Radiation Heat Transfer
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Lecture - 38
Monte Carlo Method

In the last class we had begun our discussion on Monte Carlo method.

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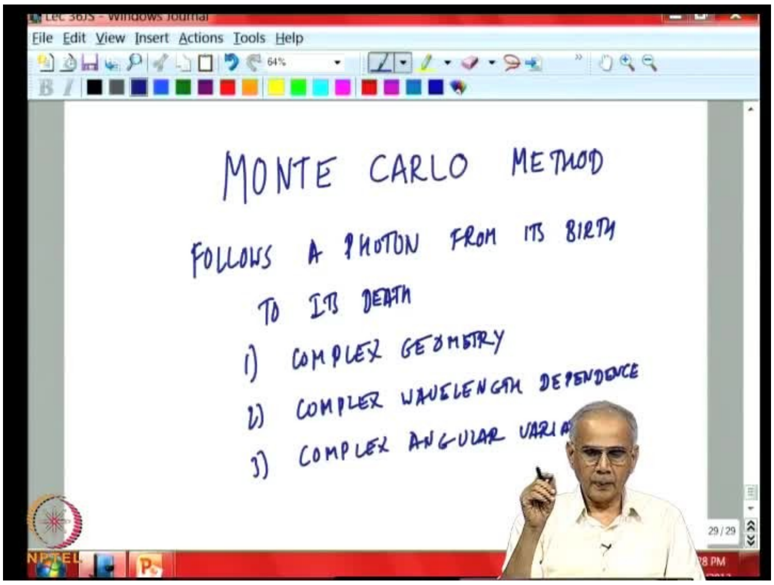

MONTE CARLO METHOD

FOLLOWS A PHOTON FROM ITS BIRTH TO ITS DEATH

1) COMPLEX GEOMETRY

2) COMPLEX WAVELENGTH DEPENDENCE

3) COMPLEX ANGULAR VARIA

This method is extremely powerful today because it uses the high computing capabilities, that we have today and also the advantages of parallel computing. Essentially, Monte Carlo follows a photon from its birth to that its death, through the path of photon once, it is emitted through the medium until it is absorbed either by the medium or by the surrounding walls.

The basic Monte Carlo method has the advantage that it can deal with complex geometry. As you recall earlier in the course we always dealt with simple geometry like parallel plates, cylinders and spheres. For those kinds of geometry conventional techniques of solving differential or integral equation is appropriate, but when we have a very complex geometry it is very difficult to solve the problem by conventional methods.

Monte Carlo method does not worry about complexity of the problem, the only challenge is, and it takes certain minimum time to set up the problem. Hence, once the problem is set up, it can handle a problem with very complex geometry without any difficulty. Essentially, it is a method which just follows the life of millions of photons through complex geometry and complex wave length dependence. If we recall it was shown that radiation transfer gases is very complex because that millions of lines.

And at the center of the line there is a strong absorption and at the wing of the line there is very weak absorption, we have to take into account all this large variations absorption between the center and wing of the line, There are millions of them and its computationally very expensive, but if you follow the Monte Carlo approach, it is somewhat easier because you can deal with the absorption, and emission of photons in parallel. You can take many, many photons in the calculation this can be followed through in parallel. That is a huge advantage..

So, of the complex angular variations, we saw that in the case of scattering. The phase function that is the probability of a photon being scattered in a given direction is very complex, with a lot of very sharp changes and that also pose a problem when we attempt to solve it, and again in the case of Monte Carlo, it can handle angular variation, wave length variations, complex geometry all that it can tackle quite easily once a problem is set up.

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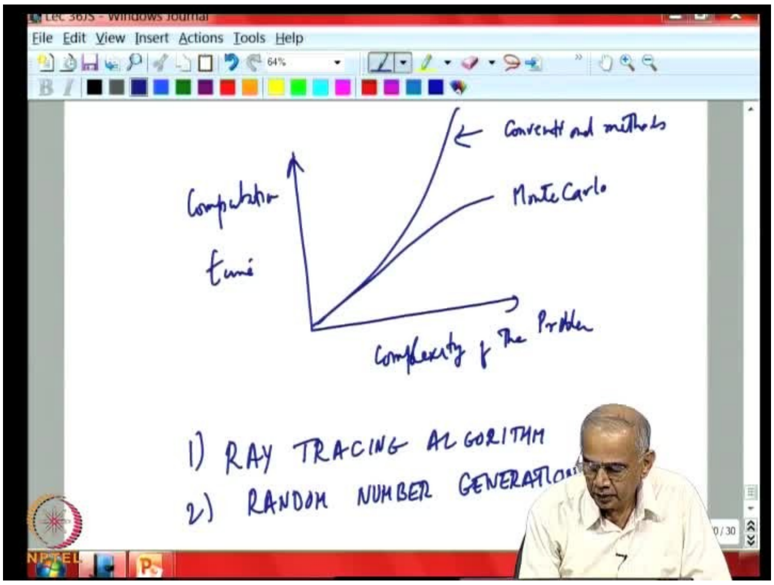

1) RAY TRACING ALGORITHM
2) RANDOM NUMBER GENERATION

As the matter of fact in general you think of the complexity of the problem on the x axis, the computational time, in the y axis, as problems become computational e time goes on increasing. Most conventional methods that solve integral and differential equations, as the complexity increases the computational time goes on increasing, but the Monte Carlo problems are usually more like this. As the problem difficulty increases the computation time does not increase as rapidly, as it is the case with conventional methods.

In Monte Carlo we do not solve any differential equation but simply follow millions of photons, from its birth to its death through changes in direction and reflection. We can see that for successfully use Monte Carlo, we need ray tracing algorithm. We should be able set-up the geometry. That is see what happens, if the photon leave is one surface, how it will change direction, where it will be intercepted. So, all that has to come from the ray tracing algorithm and today there are plenty available, we routinely use already for other purposes. This can be assumed to be available.

Second we need a random number generator. Now, this is very important in Monte Carlo because you are going to follow millions of photons through its life, we should take care to select these photons, which wave length, which angle, will have to be done through a random number generator. We need a very good quality random number generator that when you take a sample of photons these random numbers generated do not repeat. So, today again we have very powerful techniques available to ensure that, the random number generated is truly random and does not repeat itself, in short number cycles.

Now, we pointed out that Monte Carlo method involves following millions of photons. Now, we need to ask how we know that number of photons required is, not in thousands or billions. We can definitely sure that Monte Carlo method will not be very good, when you are dealing with 100 or 200 photons unless it is very simple geometry. As you go to more complex problems, the number of photons required will go on increasing, but how many photons do we really need to get a accurate answer is really not known a priori.

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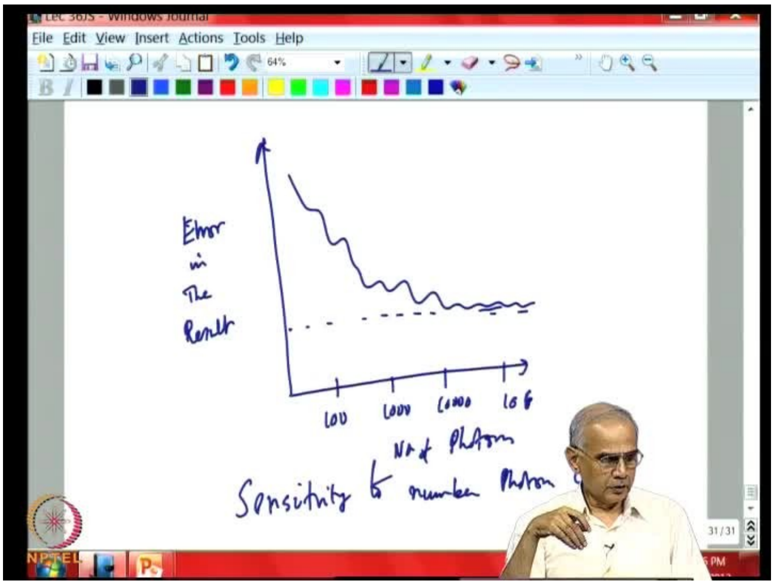

As there is an increase in the numbers of photons gradually from hundreds of photons to thousands, then we go to ten thousand we go to million. And find out what is the error in the result. This can obviously done only in a standard problem, for which we have a solution available from a different technique. We will find that as the number of photons increases, error comes down and it would not come down smoothly actually, it will may show a rather here and ultimately, it will asymptote to a low error after about may be ten thousand or a million photons.

And as we do in the case of convergence of infinite series, you want to take more, more terms in series we stop, when the result does not change, the more than say 0.1 percent. So, same thing will be done here, we will go on increasing the photons until error comes below the value that we have set. This is of course, true only for a problem for which the answer is known. We have Monte Carlo techniques and software already available that is tested on some standard problems, for which the results are quite well known. And based on that we make a judgment, about how many photons are required in a problem.

For a problem for which no result is available, for that you need to do some guess that you start with the some assumption, you will increase double the number of photons make it higher and see whether your results are sensitive to the number of photons though the sensitive analysis is absolutely essential. Especially, one must remember that one has to set their own goal for what accuracy they want 1 percent, 5 percent, 2 percent depending upon the specific situation. Then guess what should be the minimum number of photons required, for the number of photons then increase a number and see whether that is changes substantially. So, let me give example.

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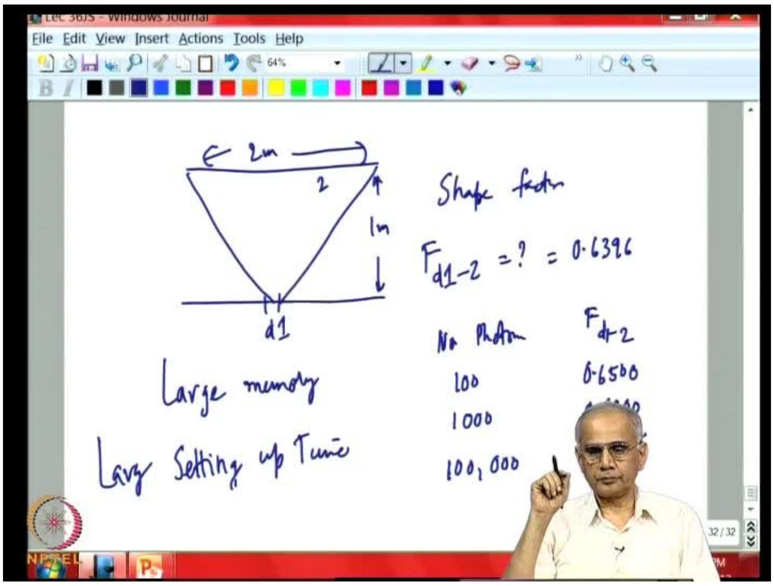

Suppose, we want to find the shape factor which we discussed earlier in the course, using standard integration procedures we start with some element here. We want to know what is fractionating leaving d 1 and reaching 2. So, for a typical example let us say about 1 meter. This is about 2 meter, this is a very simple problem for which solution is available and let us say, we know the answer is 0.6396 for this simple problem.

Now, we run the same problem with the Monte Carlo, let us say we start with 100 photons and then find what the Monte Carlo telling us. It is 100 in this case this has been done by Russel, he gets a value of 0.650. This is not bad it is a pretty close to what is obtained by an independent method, but suppose you are not satisfied the error here is of the order of 0.01 by 0.6, how about 6 percent. Let us say you are more ambitious you want to more accurate answer than you go for 1000 photon. The answer for that comes to be 0.640; this is pretty good here error is 0.0004.

The error has really has gone down substantially that you still not satisfied you can go to 1, 00,000 photons. So, at least for this problem we know that you do not need million photons, a hundred thousand photons will give us an answer accurately place, but every new problem you have to worry about, how to choose that number which will give the required accuracy, this only comes by experience by doing a large number of problems.

Also we must know that solving Monte Carlo problem on a computer, requires large memory for all the ray tracing and keeping track of all the photons. And it has a large setting up time. So, when we are dealing with a complex problem, the entire geometry has to be specified in 3 dimensions so that the ray tracing can be performed.

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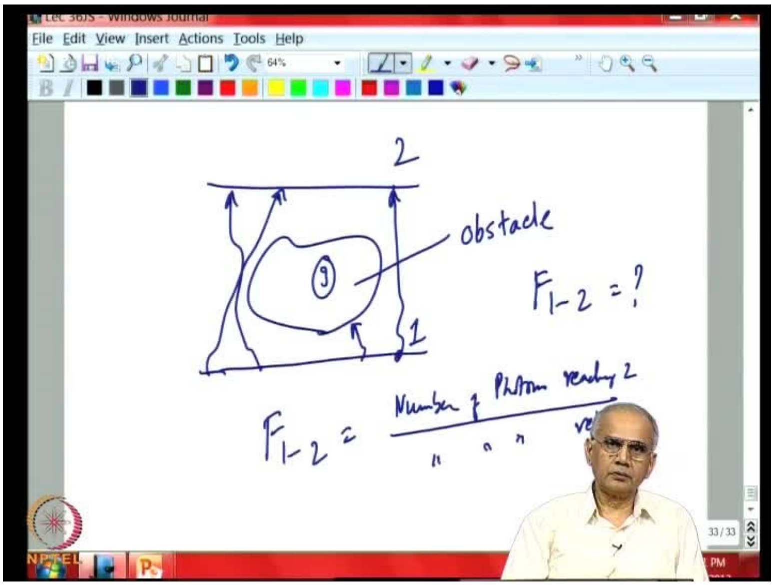

Suppose, we had a problem and we are dealing with such a problem earlier. Suppose, we had 2 objects and there is an obstacle here. We want to know what the shape factor between 1 and 2 is. This method is very difficult to do by conventional integration techniques and so, such problems are not normally done by traditional integral valuation, this problem is ideal for Monte Carlo because once the 3-D geometry is specified. The ray tracing algorithm available, we just have to choose a large number of photons and find out what fraction is able to reach surface 2, and what reaches surface 3 the obstacle.

We just have to release ten thousand, hundred thousand or million photons and find out how many photons are reaching, the surfaces 2, how many are reaching the surface 3. For example, F 1 2, as number of photons reaching 2, where number of photons released this is the F 1 2 provided, the number of photons will be large of the order ten thousand, hundred thousand and a million.

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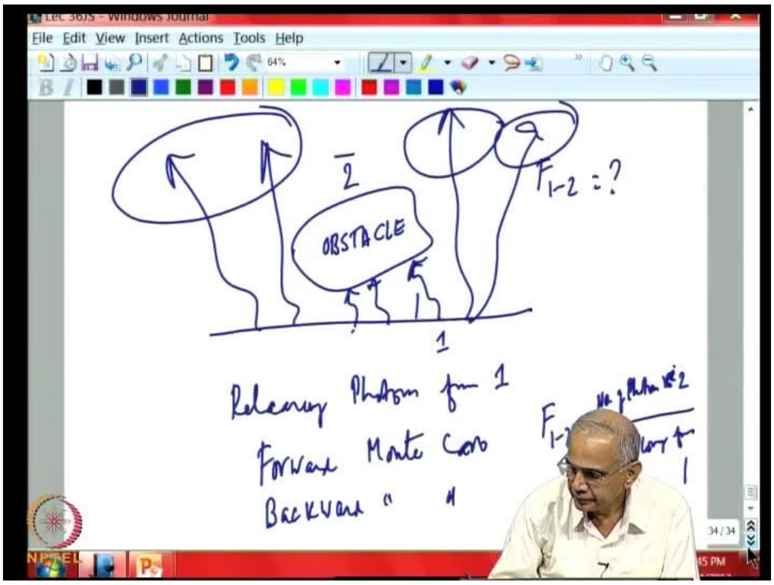

Now, suppose we have a problem with a large surface and a very small obstacle. We want to find F 1 2. Now, since surface 2 is very, very small large number of photons will not reach it. So, most of the photons you are releasing from 1 will not reach 2. This will be releasing photon from 1 will be a very inefficient method because most of photons are not going to reach 2.

This method is called the forward Monte Carlo, which involves releasing photon and seeing where it goes is not the best way to do it here since, we are interested in finding surface 2 is more efficient use of backward Monte Carlo you trace the photon backward, when the place where you want to know how many photons are arrive. So, surface show you trace photons backwards and ask very it came from. This F 1 2 will be numbers of photons reaching 2 by number coming from 1.

We have trace the photon 2 back to find out how many of them, must have come from 1. So, essentially the technique is same photons reached by 2 photons leaving 1, but here we just do not release photons from 1 and see how many reach 2, we start photons which reach 2 and ask how many of those photons came from 1. This is much more efficient because we are not tracking all other photons, which are going all over the place, which not intensive.

So, quite clearly where we use forward Monte Carlo or a backward Monte Carlo depends on a specific application, if the application involves a detector or some other object, then it is good to do backward Monte Carlo by starting from that detector, or wall which will interest you, and ask how many photons that reach there, where did that originate from, you trace it backwards. So, ray tracing of the forward or backward is not going to be different. Now, let us go through a typical Monte Carlo procedure to understand what happens.

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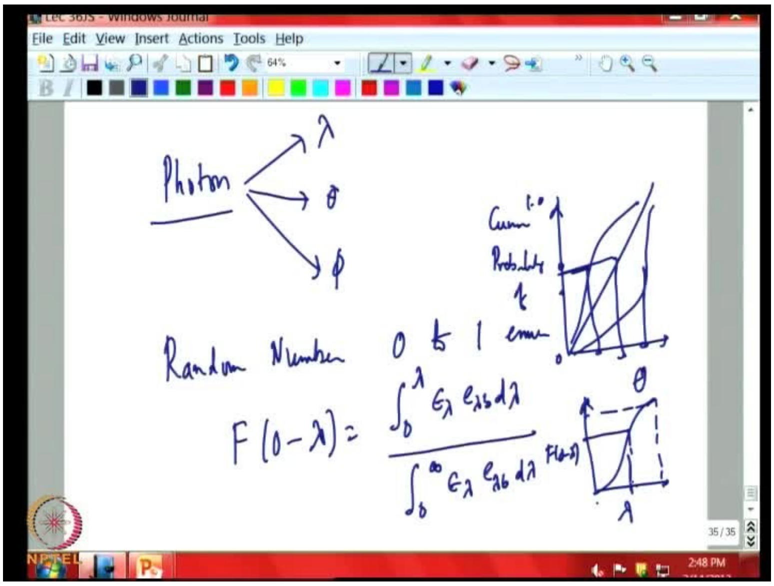

Random Number 0 to 1

$$F(0-\lambda) = \frac{\int_0^{\lambda} \epsilon_\lambda e_{\lambda b}\, d\lambda}{\int_0^{\infty} \epsilon_\lambda e_{\lambda b}\, d\lambda}$$

So, first let us think of a photon so as soon as you think of photon wave length, you have to think of its 2 angels. That is photon has got a label, it has certain wave length, certain angel theta and phi, how do you find that. We have a random rate generator and random rate generator is designed to get a random number from 0 to 1. These are available you can scale any general random rate generator and normalize it so that it is going in between number 0 and 1.

Now, let us take one example suppose, we look at a surface which emits uniformly in all directions, if that is the case the probability of emission, cumulative probability as when a theta will be a linear function because all areas, all angels are equally probable if you have a diffuse- isotropic surface. Then the cumulative probability emission with a theta will be linear function. On the other hand if the surfaces non- isotropic or non-diffuse, it may follow some other path. This is 0 to 1. We pick a number 0 to 1. So, let us say 0.6 then from 1 of these curve which depending upon which surface you are in, diffuses isotropic or non-isotropic you can find what a proper angle is, and that is angle at which that photon emits radiation.

So, fixing angle theta and phi based on the actual property of the surface as given to you can be achieved. Next, coming to wave length we know the fractional radiation emitted from 0 to lambda is. This again can be shown as a cumulative probability distribution where lambda increases comes 0 to infinity, this will go like that. Now, again we pick a random number, we would say 0.8 and for that there is a corresponding lambda. We have chosen what the lambda is. We now got a photon with a label.

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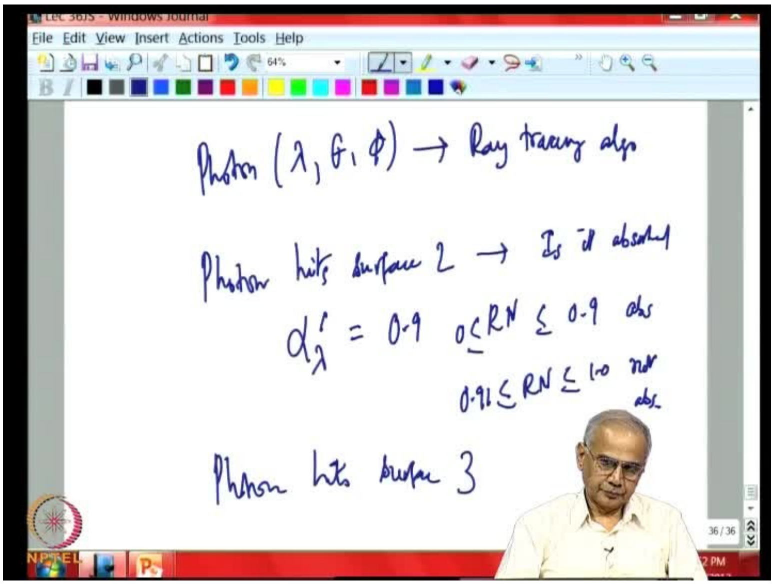

Photon $(\lambda, \theta, \phi) \rightarrow$ Ray tracing algo

Photon hits surface 2 → Is it absorbed

$\alpha'_\lambda = 0.9$ $0 \le RN \le 0.9$ abs

$0.91 \le RN \le 1.0$ not abs.

Photon hits surface 3

So, label of the photon is it has a certain wavelength of emission certain theta and certain phi. Now, you go to the ray tracing algorithm and ask whether, this photon is being intercepted by any object. So, let us first easy case of a non-participating medium. The photon is emitted by surface 1 and it keeps traveling, we are using a ray tracing software, till that photon hits another surface. So, let us say photon hits surface 2 then you need to ask a question if it absorbed. So, let us say the directional absorptivity of that object is 0.9.

We will take the view is that is the random number is between 0 to 0.9, it is absorbed if it is between 0.91 and 1 it is not absorbed. So, random number generator will determine whether that photon is absorbed or not absorbed. Let us say it is not absorbed you have a random number of 0.91. That is not absorbed so the photon is transmitted or reflected. So, let us assume that there is no transmission and hence it is reflected once it is reflected it changes direction. Now, you have to give the new direction depending on the property of that surface, the directional property is diffused specular or in-between and once you know that you can determine, what the angel is by random number generator.

To find a new direction then you have to keep going until the photon is intercepted by let say surface 3, then again you have to ask using a random number whether; this photon at this lambda is absorbed by this surface. So, once random number generator a number through which determined where it is absorbed or not absorbed let us say in this case it is absorbed. That is the end of that photons life so that photon counter is closed. This surface where it is absorbed is given that much of energy.

Now, the same process will used to follow millions of photons released from surface 1. So, at the end of the exercise you will know of the million photons that was released from surface 1, what fraction reaches surface 3 and that will determine the heat load on surface 3. This is the simple procedure for the Monte Carlo for an enclosure, in which there is no medium. This is very simple and many, many situations people will write their own software to calculate shape factor with Monte Carlo, but when you deal with a participating medium, life will become little more complicated.

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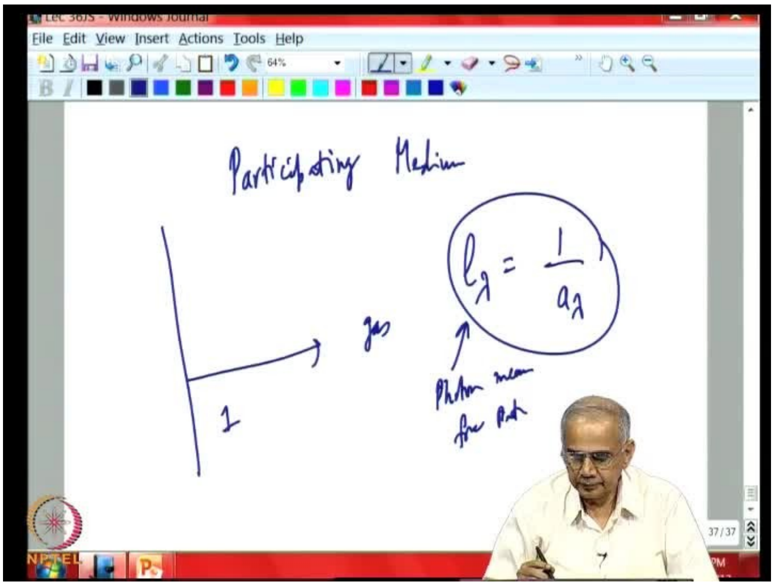

The photons leaving surface 1 is entering gas, let us say which is absorbing and scattering. So, as soon as photon leaves surface 1, you have to decide how far it will go. We have to find a photon mean free path, this gone take as 1 over a lambda for example. So, once you on the distance again as a photon leaving surface, you would keep generating random numbers to decide whether, that photon is indeed absorbed or it is scattered if it is absorbed again the story is over, you have just keep count of this energy being gain by that surface. That would conclude that photons life and hence you go for another photon. Now, with photons that are not absorbed they can be scattered.

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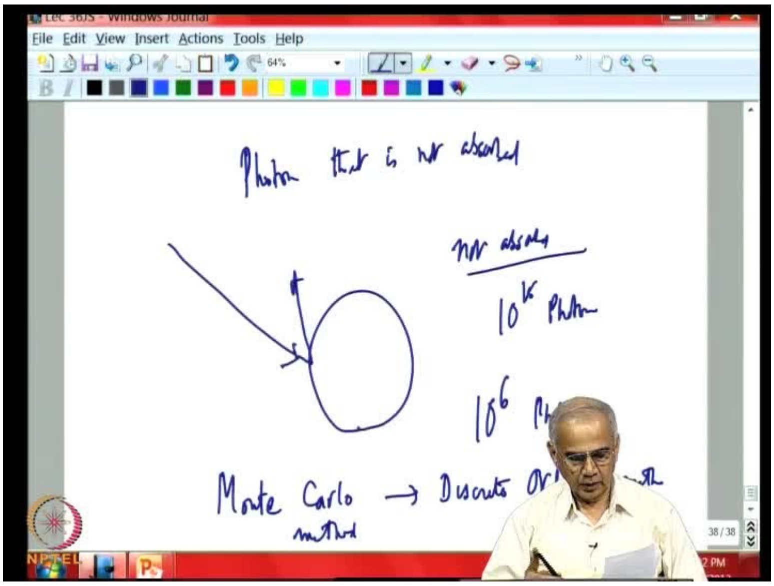

That is now a new story radiation came from some place and hit a particle that particle does not allow the photon to be absorbed, it reflects in some other direction We have to know what direction it is, what direction it has reflected. This is determined so that by random number generator using another random number. Ask yourself what is the probability for a ray to be scattered and not absorbed. This simple problem in which the photon is not being absorbed is, it is scattered and anyway scatter depends on the reflective properties of that surface. Once, it scattered again you keep following it and it is another photon and it may be absorbed or it may be reflected. This sequence of events goes on.

Although the process looks very long and tedious in terms of setting of the property, data and then the ray tracing software and integral result. Remember that this is the only methodology available to us to deal with complex geometries. Complex surface property surfaces whose absorption varies strongly with wavelength, such surfaces cannot be treated easily by traditional method like emissivity method or band mode l method.

Now, a number of techniques are available and a number of software available, to perform all these actions without the need for writing any specific Monte Carlo code. Standard codes are available you for We to calculate the amount radiation absorbed in any one surface given emission from a surface, gas interaction and finally reaching some other surfaces. Now, we mention that typically solar radiation that is emitted has about 10 to the power of 15 photons. We might wonder why people make do with a million photons.

The answer to that depends only on your sampling strategy, as most of us know today in many, many areas whether it is consumer preference or whether it is voting pattern in a democracy is determined by sample survey. And depending on your particular problem, you should be able to judge what fraction of the total photon that is emitted is enough to determine, the property that you want. So, although it is desirable that we release 10 to the power of 15 photons, it is not necessary because after certain number photons, the results will stabilize. We may get required accuracy after simulating with a million photons using truly reliable random number generator. That is the message that you have to take from various examples that what you see in the literature.

Now, we will not discuss specific details of the way Monte Carlo techniques use the process of rate ray tracing or determining the absorption of the photon. We will not going to those details, but we should point out that all the Monte Carlo techniques are undoubtedly superior to other techniques for a complex geometry. One consequence of using this kind of technique is, is not easy to determine, , what is that total error in a computation requires lot of testing, lot of expertise is built up by training the code for many, many cases to decide under what condition when need one billion photons. And when you can make do with a hundred thousand photons, or when you may require ten million photons that cannot be specified apriori it has been determined by trial and error and as you get more expertise, in running your code and getting the answer more or less right.

If that expertise can obtain in a short time then you can easily use Monte Carlo for number of applications, but before using every application one has to be sure that for that application, sufficient number of photons has been used. There are no simple thumb rules to tell you whether, a million photons are required. If you look at complex geometrical problem generally, if you look at the literature they all shows that Monte Carlo is far superior to let us say, what we discussed in the last lecture, the discrete ordinate method. So, various examples done for real life examples like boilers, glass tank furnaces, and such examples they all demonstrate that in the real life complex geometries that we encounter in engineering applications Monte Carlo is superior.

Monte Carlo generally superior to all other techniques discussion this lectures, that is discrete ordinate experts can approximation may be a finite volume method. So, as long as the geometry is complex Monte Carlo always has an advantage over traditional methods, but the decision to take use Monte Carlo must be based on how often you have to do the simulation. Now, as pointed out earlier the real difficulty in Monte Carlo is the need for setting up, the problem for a given geometry.

Now, that will take a minimum amount of time if that is large amount of time and after doing that you are going to do only one simulation then surely, it is not a very effective use of Monte Carlo. So, Monte Carlo should be used in situations where you need a large number of simulations, under variety if conditions. If that is required then it make sense to spend sufficient and effort to set up the problem, and do a lot of validations Then we were ready to do the full simulation for a variety of situations.

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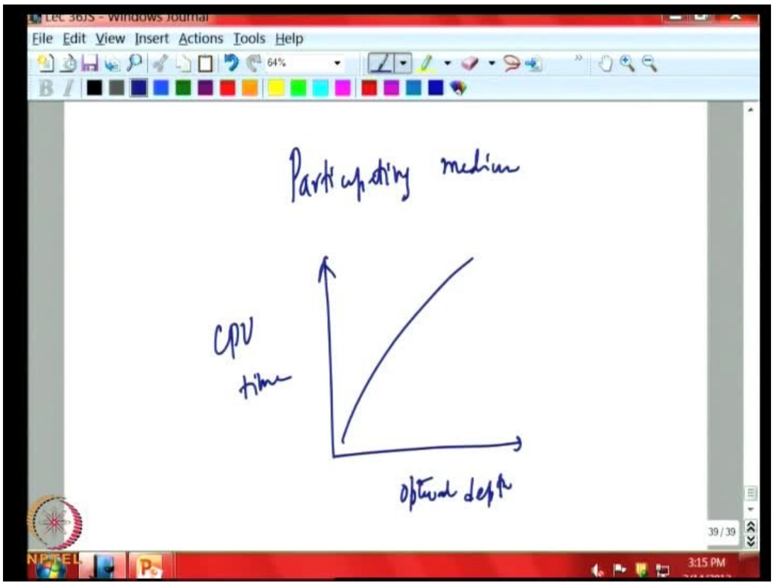

That is we must remember that in a participating medium like a gas or a particles, the CPU time required will depend on optical depth. So, optical will be increasing you will find that CPU time will be increasing This is very easy to understand, as long as you have a gaseous medium which is absorbing, scattering and emitting radiation. If it is a optically thin medium it is hardly absorbing and scattering.

The number of events that we will see inside the enclosure will be small; most of the photons will leave one wall and reach another without much change in direction or loss of photons. In such situations the Monte Carlo simulations can be done very rapidly with a few number of photons, but if there is strong absorption, emission and scattering in the medium, then there are large number events that we have simulate We does really need to simulate using a large number of photons, and naturally take you more time to do the computation.

This issue has to be kept in mind and taking up a particular problem from Monte Carlo, but it is quite clear that in the future, most radiation heat transfer problems will be done through Monte Carlo. These are going to happen because better software will be developed in the coming years; the computers are going to get faster and less expensive. And this entire means is that the desire of engineers to solve complex problems can be met and hence, the use Monte Carlo will become more and more common. Already there are books on radiation transfer which comes along with the C D containing Monte Carlo software.

Even in teaching one will see Monte Carlo being used as a familiar package, for students to understand radiation transfer because the traditional method of solving a set of basic equations in a differential or integral form will become less common. The industry is going to solve most of the real world problems using Monte Carlo. Then there is not much merit asking students to solve these problems by the traditional techniques of solving different integration equations. It may better that students get familiar with, the confidence in using the Monte Carlo software to solve fairly difficult problems, in the class. And learn about the complex behavior of radiation transfer, through a large number of simulations with a Monte Carlo code.

Now, the reason why we are saying this is, this kind of thinking is already prevalent in few mechanics where earlier, we learnt about fluid flow in complex geometries by going to the laboratory and using a wind tunnel, and water tunnel to see what kind of flow patterns are created in the real world, but today that work can be done by numerical simulation in a very short time, and in a very cost effective way. The need to go to the laboratory to learn about fluid flows is becoming less and less

In the aeronautical industry, the first aircraft which was designed and flown by the Wright Brothers had no simulation everything was done by the Wright Brothers using the practical knowledge about control about what makes an aircraft fly, and everything was done empirically nothing was done with simulation, But if we will look at the most recent aircraft it is being released in market, you will find that a large fraction of the money The effort in designing the modern aircraft, goes in stimulation because simulation is very good to tell you about errors in design or give you an insight about better design.

That is why the industry prefers to use simulation packages, if they are easy to run and learn from that what geometry what conditions will make it easier, but of course, the final design has to be tested. We need ultimately some kind of experiment to verify your computer simulations, but most of the work done through computer simulation this is now became more and more common in radiation transfer also. All the Monte Carlo will not completely supersede the traditional methods, but we believe that this will occur in the next few decades because of the power of Monte Carlo to solve any complex problem. The challenge of setting up the problem quickly, has to be met and we do expect better software will be available that the user can setup the problem very quickly.

And once that hurdle is crossed, you will find Monte Carlo being used in class rooms, in industry and any other place because it has the promise of being able to solve, the most complex problem 3 dimensional problems that we normal encounter, in radiation heat transfer in engineering. So, devices like boilers, l solar collectors, radiation through fibers media all these kinds of problems, which are 3 dimensional, complex geometry cannot be solved many conventional techniques that we have discussed in these lectures, but these lectures will not become irrelevant.

Although in practice Monte Carlo will become more and more common, but to get a physical insight into the way radiation behaves, it is necessary to solve few problems using the conventional techniques. That you get a physical feel for how radiation heat transfer behaves, we saw large numbers of examples, in this course and in which we used traditional techniques only, about the behavior of a photon being similar that of fluid, provided that photon mean free path is small compared to the scale of the problem.

And similarly, we saw that when you have very small photon mean free path we can use the differential approximation to solve these problems. So, look at the solution to complex problems through Monte Carlo various sub smaller parts of a system that behavior wants to be able to qualitatively think about its behavior, and be a related to a simple solution written in the class. So, all though all practical problems will be ultimately solved by Monte Carlo, the physical intuition that is necessary to design complex systems comes only by simple techniques that we will use to solve problems.

So, all though we ultimately desire to solve 3 dimensional problems we still can use the results from very simple and ideal geometry like flat plate, cylindrical or spherical those are necessary, in order to get feel about the real world, but the real world problems are all also all Monte Carlo, but once results come we should able to judge with this results really makes sense or not, and that intuition can begin primarily by solving some simple problems and also of course, by doing a large number of simulations in Monte Carlo. A few times most students will get a grasp of why a certain pattern of behavior is seen in certain problems.

So, although a certain technique will ultimate dominate, in view of its advantage in dealing with complex problems still so, many simple problems of infinite plate or infinite cylinder or a symmetric sphere all this cases that were discuss in these lectures are not real world problems, they are idealized problems that we did in order to get some feel for, how the real problems should behave. That when the simulations by computers are obtain and graphical display generated, we should be able to judge whether answer looks right or wrong.

And that can be primarily be done only through these simpler problems, we have done in this course which are not really directly relevant to industry settings. Now with that we close our discussion on Monte Carlo. The best way to learn Monte Carlo is really solve a large number of problems with the Monte Carlo package, and get a feel for the way Monte Carlo behaves in practice. Now, we move on to couple of other small topics, which we had not completely covered in earlier part of the course.

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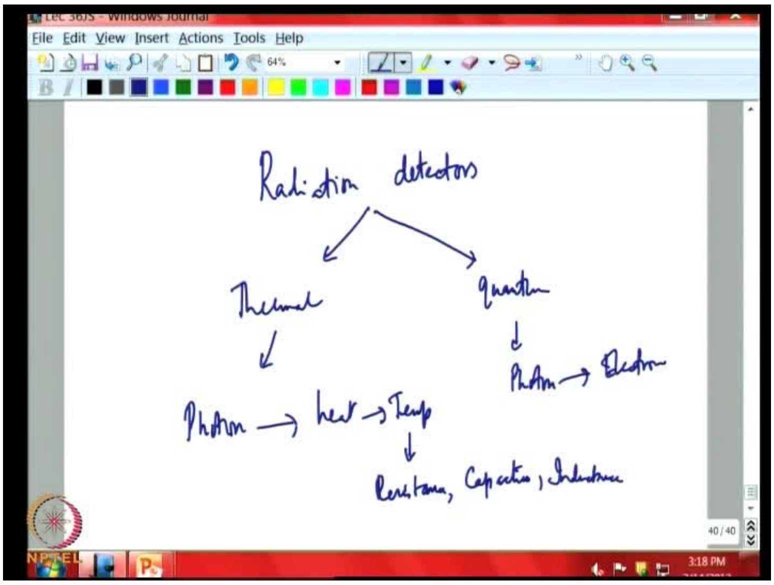

And one of them we sorted out is the radiation detectors, we had viewed covered it in one of the lectures, but not that gone through the complete. Now, there are two major types of detectors we have pointed out, one is thermal that is the radiation absorbed by the detector causes at temperature change and that leads to a property change, which can be measured. Now, the more common kinds of detectors that are now coming into the market, which are getting popular are based on quantum effects. So, with the thermal detector you converting photon to heat and hence, temperature and that temperature change can be directly measured or we can look at the change in resistance or capacitance, or inductance, induced by the temperature change.

So, large number of detectors exists, today for measuring radiation by allowing the radiation to impinging on the object to be absorbed. The temperature rise is proportional to the incoming radiation, but the more popular ones which are now coming to the market are really, the quantum detectors which convert a photon ultimately to an electron. This is a much more elegant direct method because it can be much more accurate because of each photon induces, a change in the sensor, which causes an electron one or more electrons to be released. This technique of course, will be very sensitive because it measures one photon at a time. Now, the advantage of this is that suppose, you have a very, very big source of radiation like an example we can give is detection of fires.

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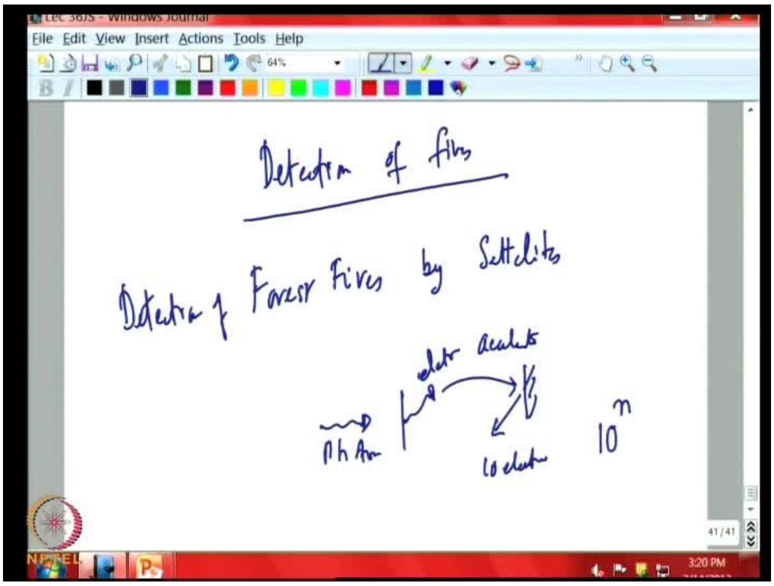

Detection of fires

Detection of Forest Fires by Sattelites

Now, today forests fire can be detected or even manmade fires are detected by satellites. Now, the amount of radiation from the forest fire, a small fire that reaches the satellite is quite small for satellites that are at 800 kilometers or even higher height and the amount of photons reaching there is very, very small. These modern sensors are able to detect them because the photons that impinging on the censors releases electrons these electrons are accelerated now, by electric field. That there is high energy then it hits another detector and for each electron that comes out here, 10 more electrons are released from another detector. We can multiply this many, many times.

If the number of photons it means 10 times and we do it. To do this 10 times you get the 10 to the power of 10. So, such large amplification means that you can detect very low levels of radiation, coming from forest fire pretty accurately; This has been used by NASA to map the world fire zones. This has become very useful understand, which are parts of the world are being destroyed or being converted through natural forest fires or manmade fires. This shows that categories, which are sensitive are very useful because they enable detect very small signals. We will continue this discussion in the next lecture.