Introduction to Atmospheric Science
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Lecture - 36
Radiative Transfer Equation (Contd.)

Okay, good morning, so we will continue with our discussion on solution to the RTE.

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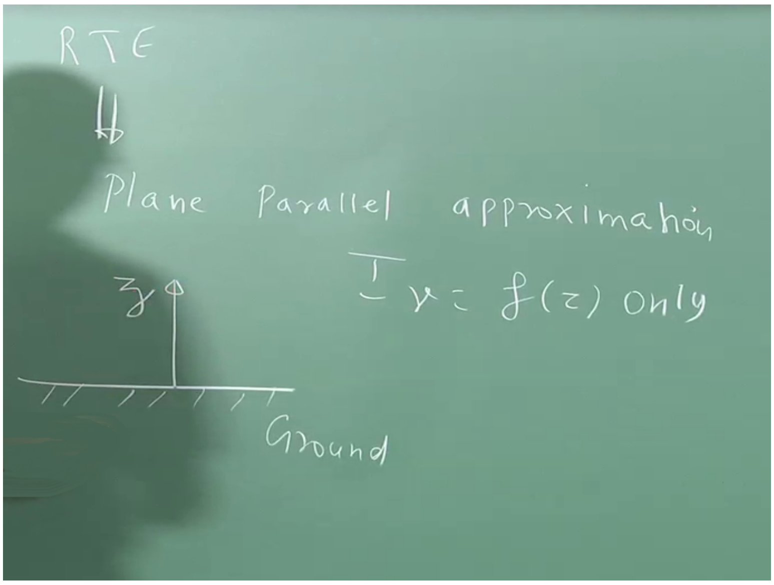

If you recall, we started with RTE okay, and then these properties could be a function of x, y, z directions, then it becomes three-dimensional and it becomes very difficult. All these properties are also varying with x, y, z then it is very bad okay. So we, okay so this plane parallel approximation makes everything a function of z only okay.

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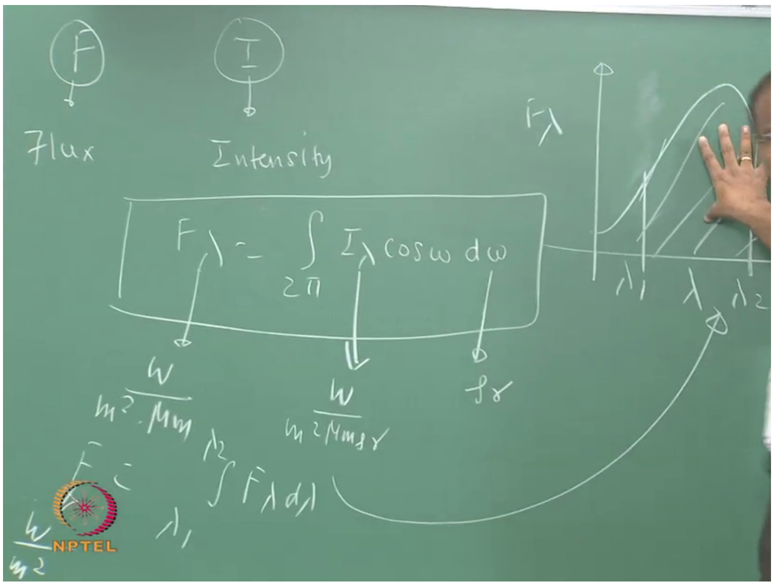

And then we talked about fluxes, what are the difference between flux and intensity?

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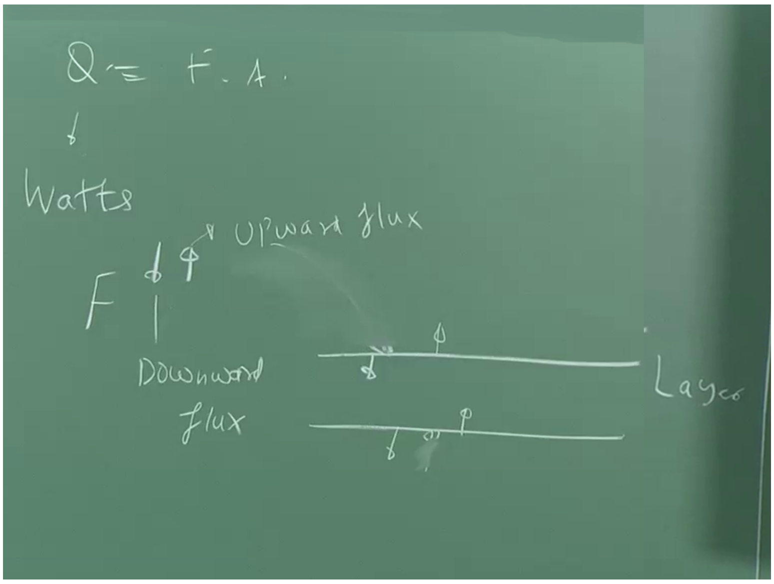

Now we can talk about some Q, this is the root right, the capital Q is very much used in heat transfer for conduction, convection, radiation. So we start from I from I we can go to F, but F is qualified by f nu or F lambda okay, it is spectral flux, then the spectral flux got watts per metre square per micrometre. You can integrate from lambda 1 to lambda 2, which is like this okay, so you can do it numerical integration.

But the F lambda is I lambda cos theta, and I lambda is treacherous, C1 lambda to the power of-5/e to the power of C2/lambda T-1, you would integrate it is not directly integrable, therefore, you will have to use numerical techniques. Because F is coming from I, I itself is coming from Planck's distribution is it okay, now we also looked at this one right. what is it that the arrow mark? This is upward correct upward flux.

What is the funda of this? Suppose you take something like this layer correct, from the bottom it can go to the top within the layer, from the top it can go to the bottom, from the top it can go to somewhere else okay, you can also receive from here, you can also receive from here, okay I think that was represented is not it. So are you able to see apart, it is not only the absorption it is also the emission okay.

So we are looking at layer wise, so we are looking at what is happening to a particular layer? Why are we interested in all this layer wise things and so on? We finally we want to be able to calculate what is the time rate of change of temperature across various layers in the atmosphere, this will lead us to, help us to find out the stability the atmosphere, and this thing and radiative budget calculation, it also useful in forecasting and so on okay. Now I where did we stop?

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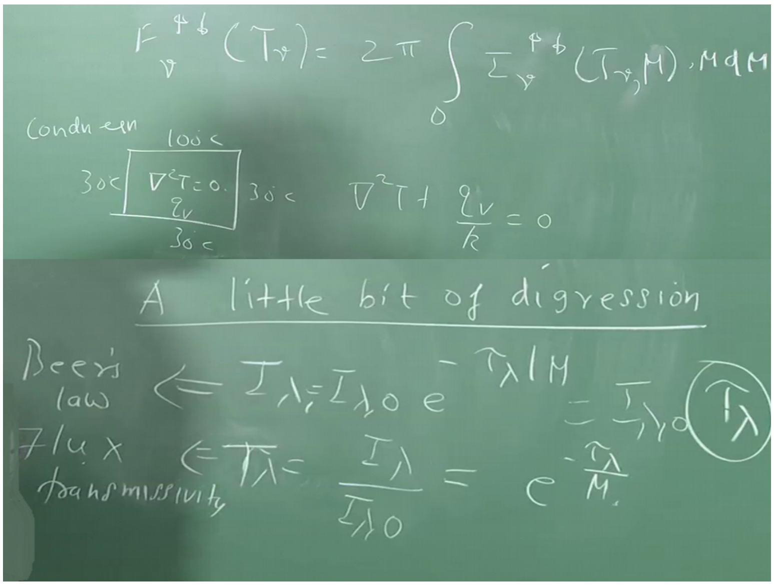

Alright, so we will be stopped here, is it right okay. What is this, is there an equation number for this last class? Why are many people absent, they went to home okay, was there number for this? No number okay. What does that equation give you? What is the use of this equation? Forget the mathematical development, what do you understand from this equation? Last week who asked me a doubt tau I of mu, is it okay.

Let us get back to this, what does the equation give you, if I know what I nu is okay, and if I put that mu is cos theta, again if I am able to integrate either analytically or numerically I will be able to get the flux, so from intensity I will be able to get the flux, this is the intensity flux, intensity flux relationship equation okay. That I nu itself has to come from what? Not only Planck's law, I nu if it is just one layer of a, if it is just one atmospheric layer who is free.

Suppose this is the cell phone and this is your I nu, then there is no problem at all, but suppose this cell phone is a layer in the atmosphere there are layers above it and layers below it solar radiation is coming in, earth's infrared radiation is coming up, this fellow himself will be reradiates infrared radiation. Therefore, how do you solve it? It is not a hopeless case, to solve it you have to solve dA lambda/dS+ kappa lambda I lambda is=whatever, for that you need to know that kappa lambda or kappa nu.

So if you solve that equation of transfer, you will be able to get that value of I nu, if you shove it into this, you will be able to get that f of nu. Then if you have an instrument which will detect this radiation between nu to nu+ d nu, then you put it into that equation, in this equation and get so many watts per metre square, this will be the watts per metre square captured by the sensor which you are placed above the earth satellite.

This can be designated frequencies which will be decided by the atmospheric scientist and the Remote Sensing people together after looking at the electromagnetic spectrum and how the radiation is absorbed at the top of the atmosphere under the ground, where there are absorption bands where are the transmission bands.

Based on this, we will have a multi-channel or a multispectral infrared sounder, infrared imager, microwave sounder, microwave imager and so on, with which we will be able to what is called reconstruct the vertical profile of the atmosphere. That means what? We should be able to reconstruct the temperature and humidity profile without launching balloons all over the place or in the absence of this, you will report only climatological data.

Generally, in Chennai in November, temperature from 0 to this thing 10 kilometres will be like this. If you do not know anything you would say that. Next step is to have a radius one day, but that radius one day only based on Meenambakkam and Nungambakkam, you cannot say that the same thing will be evaluate in Anna Nagar or Besant Nagar or OMR or whatever. If you have high resolution satellite, then a 2 kilometre by 2 kilometre, 3 kilometer by 3 kilometre you can say it with confidence.

That is what satellite meteorology has achieved, so meteorology need not be only to making measurements on the playground or making measurements in the ground having rain gauges having balloons, meteorology can also be done by having satellites, that is satellite meteorology. Meteorology can also be done by cracking the CFD problem in the atmosphere or the radiation problem in the atmosphere, that can be computational meteorology, or numerical weather modeling okay.

In that numerical weather modeling it is predominantly the winds and this thing and moisture and energy and all this which are included, all these which are considered but radiation will come as the force term just like your Laplace equation, you remember your Laplace equation.

So I am digressing quite a bit okay, if you want to solve the conduction equation 100 degree centigrade, 30 degree centigrade, 30 degree centigrade, 30 degree centigrade.

There can also Electrical engineers, who are the Electrical engineers here? Okay, so you can also be del square phi=0, where phi is your potential electric potential. Civil engineers, he can also be the stream function, who are the Civil Engineers here? He can also be the stream function and this thing under water seepage problem and all that porous media all that okay. So it can be anything, now the equation is del square T=0.

If I have volumetric heat generation which can occur because of biological reaction or because of an exothermic reaction or because heat is generated by nuclear fuel rod, this equation will get converted to del square T+ qv/k=0, where k is thermal conductivity of the medium, and qv is the volumetric heat generation rate. Likewise, so this is the energy equation for this problem, the energy equation for the atmosphere will be u dou T/dou x+ v dou T/dou x+ w dou T/dou x all those things which is coming from your Navier Stokes equation.

In that equation the source term, you can add the radiation term as a source term, if you want to calculate the source term you have to solve the equation of transfer, that is the radiation CFD coupling. So the numerical weather prediction is basically the CFD applied to the atmosphere, why cannot we just use fluent? Fluent is not capable of handling that because the co-ordinate system is different versus this spherical co-ordinate system.

We use what is called the Mercator projection and so on, the earth is also not a perfect sphere, so we have to do some modification, and this fellow is also rotating, once he is rotating you have to take care of the Coriolis component of the earth's rotation which will be another term in the momentum equation and so on, then you add these source terms and then you are done. So it is very, very difficult, if you start programming it you will not finish your PhD.

So many people have done it, and the body of literature is available and community software are available. So these are numerical NWP models, numerical weather prediction model, you would understand one of these models pick them up and start solving. And their models are at various scales, there can be a model which can solve at 400 kilometers by 400 kilometers resolution which will be global model.

Or you can have a mesoscale or regional model, where you solve only over the Bay of Bengal or the Arabian Sea or the Indian subcontinent for which you require boundary conditions, who will give you boundary conditions? Boundary conditions are to be updated with the time. So there is a global model which is always running, the Yankees are running, the Americans are running, so on a 400x400.

So on your domain whatever is the result for 400x400 dynamically once in 6 hours take that boundary condition and start with some initial condition and do your local stuff, and you can add your own masala by putting your own model for Indian monsoon there is that and run, and then you keep on reducing the domain if you are able to run it on one kilometre by 1 kilometre, your Paramour where we go or supercomputer you run.

And then you validate with your radius on this and that thing, and once cyclone passed cyclone you prove that your model is working well, I have now cracked the problem of cyclone. From tomorrow onwards, any cyclone is formed 72 hours in advance or 48 hours in advance, I can predict it, the 5 minute goes on numerical weather prediction okay. So for the numerical weather prediction, the source term radiation is important.

Radiation is important separately without numerical weather prediction, because from satellite data we know whether forests have degraded, whether there is a afforestation and deforestation, weather in a particular place there is a fire okay. Civil engineers they want to use land use, land use habitation, so many things can be studied using this okay. And you can reconstruct the profile of the atmosphere and so on.

And you can see is over 30 or 40 years or 50 years, and look at climate change, climate change cannot be studied with the satellite data for 5 years. So now satellite data is available for 30 to 40 years, people are saying we can make some inroads into this, because climate means we have to look at hundreds of years', decadal variation minimum decadal variation century wise variation epochal variation and so on, are you getting the point.

So this is not 2 minute courses on climate change, which will come Friday we will look at the climate change. Now this equation is clear, it gives the relationship between intensity. Now this is hard to integrate even though an expression for I nu is available, this mu d mu because this I nu, you know some e to the power of- will come, for the case of pure absorption the e to the power of-kappa lambda is coming tau nu is coming which is the Beer's law correct.

Therefore, if e to the power of-*cos theta d of cos theta, it is actually cos theta sin theta d theta d phi, the d phi I have integrated and put taken the 2 pi out that is why the double integral has become a single integral, how many of you did not recognize that, so everybody is with me the solid angle is double integral, the d phi 2 pi we have taken out, so we made it single integral 0 to pi/2, I made 0 to 1 out, cos theta is mu, sin theta is that b mu and then -01 1l0, all those things we have done.

Now we have to use we have to introduce the term called flux transmissivity, so let us do a small excursion. So a little bit of digression, so this Beer's law correct, so tau lambda=, what is the intensity of I lambda after it has travelled through a distance of S compared to the, what is the I lambda after the I lambda 0 has travelled through a distance of S as opposed to the I lambda when it has not travel through any distance of S=0, which corresponds to I lambda 0.

So this tau lambda is called the flux transmissivity, and it is always expected to be for a pure absorption case this value will be, first of all is it a dimensional or dimensionless? Dimensionless, it has a value between it absorbs everything what is the value then 0, it allows everything: 1, very good. So these are asymptotic limits, so the tau lambda is called the flux transmissivity, what is the value from this equation? What is the expression? e to the power of very good.

So there is a conflict okay T lambda, tau lambda is appearing on the left side okay, there is issue with that okay. Chaitanya you are happy? How do you put tau dash something, the point is made right?

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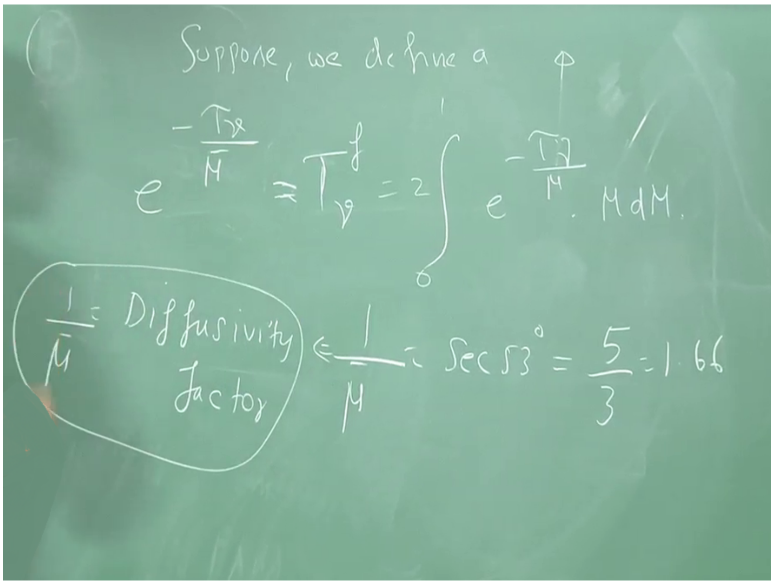

Similarly, suppose we define tau of nu flux transmissibility, I define something like this, so it is hard to, it is confusing and this one we make it tau nu, not following. Why am I putting this 2 to the power of, because I expect that when I nu, and I simplify that e to the power of-term will come inside that alright, that is why I am doing this. So I say that I define an equivalent flux transmissivity.

I define such a way that this whole expression can be replaced by e to the power of-tau nu/mu bar, where mu bar is an average zenith angle and the values of this expression or the intensity at this average zenith angle is representative of the integral of 2 of 0 to 1 mu d mu and all that, are you getting the point? Very good. How many of you not understood anything? What? This you can say correct, okay I will use the cell phone again okay.

Let us solve a heat transfer problem, this is my hand is the base it is generating heat 100 degree centigrade here, I want to cool it this assume that the cell phone is a metal. I put I attach it like this I attach several fins, that means it will take the heat from this metal then it will cool surface areas increase, once you have surface area, what do you do that for coffee and tea? Surface area is increase more than what is just in the, have you seen okay.

So here to the surface area is increased okay alright, now the temperature will decrease from here to here that means from here it is 100 degrees, from here to here it decreases, but the temperature may also change in this direction okay. Now what I am saying is in the other direction, if in the fin in this direction it is decreasing like this okay, in this direction suppose it is varying like this, what I am trying to say is I have to measure the temperatures.

And we are already putting some thermocouples and measuring, it will be very costly affair for me to put for several rows across the other direction also, and then take a global average. What I will say is I will numerical integrate the temperature at any section x in the direction if this is y in this direction I will integrate, and then I find out here at what height the local temperature is very close to the average temperature in that direction.

Then I will use only, so I will put instead of putting thermocouples in the middle, I will put thermocouples at that height where the local temperature is a representative of the average in the other direction, like that is their average zenith angle, that is a point, is now it is clear okay. That average zenith angle is 53 degrees 1/mu bar=sec 53, what is sec 53? Do not say1/cos 53, I want the value.

There are 2 people Ram and Sam, Ram is asked where is your house my house is next to Sam, Sam where is your house? Next to Ram, then how do you solve the problem? What are your coordinates, you have to fix the coordinates right, how can you measure the speed of the train when you are inside the train? Do not say sir will I look at some; I mean you have to be an observer standing outside is not it. What do you say Sujay? Okay.

Now let us not get into philosophy, so the next logical question is how do you study the mind? If you are both the observed and the observer okay, this will be edited or let it be, it is okay right. So now this 1/mu bar=sec 53 is 1.66. In other words, the flux transmissivity of a layer please write down this, in other words you need not write. The flux transmissivity of a layer is equivalent to the intensity of a parallel beam of radiation which is transmitted and passing through the medium its zenith angle of 53 degrees okay.

This1/mu bar in radiative heat transfer is a frequently referred to as the diffusivity factor, maybe I think somebody who studied solar radiation they might have given diffusivity factor, Srinivasa Reddy must have taught, you vaguely remember or diffusivity factor is that 1.66 takes care of the directional effect okay. So applying the concept of diffusivity factor applying this concept we can simplify that.

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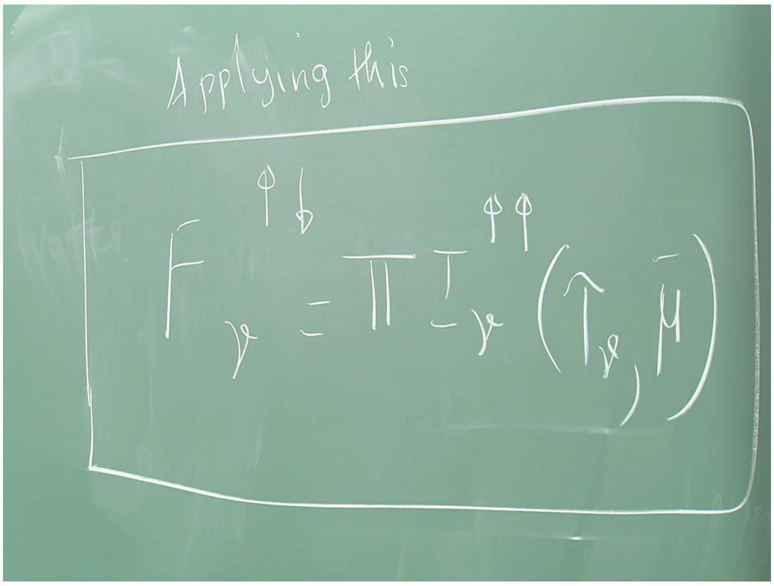

Please note there is one 2 inside this, there is one 2 inside, so there is one 2 here, so that has to be taken care of. So is it okay very good, what is the simplification we have done? You tell me what the value of I nu is, I will give you the value of f nu approximately by using the sec 53. What is that I nu itself? It has to be the equation of transfer has to be solved, Rowan is it clear okay. So this is the progress we have achieved so far alright.

Now I told you with this you can integrate over the wave number, integrate over the wave number means we integrate over a wavelength interval. Why do you want over a wavelength interval? I gave you the story of Remote sensing, where you want to integrate over some channels, 10.5 micrometre to 10.8 micrometre, 6.7 micrometre to 6.8 micrometre, and you design a sensor, no sensor can be designed for capturing only at 6.77 micrometre and 6.76 it will be 0, 6.78 it will be 0, it will not be like this.

So there will be I told you SRF spectral response function, we saw about Lorentz broadening and the pressure broadening, all these stories we have already seen okay. Now we have to solve a very important problem in the atmospheric radiation, how to get the vertical profiles of the radiative heating rate with this development?

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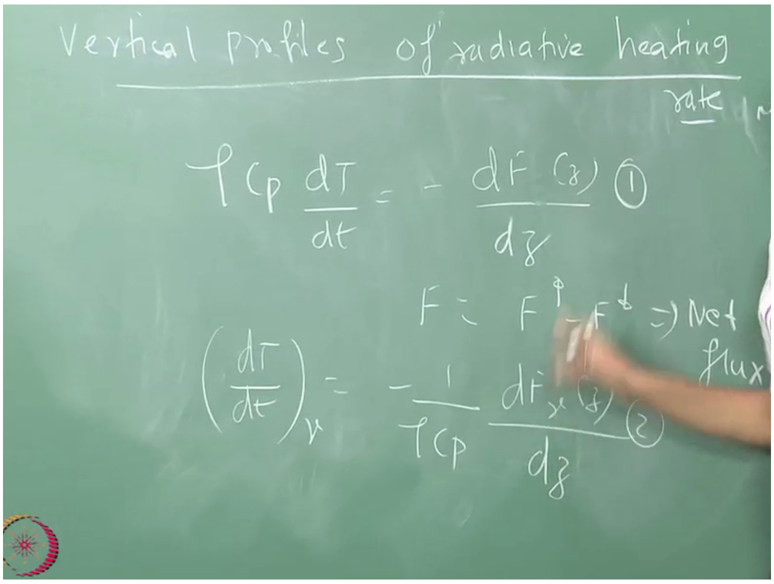

Okay, so if you take a layer of the atmosphere, the rate of change of this will be= correct, now you can take it as F, F is let us call it 1. What is rho? Density of air, CP is the specific heat capacity of air. So now we can find out what is the rate of change of temperature with respect to time for a unit wavelength interval or unit wavenumber interval. Sir, what are you doing good question.

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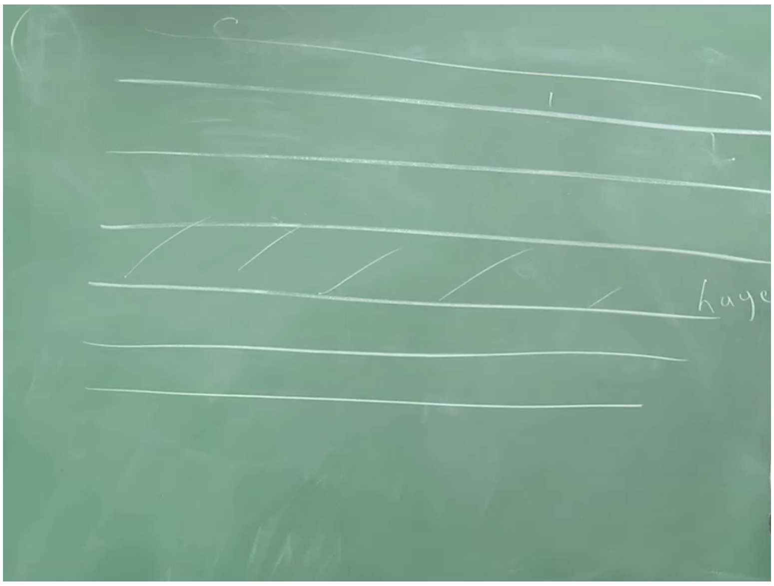

Okay, what I am doing is like this atmosphere is like this, I am doing a particular layer, in a particular layer I am trying to find out what is the rate of change of temperature with time, like that if I do it for all the layers I can get the vertical profile of the radiative heating rate. Then I can figure out what is this profile separately for various constituents I can do for Carbon dioxide, for water vapour, for Ozone and so on.

Separately, for various wavelengths I can do short wave long wave, then I can integrate and find out over all part of the spectrum. Finally, you will realize that the temperature will decrease in the troposphere, otherwise if all these calculations will be wrong, so the temperature will decrease in the troposphere, and the atmosphere has to be radiative equilibrium in the, which portion of the atmosphere?

Temperature does not decrease with for at least some portion, why is it called stratosphere? There is no temperature change for some region in that region it has to be radiatively it has to be at radioactive equilibrium, what is that mean? F up will be=F down, if F up is !=F down, the temperature will change depending on which is more powerful going out or incoming okay, this is the problem we are going to solve.

So all these developments critically hinge on your ability to get F, F is already related to I, I comes from the equations of transfer. Now we can take one layer, and if we are working only in the infrared part of the spectrum, we are going to simplify that I nu, once you simplify that I nu, we have a simple expression. What is this? This is a very simplified approach to solve this problem, or you can say to hell with all this.

For every nu I will calculate the kappa nu and this thing, and I used the most powerful supercomputer and solve it, that is also okay. But we are now trying to see just like lumped capacitance method, where we say the whole body is at one temperature, like that we are trying to do a simplified treatment. But please remember that even this simplified treatment gives a reasonable accuracy to the heating rate problem in the atmosphere okay alright.

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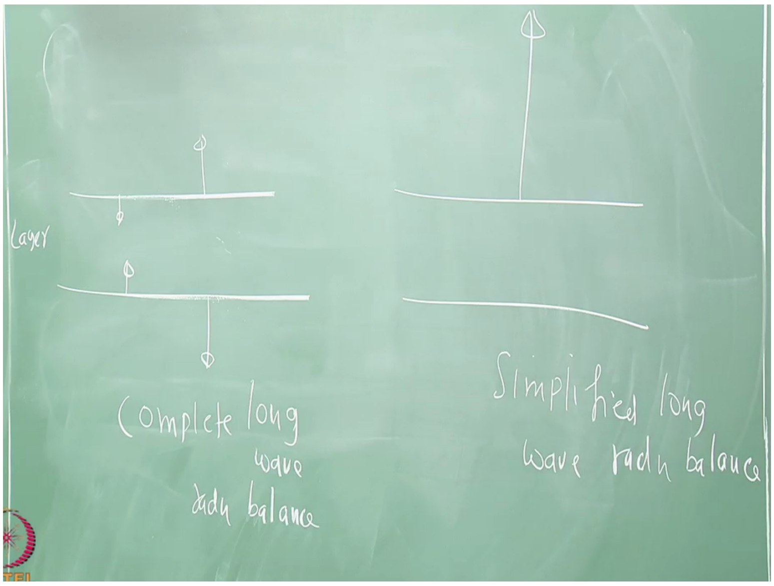

Now what is the next step, now I have to do the I already do this right. So if it is one layer some radiation is going to outer space from this, so the next layer. Now there is something incoming to this, from bottom something is going up, and it can also do something to the other layers. So now sir, why are you considering long wave? I am considering long wave to kick the solar radiation out, because I am doing only infrared remote sensing.

We derived the equation of transfer for the infrared, so an infrared part of the spectrum the contribution by solar radiation is very less correct, that we have already seen okay. So it is the cooling problem for every layer, it is a cooling we can consider it as the cooling problem, and we simplify all this, and we say there is only one flux going outside. So if you have this simplification then what happens?

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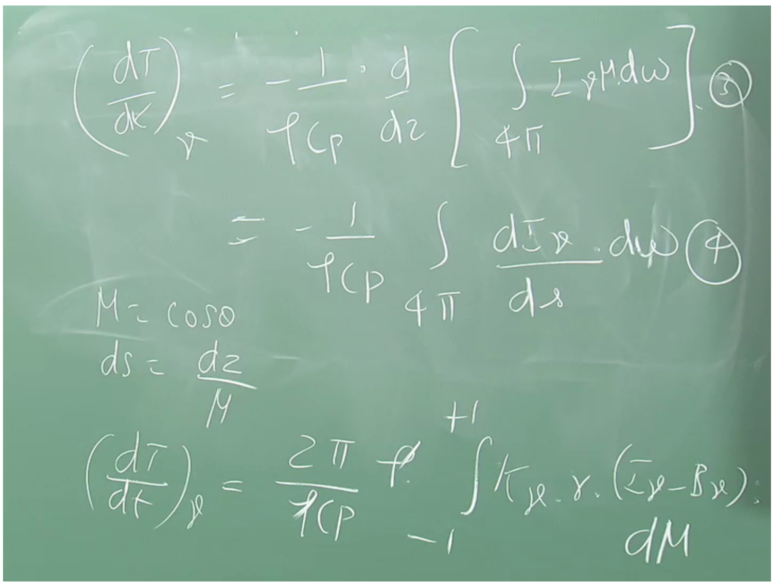

So dT/d tau= d/dz agree, there will be mu right mu d, do not get worry too much. I wrote that I kept my dT/d tau time I took the rho CP outside I retain the -, I took the d/dz then I wrote the expression for F. Instead of 2 pi suddenly I made it 4 pi, because I am considering the full sphere, you can you can have consideration for that layer something can be from the top, something can be from the bottom also, is that okay.

Now omega, I nu*mu is basically I nu*cos theta that is for the flux to this thing, then d omega solid angle will be there. Between 3 and 4, I did something, what? I swap the places of integral and the derivative that is Leibniz's rule correct. I used the Leibniz's rule, one more thing I did, I kicked the mu out, because I now considered S= by mu, I did many things okay. Therefore, dI nu/ds=kappa nu nu I nu-B nu/d mu that is coming from the radiation of transfer.

Then I did something omega consists of cos theta sin theta d theta d phi, or sin theta d theta d phi, that d phi if I integrate I can take out some 2 pi out. Then what I did was that the rho can be taken out, rho rho will get cancel, then the cos theta I put it as, sin theta I can put it as d of cos theta d mu, I change all these alright. Do not ask me why 4 is not coming, you check out that 2 is correct okay.

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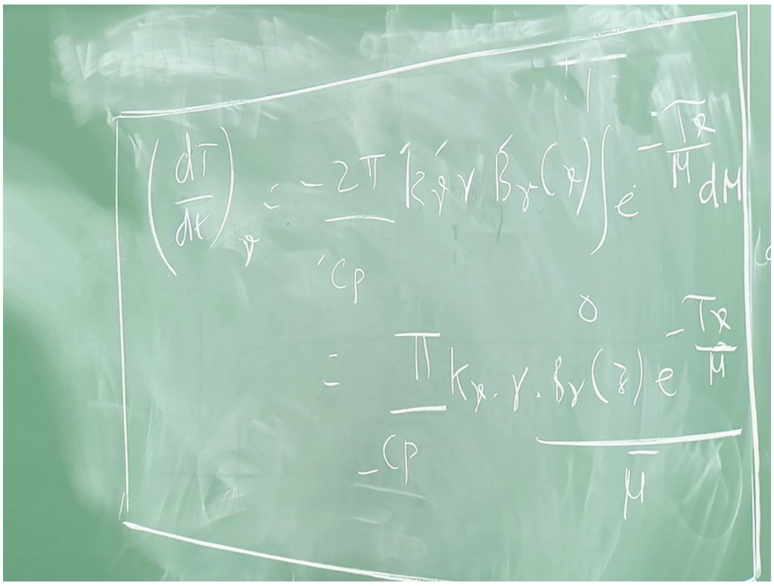

So -2 pi/CP, CP is coming right okay. Here, I am using this approximate watch this carefully otherwise you will forget you lose track of this, there are so many exchanges which are taking place, finally we make an approximation that all these internal exchanges are cancelling, from this layer only there is an outward flux which is going out, which is basically dependent only on the emission.

Therefore, in the I nu-B nu, this is I nu can be completely cancelled because I nu coming in=the I nu going out, but it is B nu is there because by virtue of its temperature being >0 kelvin, that will exists so that - will come. And then this is very nice, that is 0 to 1 e to the power of-tau nu/d mu just now we saw it can be replaced by the flux transmissivity, which is the sec 53 okay. So if it is like this, then this is=, you can check it with Excel.

So for every layer for every frequency or wave number, if you know what the kappa nu is, if you know the temperature okay, you can get B nu, you know this is not nu this is r, what is the r is your some concentration, what is the particle density, particle density or what is it, so right hand side CP you know, k nu you know, B nu you know, this one is the flux, this thing and therefore, you can actually find out this mu bar is 1/sec 53.

It is possible for you to calculate the dT/d tau of nu in the infrared part of the spectrum, this can also be done for if you nu kappa nu, each of these gases you can do H_2O, CO_2. And then for the complete gas mixture of the atmosphere also we can do. Now we are not going to solve it, somebody has already done the simplified form, in the next class, it is already late, in the next class I will show you the vertical profiles okay.