Introduction to Atmospheric Science
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Lecture - 33
Physics of scattering, emission and absorption

Good morning, we will continue with our study of radiative transfer, radiative process in the atmosphere, so we looked at the definitions and the basic laws, okay, so, we defined what is intensity, we looked at solid angle, difference between plane angle and solid angle, the spherical coordinate system, then we looked at the radiation laws essentially the Planck's blackbody distribution.

How it is derivative and making it stationary, lead us to the Wien's displacement law, which states lambda max T:898 micrometer kelvin, okay, and then the integral of that gives you the Stefan Boltzmann law, which gives the blackbody emissive power and then using the concept of the solid angle sigma T to the power of 4 and lambda max T:898 micrometer kelvin. We solved gamut F problems ranging from what will be the colour temperature of the sun.

We figured out that if you take the maximum to be 0.475 micrometer the colour temperature turns out to be 6100 kelvin, but the sun’s blackbody distribution does not correspond to that of a blackbody therefore there will be some minor variation, but for all practical purposes unless otherwise stated in this course this temperature of the photosphere is 5800 kelvin, alright, then involving all these concepts of sigma T to the power of 4, solid angle and all that.

We solved some problem where we found the earth’s temperature from the sun’s temperature, sun’s temperature from the earth’s temperature and so on. Now it is time for us to go little deeper and what happens in an atmosphere which has got absorbing and emitting gases, what happens to the fate of radiation which goes through a gas which is participating, that means which will participate in the absorption, emission and scattering.

So this is quite different from air in this room which does not participate because radiation can go from one wall to the other wall, then this is called surface radiation, okay, because air is radiatively transparent, which will just let the radiation pass, but you have got water vapour in that and the carbon dioxide, methane and all this, these are all are strongly absorbing gases, can also scatter, that means scattering is reflection from a volume.

Reflection is generally from a surface, scattering is from a volume and this scattering need not take place equally in all the directions. If it is equal in all the directions it is called isotropic scattering. If it is not the same in all the directions it is anisotropic scattering, then if it is anisotropic and it is wavelength dependent, so if it is wavelength dependent and direction dependent and so on.

But you are considering 3 dimensional radiative transport in the atmosphere then it is a very, very complex, so very, very tough problem to solve, because unlike your CFD problem where you are solving for, you get the temperature and velocity, here you have to solve the radiative transfer for every wavelength and so it is going to be very, very difficult, so there are some programs we do what is called line by line calculations.

Line by line calculations mean I will solve the radiative transfer for 1 micrometer, 1.01 micrometer, 1.02 micrometer, why should I do that because in the last class I told you, the absorption spectrum is such a (()) (03:25) function, it is not same, it is not equal for all wavelengths, are you getting the point, so different parts of the spectrum carbon dioxide water vapour they play mischief.

So you will have to solve this and then if you are interested in finding out what is the total radiation coming out in the IR, total radiation in the shortwave and all that you integrate between the 2 limits lambda 1 and lambda 2, then you will do that integral lambda 1 to lambda 2, i lambda cos theta d omega, d lambda whatever, i lambda, cos theta, d lambda okay, so it gets messier and messier.

Therefore, some approximations have to be done if you have to treat radiation in the atmosphere. Usually we treat radiation to be 1 dimensional, what does it mean, the atmosphere is only a few 10s of kilometers, the radius of the earth is 6370 kilometer. The height of the atmosphere maximum we see that after 50-60 kilometers it becomes very ratified.

Assume it to be 80 kilometers, so 80/6300 is very less, therefore we will say that all the radiative processes, the variation of the intensity in the other 2 directions that is except the height, the other 2 coordinates they are not significant, this is a very crucial approximation we make in the atmosphere which is called the plane parallel approximation, we will look at this a little later, I am just giving you overview.

So with this plane parallel approximation, the equation of radiative transfer becomes a one dimensional equation, just because it is a one dimensional equation, it does not mean that it is easy, it is what I say it is deceptively innocuous because it has to be written for every lambda and you will have to work it out for every lambda, for every lambda you should know what is the absorption, scattering and absorptivity, emissivity and the scattering coefficient and so on, which will come from spectroscopy.

So the chemistry and physics they are involved, so you need a database to evaluate all this and for example if somebody wants to do planetary science in other atmospheres, some of you may be interested know you want to study what is happening in the atmosphere of Mars, Mercury or Venus or somebody wants to study that here then who will give the data for that? you need to know the composition of those gases in those atmospheres.

And then you need to get the properties and all that so that is the, we cannot just a free body diagram and solve and wow, upward force = downward force, it cannot be made so simple like that, okay, now we will have to go little deep in to the physics of scattering, absorption and emission processes, okay, which will logically lead us to the governing equation which is the RTE equation.

Radiative transfer equation, some people, see we should not say I will repeat again know, you say it again or you repeat? You got it, so I should not say RTE equation, I should say RT equation or RTE, you go the point, people who have not got the point are in delicious slumber.

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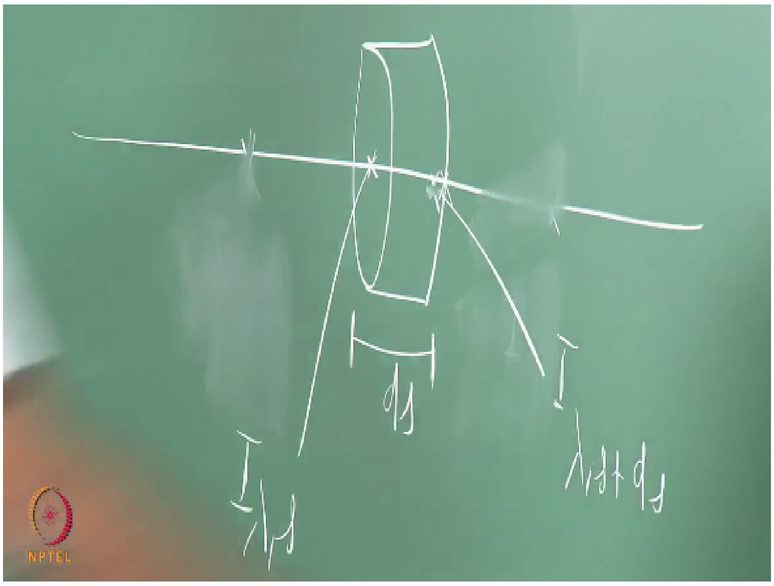

So today it will be a little bit dry, it will be theoretical, but there will be some silver lining is some part in today's class we will see why the sky is blue, so the answer lies in the study of scattering and all that, okay, some place we will try to answer that question by solving a problem, so consider a small control volume, 3-dimensional, so this ds, radiation is going through that, this is a small gas volume, we call it as elemental gas volume, okay.

The intensity here is I lambda at s, the intensity here is I lambda s. The I lambda the intensity which is coming out so this is the direction of the radiation, it is badly drawn, you draw a straight line properly. So whatever is coming out of the control volume is equal to whatever is entering plus, correct, this form simple Taylor series neglecting higher order terms, okay.

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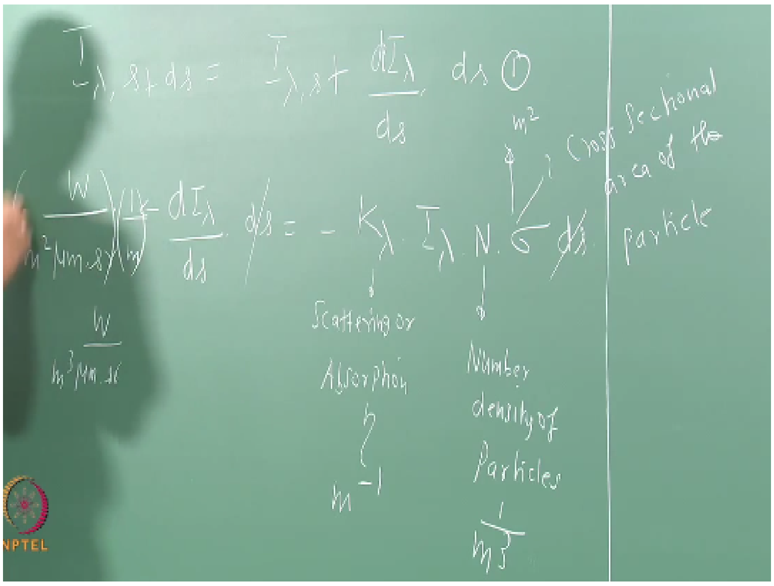

What do you think this d I lambda/ds * ds will be? d I lambda/ds is the rate of change of intensity with distance, ds is the local coordinate is s, okay, so this now you have to know that this is –kappa lambda, I lambda, N * sigma please note this carefully so the d I lambda/ds * ds is –I lambda, kappa lambda * ds right. What is this, scattering or absorption efficiency, this is the intensity, this is the number density, number density of particles in the atmosphere.

This is the cross sectional area of the particle, so let us cancel out the ds, this is got meter -1 so this is 1/meter cube, this is meter square, what are the units of I lambda, watts/meter square micrometer steradian * 1/meter. Let us come here 1/ meter, scattering or absorption let us figure out, I think I made a mistake, let us figure out, so watts per meter square micrometer steradian n is 1/meter cube sigma is meter square, correct.

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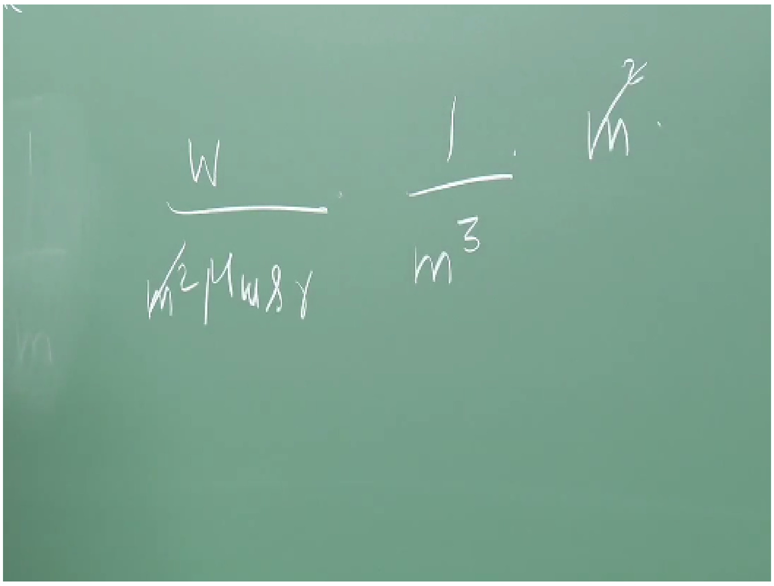

What do you get? watts per meter to the power of 4 micrometer steradian, it is not balancing, so this scattering efficiency should be dimensionless, correct, then it is watts per meter cube micrometer steradian, okay, now what is important is.

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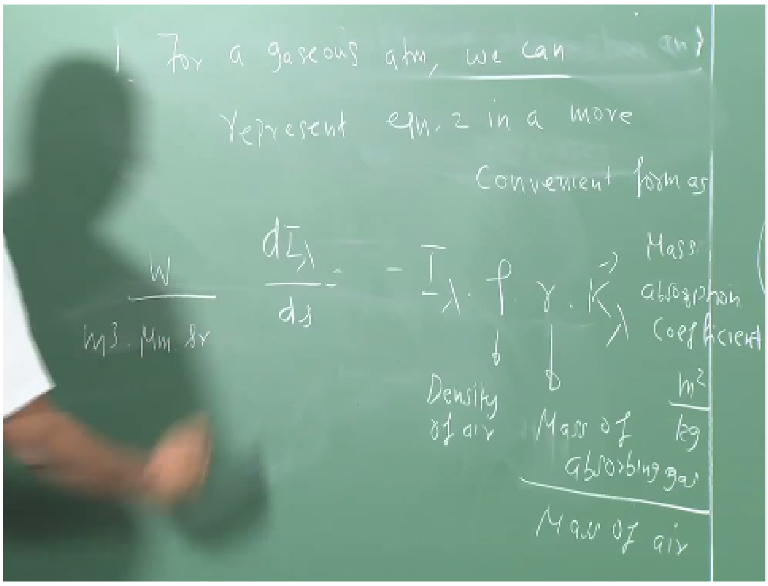

For a gaseous atm, we can represent eqn. 2 in a more convenient form as

$$\underset{\frac{W}{m^3 \cdot \mu m\, sr}}{} \quad \frac{dI_\lambda}{ds} = -I_\lambda \cdot \underset{\substack{\downarrow \\ \text{Density of air}}}{\rho} \; \underset{\substack{\downarrow \\ \frac{\text{Mass of absorbing gas}}{\text{Mass of air}}}}{r} \cdot \underset{\substack{\text{Mass absorption Coefficient} \\ \frac{m^2}{kg}}}{\kappa_\lambda}$$

For a gaseous atmosphere we can represent equation 2 in a more convenient form as, rho is density of air, r is mass of, now this is mass absorption coefficient, now please check the dimensional consistency of this, left hand side is, right side is,

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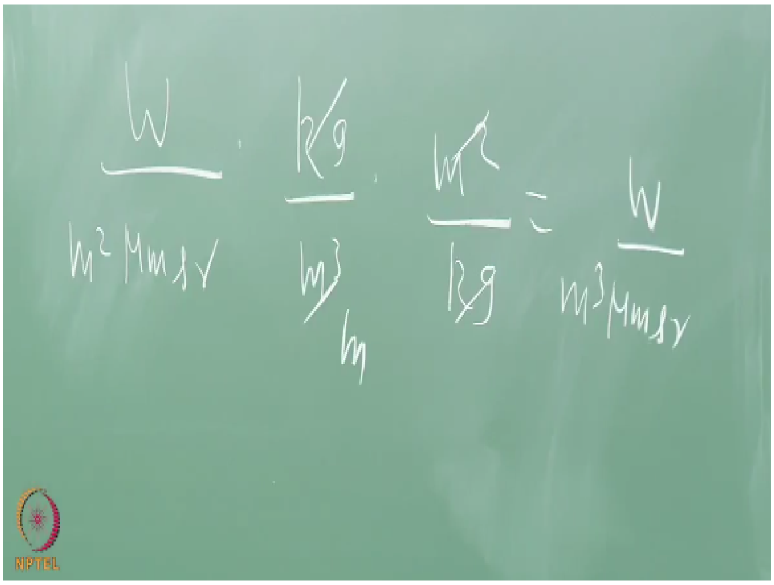

$$\frac{W}{m^2 \mu m\, sr} \cdot \frac{\cancel{kg}}{\cancel{m^3}_{\,m}} \cdot \frac{\cancel{m^2}}{\cancel{kg}} = \frac{W}{m^3 \mu m\, sr}$$

Are we doing fine? So I am happy, I think it is fine now, density of air you will always know right, what is r? mole fraction, no, no, mass fraction, kappa lambda, problem, you have to get kappa lambda from data books or from database or from spectroscopy and so on, but you know it for an individual gas, okay, if you know for one gas it is possible for you to write this and then you integrate it between limits that we will see little later.

But first we will have to formulate the problem. The first part is formulation of the radiative transfer problem in the atmosphere, now. So this rho r kappa lambda is called the volume scattering, sometimes it is also called the extinction coefficient, why? So some radiation is coming like this then it comes I lambda + d lambda, so the rate of change of this I lambda with ds depends on how much of this radiation is absorbed and then if it having a temperature more than 0 kelvin this will also emit radiation according to the Prevost law.

Then if the particles and if there is a particular relationship between the wavelength of the radiation and the size of the particle, then it can scatter also and it can scatter in various regimes that we will see little later, so whatever is coming out will be after all these processes are finished, these processes could be one of the three, scattering, absorption, there is no emission.

In this case we have considered is weak emission, because emission term will add to this I lambda, are you getting the point. Now I am saying that d I lambda/ds is minus that means I am not considering, I am considering the case of a weak emission or no emission, so there are only 2 processes, which attenuate or reduce the intensity as it passes through the gas volume, this could be reflexion, it is just reflected off, which means it is scattered or it is absorbed.

So whatever is coming out will be less than whatever is entering, that this is the most difficult thing to calculate for various gases at various wavelengths, it is not one value, it is not one value like thermal conductivity of water, thermal conductivity of water is 0.6 watts per meter per kelvin you close the problem with that, you solve any convection problem with that or thermal conductivity of mild steel okay 40 watts per meter per kilo you solve the problem.

The radiative transfer problem cannot be solved like this, you understand and if there is emission, I am just giving you a sneak peak, if there is emission, that emission will be governed by the Planck’s law and so on, okay, or if you for example consider this to be integrated over all the wavelengths then you will have dI/ds - this absorption term + the emission term. The emission term will have sigma T to the power of 4.

Because you know the Stefan Boltzmann law, okay, then if you have to integrate, then you have to integrate okay, across some area or volume or something like that so you will have differential terms on one side of the equation and integral terms on the other side of the equation which makes it as what is called as integro-differential equation, which is very, very difficult to solve.

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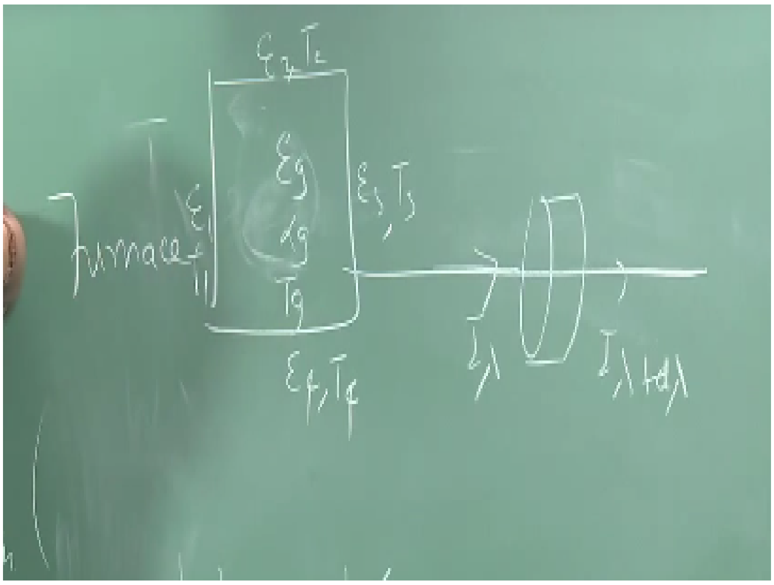

Okay for example in the gas volume it is enclosed here, furnace, so there may be some emissivity like this. There is a gas who is having emissivity, who is having absorptivity, who is having a temperature of Tg then the dI lambda/ds all these applicable here, then there is radiation from here to here, here to here, here to here for which the view factor, solid angle all these things will occur.

Therefore, this contains both surface radiation and gas radiation, so this will become an integral equation, if you want to solve for the temperature distribution then we may use some conduction or convection apart from this radiation, then finally you will end up with what is called integro-differential equation, which is very, very difficult to solve, okay, and these complications are there, right, it is function of lambda and so on.

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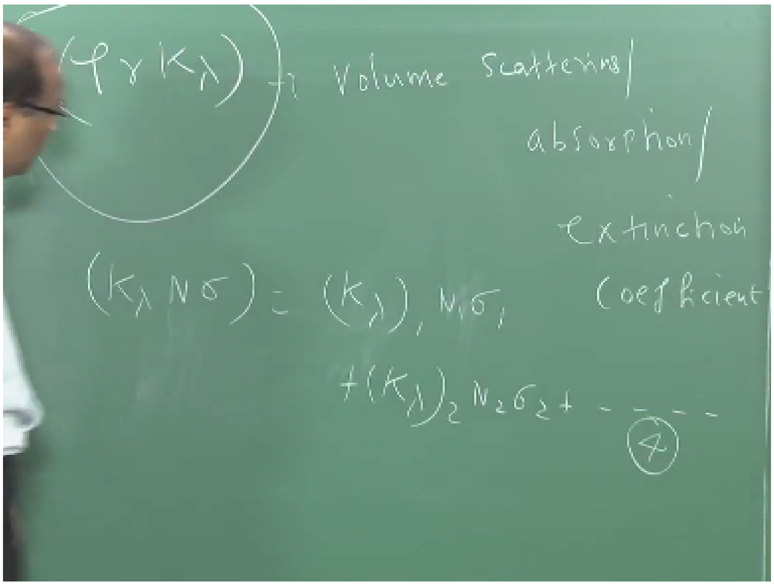

Now this kappa lambda N sigma this is the important assumption we are making, what is this? can you understand this, I am making a very, very important assumption here, what is this term, this term actually, this term is responsible for reducing the I lambda is not it, okay, so if the gas is the mixture of several gases we are saying that the contribution from each gas is additive, that is an important assumption.

So you can find out the sigma 1 kappa lambda N1 and all that for individual gases, find out the individual contributions, add, that will be the overall contribution, that means we are neglecting the cross effects or the interaction effects between various, so how is that possible and all that to a large extent this is valid, okay, so contribution of various gases and particles are additive and not addictive. They are additive okay.

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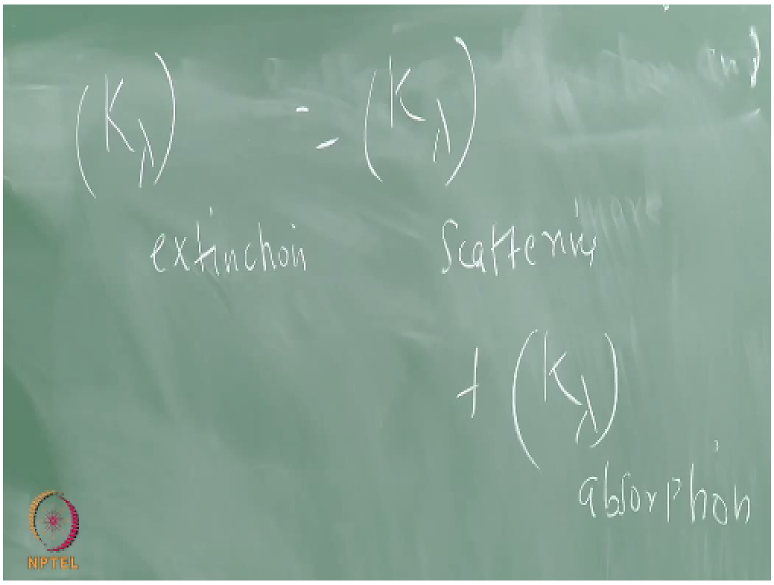

Then comes the important relationship, so if you know the kappa lambda for absorption and kappa lambda for scattering you add these 2 and together you call it as kappa lambda of extinction. For a no scattering case the kappa lambda of the extinction will be the same as the kappa lambda of absorption. For a heavily scattering and low absorbing case kappa lambda scattering will be equal to the kappa lambda of extinction will be equal to kappa lambda of scattering.

But emission is not included here because we are talking about extinction, what are those things which reduce the I lambda, emission will increase the I lambda, so it will be in addictive term, instead of –I lambda kappa lambda there will be some more term, one more addictive term, Kirchoff's law will come. You know Kirchoff's law, Kirchoff's law will connect emissivity and absorptivity that I will come to in the next class, not now, okay.

Now we will have to look at some scattering, I want to show something, okay. The title should be scattering by air molecules and particles.

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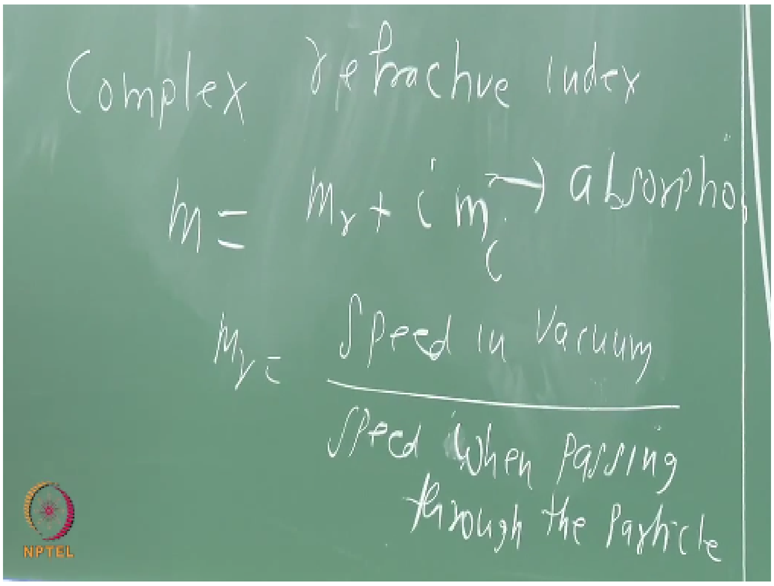

Complex refractive index

$m = m_r + i m_i \rightarrow$ absorptio

$m_r = \frac{\text{Speed in Vacuum}}{\text{Speed when passing through the Particle}}$

Scattering is such a big subject, we can have a course on that in a radiative transfer, we can offer a course on radiative transfer of the atmosphere for which some 25 hours we can spend on scattering okay, so in 1 or 2 hours we will see what best we can do. There are several theories like say Rayleigh scattering, Mie scattering and so on okay, what did I say, and particles. Scattering is reflexion from a volume.

Scattering is direction dependent and scattering is also wavelength dependent. Therefore, getting scattering properties is a nightmare if you want to solve a radiative transfer problem, for example when we are solving the microwave radiative transfer which our group has been doing for the last 12 years, then for each wavelength if you have to calculate this scattering properties it is anisotropic so it is extremely difficult why?

Not only are the properties wavelength dependent all the raindrops are not of the same size, so there is a size distribution of the raindrop also, which will be Marshall-Palmer distribution or this thing or gamma distribution and so on, so you have to assume a way of distribution and then you will look at how many particles, how many sizes you are taking, for example we assume some 100 sizes.

For each of the sizes we have to find out the scattering coefficient, then use what is called the single scattering assumption that means there is no interaction then find out for each of these sizes what is the scattering and then kappa, lambda, N1, sigma, 1/kappa lambda, N sigma 2 and all that you add it and then get the overall scattering for one wavelength, then if you are solving the microwave radiative transfer for 5 wavelengths depending on your sensors.

What ISRO or NASA is doing then you will have to find out this scattering coefficient for each of these wavelengths and as many number of wavelengths as there are sensors on your satellite. So remote sensing calculations are radiative transfer calculations can be very, very formidable and it involves the databases, it involves TDS calculations, times consuming calculations and so on, okay.

Now we will have to look at something called x. Consider spherical particles in the atmosphere, it is radius is r, the lambda is the wavelength of the radiation, we want to look at the scattering of radiation of wavelength lambda by a spherical particle whose radius is r, there is a size parameter which is called x, okay, now we have to define a complex refractive index m equals, so this refers to absorption.

Speed means speed of light by speed in particle, okay, electrical engineers are happy right, once you see I, who are the electrical engineers here, the moment you see I you, some movie I is going to come. Okay. Mechanical, chemical, civil all when we see I we get nervous, correct. I will give you a present slide so you try to copy as much as possible or I can post it to you on this thing.

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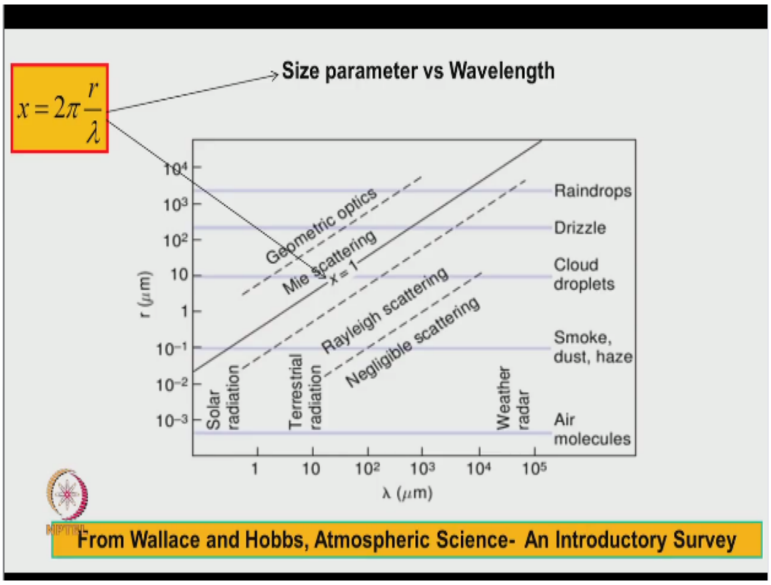

Size parameter vs Wavelength

$x = 2\pi \frac{r}{\lambda}$

From Wallace and Hobbs, Atmospheric Science- An Introductory Survey

Try to just copy little bit of this, so size parameter versus wavelength so this again from the book Wallace and Hobbs on Atmospheric Science-An Introductory Survey. So you can have no scattering, negligible scattering and heavy scattering. So the scattering cannot be, so to cut a long story short, the regime of the scattering and whether scattering is to be considered or not, just depend only on the wavelength or on the size of the particle.

It depends on the parameter called x, that x is 2 pi r/lambda. So you first calculate your x and then figure out your r and figure out your lambda and find out in which portion of the graph you are in, depending on that you will have to handle the scattering for that regime. For example, for very low values of lambda and very low values of x, should you worry about scattering?

Very low values of lambda, 0.4 to 0.7, solar radiation, let me explain this okay, see, 0.4 to 0.7 is solar radiation, lambda but the m, look at this graph very carefully, it is a log scale on both the x and y axis correct, because it shows decadal variation 1, 10, 100 okay, and this r is also the radius of the particle, now consider solar radiation, air molecules size is like this, the x is very small, x is 1 is here, size parameter, x is lower here, x is higher here, so I have put x = 2 pi r/lambda.

So x is increasing in this direction. So if you look at this the size of the particles, air molecules less than 10 to the power of -3 micrometer. Smoke, dust and haze are 0.1 micrometer, cloud droplets can be 10 micrometer to 100, drizzle is 100 micrometer, rain drops can be millimeters, you know that, you do not require somebody to teach you, okay, now this solar, in the solar radiation which is incoming air negligible scattering.

But once you have smoke, dust and haze, some air, the Rayleigh scattering will be like this, so some amount of Rayleigh scattering will take place, okay, but the moment you are looking at the cloud droplets and this thing and so on, so the game changes. Let us go to microwave, what did I say, what is the lambda for microwave? Please flip back 10 to the power of 3 more than 100 micrometer right, okay.

More than 100 micrometer, so if you say in the microwave part of the spectrum you are here, correct, drizzle rain, I want to find out what is the radiation coming in the microwave part of the spectrum in a raining atmosphere, I will have to use geometric optics or Mie scattering theory right, which is quite tough involved okay, so depending on the size of your particles, depending on your lambda you will have to apply different scattering theories, is that okay.

So this is basically the sum and substance of this, now particles with x much much <1 are relatively ineffective at scattering radiation, so this is called the Rayleigh scattering regime, which is indicated in the picture.

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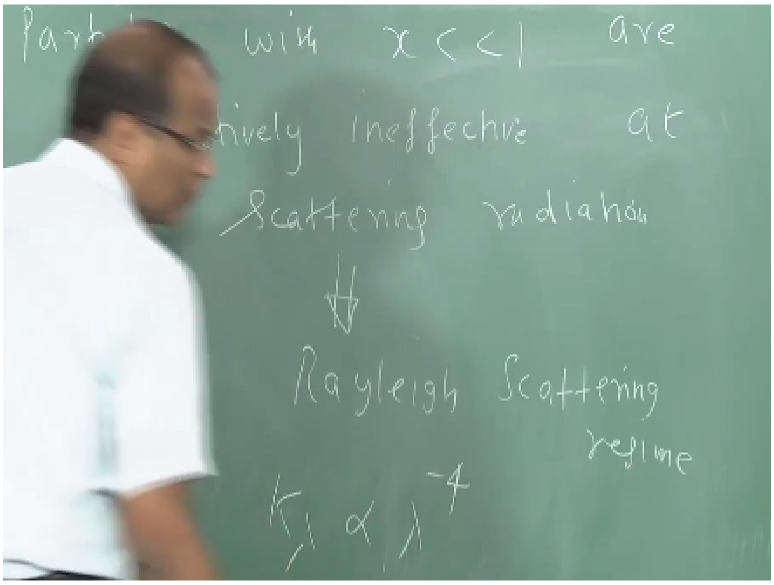

People have done studies to show that kappa lambda varies lambda to the power of -4 for Rayleigh scattering plus scattering is isotropic it is same in all the directions, okay. Let us solve a problem now. Problem #47, estimate the relative efficiencies with which red light (lambda = 0.64 micrometer) and blue light (lambda = 0.47 micrometer) are scattered by air molecules, okay.

So if you extrapolate first what is the regime? Air molecules are here, okay 0.4 to 0.7 if you extend this line what regime is applicable, do not say it would not be scattered and everything is 0, Rayleigh scattering, okay, now I told you for Rayleigh scattering the kappa goes lambda to the power of -4, so I just want you to find out kappa of red/kappa of blue, and comment, is it clear.

The absorption coefficient is the function of wavelength goes as lambda to the power of -4, now first part of the story is for air molecules and visible part of the radiation which part of the scattering is working, okay, so this is air molecule, see, watch, we are looking at air molecules, so you have to look at this line, okay, then you are looking at 0.4 to 0.7, so you are looking at this.

So this is the line which is coming, so it comes under which scattering, Rayleigh, how does Rayleigh scattering, the kappa vary, the scattering coefficient vary with wavelength I just gave you the relation, so find out kappa lambda of red/kappa lambda of blue, please do it.

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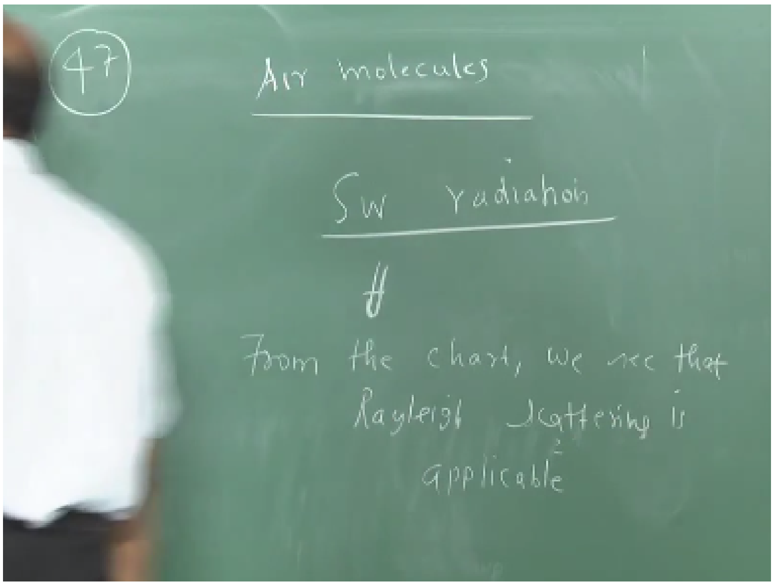

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Air molecules

SW radiation

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From the chart, we see that
Rayleigh scattering is
applicable

Problem is #47, from the chart we see that Rayleigh scattering is applicable.

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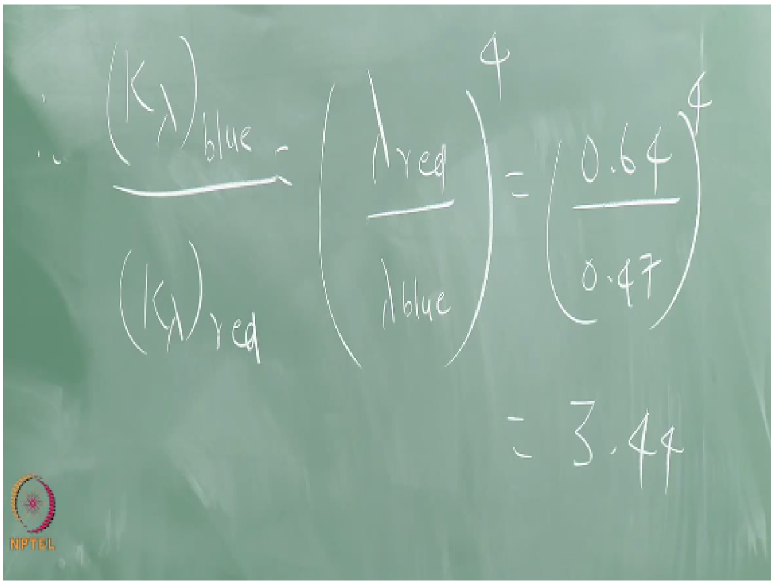

$$\therefore \frac{(K_\lambda)_{blue}}{(K_\lambda)_{red}} = \left(\frac{\lambda_{red}}{\lambda_{blue}}\right)^4 = \left(\frac{0.64}{0.47}\right)^4$$

$$= 3.44$$

So this is 0.64, what did I say, okay, hence there is a domination of scattering by the wavelength blue colour in the atmosphere compared to red, this is evidenced, the proof of the pudding is in the eating this is evidenced by the blueness of the sky particularly when it is smoke free and it is free of aerosols, okay, the next question should come, why is it not violet? You will ask know, violet, indigo, blue right.

If that is so red to violet will be even higher why is the sky not violet in colour, the human eyes are incapable of detecting this violet too much okay, the blue is the preferred, we are able to see blue more clearly, because of that the sky is not violet in colour, does not appear to be violet in colour, okay, now, let us go to the next regime, we will speak for another 5 minutes and we close.

We want to go to the Mie scattering regime, is this clear now? Why are the oceans blue in colour then, “Professor - student conversation starts” yah, yah, I think so right, but sometimes it is green also, I do not know, then change your glass “Professor - student conversation ends.”

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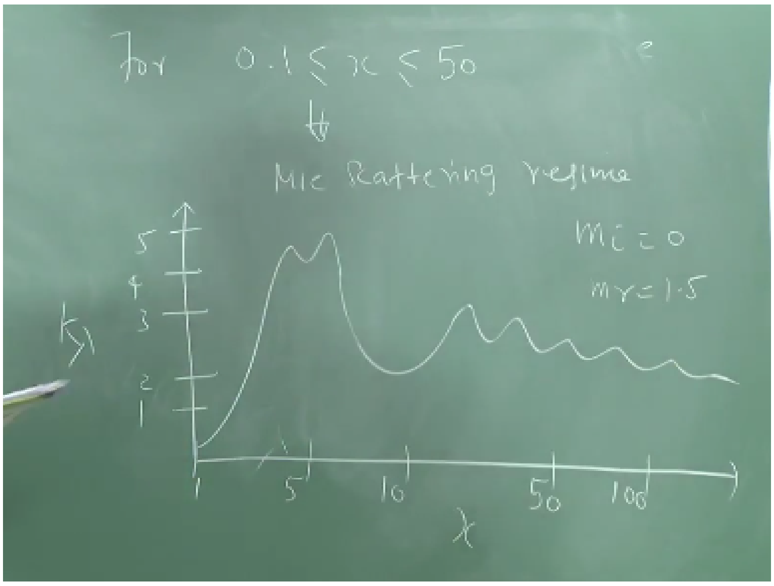

So, for 0.1 </= x </= 50 okay, that regime, this regime the Mie scattering is applicable, okay, so the refractive index is 1.5 and mi 0 means there is no absorption for this case, for a sample case it is worked out, so it works out like this, so you can see that once you have Mie scattering which is applicable for raindrops, the analysis becomes difficult. It is not simply lambda to the power of -4.

For every lambda it appears to have a value is not it. So for 0.1 < x </=50 for a chosen value of let us say mi = 0 and mr = 1.5 please note down the scattering efficiency has a damped oscillatory behaviour, it has a damped oscillatory behaviour, there appears to be a mean value of 2. What can you say further about this? as lambda keeps on increasing, this kappa lambda seems to attain a steady state value of 2, okay.

Then we will have to look at some absorption by particles and all that we looked at scattering, then we looked at absorption by particles and then we will close. As you can see I cannot ask many problems in scattering and all that because it is non-isotropic then kappa lambda you will have to do integrations, it has to be a take home assignment, we cannot simply solve problems involving scattering in the Mie scattering regime it is very, very difficult, okay.