Introduction to Atmospheric Science
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Lecture – 35
Radiative Transfer Equation – Derivation

So in today's class we will derive the equation of transfer and look at some solutions to the equation of transfer to the fundamental importance in atmospheric science particularly because radiation is a source term in the dynamic model and radiative transfer is also very, very important in satellite remote sensing and also we would like to know from the equation of transfer and the energy equation we would like to know the dt/dz in the atmosphere.

And also we want to know the heating profiles in the atmosphere and all that with which we will be able to predict. So, the equation of transfer.

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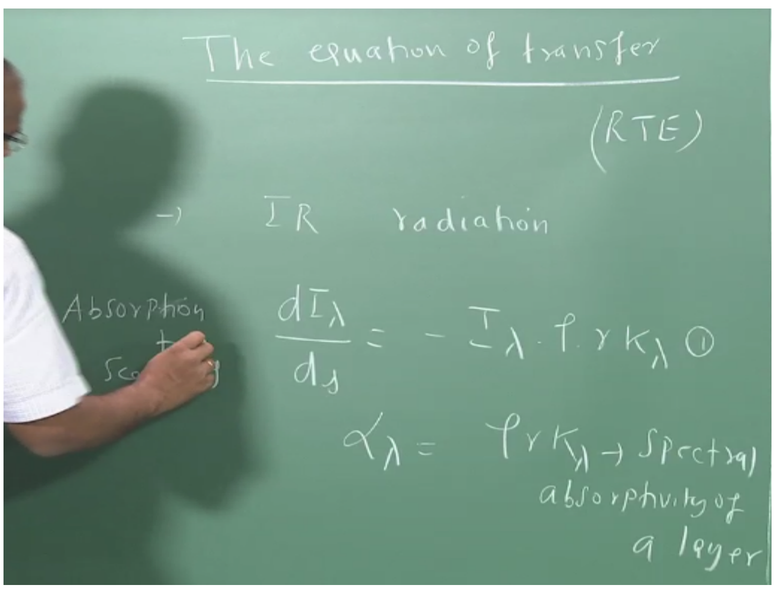

It is called the radiative transfer equation. So let us derive the equation for the infrared radiation because it is of importance to us in remote changing. So we already saw this in the last class d lambda - ds is - . This is the extinction or the attenuation suffered by radiation as a consequence of absorption and scattering by the atmosphere. Alpha lambda this is the spectral absorptivity. Spectral means monochromatic or particular wavelength lambda of a layer means layer in the atmosphere.

Suppose it derivative my radiation class for mechanical engineering students I will not say for I will say layer for me will be like this.

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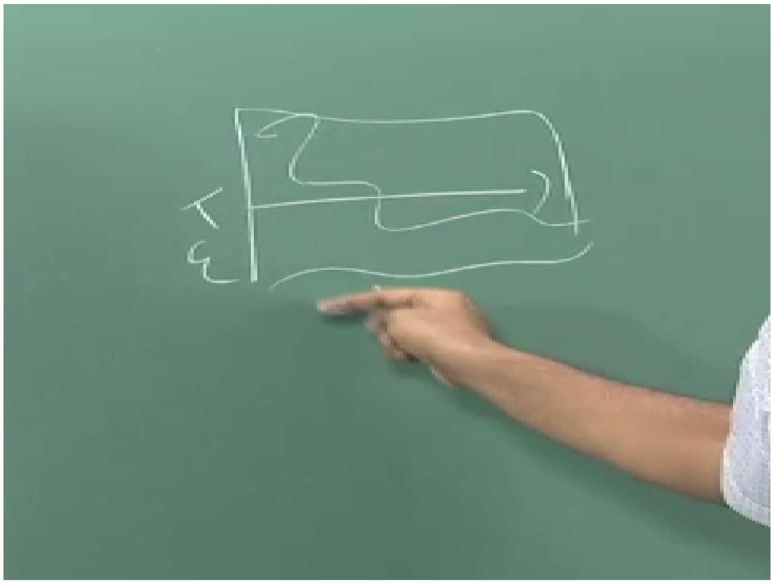

There will be heat at wall it does an emissivity and finding out how the radiation is spreading this could be a furnace, combustion chamber and so on, but layer is different here. In atmospheric science, the layer of the atmosphere. So this is isotropic scattering.

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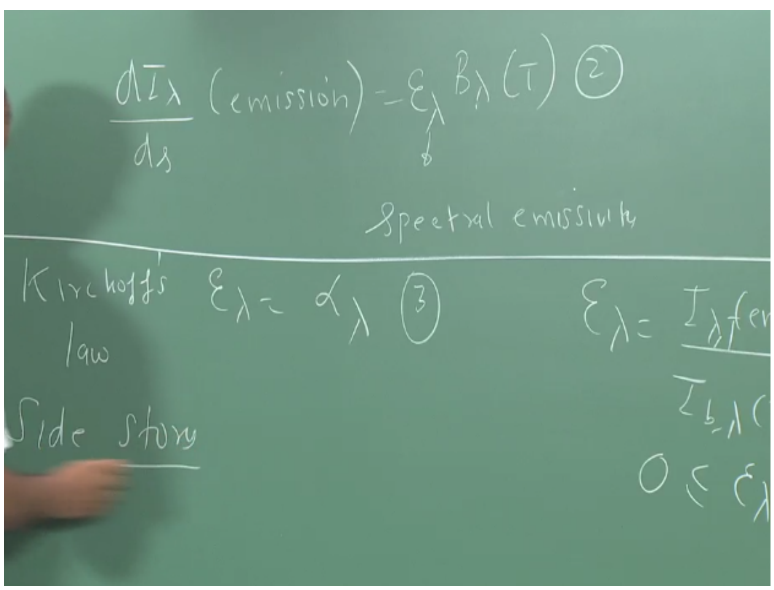

So the d lambda is basically if we look at the emission part which we have not considered so far the intensity will change with respect to s by epsilon lambda b lambda where epsilon lambda is a spectral emissivity. So what is the problem? We have one more property. So we will have to get information on this spectral absorptivity. So what is the connection between these 2. How do you think you can establish a connection between these 2? Let me take that.

You understand my question? What you can establish the connection between the 2. You know properties and determine. So it is going to be a problem for you. How do you deal with that? So I am going to say is this from Kirchhoff's law. I am just looking through the notes to see whether I am using kappa lambda or alpha lambda. So that I do not want that is why it is fine. Now kappa lambda is something is kappa lambda is what.

What is the definition for kappa lambda absorption coefficient? So this is absorptivity. So this absorption coefficient so I just got a doubt whether I am missing up with. So it is fine. Kappa lambda * rho * is fine. How do you know it is correct? What is this is the d a lambda/d = b lambda * T there is no need that the atmosphere must be a black body. If it is not a black body, it should be multiplied by an efficiency factor so epsilon lambda is nothing.

But I lambda emission/Ib lambda of T at a particular temperature at a wavelength what is the emission the spectral emission from actual surface or volume or whatever/whatever is emitted by a black body at that temperature and that wavelength. So it is a non-dimensional parameter which varies between 0 and 1. If it becomes 1 it becomes a black body. If it is 0 there is no longer under consideration, but now if you look at radiation falling on an object. So now you are going to side story. Let us get back to the equation of transfer after a couple of minutes.

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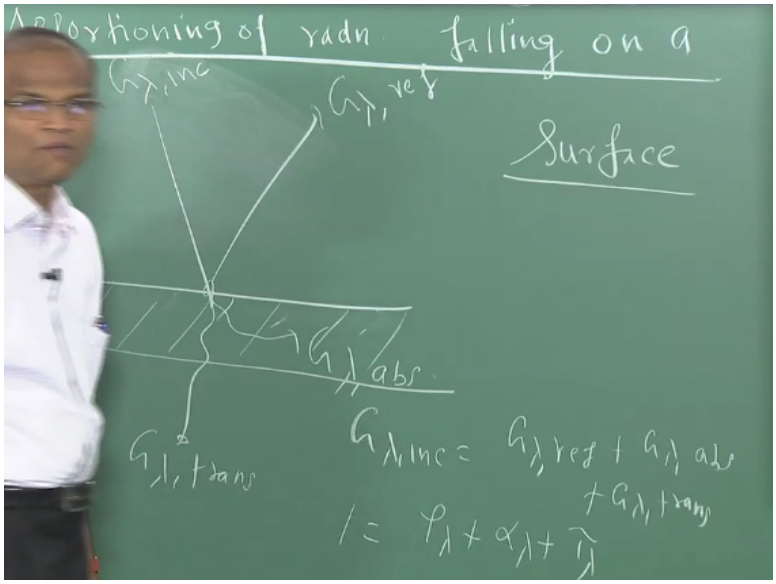

Now we already saw if radiation is incident G lambda incident this could be G lambda reflected. G lambda absorbed. It comes out the layer the G lambda transmitted. If radiation is falling on the duster what will be G lambda transmitted, 0 but it is a glass it will be nonzero. Atmosphere will allow. The atmosphere is completely transfer the view has gone you cannot see the sunlight. So even with clouds it will allow and clouds do not cover all the time cover the atmosphere all the time.

So, if you look at radiation falling on a surface, apportioning of radiation falling on a surface. So by energy balance dividing by G lambda incident throughout you get rho lambda spectral reflectivity + alpha lambda spectral absorptivity + tau lambda spectral transmissivity = 1. This is basically what is happening to the incoming radiation. It has got nothing to do with the black body characteristics of a particular body or what is its temperature and so on.

Though, its absorption can be related to the temperature. It is more dependent on the temperature of the surface which is gaining the radiation, but now emission is because of rotational translation, vibration energy of the molecules because of provokes law anybody can be more than temperature > 0 Kelvin will emit radiation therefore this emission and absorption are completely different processes.

So it is very difficult to logically establish a relationship between imaging and absorption. So we will have to take recourse to thermodynamics or you have to take recourse to experiments but such a relationship can also be explained figured out experimentally or empirically through experiment. Now let us see a simple proof of the Kirchhoff's law.

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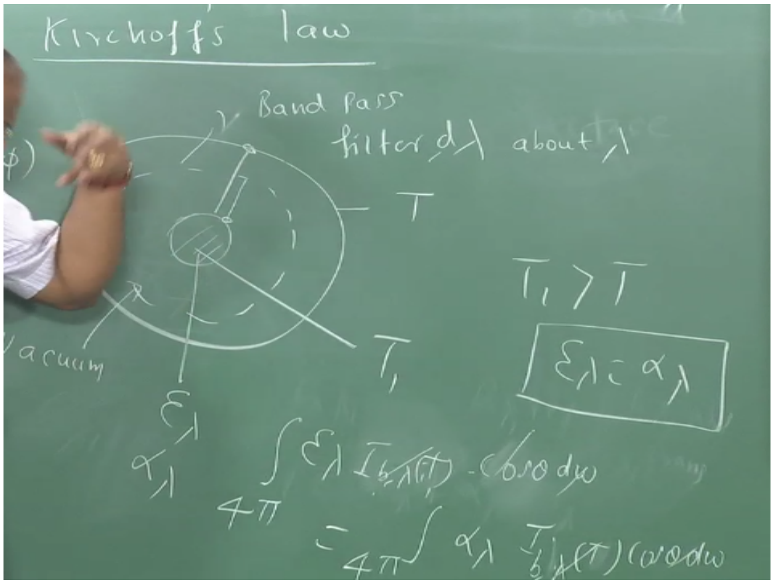

Consider an isothermal enclosure completely evacuated. It is at temperature T. Now I place a body which has a got a spectral absorptivity of epsilon lambda, spectral emissivity of spectral lambda and spectral absorptivity of alpha lambda. Initially it is at a temperature T1. Situation clear. a holosphere evacuated, it is maintained at a temperature of T how circulating hot water, cold water, something I do something. Now I put a body in the middle.

How do you put the body do not ask me that I make a hole, I put it enclose the whole I mean let us worry about it is hypothetically experiment? Now that body is initially hot even it is > t. What do you think will happen after sometime? Equilibrium will be reached. Suppose this is a small body and large enclosure, both will reach a temperature of T otherwise some temperature in between that depends on MCM, 1C1, MC1, M1CP1, M2CP2.

Do not get worried about all those thermodynamic calculations. Let us assume that the body reaches a temperature of T at that point and time. The net radiation on that body has to be 0 or otherwise it will emit or absorb radiation. It will receive or sent back radiation to a body at the same temperature, which will violate the second law of thermodynamics. Now let me put band pass filter.

This band pass filter allows radiation from this body to reach the enclosure only if it is in a narrow interval of d lambda above lambda and both d lambda and lambda are under my control. I can keep d lambda very small and lambda you can change from 0.1, 02 whatever ultraviolet you name it. Anyway I am not going to do that experiment. It is all of a thought experiment. So I can have a band pass filter for any d lambda about lambda.

Now only radiation in that band will be allowed to reach whatever radiation outside this band will simply come back to this. So under these conditions, from the second law of thermodynamics we know that any net, the net rate of radiation exchanges from the body to the surroundings or to the enclosure has to be 0 therefore, whatever it is emitted by the body which is given by epsilon lambda, I lambda of T d omega.

So cos theta factor will come over the full solid angle will be equal to whatever is absorbed also it will absorb only in the same band pass filter. Anything this will obstruct anything outside that band. Now the whole point is after it is equilibrated so this is T1 after it has equilibrated it is also Ib lambda of T. It is actually epsilon of Ib lambda * T1 * alpha lambda * Ib lambda of T because we are already putting epsilon lambda * Ib lambda.

Now what has happened is this T1 has become T. So now I can repeat the experiment, add infinitum for other values of lambda and I can prove that epsilon lambda = alpha lambda. If I take it from lambda 0 to infinity, I can also prove that epsilon = alpha which is the Kirchhoff's law. I can make it threateningly formal by saying that I am allowing radiation only in a particular direction of theta and phi and I can prove that.

So the directional spectral emissivity = directional spectral absorptivity this is the most general form of Kirchhoff's law. Already we are assuming that it is diffuse that means a direction dependence is gone and all that. I do not want to go so deep because our goal is not to study more about Kirchhoff's law. Our goal is to study the equation of transfer. Now later on you can proof that even whether this enclosure model it can be proved it is proved from thermodynamics.

It is proved it is so intuitive that epsilon lambda must be = alpha lambda. The major advantage is there is no need to measure epsilon lambda separately that alpha lambda which you got by kappa nu I nu. Yesterday, we saw all that Doppler broadening, Lorentzian broadening and all that. With that HITRAN database, MODTRAN database, LOWTRAN database you will be able to get absorptivity. Now use the Kirchhoff's law and then kill the emissivity by saying equating emissivity to absorptivity. Side story is over.

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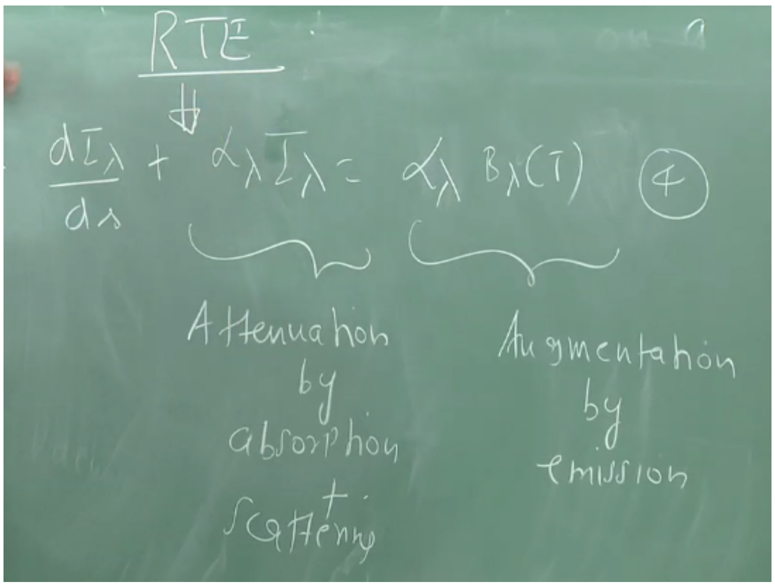

Therefore, this is more likely correct. Right side, the source term, emission term. The first term on the left hand side, the rate of change of radiation intensity with distance. The second term gives you the absorption or the attenuation and the right hand side is the second term is the decrease of I lambda because of extinction by scattering and absorption the third term is the augmentation by emission. I made a mistake here. I did not use the Kirchhoff's law; it did not make a mistake. I did not use the Kirchhoff's law and what do you do the right hand side.

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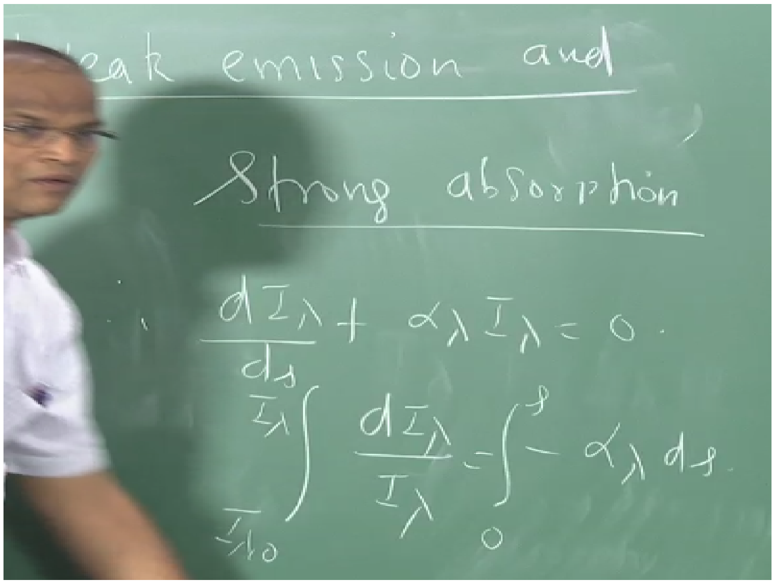

Consider the case of weak emission and strong absorption which means this is called RTE, radiative transfer equation or the equation of transfer or the RTE equation. Let us consider the case of weak emission and strong absorption so the right hand side term vanishes therefore integrating from 0 to s.

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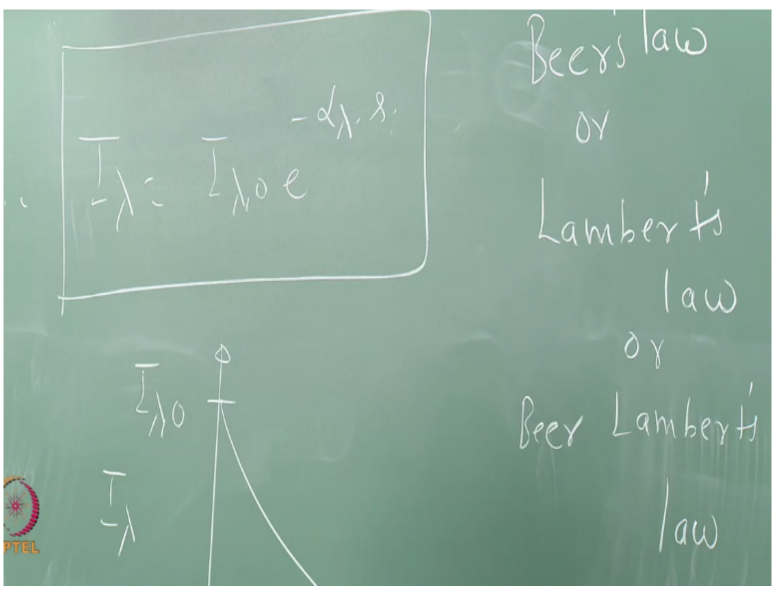

So the case of weak emission and strong absorption the radiation intensity the spectral radiation intensity exponentially decays with distance. Now this is called the Beer's law. So if you recall I should not block this camera. So if you recall then you look at the first chapter and the second chapter I told you that the ocean.

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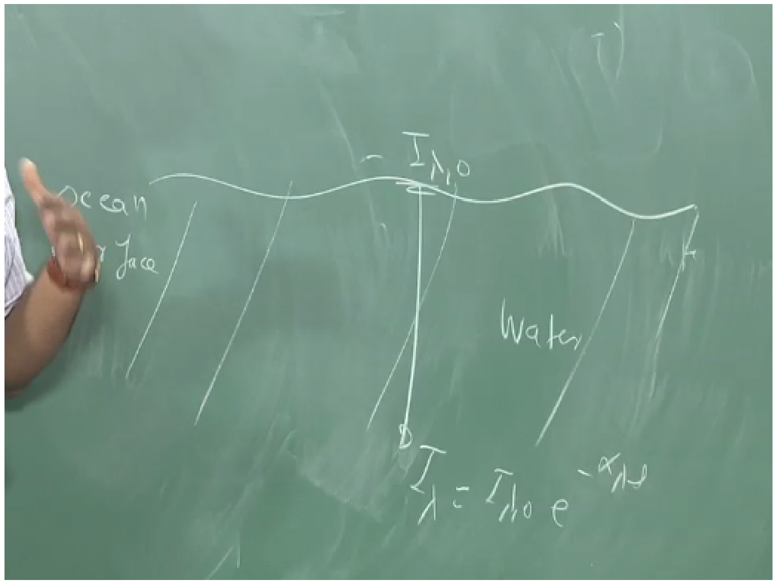

If you go to the ocean what is the big absorption of radiation. I lambda0. So the absorption of radiation exponentially decays with depth therefore little light is available at the bottom layers therefore photosynthesis it is not possible to do photosynthesis after the 100 meters or so and how do these organisms live here. So some fellows are very active in photosynthesis and then they decay and die.

These organisms will eat these fellows and then there are some up willing in some places and this will go up comes down so these churning is taking place and also because I lambda0 is > I lambda the temperature at the surface is more the temperature below so therefore less dense water is available at the top this makes it is a stable situation it is responsible for also for a stable climate and so this Beer, Lambert's law is very, very crucial. What is the unit of this alpha lambda as we have I do not know whether per meter.

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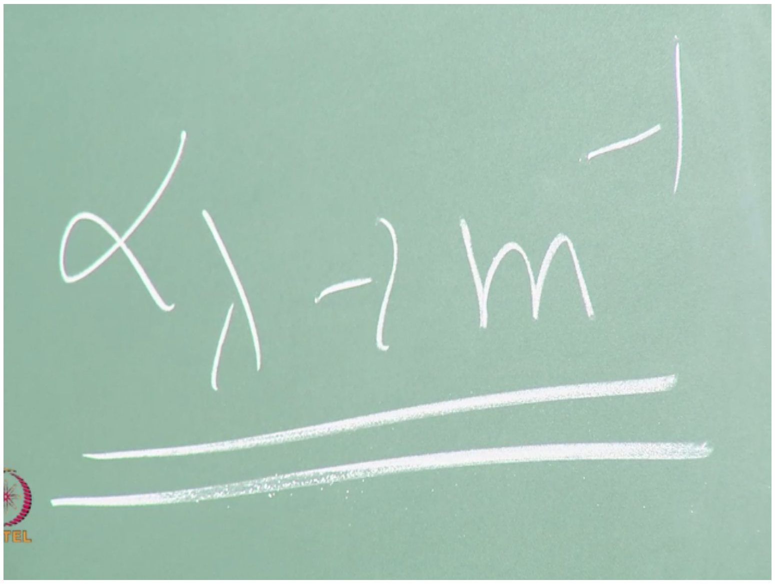

This alpha lambda in your actual engineering radiation alpha lambda is dimensionless, but in atmospheric radiation and gas radiation this alpha lambda that is going on in my mind for the past 3 minutes whether I made a mistake it is fine. Meter - 1 * meter dimensionless. e to the power it has to be. This is only an asymptotic case with the actual case imaging has to be considered. Then you will watch the fun. It becomes mathematically very involved to solve it.

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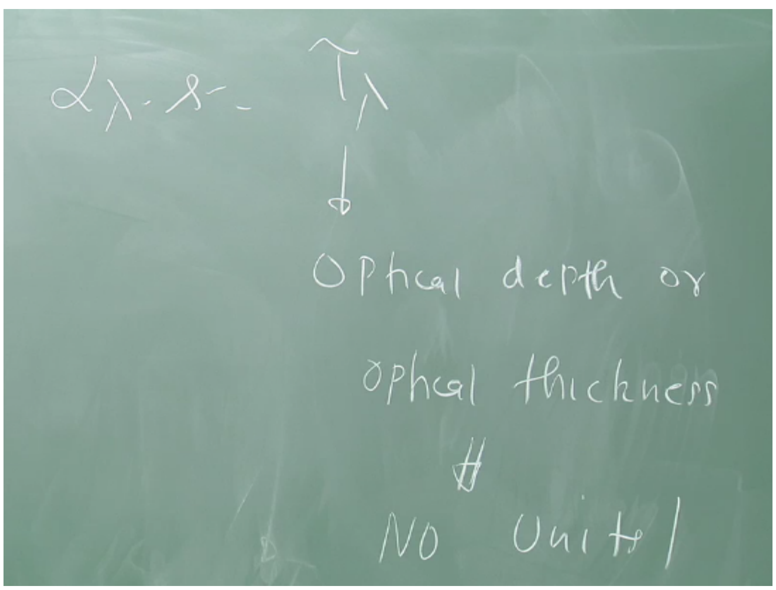

Tau lambda is called the optical thickness or optical depth. No units. It is dimensionless.

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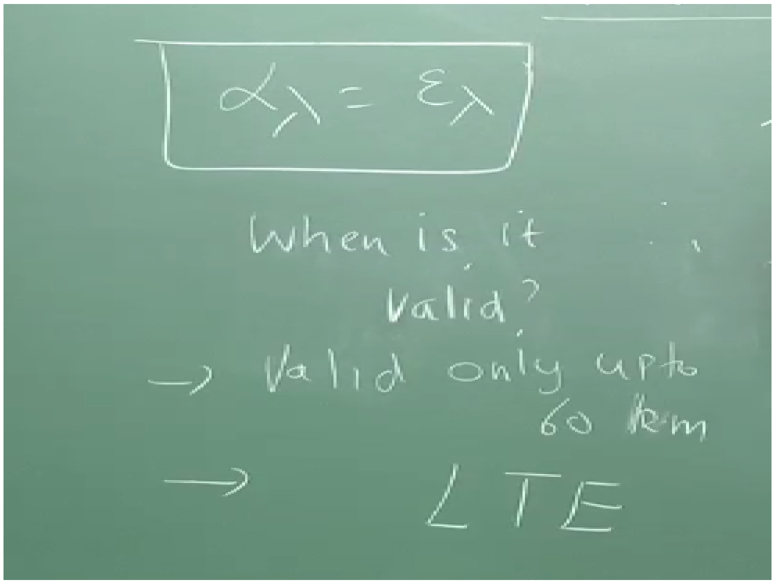

The question comes alpha lambda = epsilon lambda when is it valid. So please take down Kirchhoff's law is valid when frequency of molecular collisions is much more > the frequency of the radiation. Is it clear? The frequency of the collision is much more than the frequency of the radiation which is characterized by a C, nu, lambda all that. When is it valid? When is the frequency of molecule or where is the frequency of molecule collisions very high in the atmosphere.

Where is high density? So in the lower layers of the atmosphere it is valid so this Kirchhoff's law is valid only up to 60 kilometers. When we say that the frequency of the molecular collision is much > the frequency of the radiation a particular criterion is satisfied and that is called local thermodynamic equilibrium. We say Kirchhoff's law is valid only when the frequency of the molecular collision is much, much larger than the frequency of the radiation.

This happens when the pressure is sufficiently high so that the atmosphere is not rarified and the atmosphere is not rarefied up to the first 60 kilometer. After 60 kilometers it is not valid. Therefore, we say for the first 60 kilometers local thermodynamic equilibrium exists. LTE exists. Beyond 60 kilometers what you do. We have to do molecular simulation. You cannot define something like a pressure because continuum is not valid.

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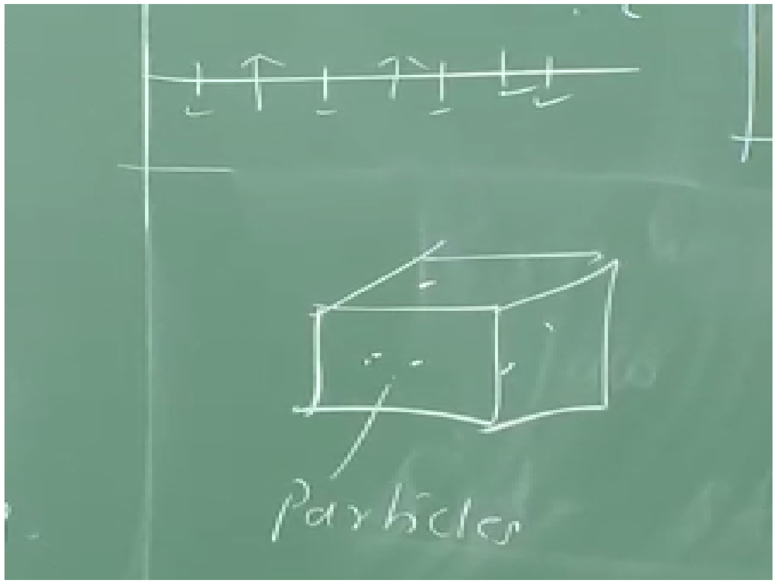

What is continuum? Consider a container, a volume consisting particles. Constantly particles enter the control volume and leave the control volume because there is statistical nature of distribution of the energy of the particles the particles are vibration, translating, their kinetic energy, potential energy all that the particles will be moving. Suppose on a particular height particles are continuously moving.

If there are 1 million particles and some 100 or 200 particles leave it will not make a big different, but if there are 4 particles if 2 particles leave then it is coming therefore it is very difficult to define average property temperature, pressure, density, that means the continuum hypothesis fails then you have to do molecular level simulation. Imaging doing molecular level simulation the whole of the atmosphere with the normal simulation itself.

We are not able to do it is very, very futuristic and 100 years later all laptops can solve all these problems I do not know. Then we do not know what we will be teaching that is a different story. Right now, the continuum hypothesis helps us to do all these things. I am able to define something called spectral absorption, spectral emissivity and equate it and so on. So LTE exists. Local thermodynamic equilibrium.

In ionosphere and all that it is not our consideration some physicists may be of interest they may have interest in ionosphere and all. Then they may have to do some other formulation to handle. Now I am going to pronounce this Marius will correct me.

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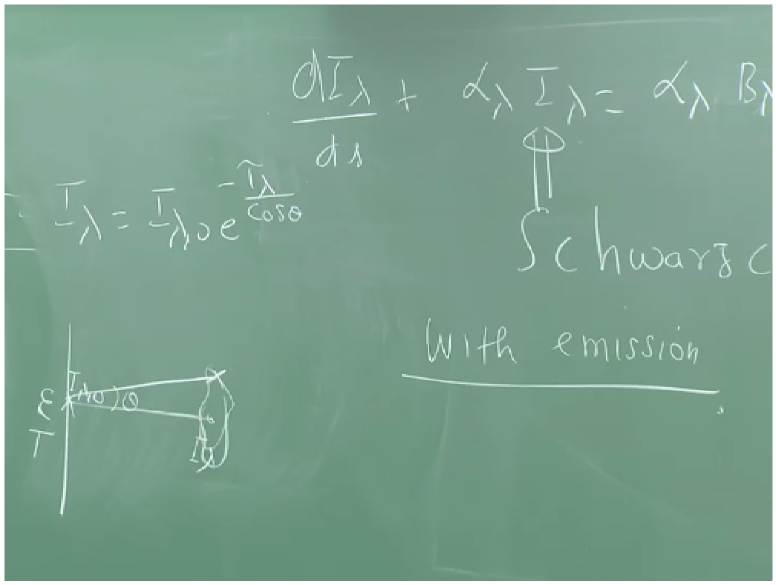

Yah please help me. Schwarzschild's. Schwarz is black, and child's solider. Black amour. So this is the Schwarzchild's equation obviously this is German he is happy now. So this equation is called the Schwarzchild's equation. Now this solution what we have done so far is only for the no emission case. With emission so we will have to do the solution now. Is it okay tau lambda? Suppose this is the body. It has got epsilon and T.

So we are looking at this is I lambda 0 what is I lambda. Suppose you are looking at some place here then the angle is theta then this will be why sir it should be tau lambda * cos theta it is not that way. Please watch. Radiation from here is traveling all the way up to here. There is a gas volume. That fellow is playing mischief. He is reducing the radiation. So this I lambda is lower compared to I lambda0 by the factor e to the power of minus.

E to the power of minus is always varying from 0 to 1 agreed. Now this straight path theta the Zenith angle is 0. The straight path it travels only a distance this much yes. When it travels, when it goes through a slant path it is Zenith angle of theta it travels a distance = s/cos theta which has to be more than the s itself.

Therefore, it suffers more when this Zenith angle is nonzero and theta = 0 is called a Nadir. In the satellite also Nadir viewing. Nadir viewing means you are viewing like this. If the angle is like this the radiation has to go through a longer distance it will suffer more. So and also in atmospheric science.

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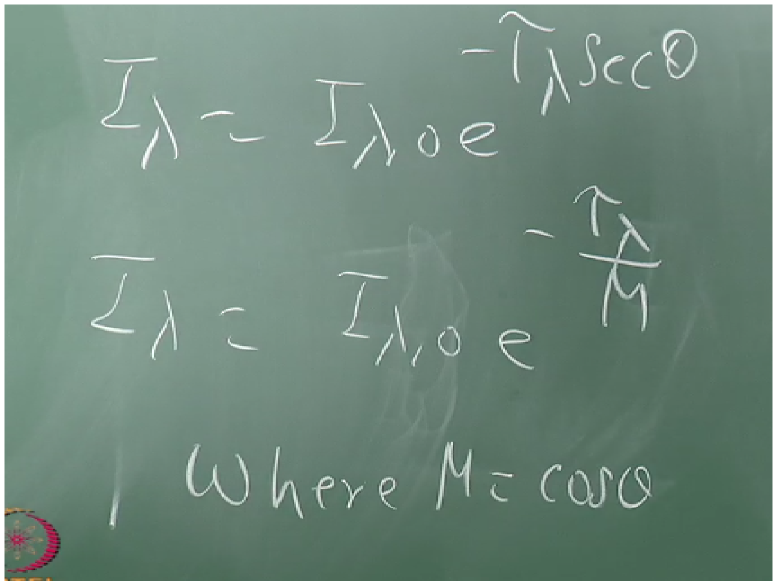

So people will call it different ways. tau lambda * secant theta/mu is this clear. If you look at different books nomenclature is different, but funda is the same for all. Now let us already I have erased it with emission we will see the solution.

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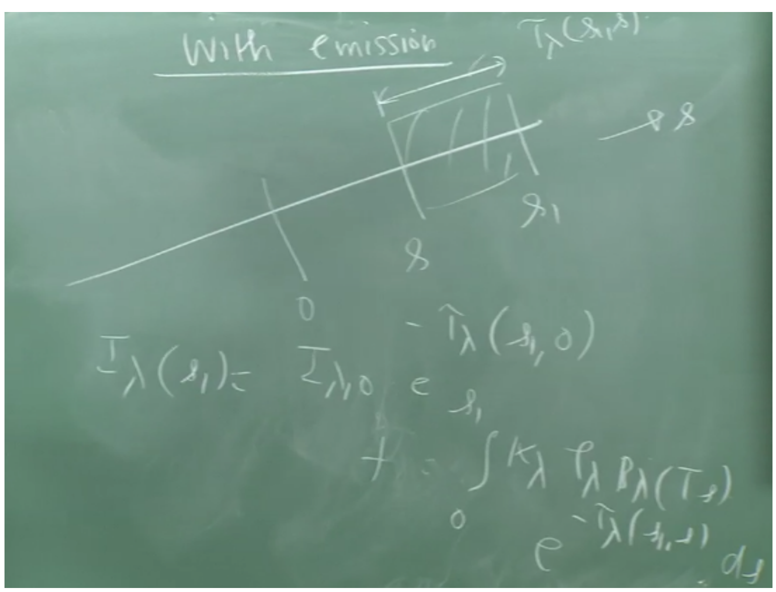

I have to draw a picture now. This is some direction S. 0, s, s1. s is basically local coordinate. We are not getting stuck to xy or whatever. Now the optical depth is tau lambda. The optical depth of that layer is tau lambda s1 to s that is between s1 and s. Now, e to the power of kappa lambda * tau lambda, B lambda. please take it down. This is the solution to the equation of transfer using the integral term. So it consists of 2 terms.

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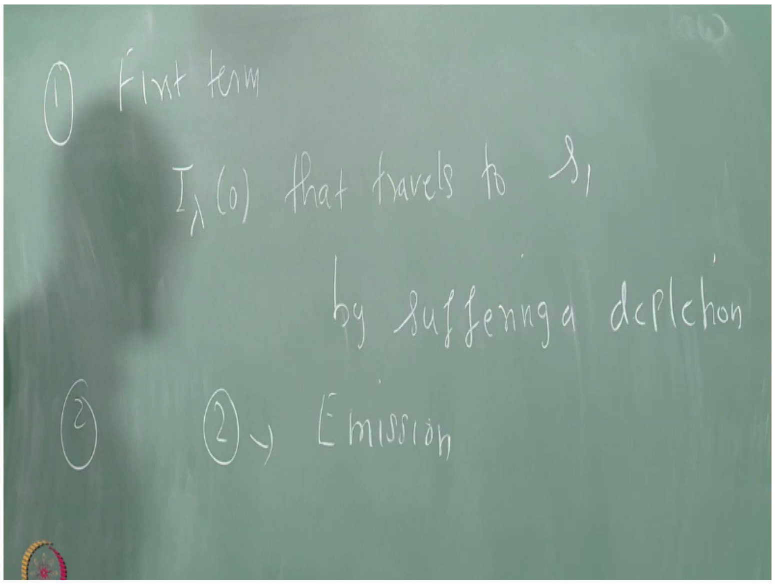

So the first term is I lambda of 0 that travels all the way up to S1 by suffering a depletion which is given by the exponential factor. The second term is emission which increases the I lambda. Please note that the integrating factor, integrating kappa lambda rho lambda all that you know. What is kappa lambda, rho lambda, B lambda this is basically epsilon lambda which is = alpha lambda under some cases. e to the power of - tau if we integrate it we will get back.

But e to the power of - tau is applicable from s2, s1 only because we are taking emission in that layer start and look. First time when I studied, I also had the same feeling radiation is difficult. Do not worry about that. See from 0 it is starting. From 0 from I lambda 0 we are trying to see how much it has suffered here, but now I am looking at only a small layer from S to S1, the emission must not be considered for the full because I am taking layer wise.

So what comes here is basically what has started from here and travelled all the way and they suffer a depletion, but what is the local plus what is the local emission there. That local emission

I am taking s1 to s that is all. **“Professor - student conversation starts”** But I am not taking ds, so s1 I am saying. **“Professor - student conversation ends”** Now there is some very important assumption which we have to see.

So the first term you have got the tau lambda from 0 to s1. The first term the tau lambda from 0 to s1 in the second term it is from s to s1 I have already explained why it is. If you have not understood this, then you have not understood today's lecture. Whatever arrives at that point even though it has suffered + whatever is locally augmented in that layer? Now we have to look at what is called the plain parallel approximations the most important approximation in radiative transfer. You will be wondering what type of question I can ask in the exam.

I can give the alpha lambda, I can give I lambda0, I can ask you to integrate. I can give you the kappa lambda so many meter square per kg and this thing. What should be the depth of the layers such that the radiation is coming out is half of I lambda, I lambda0. What will be the thickness such that I lambda = 35% of I lambda0, some silly questions like that I can ask. and if the depth, the straight question it will be if the depth is some 10 cm, kappa lambda is this much, the rho is this much this is this much what is I lambda, given I lambda0.

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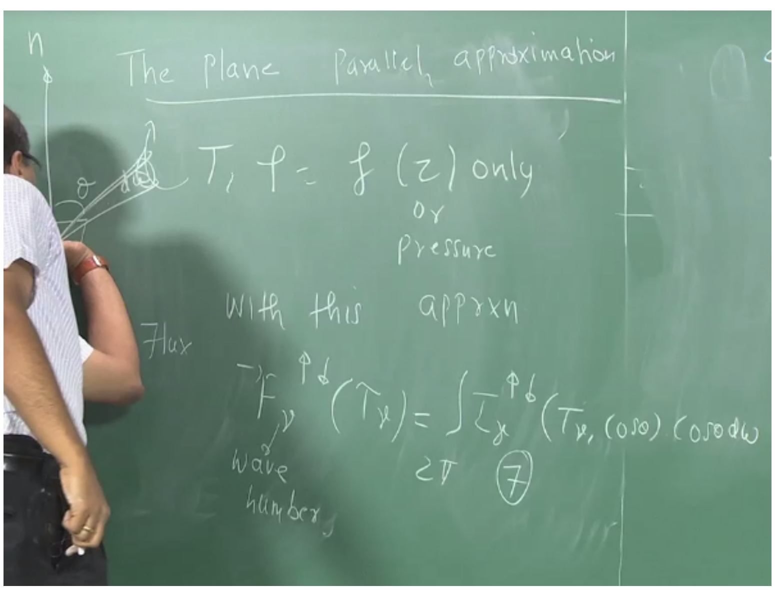

Now, the plane parallel approximation. Now compare the radiation intensity for a straight path and theta = 0 as suppose to a slant path that theta = 60. What will be the I lambda in a directing of 45 degrees. I can ask that type of question. The plane parallel approximation. As I told you the radius is 6,370 kilometers. The atmosphere is only 80 to 100 kilometers then therefore it is perfectly legal for us to assume that all the properties are function only of the z coordinate so which means.

So this approximation this is called the plane parallel approximation. What is that? **"Professor - student conversation starts"** emission in all directions so (()) (42:25) In that path only. You have not understood I can see from eyes. You are asking the right question it is not the flux. He is asking the right question sir what happens if it is this path and the path is changing I gave for 1 straight path other path I have to put the mu, cos theta, cos theta * d omega and then I integrate over 2 pi or 4 pi that is coming.

That is why calculation of flux becomes you get fed up with me. I am obstructing huh? Akhil is part of the NPTEL. Is it okay. I am just doing for 1 path that is coming now. So this is a good question. **"Professor - student conversation ends"** So with this approximation nu is my wave number. This is the flux. Now what is the equation number we started know. Today we started with 1, 2, 3. Please tell me some number. For this. 6. Now this is not frequency this is wave number. So f is flux. Wave number.

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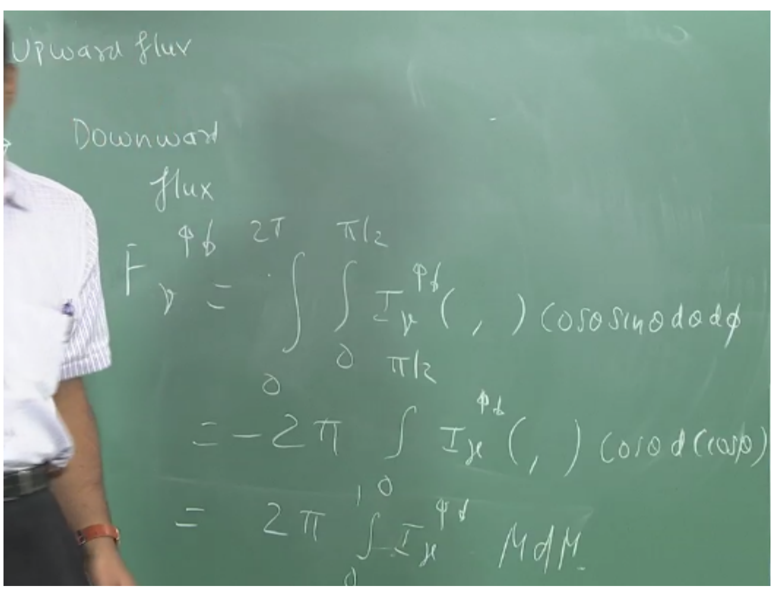

This is upward. You have to calculate for a particular layer both upward and downward flux. Is it not. So now you fill this. Why that cos theta coming the doubt will come because it is always dA cos theta. So the cos theta will come. Now for the Azimuthal angle we can neglect the variation with respect to the azimuthal angle so this will become. It is always da cos theta d cos theta, df cos theta is - cos theta. That is okay. So Mu d mu.

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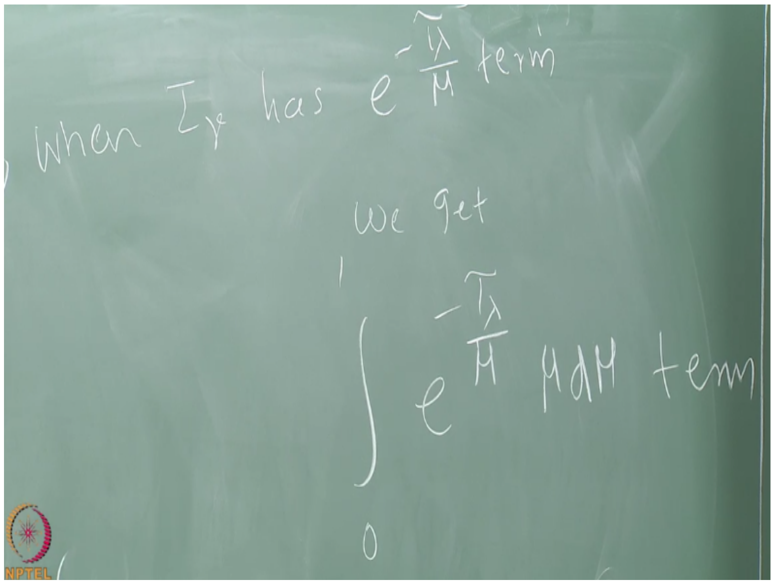

The whole problem is we get the term e to the power of - tau lambda/mu d mu. What happened? t nu cos theta it is will change with the angle. t nu cos theta and then into cos theta. Now to cut a long story short e to the power of tau/mu * dmu is not possible to integrate ay problem. What is the problem how we get this? This is a long procedure that flux to this thing is from intensity you have to get the flux what is the problem.

Leave this I mu + or - if there is any intensity from intensity you have to multiply by the solid angle because over the complete solid angle.

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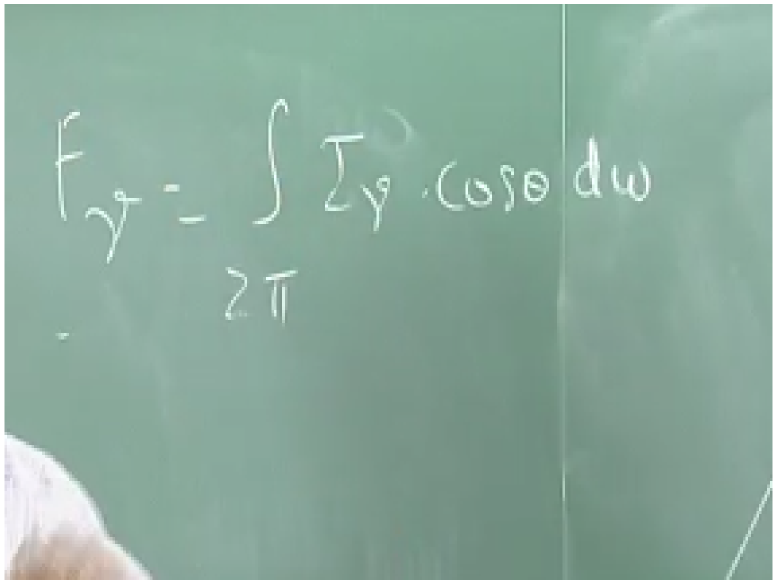

I told you in 1 class. So we start from this, this is the definition. Over the 2 pi solid angle I nu * cos theta is because of the dA cos theta term because the intensity has got the directional component. The flux is basically watts per meter square and so on. The intensity is to take care of the directional effects of radiation that is why it has got the dA cos theta coming. In the direction theta phi, normal to and the area normal because of the definition always when you are converting that I into this thing that cos theta term will come.

You cannot integrate this d omega, cos theta sin theta d theta d phi will come if you still have a doubt what my conduction and radiation in YouTube. I have explained this fully in one lecture.