

**Introduction to Atmospheric Science**  
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**Lecture - 41**  
**Atmospheric Dynamics**

Okay so good morning so welcome to the last class of this course. So, thus far we have been looking at various aspects of atmospheric science. The first few weeks we looked at the earth system various components right the cryosphere the mantle the oceans land and all that okay. And then we then we spent a considerable amount of time in atmospheric thermodynamics where first we started finding various quantities. The most important of the hydrostatic equation.

Okay the hydrostatic equation which gives the relationship between pressure and height and with which we are able to use pressure as a proxy for height and you define a scale height for the atmosphere 7.5 kilometers. And then we went into moisture thermodynamics where we define related to humidity specific humidity mixing ratio and all that then we went to this Skew T in P chart and then we spent a lot of time on that potential temperature.

This temperature and all that right well potential temperature, wet bulb temperature dew point and we worked a lot of problems we worked out lot of problems. We looked at various phenomena one interesting phenomena to be studied was the Föhn effect it is the very cold wind on one side cold wet on one side rises over the mountains sheds this moisture comes out as hot and dry okay. So, we have seen all that and then we went into the second law.

We went to the second law of we went to the second law of thermodynamics looked at entropy and all and the most important thing was the Clausius Clapeyron equation. The Clausius Clapeyron which let you look at the change in vapor pressure with respect to temperature and I also asked a problem in one of the quizzes. I think Mount Everest and what Mount Everest what will be the boiling point and water and stuff like that.

You can after that we went into a very important chapter namely radiation. Atmospheric radiation we looked at the wave theory the  $E = h\nu$  and we looked at the electromagnetic spectrum.

And then we looked at emissivity absorptivity and then we looked at the earth as a whole calculating the equilibrium temperature and all that and then we entered the equation of transfer.

Which is very much useful in remote sensing then 1 dimensional plane parallel approximation where everything is a function only of Z then layer average then we looked at vertical profiles of heating rate. Then we found out what are the cancellation effects taking place in the troposphere what are the the cancelling effort taking place in the stratosphere. But that is because of that only there is a net cooling in the troposphere.

At the end of the course if somebody asked why the troposphere is cooling you have a scientific answer to that okay and in the stratosphere everything cancels out. Then we also look that continuing on this equation of transfer. We looked at the Doppler broadening and the pressure broadening and then how can integrate over wave number and if you have a sensor on the top of a satellite how we can capture the radiation coming from clear sky.

How we can capture the radiation through the clouds and how we can do remote sensing basically three types visible remote sensing which is available only during day time. You take pictures you decide whether it is a cloudy day or a clear day then you go to infrared camera. You can get temperature on the cloud top if we can get the temperature on the cloud top. Let us the temperature is -40 degrees centigrade and the surface is 30 degree centigrade.

We assume of lapse rate of 6 you can actually find the height of the cloud globe then you are now into meteorology. And you figure out whether it is a cirrus cloud or the cumulus or the cumulant the cumulonimbus called the CB. The cumulonimbus cloud is a rain cloud usually at a substantial height and we also I talked about the I also briefly mentioned about the anvil okay the cumulus and the cumulonimbus anvil right.

Only pilots will not go into the cumulonimbus clouds and although briefly sometime I mentioned we did not go deep into this. Okay so then there is a microwave remote sensing? The microwave remote sensing the microwave has the capacity to penetrate through clouds but the microwave is  $=h\nu$  is very small. So, current levels of technology do not allow you to put the microwave on a

geostationary satellite y what is the advantage of geostationary satellite.

Suppose this is India so you can have your this is my inside 3d okay so at 36000 kilometers. I can make it always focused like this. So, 24/7 it will give kalpana INSAT 3d all that but a but the microwave I cannot put a sensor and pick up the radiation here. So, the microwave will be at a height of 800 kilometers. Then it will go like this that is an it goes like this it is a polar orbit if it is like the like this it is equatorial orbit.

It will go like an inclined orbit then depending on the height you would figure out. How many times it will cross Chennai or India 2 times a day or 3 times a day 4 times a day okay so they are called Leo lower orbit okay. So, the remote sensing itself we can offer a separate course and all that so then we looked at radiation and climate. Okay the equilibrium temperature of the earth 255 kelvin.

The mean surface temperature is 288 and so on and then we solve some problems. We looked at what is called the radiative forcing 1 kelvin 3.76 watts per meter square and then we looked at climate sensitivity and climate feedback. So, there is a gain called G which is  $\lambda/\lambda_0$  sensitivity with feedback/sensitivity without feedback. We figured out the feedback for whatever clouds clouds what is the net feedback clouds is 0 okay.

So, and then yesterday we ran through a presentation where we looked at various forcings. What about ice age what how do we prove that much of the climate change in the last century is attributable to man or anthropogenic. We finally proved that it is it is virtually certain or it is most likely that the last 50 years or 100 years the change in the global climate and the mean sea level rise.

And all these are attributable to anthropogenic causes and then if you extrapolate what is the situation in the year 2100 for various scenarios. These are called as a1 a2 a3 a4 or b1 b2 in the normal nature of IPCC the intergovernmental panel on climate change they regularly do some assessment reports where large number of scientists work out based on the research and then based on various meetings they release the report.

Now the 4th report have also come I am not I do not have time to discuss that. My presentation is based on the third report you can see that there are projections varying from 0.5 to 3 levins change. The worst is 3 kelvin the worst is 3 kelvin the best is 0.5 that is committed change the carbon dioxide will stay at 390 but that is also not going to happen there is a committed change there is a worst case scenario.

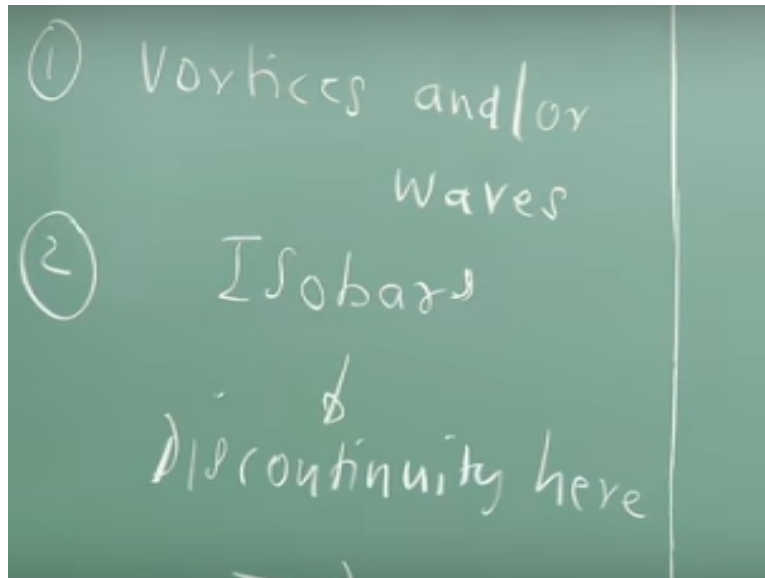
So, let us be aware of this and then let us do mitigating measures there is no point in this or say spreading Casandras saying that predicting tomorrow doomsday and all that as engineers they will ask what are you so what or what is new what is your solution. The solution cannot be given only by engineers because it is a geopolitical, social many things are involved if you ask people to not use their cars it is not going to work.

So, you need to find fuel cell and then they will say how to do it like a petrol bunk do you have petrol stations to refill this thing. Now you cannot run 400, 300 miles with the fuel cell because it is really heavy and so there are lots of technological barriers to be overcome. So, we have to go slow now the Prius the Toyota Prius this is a hybrid vehicle right so whenever you break regenerates okay.

So, there are some solutions carbon dioxide capture sequestration ion improving the efficiency of power plants going solar renewable. So, the various technologies with which you need that 3 degrees raise by 2100 you can bring in down okay that is engineers can give technical advice. But this is ultimately a call which has to be taken by society the policymakers and all that and so we leave it at that stage.

Because at 1 stage science will stop other things will the science if for example when it comes to war you will not think about optimization. When you think about launching a satellite. You will not minimize the cost decide whatever be the cost it has to work right. So, this optimization the best possible scenario all that will be only in normal conditions okay so let us not worry about that now were going to look at atmospheric dynamics.

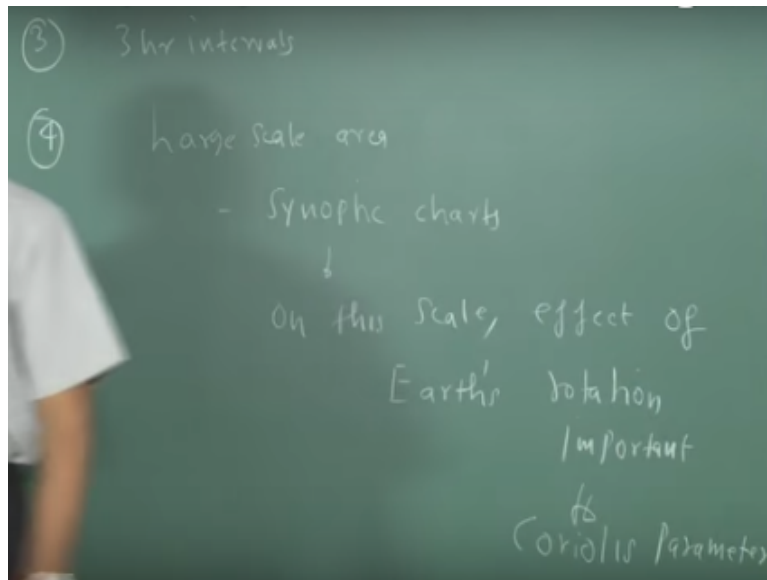
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So, if you look at daily weather charts if you look at daily weather charts you have you must have seen on the internet or TV vortices and waves isobars contours of constant pressure and a discontinuity. How many of you have the next have the next class can you go at 08:55 okay last I will try to finish this discontinuity here is called a front. It is like a shock wave in fluid mechanics the discontinuity.

For example, if you have a cyclone you have seen these are isobars if it is 1013 here it will be 970 or 980 lower the pressure here pressure difference is more. So, you know that higher pressure gradient more will be the velocity okay so if there is a discontinuity in the for example if there is some discontinuity here not in a cyclonic in another situation. We call it as a warm front cold front all that we are not getting deep into that.

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Usually charts are prepared 3 hour intervals and large and so they are called synoptic charts. So, much of conventional meteorology is called synoptic meteorology where the starting point is pressure temperature humidity from various stations you release forecast once in 3 hours. And then you put it on a large chart and then you consider a large scale that is synoptic meteorology whether it will rain in Adyar in the next 3 hours is not synoptic meteorology.

That is very accurate whether it will rain in a football stadium is not conventional meteorology that is high precision. Now casting 0 to 6 hours in a very small region I want to know then technology is different you need a radar, satellite. Radar everything but this if you want to have over a large scale then this is conventional meteorology what is the problem with this on this large scale the effect of earth's rotation is very important.

So, you cannot use a simple xy coordinate not even this spherical coordinates without taking the earth's rotation. The earth's rotation will come in the governing equation which makes the CFD part of the problem complicated you are getting the point effect of earth's rotation important okay so this will come under Coriolis parameter. So, this needs to be included in the governing equations.

Okay next I am sorry I am going very fast dynamics it itself is a 40-hour course in atmospheric science department and in 45 minutes whatever we can really I will try to simplify as much as

possible without glossing over the fundamental concepts okay.

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The image shows a chalkboard with the following handwritten text and equation:

Gov. eqn.

$$\text{N-S} \leftarrow \frac{DV}{Dt} = g' - \frac{1}{\rho} \nabla P + F' \quad (1)$$

ems.

Frictional force

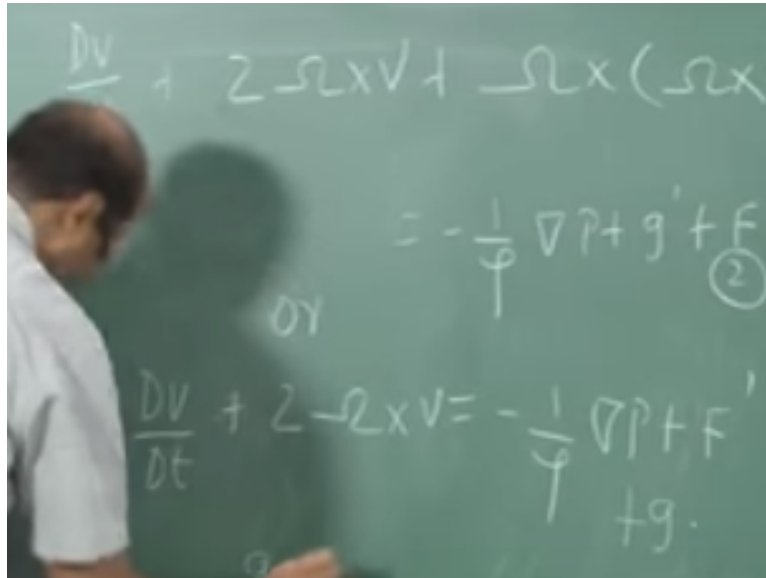
So, the governing equations will be D is the material derivative the rate of change of velocity city you are all know that this is the Navier stokes equation in vectorial form correct. I hope all of you are able to recognize it is the ns equation. So, this is the frictional force this frictional force you have to put  $\mu \frac{du}{dy} + \mu \frac{dv}{dy}$  this is still Stokes hypothesis to  $\frac{2}{3} \mu \frac{d(\text{div } \mathbf{u})}{dy}$  and all that and resolve that.

That cannot be done for the atmosphere because the scale is 30 kilometers or 40 kilometers you can do it for fluid flowing in a pipe so that is a difference. So, you put some bulk representation or some approximate formula for f- okay. If you so you cannot do full numerical simulation like what you are doing for example there is a closed cavity. I want to do full 3 dimensional natural convection situation.

For example, 3 dimensional simulation of flow in this room it is possible to do with modern computers but from Chennai to Singapore and from Sri Lanka to Burma on that domain if you want that resolution that is not possible you are able to understand okay. Then each point you will have millions and millions and millions of nodes the computer will it will just take too much time on it see the memory will not be sufficient.

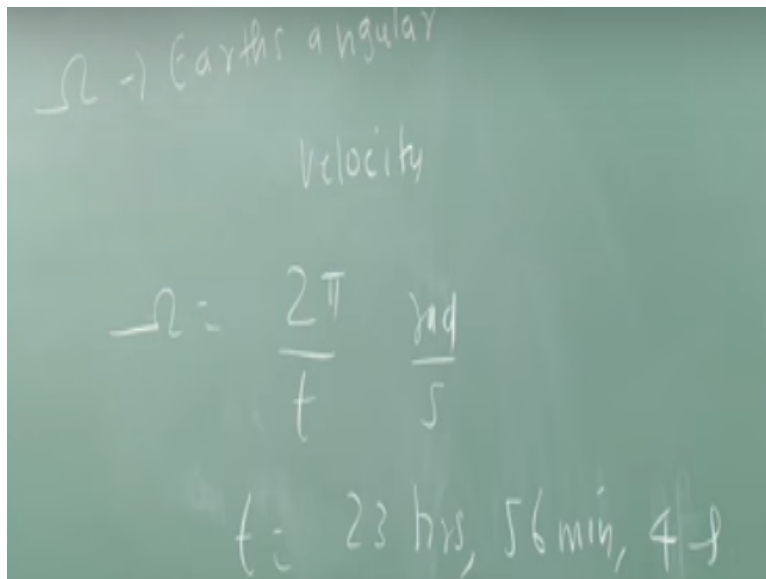
Okay this is for a fixed frame of reference but we need to have a rotating frame of reference correct.

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What is this?  $1/\rho$  that is a curl of the velocity vector right curl of  $v$  okay so  $r$  okay where okay I just checked the exactness of this so that we do not make mistakes yeah that is okay okay so this is a gravity term what is  $\omega$ .

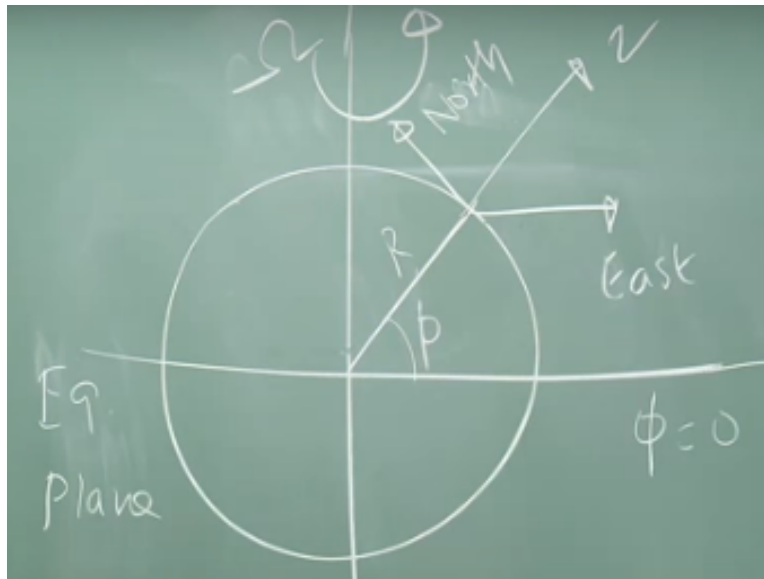
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Okay just calculate this time period is please get  $\omega$  I will just got it yeah sorry  $7.29 \times 10^{-5}$ . So, at the end of the course please remember this value the angular velocity level is  $7.3 \times 10^{-5}$  to the power of radian per second. So, just remember this value okay.



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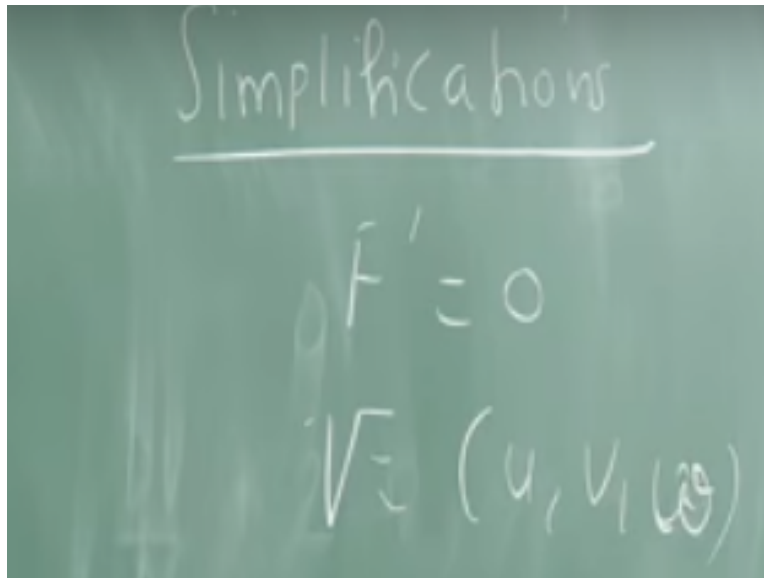


So, I will just draw a picture okay this is the azimuthal angle. So, there is nothing there is no sphere here so this is the earth you cut it here in the equator that is called the equatorial plane okay this is the equatorial plane this one okay that is equatorial plane okay now this  $R$  this is the azimuthal angle. So, this fellow is rotating like this so that I call it as  $\omega$  this fellow is spinning like this spinning like this and then all coordinate system is a little  $Z$  is  $Z$  is here.

So, this I call it as north east north east and  $z$  okay so what are the 3 axis,  $z$  is the local vertical okay the  $xy$  plane is tangent is tangent to the origin okay it is tangent to the surface at the origin okay but the problem is coordinate system like this is valid only near the equator as you go up so some dead end will take place okay. So, something has to be done before that I will give you the what is a curl of the velocity of the vector  $ijk$   $\cos \phi \sin \phi$ .

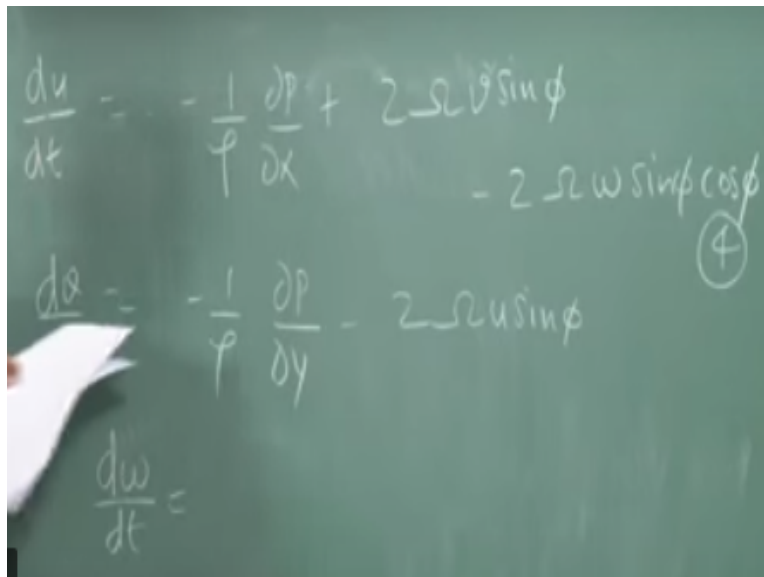
And I will give take the 3 components as  $uvw$  do not confuse this  $v$  with this  $v$  if you want you can do like this what is  $\phi$ ? I have already defined here we want to call it as a model of the earth. Some model of the coordinate system and what we understand from this if you have a north east and  $z$  coordinate and the earth is spinning and the radius is continuously changing from the equator to the pole it is better to go for a spherical coordinate system okay.

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A spherical coordinate system is better than simplification the equations the equations which we saw are formidable to solve okay so we need some simplifications on this. First we said the frictional force is 0 the approximation 1 is okay I mean we have to you have to move on right. So, the friction is set to 0 and v then we get the 3 equations.

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So,  $du/dt$  we can spend hours deriving this but we do not have time, what is the equation number for this 4 5 4 okay so do not get scared we are not going to solve this and in 20 minutes we cannot do much right. So, I am going to simplify further and further and further but I want you to know this because the rate of change of velocity with respect to time for each of the 3 components okay okay.

Now look at what I am going to say u velocity we sorry v velocity w velocity you looked at you studied winds and all that you know the sea breeze land breeze you looked at. Out of this uvw in the atmosphere which is negligible ah this one if you have if you have 15 or 20 meters then all of you will go up and down. So, this w is only a few centimeters per second okay. So, these are the simplifications. First simplification okay so certain terms involving w can be knocked off agreed.

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$$2\Omega w \cos\phi \ll 2\Omega v \sin\phi$$
 Not valid at very low latitudes  

$$2\Omega u \cos\phi \sim 10^{-9} \frac{m}{s^2}$$

$$\ll g \sim \frac{10}{s^2}$$

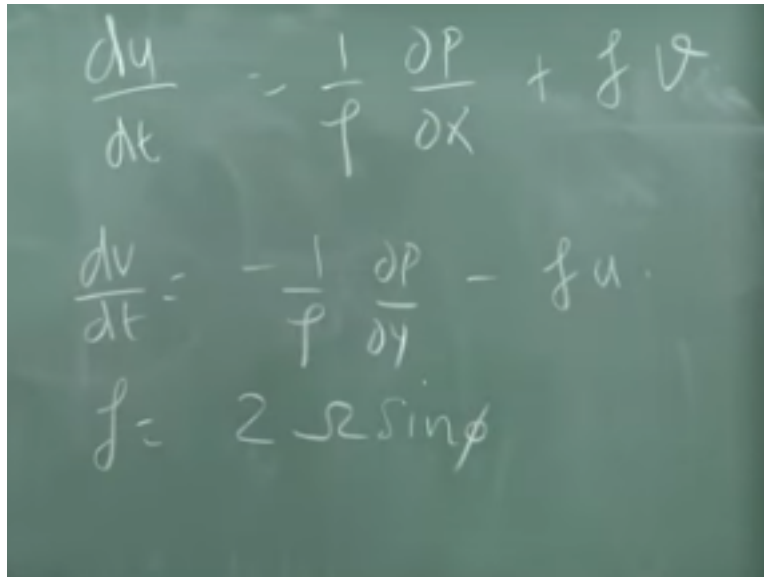
Therefore  $2\Omega w \cos\phi$  is much much  $<$  the  $2\Omega v \sin\phi$  but when will this cup I mean when will this fail when the  $\cos\phi$  when the  $\phi$  is  $v$ . When the  $\phi$  is very small  $\cos$  is very high  $\sin$  is very low therefore  $w$  is much much  $<$   $v$  the terms will become comparable. So, near the equator this will cup because the  $\phi$  is with respect to the equator. So, strictly getting the point for this  $\cos$  will become very close to 1 this is very close to 0.

So, this so is not valid near the so let us keep it or let us remember this **“Professor - student conversation starts”** The straight line the term in the x coordinate of needed when we wrote  $du/dd$  sorry when you wrote  $du/de$  the term was  $2\Omega v \sin - 2\Omega w \cos\phi \sin\phi$   $2\Omega u \cos\phi \sin\phi$  one second let us run through this I will get back to this problem **Professor - student conversation ends”**. Okay except at very low latitudes.

Then  $2\Omega$  see whether this make sense. So, this is much much  $<$   $G$  which is 10 meters per

second square. Now I will write the equation if I made a mistake there you help me correct it so that the people watching the lectures are getting it right. Okay now with all this are you getting the point  $2\omega u \cos \phi$  it is  $10$  to the power of  $-4$  meters per second squared but  $G$  is this thing. So, with respect to  $G$  therefore these equations become.

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$$\frac{du}{dt} = -\frac{1}{f} \frac{\partial p}{\partial x} + f v$$

$$\frac{dv}{dt} = -\frac{1}{f} \frac{\partial p}{\partial y} - f u$$

$$f = 2\omega \sin \phi$$

Okay of course but the  $w$  term does not have the Coriolis component **“Professor - student conversation starts”** Yeah Rajesh, please tell me whether any changes are made in the equation so this is called the Coriolis parameter important thing is  $du/dt$  is a function of  $dp/dx$  and  $f$  of  $v$   $dv/dt$  is a function of  $dp/dy$   $f$  of  $u$  so it is coupled okay. Now you write some numbers for this 5 6. So, we look at what is called the geostrophic approximation **Professor - student conversation ends”**.

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Coriolis

Geostrophic Wind

$$U_g = -\frac{1}{f \rho} \frac{\partial p}{\partial y}$$

$$V_g = -\frac{1}{f \rho} \frac{\partial p}{\partial x}$$

force = Pressure force

Equation 5 and 6 what do they say leave the mathematics equations 5 6 equations 5 and 6 what do they say the inertia term is balanced by the sum of the pressure term and the Coriolis term so we are looking at asymptote or special conditions of this equation. If the inertia term can be negligible then the pressure will be in balance with Coriolis that is the geostrophic I come again if the inertia term is very small the pressure term will be in balance with the Coriolis.

So, why are you doing all this sir question comes no way and what are you doing only you wrote all these equations simplifying simplifying all this what is the beauty suppose this inertia term is difficult to evaluate  $\frac{du}{dt}$  and take derivative and all now this is set to 0 okay I know the Coriolis term right okay so if I know what is the pressure distribution I can get the velocity field I can directly calculate the velocity field from the pressure.

Which is very important for synoptic meteorology without requiring a supercomputer but you can ask me sir when will it fail we can work out something called the Rossby number for a particular value of Rossby number this will not work okay. So, the geostrophic approximation is set inertia to 0 so I can get the 2 winds from the pressure field with the geostrophic wind. Okay so we define a new number called  $R_o$ .

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$$Ro = \frac{U^2}{f_0 \cdot U} = \frac{U}{f_0 L}$$

Rossby no

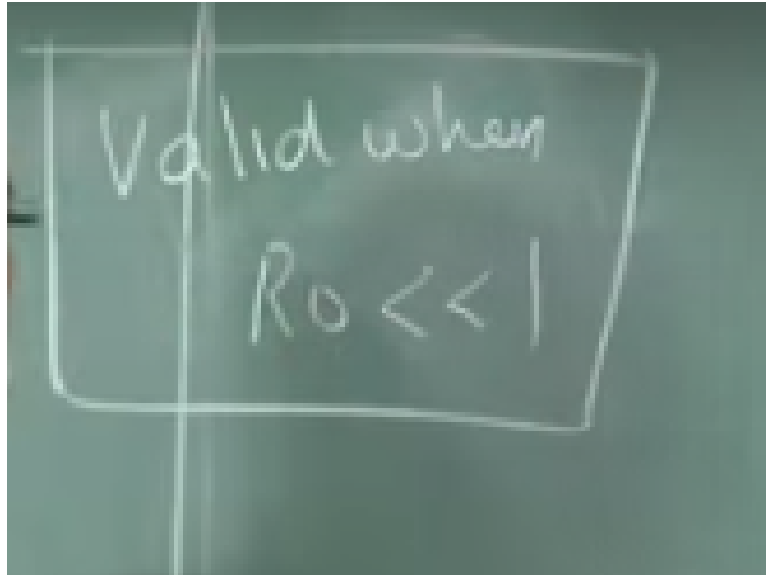
Mid-latitudes  
 $U \sim 10 \text{ m/s}$ ,  $f_0 = 10^{-4}$

Called the Rossby number okay so mid latitudes  $10$  to the power of  $-4$  yeah please calculate the Rossby number for mid latitudes what is that  $f_0$   $10$  to the power of  $-4$   $7.29$   $10$  to the power of  $-5$  approximating it is all right now  $0.7 \cdot 10$  to the power of  $-5 \cdot \cos \phi$  that is cupping I mean not  $0$  I mean so what is that Rossby number  $0.1$  correct. So, if the Rossby number is small geostrophic approximation is valid.

So, what is the take away from this if you are looking at large scale winds in the mid-latitudes without solving a detailed CFD equations you can get a handle on the wind speed up proximately approximately take it to take a twister or a tornado take a tornado  $L=100$  meters what will be the Rossby number very high geostrophic approximation will fail. We cannot use for a small system like the tornado or a tornado twister and so on.

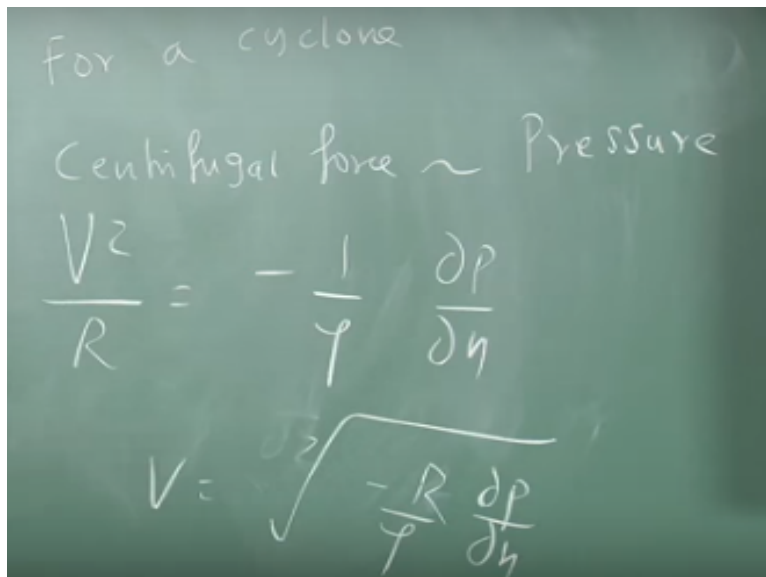
But looking at a large scale geostrophic approximation is good without the computer if somebody gives you pressure I can tell you approximately the velocity  $7$  meters  $8$  meters  $10$  meters per second it will not be way off it is a first cut approximation. Now for these small systems what should we use we use what is called the cyclostrophic approximation that will be the last thing we will be seeing.

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Not valid regardless of the system it is not valid anywhere near not valid to any system near the equator. In the mid latitudes also do not use it to calculate the wind speed of a tornado it will give wrong values understand.

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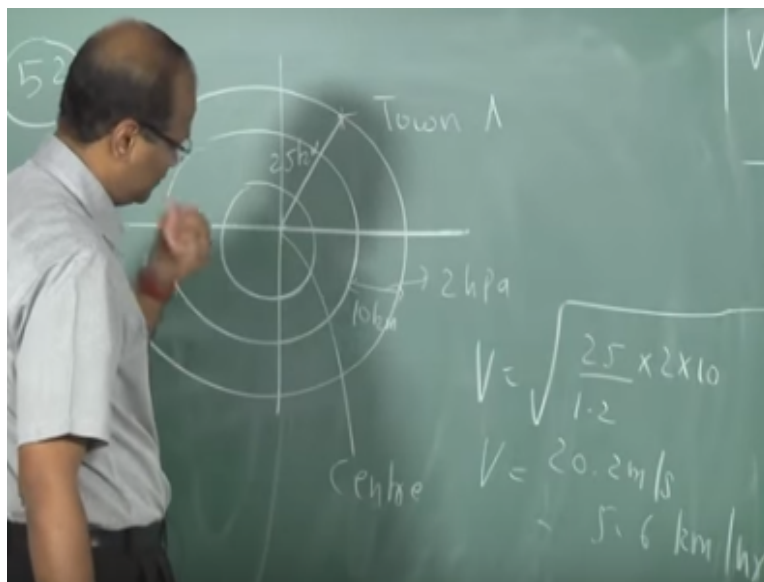


The last thing you are going to study is cyclo strophic approximation cyclo strophic approximation is at lower latitudes  $f$  is small for a system like a cyclone okay where the centripetal force of the centrifugal force is balanced by pressure yes very good the centrifugal force is balanced by the pressure. Then the equation okay with a normal pressure gradient  $dp/dm \cdot 1/\rho$  the minus is to take care of that the  $dp/dm$  is negative.

So, It will give you the velocity R is opposite of the  $\frac{dp}{dm}$  sorry  $v$  okay. The last problem of the course problem number anybody of course okay the last problem for the course problem number 52 okay problem number 52. Estimate the wind speed in a cyclone estimate the wind speed in a cyclone at town A located 25 kilometer located 25 kilometer radially from the center of a cyclone.

Isobars are circular and equidistant isobars are circular and equidistant and are drawn at intervals of 2 hPa. The distance between two isobars being 10 kilometers.

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The distance between 2 isobars what is the distance that I gave you yeah please calculate the maximum speed calculate the wind speed. So, you know  $\frac{dp}{dm}$  density you take it as 1.2 density you can take it as 1.2 kilogram per meter cube how much is it 20 meter per second okay let us do this  $v=rs$   $25 \times 10$  power divided by. **“Professor - student conversation starts”** So  $v=$ , where is the density okay 20.2 what is the speed in kilometer per hour good speed yeah 5.6.

Now if you take the radius to be 50 kilometers and this is 4 for example let us take this is 1 the other 1 is for every 10 kilometers it is 5h power what is  $v^2$  and do it for town B at 50 kilometers town b is at 50 kilometer right 4 h power for every 10 kilometer that is  $v^2$  what is it. So, as the isobars get crowded as isobars get crowded the gradient increases. So, the cyclone become more and more powerful the winds will be higher. **Professor - student conversation ends”**.



I leave it as an exercise to you if you use the geostrophic approximation you will get an absurd answer you have that  $u = 1/f$  of  $\rho \cdot \frac{dp}{dy}$  if you do that take the  $\frac{dp}{dy} = v \frac{dp}{dy}$  use the Coriolis you will get some silly answer okay because you are not supposed to use the geostrophic approximation for this. So, that brings us to is it okay so that brings us to the end of the course.

So, I already summarized the course in the beginning itself. So, in this 40 hours we looked at various components of the earth system then the thermodynamics and then the radiation in climate radiative transfer climate science and atmospheric dynamics. The takeaway is basically climate is whether average over time, weather is largely dependent on many things like not any solar radiation but winds and other phenomena.

The climate is largely determined by radiation last 100 years the climate of the earth is definitely changing and is more like it is virtually certain that it is because of anthropogenic causes. There are some mitigation measures which you can take with which you can reduce or mitigate this increase various tools are available for you to study the weather. Various tools are also available for you to study rain, tornadoes various storms.

And with the help of numerical modelling things and satellites we have reached a stage where we are able to predict next 6 hours accurately what happens that is called now casting next 72 hours what happens is called forecasting and we are also able to do seasonal forecast of monsoon then and long term forecast is basically climate sensitivity to climate change studies and so on. So, it is relatively new discipline <100 years old but lot of excitement is there.

Because it can combine analytical methods, numerical methods and experimental techniques and satellites and so on. So, if any of you interested in going further studies you are encouraged to take advanced courses research do your research or project up. Because it is directly related to the environment. Thank you very much.