you a small table I will give you the value of small z capital Z and g for various heights you will realize that this 9.81 will slowly increase or decrease with height? It will decrease with okay. I do not have to tell you why.

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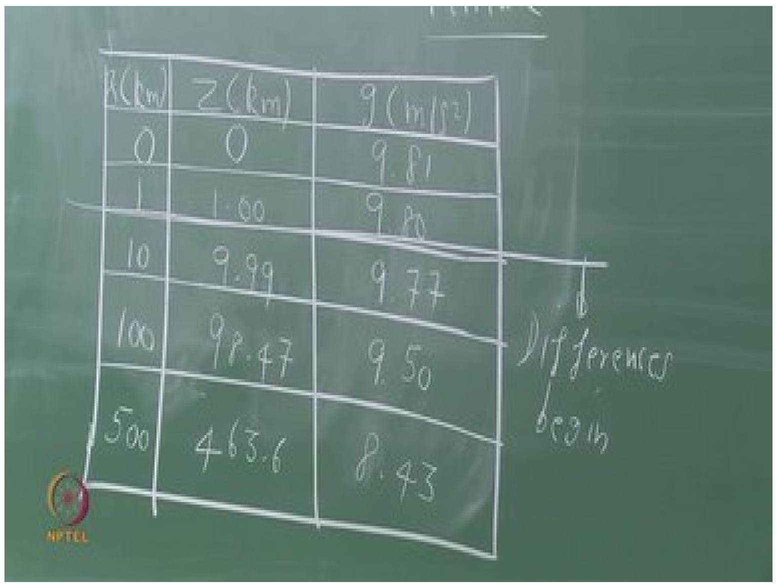

So, some representative place the measurements have been done and some calculations okay, z in kilometer, z is also in kilometer g is in meters per second square 0, 0, 9.81, 1, 10, 100 and 500 kilometers 1.00, 9.99, 98.47 you can see it starts varying first two entries are already up to 10 kilometers then it starts changing. So, 9.80, 9.77, 9.50, 8.43 okay, so height of 500 kilometers the g can reduce reduces to value 8.4 meters per second squared.

So, if you are if you are doing calculations involving 100's of kilometers of atmosphere a value of 9.81 is not correct if your attention is restricted to lead to the troposphere then it does not make much difference right. It is still 9.8, 29.8 you can use that. So, what is your consideration noticing you look in jet weather events and first was 10 kilometers even just take you can pack all this. Let us take it as g naught all right.

Now z2 - z1 is something which we have to z2 - z1 is something which we have to obtain now. So, we will have to do some derivation we have to go through some mathematical steps. Let us do that now is it clear up to this stage, you can see that so differences begin after some height.

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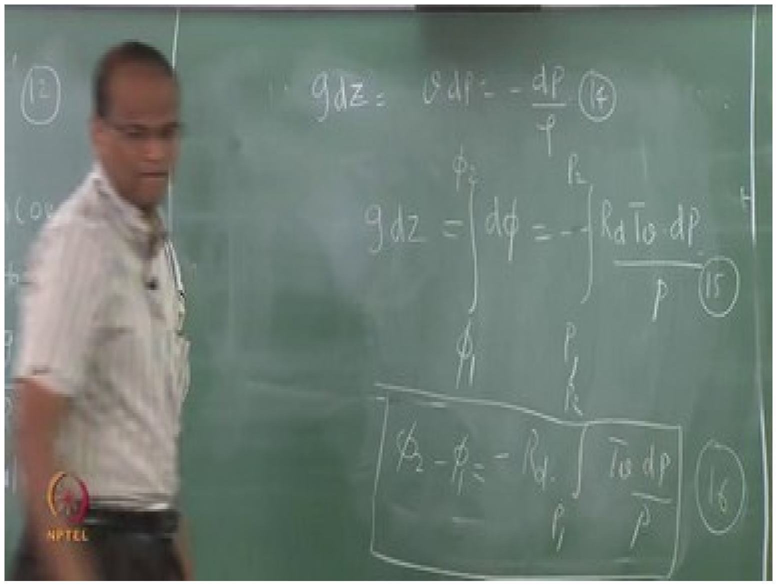

So, let us work with dou P by dou Z Rho inconvenient to use please give some equation number to the okay. And Rho is hardly measured, so the challenge in advanced exchange is to get rid of the Rho, it is so fond of Rho in chemistry. But where to get rid of the Rho in atmospherics okay so please help me how will you remove the Rho ideal gas equation okay. We will remove the Rho through the ideal gas equation P by which R and which TR and T is very good.

So, just now the last class we have seen the concept of virtual temperature so it should be P is the same but R is a basic because moist air there can be Rd is different from RT. So, if you both T and v are there dry air is there is moist air then whatever is present then we do not know which one to use. So, that is why we discussed we articulated the concept of the virtual temperature and so we use Rd and Tv is it okay.

Now please start working on this I want to get Z2 - Z1 okay what is gdz said is also = okay, so equation 13 okay I call this equation 14 all right. So, sorry no wait I am thinking about something else so now dt by dz is 0 okay, just simplify this what will be the next step. No gdz =

d Phi okay, so this is Rd - or you can read dp by P that okay that is fine 15, no, is it ok. So, Phi 2 - Phi 1 - is the – there? Yes.

I will answer your question just hang on ok we will complete this Rd what will happen to the Tv there is some g, no fine Rd, he wanted to keep the Tv inside we will keep it no problem. See Tv is having the Pv - and P, so it is dangerous to; is that answers your question right. But that PV dash by P is a small quantity. So, it you have to take a call and whether you have to retain it or not okay. So, this is correct did I miss this - I have miss out okay alright.

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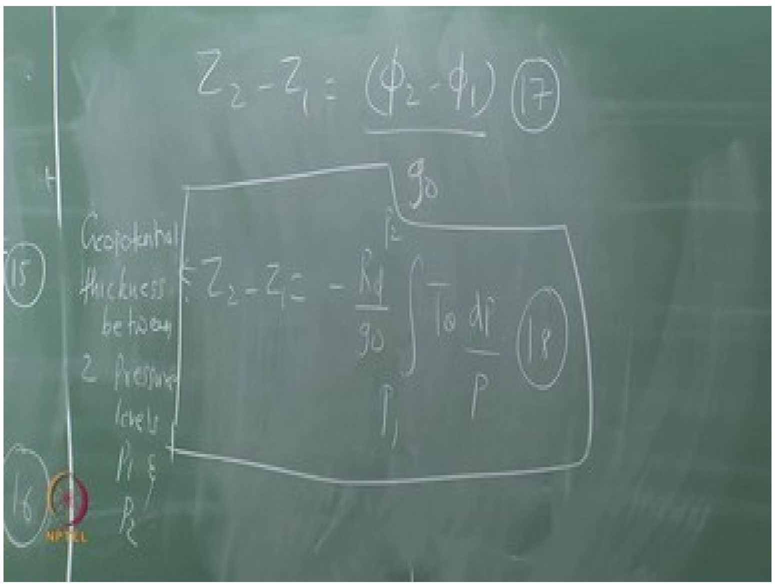

Then Z2 - Z1 that is a goal right, what is that I am integrating Li let us say the exit very complicating things it is fine what is the mistake in this you tell me. You want to find out that Z that is why whatever you; you are putting all the other things on the right hand side, there is no problem ok. So, Z2 - Z1 okay there is an important relation. So, Z2 - Z1 is the geo potential thickness between two levels P1 and P2.

There are two pressure levels P1 and P2 in the atmosphere Z2 - Z1 is the geo potential thickness between two pressure levels. For an isothermal atmosphere if you consider that the Tv and T are sufficiently close to each other that is it is not 5% water vapor edges 1 or 2 % whatever then it is possible for you to pull out the Tv out or out of the integral in which case the Tv can be replaced by T then you will have Z2 - Z1 = Rd into T by g naught into dp by this thing okay.

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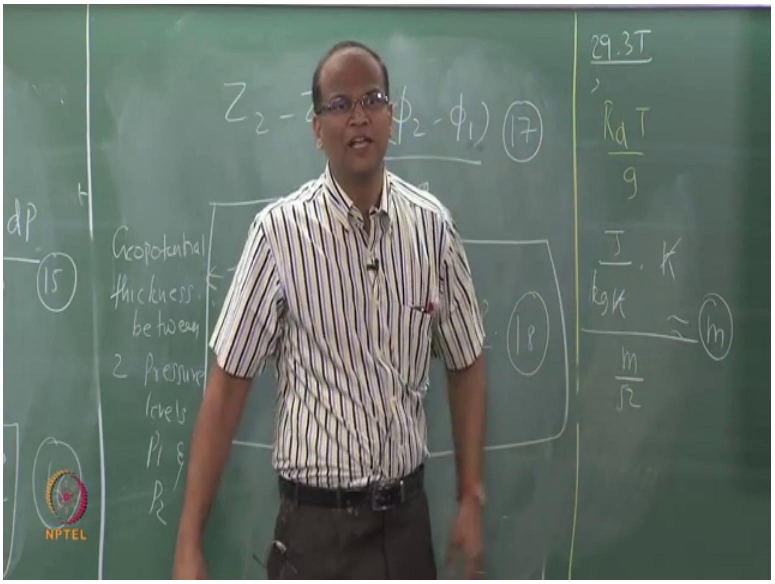

That R into T by g that R into T by g is very interesting I will just explain that concept and stop will continue in the afternoon. So, if you make it as if it is an isothermal atmosphere the temperature is not changing in, we can take this out of the integral and then I say that Tv and T are very close to each other then I make it Rd T by g. What will be the units of Rd, Joules this is, what is kilogram meter per second square? Joules is it correct.

Is it non dimensional okay, so this is kilogram is Newton right, so you are getting okay, it is non dimensional. Now is it okay no just hang on kilogram meter per second square is what okay, let us do this I think I messed this up here, ah so it will have finally meter. That is actually hits the scale height of the atmosphere which we use a 7.5 delimitation one of the earlier problems.

So, this Rd T by g naught is very interesting 287 by 9.8 into T this will actually turn out to be we will do it in the next class 29.3T and between the stratosphere if you take the average of troposphere and stratosphere T will be 255 Kelvin. So, if you multiply 255 by 30 you will get something like 7,500 meters and 7.5 kilo meters is the scale height of the atmosphere. Then if you have the scale height of the atmosphere, you will get a very beautiful relationship that P by P naught is e to the power of - z by h.

That is the equation which we started out in the earlier classes but now we have formally derived it from first principles starting from the hydrostatic equation definition of geo potential the definition of geo potential thickness interaction of virtual temperature and making a correction for the virtual temperature and then introducing this concept of scale height.